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MORE ON ELECTRIC AND MAGNETIC FLUXES IN $SU(2)$

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The free energies of static charges and center monopoles are given by their fluxes. While electric fluxes show the universal behaviour of the deconfinement transition, the monopole free energies vanish in the thermodynamic limit at all temperatures and are thus irrelevant for the transition. Magnetic fluxes may, however, be used to measure the topological susceptibility without cooling.

In the pure $SU(N)$ gauge theory without quarks 't Hooft's gauge-invariant electric and magnetic fluxes describe, respectively, the effect of a static fundamental color charge and a center monopole in a finite volume.[1] The partition function of a certain amount of electric/magnetic flux yields the free energy of a static electric/magnetic charge with boundary conditions to imitate the presence of its 'mirror' (anti)charge in a neighboring box along the direction of the flux. To measure the free energies of fluxes one imposes, in a first step, 't Hooft's twisted b.c.'s to fix the total numbers modulo N of $\mathbb{Z}_N$-vortices through the 6 planes of the 4-dimensional Euclidean $1/T \times L^3$ box. In $SU(2)$ for example, twist in one plane corresponds to an ensemble with an odd number of $\mathbb{Z}_2$-vortices through that plane. It differs by at least one from the periodic ensemble with an even number; and their free-energy difference is what it costs to add one such vortex to the system.

Qualitatively, the vortices lower their free energy by spreading in the plane of the twist. At $T > 0$ we thus distinguish between two types:

Temporal twist in a $1/T \times L$ plane is classified by a vector $\vec{k} \in \mathbb{Z}_N^3$ parallel to the L-edge. With increasing temperature T, the vortices are squeezed in such a plane more and more. They can no-longer spread arbitrarily and this is what drives the phase transition. In the thermodynamic limit, their free energy approaches zero (infinity) for T below (above) T_c.[2,3]

Magnetic twist is defined in a purely spatial plane. It fixes the conserved, $\mathbb{Z}_N$-valued and gauge-invariant magnetic flux $\vec{m}$ through that plane. Since the vortex can spread in such a plane independent of T, its free energy, or that of a static center monopole, vanishes for $L \to \infty$ at all T.[5]

The partition functions of fixed units of electric and magnetic fluxes, $\vec{e}, \vec{m} \in \mathbb{Z}_N^3$, which we denote by $Z_e(\vec{e}, \vec{m})$, are obtained as 3-dimensional $\mathbb{Z}_N$-Fourier transforms, w.r.t. the temporal $\vec{k}$-twist, of those with twisted b.c.'s, $Z_k(\vec{k}, \vec{m})$. Purely electric flux yields the free energy of a static fundamental charge $\propto -\ln Z_e(\vec{e}, 0)$ in a well-defined (UV-regular) way.[4] Measuring in $SU(2)$ the ratios of different Z_k's (Z_e's) for the various $\vec{k}$-twists (electric fluxes) we demonstrated their Kramers-Wannier duality.[2] This duality is the analogue in $SU(2)$ of that between the Wilson loops of the 3d $\mathbb{Z}_2$-gauge theory and the 3d-Ising spins reflecting the different realizations of the 3-dim. *electric center symmetry* in both phases.

There is no analogue of magnetic flux in the 3d-spin systems. Changing the spatial $\vec{m}$-twist is a gauge-invariant way of introducing one more static center monopole. However, the monopole free energy is exponentially suppressed with the spatial string tension σ_s and thus tends to zero, in the thermodynamic limit, at all temperatures. At T_c, for example, our data yields $\sigma_s = (2.2 \pm 0.2) T_c^2$ consistent with the zero temperature $SU(2)$ string tension.[5] We find $Z_k(\vec{k}, \vec{m}) \rightarrow Z_k(\vec{k}, 0)$ for all twists, and $Z_e(\vec{e}, \vec{m}) \rightarrow Z_e(\vec{e}, 0)$ for all fluxes, with $L \rightarrow \infty$ at any T. Magnetic fluxes are thus irrelevant for the phase transition. The corresponding 3-dim. *magnetic center symmetry* remains unbroken, and center monopoles always 'condense'.

Nevertheless, magnetic twist can be used to fix the fractional content of the topological charge $Q = (\nu + \vec{k} \cdot \vec{m}/N)$ (with $\nu \in \mathbb{Z}$). In $SU(2)$ it is half-odd integer for $\vec{k} \cdot \vec{m} = 1 \bmod 2$ and integer otherwise. This may be used to extract the topological susceptibility from differences, in the finite volume, between the integer and half-odd integer sequences of topological sectors. Recall that with twisted b.c.'s the θ-sectors can be represented as

$$Z_\theta(\vec{k}, \vec{m}, \theta) = \exp\{-F_\theta(\vec{k}, \vec{m}, \theta)/T\} = \sum\nolimits_\nu e^{-i\theta(\nu + \frac{\vec{k}\vec{m}}{N})} \, Z_\nu(\vec{k}, \vec{m}, \nu) \; , \quad (1)$$

from which for the topological susceptibility χ, one obtains,

$$\chi \, L^3 = \frac{d^2}{d\theta^2} \, F_\theta(\vec{k}, \vec{m}, \theta)\Big|_{\theta=0} = T \, \frac{\sum_\nu (\nu + \frac{\vec{k}\vec{m}}{N})^2 \, Z_\nu(\vec{k}, \vec{m}, \nu)}{Z_\theta(\vec{k}, \vec{m}, 0)} \; . \quad (2)$$

We expect χ to result from a density of (more or less) localised objects. Local observables should, however, be insensitive to b.c.'s, at least to magnetic twist which we may introduce without cost for sufficiently large L. We therefore assume that the partition functions factorize $Z_k(\vec{k}, \vec{m}) \equiv Z_\theta(\vec{k}, \vec{m}, 0) = \sum_\nu w(Q) \widetilde{Z}(\vec{k}, \vec{m})$, with a form of $w(Q)$ that does not depend on the b.c.'s and a Q-independent factor $\widetilde{Z}$ that does. The ratios with odd/even $\vec{k} \cdot \vec{m}$, shown for various L in Fig. 1, are fairly independent of the

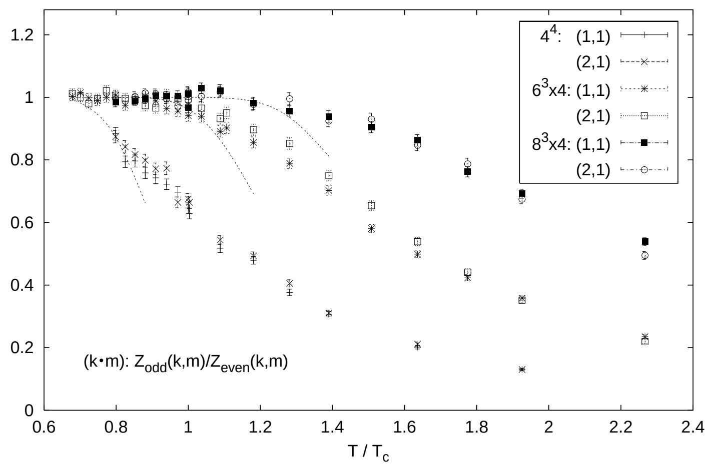

Figure 1. Ratios of partition functions of half-odd integer/integer topological sectors.

magnitude of the twists thus confirming this factorization which predicts for them the unique form $\sum_\nu w(\nu + 1/2)/\sum_\nu w(\nu)$. Inserting a Gaussian topological charge distribution $w(Q) = \exp\{-Q^2/(2\Delta)\}$, as a first approximation to a more realistic one, we obtain the fits shown in Fig. 1 together with the rough estimate, $\chi = \langle Q^2\rangle/(L^3/T) \approx \Delta/(L^3/T) \approx (156(5)\text{MeV})^4$, for $\Delta \gg 1/4$. Since $\Delta \propto L^3/T$, the last condition fails for smaller volumes (larger T). Then, $\langle Q^2\rangle \equiv \sum_\nu Q^2 w(Q)/\sum_\nu w(Q) \neq \Delta$ (but $\langle Q^2\rangle/\Delta \to 0$ as can be verified by Poisson resummation). This limits the applicability of the estimate to $T \lessapprox \{0.8, 1.1, 1.3\}T_c$ for the $LT = \{4/4, 6/4, 8/4\}$ lattices.

Larger spatial lattice sizes are required to extend the temperature range of the procedure upwards and to reliably extract a temperature dependent topological susceptibility in a range around T_c. Encouragingly, our estimate for χ is of the right order, it extends to $T > T_c$, and our preliminary analysis suggests that the topological susceptibility can be extracted without cooling, at least in principle, by comparing via twisted b.c.'s sectors that differ only by the fractional part of their topological charges.

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1. G. 't Hooft, *Nucl. Phys.* **B153**, 141 (1979).
2. Ph. de Forcrand and L. von Smekal, *Phys. Rev.* **D66** (RC), 011504 (2002).
3. L. von Smekal, with Ph. de Forcrand, in *Confinement, Topology and other Non-Perturbative Aspects of QCD*, Eds. J. Greensite and S. Olejnik, NATO Science Series, Kluwer (2002), pp. 287 - 294, [hep-ph/0205002].
4. Ph. de Forcrand and L. von Smekal, *Nucl. Phys.* **B106** (PS), 619 (2002).
5. L. von Smekal, with Ph. de Forcrand, for *Lattice 2002*, hep-lat/0209149.