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Infrared behavior of gluons and ghosts in ghost-antighost symmetric gauges

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To investigate the possibility of a ghost-antighost condensate, the coupled Dyson-Schwinger equations for the gluon and ghost propagators in Yang-Mills theories are derived in general covariant gauges, including ghost-antighost symmetric gauges. The infrared behavior of these two-point functions is studied in a bare-vertex truncation scheme which has proven to be successful in the Landau gauge. In all linear covariant gauges the same infrared behavior as in the Landau gauge is found: The gluon propagator is infrared-suppressed whereas the ghost propagator is infrared-enhanced. This infrared singular behavior provides an indication against a ghost-antighost condensate. In the ghost-antighost symmetric gauges we find that the infrared behavior of the gluon and ghost propagators cannot be determined when replacing all dressed vertices by bare ones. The question of a BRS invariant dimension-2 condensate remains to be further studied.

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I. INTRODUCTION

A large body of experimental data supports the general belief that quantum chromodynamics (QCD) is the correct theory of strong interactions. Nevertheless we are left with the task of understanding the physics of hadrons, and hereby in particular the mechanisms of confinement and spontaneous breaking of chiral symmetry. Gaining such insight requires reliable nonperturbative treatments of QCD. Hereby Monte Carlo lattice calculations provide a rigorous nonperturbative approach to QCD. They have the advantage of fully respecting gauge invariance independently of the size of the lattice used. On the other hand, the extraction of the continuum values of physical observables from the lattice data requires a careful study of the scaling regime. The observed scaling behavior, however, will be in general contaminated by finite size effects. With respect to studies of the confinement mechanisms this is problematic: As infrared singularities are expected to occur in QCD there is a definite need for a continuum-based nonperturbative approach.

To this end we note that the Schwinger-Dyson equations of QCD can address directly the infrared region. They provide genuine nonperturbative information and are at the same time fully formulated in the continuum theory. Such an approach is, however, less rigorous than lattice calculations in the sense that truncations of the tower of coupled equations are necessary in practical calculations. Justifications for such truncations can be given on the basis of general principles such as e.g., a restriction to the first Gribov region, see Ref. [1] and references therein. Nevertheless, the validity of the employed truncation is finally judged by comparing its results with either the results of Monte Carlo calculations or experiments. The latter is easily possible as the Schwinger-Dyson approach has been successfully applied to the description of hadron phenomenology, see, e.g., the recent reviews Refs. [2,3] and references therein. Furthermore, despite recent progress by improved lattice algorithms, and despite the increasing computer time available for lattice calculations, including dynamical fermions is exceedingly cumbersome and finite baryon densities are hardly accessible in realistic SU(3) lattice simulations. On the other hand, dynamical fermions and finite baryon densities can be relatively easily treated in the Schwinger-Dyson approach to QCD.

In recent years the fundamental Schwinger-Dyson equations of SU(N) Yang-Mills theories have been solved explicitly in certain approximations yielding gluon and ghost propagators [3–5,7–9]. In these calculations, carried out in Landau gauge, vertex functions constructed from appropriate Slavnov-Taylor identities as well as bare vertices have been employed. The results proved to be qualitatively similar among each other and agree well with recent lattice calculations [10–14] for both the gluon and ghost propagator. The common, though gauge dependent, result of both approaches is an infrared suppressed gluon propagator and an infrared enhanced ghost propagator. Furthermore, the inclusion of dynamical quarks does not alter the infrared behavior of gluon and ghost propagators and leads to only slight modifications for nonvanishing momenta for the number of light flavors $N_f \leqslant 3$ [15]. These results especially imply that the ghosts take the role of the long-range correlations in the theory. Such a behavior is in accordance with the Gribov-Zwanziger horizon condition, see, e.g., Ref. [7] and references therein, and the Kugo-Ojima confinement criterion, which in Landau gauge includes the statement that the ghost propagator should be more singular than a simple pole [16].

The central assumption in the Kugo-Ojima confinement scenario is the invariance of the measure of the functional integral under Becchi-Rouet-Stora (BRS) transformations

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and the existence of a nilpotent BRS operator [17]. The most general Lorentz invariant and globally gauge invariant Lagrangian of dimension 4 that can be constructed under this assumption has been derived in Ref. [18]. In addition to the structure appearing in ordinary linear covariant gauges, the Lagrangian contains a second gauge parameter which controls the symmetry of the Lagrangian under ghost-antighost interchange. Furthermore, a four-ghost interaction term is present. We will use this Lagrangian as the starting point of our investigation.

Our main interest in this paper will be to explore the situation in these general covariant gauges. Away from the Landau gauge limit the connection between the Kugo-Ojima confinement criterion and the infrared behavior of the ghost dressing function is far from obvious. In particular, the question might arise whether it is possible that the infrared dominant role of the ghost dressing function, seen in the Landau gauge, is assumed by other degrees of freedom like the longitudinal gluons in other covariant gauges. As a matter of fact, infrared dominance of longitudinal gluons is seen if stochastic quantization is used instead of the Faddeev-Popov quantization [19]. Furthermore, calculations based on many-body techniques provide evidence that in Coulomb gauge (employing the usual Faddeev-Popov quantization) the ghosts and the Coulomb gluons are both infrared-enhanced [20]. This latter picture for Coulomb gauge QCD obtains (at least partial) support from lattice [21] and renormalization-group calculations [22]. Care has, however, to be taken as the Coulomb gauge limit is highly nontrivial, see, e.g., [23]. On the other hand, the benefit of the Coulomb gauge is obvious. The time-time component of the gluon propagator and the heavy quark potential fulfill a strictly valid inequality [22,24] with the Coulomb string tension being several times larger than the asymptotic one [25]. Even more important, quark confinement directly results from infrared-enhanced Coulomb gluons, see, e.g., Refs. [26,27] and references therein. Instead of exploring the correlation functions in noncovariant gauges, in this paper we will study Green's functions in covariant albeit nonlinear gauges.

Ghost-antighost symmetric gauges are of special interest when investigating the possibility of a BRS invariant condensate of dimension 2 in QCD. Such condensates occur in the operator product expansion of the gluon propagator [28–30], bear some relation to the Gribov problem [31], may result in gluon mass generation [32], and may be important for confinement in general [33,34]. Hereby it has been clarified recently that these condensates are highly nonlocal [35,36] and that they are only BRS invariant after eliminating the Nakanishi-Lautrup field via its equation of motion [37]. This kind of restricted BRS invariance has been called ''on-shell BRS invariance'' and can be related to a residual gauge symmetry after gauge fixing.

The solutions of the gluon and ghost Dyson-Schwinger equations in a Landau gauge provide a somewhat different picture: Whereas the operator product expansion of the gluon propagator requires such a dimension-2 condensate, its interpretation with respect to a gluon mass is made impossible by the gluon propagator's infrared behavior $D(p^2=0)=0$ instead of $D(p^2=0)=1/m^2$. Also the highly infrared singular ghost propagator excludes a ghost mass and/or a ghost-antighost condensate. Therefore the question arises whether in general ghost-antighost symmetric gauges of the infrared behavior of the propagators can be interpreted in terms of gluon and ghost ''masses.''

This paper is organized as follows. In Sec. II we summarize some properties of the general Lagrangian given in Ref. [18] and outline the derivation of the coupled set of Dyson-Schwinger equations (DSEs) for the ghost and gluon propagators. As the Lagrangian contains a four-ghost interaction a rich structure in the ghost DSE emerges which closely resembles the one already present in the gluon equation of ordinary linear covariant gauges. In Sec. III, we employ a truncation scheme that has proven to be successful in the Landau gauge and study in particular the infrared behavior of the ghost and gluon dressing functions for general values of the two gauge parameters. Furthermore, we show that in the ghost-antighost symmetric gauges the contributions of the genuine two-loop terms (generalized squint and sunset diagram) in the gluon and the ghost DSEs must be properly taken into account in the infrared. In the linear covariant gauges no such terms are present in the ghost DSE, and self-consistent results can be obtained assuming the two-loop terms in the gluon equation to be subleading in the infrared [42]. In general ghost-antighost symmetric gauges, on the other hand, the bare-vertex truncation is insufficient to clarify the infrared behavior of the gluon and ghost propagators. In Sec. IV we will provide numerical solutions for the DSEs in the Landau gauge limit of the ghost-antighost symmetric case of the Lagrangian and recover the solutions found in [9] from a different direction in two-dimensional gauge parameter space. In the last section we give our conclusions. Technical details are deferred into four Appendixes.

II. THE DYSON-SCHWINGER EQUATION FOR THE GHOST PROPAGATOR

A. Renormalized double BRS symmetry

The most general Lagrangian of dimension 4 that is Lorentz invariant, globally gauge invariant, invariant under BRS- and anti-BRS-transformations, Hermitian, and omitting topological terms, is [18]

$$\mathcal{L}=\frac{1}{4}F_{\mu\nu}^2+\frac{(\partial_\mu A_\mu)^2}{2\lambda}+\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{\lambda}{2}(\bar{c}\times c)^2 - i\frac{\alpha}{2}D_\mu\bar{c}\partial_\mu c - i\left(1-\frac{\alpha}{2}\right)\partial_\mu\bar{c}D_\mu c. \tag{1}$$

The field strength tensor and the covariant derivative are defined as

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a-gf^{abc}A_\mu^b A_\nu^c,$$
$$D_\mu^{ab}=\partial_\mu\delta^{ab}+gf^{abc}A_\mu^c, \tag{2}$$

and the abbreviation $(\bar{c}\times c)^a=gf^{abc}\,\bar{c}^b c^c$ is used. Note that both ghost and antighost fields, $\bar{c}$ and c, respectively, are chosen to be Hermitian, $c^\dagger=c$ and $\bar{c}^\dagger=\bar{c}$. This is necessary

to maintain the Hermiticity of the Lagrangian for all values of the gauge parameters λ and α, see, e.g., [17] and references therein. Furthermore, we work in Euclidean spacetime.

From the two gauge parameters of the Lagrangian, the first one, λ, is the usual parameter of linear covariant gauges, whereas the second one, α, controls the symmetry properties of the ghost content. For the cases $\alpha=0$ and $\alpha=2$, one recovers the usual Faddeev-Popov Lagrangian and its mirror image, respectively, where the role of ghost and antighost have been interchanged. For the value $\alpha=1$, the Lagrangian is completely symmetric in the ghost and antighost fields.

In Ref. [18] it has been shown that the S matrix of the theory is invariant under variation of the gauge parameters λ and α. Therefore, gauge invariance of physical observables is ensured. One-loop calculations confirm in particular the independence of the first nontrivial coefficient of the β function from the gauge parameters.

Furthermore, the existence of a renormalized BRS algebra has been proven [18], thus the theory given by Eq. (1) is multiplicatively renormalizable. From one-loop calculations, one finds that the Faddeev-Popov values of the gauge parameters, $\alpha=0$ and $\alpha=2$, are fixed points under the renormalization procedure. The same is true for the ghost-antighost symmetric case $\alpha=1$. The case of the Landau gauge, $\lambda=0$, corresponds to a fixed point as well, because the constraint $\partial_\mu A^\mu=0$ is not affected by a rescaling of the gluon field.

To be specific, the renormalized BRS (s_r) and anti-BRS ($\bar{s}_r$) transformations are given by

$$
\begin{aligned}
&s_r A=-\tilde{Z}_3 D_r c, \quad \bar{s}_r A=-\tilde{Z}_3 D_r \bar{c},\\
&s_r c=-\tilde{Z}_1\frac{1}{2}(c\times c), \quad \bar{s}_r c=-\tilde{Z}_1\frac{1}{2}(\bar{c}\times\bar{c}),\\
&s_r \bar{c}=B-\frac{\alpha}{2}\tilde{Z}_1(\bar{c}\times c),\\
&\bar{s}_r c=-B-\left(1-\frac{\alpha}{2}\right)\tilde{Z}_1(\bar{c}\times c),\\
&s_r B=-\frac{\alpha}{2}\tilde{Z}_1(c\times B)-\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{1}{2}\tilde{Z}_1^2[(c\times c)\times\bar{c}],\\
&\bar{s}_r B=-\left(1-\frac{\alpha}{2}\right)\tilde{Z}_1(\bar{c}\times B)+\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{1}{2}\tilde{Z}_1^2[(\bar{c}\times\bar{c})\times c].
\end{aligned}
\tag{3}
$$

Here $D_r=\partial-Z_3^{1/2}Z_g(A\times)$ is the covariant derivative in the adjoint representation, with color and Lorentz indices suppressed. Note that the Nakanishi-Lautrup auxiliary field B can be eliminated from the BRS-transformations by using its equation of motion. The corresponding BRS-transformations are called ''on-shell.'' Note furthermore that the application of the BRS-operator $s_r(\bar{s}_r)$ on a field increases (decreases) the ghost number by $+1$ (-1), thus we can assign the value $N_{FP}=+1$ $(N_{FP}=-1)$ to the (anti-)BRS-operator itself. The BRS-operator and the anti-BRS-operator are nilpotent and related by $s_r\bar{s}_r+\bar{s}_r s_r=0$. These properties are, however, lost when considering ''on-shell'' BRS-transformations.

The Maurer-Cartan conditions, in addition to the forms of $s_r c$ and $\bar{s}_r\bar{c}$, for ghosts c and antighosts $\bar{c}$ in a ghost-antighost symmetric formulation, thereby require [18]

$$
s_r\bar{c}+\bar{s}_r c+\tilde{Z}_1(\bar{c}\times c)=0. \tag{4}
$$

The correspondence between the bare Lagrangian and its renormalized version including counterterms is given by the following rescaling transformations:

$$
\begin{aligned}
&A_\mu^a\to\sqrt{Z_3}A_\mu^a, \quad \bar{c}^a c^b\to\tilde{Z}_3\bar{c}^a c^b,\\
&B^a\to B^a/\sqrt{Z_3}, \quad g\to Z_g g,\\
&\alpha\to Z_\alpha\alpha, \quad \lambda\to Z_\lambda\lambda,
\end{aligned}
\tag{5}
$$

where five independent renormalization constants Z_3, $\tilde{Z}_3$, Z_g, Z_α, and Z_λ have been introduced. Furthermore, four additional renormalization constants are related to these via Slavnov-Taylor identities,

$$
\begin{aligned}
&Z_1=Z_g Z_3^{3/2}, \quad \tilde{Z}_1=Z_g\tilde{Z}_3 Z_3^{1/2},\\
&Z_4=Z_g^2 Z_3^2, \quad \tilde{Z}_4=Z_g^2\tilde{Z}_3^2.
\end{aligned}
\tag{6}
$$

Note, however, that contrary to standard Faddeev-Popov gauges $Z_\lambda\neq Z_3$, e.g., at one loop (MS scheme), one has [43]

$$
Z_\lambda=Z_3-\frac{g^2}{16\pi^2}\frac{1}{\epsilon}N_c\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\lambda. \tag{7}
$$

The gauge-fixing part of the Lagrangian (1) can be written in the following three equivalent ways:

$$
\begin{aligned}
\mathcal{L}_{GF}&=\frac{i}{2}\frac{1}{Z_3\tilde{Z}_3}s_r\bar{s}_r(Z_3 AA+iZ_\lambda\tilde{Z}_3\alpha\lambda c\bar{c})\\
&\quad+\frac{Z_\lambda}{Z_3}(1-\alpha)\frac{\lambda}{2}s_r(\bar{c}s_r\bar{c}) \qquad (8)\\
&=is_r\left[\bar{c}\left(\partial A-i\frac{Z_\lambda}{Z_3}\frac{\lambda}{2}B\right)\right] \qquad (9)\\
&=iB\partial A+\frac{Z_\lambda}{Z_3}\frac{\lambda}{2}B^2+\frac{Z_\lambda}{Z_3}\tilde{Z}_1^2\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{\lambda}{2}(\bar{c}\times c)^2\\
&\quad+i\tilde{Z}_3\left[\left(1-\frac{\alpha}{2}\right)\bar{c}\partial D_r c+\frac{\alpha}{2}\bar{c}D_r\partial c\right]. \qquad (10)
\end{aligned}
$$

This is verified by direct calculation via the transformations defined in Eqs. (3). In the form of Eq. (10) the gauge fixing Lagrangian shows that the renormalization constants introduced in Eq. (3) correspond to the replacements of bare by renormalized quantities as given above.

We may rewrite the gauge fixing Lagrangian of Eq. (10) once more,

$$\mathcal{L}_{\mathcal{GF}} = iB\partial A + \frac{Z_\lambda}{Z_3}\frac{\lambda}{2}B^2 + \frac{Z_\lambda}{Z_3}\tilde{Z}_1^2\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{\lambda}{2}(\bar{c}\times c)^2 + i\tilde{Z}_3\frac{1}{2}(\bar{c}\partial D_r c + \bar{c}D_r\partial c) + i\tilde{Z}_1(1-\alpha)\frac{1}{2}\partial A(\bar{c}\times c). \tag{11}$$

This emphasizes the role of the gauge parameter α. In this form, the only term not symmetric under Faddeev-Popov conjugation, $c\to\bar{c}$ and $\bar{c}\to -c$, is the last one (which is antisymmetric with respect to Faddeev-Popov conjugation). It vanishes for $\alpha=1$. With the current (real) Hermiticity assignment for ghost and antighost fields, the Lagrangian is Hermitian for all α, and it reduces to the standard Faddeev-Popov form for $\alpha=0$. We could also introduce Hermitian adjoint ghost and antighost fields, with the assignment $c^\dagger=\bar{c}$, via the Caley map [44]. This would then lead to

$$\mathcal{L}^{cc}_{\mathcal{GF}} = iB\partial A + \frac{Z_\lambda}{Z_3}\frac{\lambda}{2}B^2 - \frac{Z_\lambda}{Z_3}\tilde{Z}_1^2\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{\lambda}{2}(\bar{c}\times c)^2 + \tilde{Z}_3\frac{1}{2}(\bar{c}\partial D_r c + \bar{c}\,D_r\partial c) + \tilde{Z}_1(1-\alpha)\frac{1}{4}[\bar{c}(\partial A\times\bar{c}) - c(\partial A\times c)]. \tag{12}$$

While this form of the Lagrangian, which we will not use further herein, is still Hermitian, it no longer reduces to the form of standard Faddeev-Popov theory for $\alpha=0$. Thus the Faddeev-Popov Lagrangian is only consistent with Hermiticity for the choice of real ghost fields [17]. With complex conjugate ghost and antighost fields, additional terms for $\alpha=0$ survive (which are absent in standard Fadeev-Popov gauges). Only for $\alpha=1$ do both versions, with Hermitian real or complex conjugate ghost pairs, have the same Lagrangian and may be interchanged arbitrarily.

B. Ghost and antighost Dyson-Schwinger equations

Without invariance under Faddeev-Popov conjugation, i.e., without ghost-antighost symmetry ($\alpha=1$ or $\lambda=0$), we have separate ghost and antighost DSEs which are *not* identical. Consider the following representations of the ghost (antighost) derivatives of the action (for brevity we indicate by subscripts the space-time arguments of fields):

$$\begin{aligned}\frac{\delta S}{\delta c^a_x} &= \tilde{Z}_3 i(\partial D_r\bar{c})^a_x + \frac{Z_\lambda}{Z_3}\tilde{Z}_1\left(1-\frac{\alpha}{2}\right)(\bar{c}\times B)^a_x \\ &\quad - \frac{Z_\lambda}{Z_3}\tilde{Z}_1^2\frac{\alpha}{4}\left(1-\frac{\alpha}{2}\right)[(\bar{c}\times\bar{c})\times c]^a_x \\ &= \tilde{Z}_3 i(\partial D_r\bar{c})^a_x - \frac{Z_\lambda}{Z_3}\lambda\, s_r B^a_x \\ &= -is_r\left(\partial A^a_x - i\frac{Z_\lambda}{Z_3}B^a_x\right),\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\delta S}{\delta \bar{c}^a_x} &= \tilde{Z}_3 i(\partial D_r c)^a_x + \frac{Z_\lambda}{Z_3}\tilde{Z}_1\frac{\alpha}{2}(c\times B)^a_x \\ &\quad + \frac{Z_\lambda}{Z_3}\tilde{Z}_1^2\frac{\alpha}{4}\left(1-\frac{\alpha}{2}\right)[(c\times c)\times\bar{c}]^a_x \\ &= \tilde{Z}_3 i(\partial D_r c)^a_x - \frac{Z_\lambda}{Z_3}\lambda\, s_r B^a_x \\ &= -is_r\left(\partial A^a_x - i\frac{Z_\lambda}{Z_3}B^a_x\right).\end{aligned} \tag{14}$$

The two DSEs then follow readily from

$$\left\langle \frac{\delta S}{\delta\bar{c}^a_x}\bar{c}^b_y\right\rangle = \left\langle c^b_y\frac{\delta S}{\delta c^a_x}\right\rangle = \delta^{ab}\delta_{xy}. \tag{15}$$

Of course, they are related by Faddeev-Popov conjugation $\mathcal{C}_{\rm FP}$ which interchanges the two. In particular,

$$\begin{aligned}&\mathcal{C}_{\rm FP}c=\bar{c},\quad \mathcal{C}_{\rm FP}\bar{c}=-c,\\ &\mathcal{C}_{\rm FP}B = B+\tilde{Z}_1(1-\alpha)(\bar{c}\times c),\quad \mathcal{C}_{\rm FP}A=A.\end{aligned} \tag{16}$$

The transformation of the Nakanishi-Lautrup B-field follows from compatibility with BRS/anti-BRS invariance and

$$\bar{s}_r = \mathcal{C}_{\rm FP}s\mathcal{C}^{-1}_{\rm FP}. \tag{17}$$

On the level of the BRS and anti-BRS transformations we can have this form of Faddeev-Popov conjugation for arbitrary α. However, it is relatively easy to verify that the Lagrangian, i.e., the measure of the theory, is not invariant under $\mathcal{C}_{\rm FP}$ and thus ghost and antighost DSEs are not identical, unless $\alpha=1$ or $\lambda=0$: With the above Faddeev-Popov conjugation rule for the B-field, the sign change in the last term of Eq. (11) is exactly compensated by the first term,

$$\begin{aligned}&iB\partial A + i\tilde{Z}_1(1-\alpha)\tfrac{1}{2}\partial A(\bar{c}\times c)\\ &\overset{\mathcal{C}_{\rm FP}}{\to} i[B+\tilde{Z}_1(1-\alpha)(\bar{c}\times c)]\partial A - i\tilde{Z}_1(1-\alpha)\tfrac{1}{2}\partial A(\bar{c}\times c)\\ &= iB\partial A + i\tilde{Z}_1(1-\alpha)\tfrac{1}{2}\partial A(\bar{c}\times c).\end{aligned} \tag{18}$$

In this way, the violations of Faddeev-Popov conjugation invariance can entirely be moved into the term $\propto \lambda B^2$, and they thus obviously disappear in the Landau gauge $\lambda = 0$. On the other hand, in the more general ghost-antighost symmetric case, with $\alpha = 1$ and $\mathcal{C}_{\rm FP} B = B$, the theory does have the invariance under Faddeev-Popov conjugation for all λ and we can then immediately conclude that expectation values of $\mathcal{C}_{\rm FP}$-odd operators vanish.

Let us now look at one of the ghost DSEs, e.g., from Eq. (14) we obtain

$$\delta^{ab}\delta_{xy} = \left\langle \frac{\delta S}{\delta c_x^a} \bar{c}_y^b \right\rangle = \tilde{Z}_3 \langle i(\partial D_r c)_x^a \bar{c}_y^b \rangle - \frac{Z_\lambda \lambda}{Z_3} \langle (s_r B_x^a) \bar{c}_y^b \rangle . \tag{19}$$

For the second term on the right-hand side we write

$$\begin{aligned} \langle (s_r B_x^a) \bar{c}_y^b \rangle &= \langle s_r (B_x^a \bar{c}_y^b) \rangle - \langle B_x^a (s_r \bar{c}_y^b) \rangle \\ &= -\langle B_x^a B_y^b \rangle + \tilde{Z}_1 \left(1 - \frac{\alpha}{2} \right) \langle B_x^a (\bar{c} \times c)_y^b \rangle , \end{aligned} \tag{20}$$

where we have used that expectation values of total BRS variations vanish. For the B-field correlations, and with its equation of motion $Z_\lambda \lambda \; B = i Z_3 \partial A$, one furthermore has

$$\frac{Z_\lambda \lambda}{Z_3} \langle B_x^a B_y^b \rangle = \delta^{ab}\delta_{xy} - \frac{Z_3}{Z_\lambda \lambda} \langle \partial A_x^a \partial A_y^b \rangle . \tag{21}$$

Inserting Eqs. (20) and (21) into the ghost DSE (19), we arrive at

$$\begin{aligned} \frac{Z_3}{Z_\lambda \lambda} \langle \partial A_x^a \partial A_y^b \rangle &= \tilde{Z}_3 \langle i(\partial D_r c)_x^a \bar{c}_y^b \rangle \\ &\quad + i \tilde{Z}_1 \left(1 - \frac{\alpha}{2} \right) \langle \partial A_x^a (\bar{c} \times c)_y^b \rangle . \end{aligned} \tag{22}$$

In the last term herein we inserted the equation of motion (e.o.m.) for the B-field again. This term is odd under Faddeev-Popov conjugation and thus vanishes in the ghost-antighost symmetric case $\alpha = 1$, as asserted above. We thus have the important form of the ghost DSE in the Faddeev-Popov symmetric formulation (in which there is only one such DSE),

$$\frac{Z_3}{Z_\lambda \lambda} \langle \partial A_x^a \partial A_y^b \rangle = \tilde{Z}_3 \langle i(\partial D_r c)_x^a \bar{c}_y^b \rangle . \tag{23}$$

Note that we obtain the same equation for standard Faddeev-Popov theory ($\alpha = 0$). The important difference from the standard form of the ghost DSE is given by

$$Z_3 \langle \partial A_x^a \partial A_y^b \rangle - Z_\lambda \lambda \, \delta^{ab}\delta_{xy} , \tag{24}$$

which vanishes in the usual Faddeev-Popov theory. For general α, however, the Slavnov-Taylor identities are modified also and this contribution no longer needs to vanish as we will see at the end of this section. Before that, we give a convenient (symmetrized) form of the ghost DSE valid for arbitrary α without ghost-antighost invariance. Note that we could equally have started from the ghost derivative in Eq. (13) and $\langle \; c_y^b (\delta / \delta c_x^a) S \; \rangle = \delta^{ab} \; \delta_{xy}$. This would lead us to the Faddeev-Popov conjugate of Eq. (22) [obtained from Eq. (22) with $c \to \bar{c}$, $\bar{c} \to -c$, and $\alpha \to 2 - \alpha$]. Adding the two, we obtain a Faddeev-Popov symmetric version in the place of Eq. (22),

$$\begin{aligned} \frac{Z_3}{Z_\lambda \lambda} \langle \partial A_x^a \partial A_y^b \rangle &= \tilde{Z}_3 \frac{1}{2} [\langle i(\partial D_r c)_x^a \bar{c}_y^b \rangle + \langle c_y^b i(\partial D_r \bar{c})_x^a \rangle] \\ &\quad - i \tilde{Z}_1 \frac{1}{2} (1 - \alpha) \langle \partial A_x^a (\bar{c} \times c)_y^b \rangle . \end{aligned} \tag{25}$$

Just as we have a doubling of ghost DSEs, in the absence of Faddeev-Popov conjugation invariance, we also have a doubling of Slavnov-Taylor identities. As the result of one such new Slavnov-Taylor identity we will derive below that

$$\tilde{Z}_1 \frac{1}{2} \langle \partial A_x^a (\bar{c} \times c)_y^b \rangle = \tilde{Z}_3 \frac{i}{2} [\langle i(\partial D_r c)_x^a \bar{c}_y^b \rangle - \langle c_y^b i(\partial D_r \bar{c})_x^a \rangle] . \tag{26}$$

This allows us to write for the ghost DSE (25) and general α, finally,

$$\begin{aligned} \frac{Z_3}{Z_\lambda \lambda} \langle \partial A_x^a \partial A_y^b \rangle &= \tilde{Z}_3 \left[\left(1 - \frac{\alpha}{2} \right) \langle i(\partial D_r c)_x^a \bar{c}_y^b \rangle \right. \\ &\quad \left. + \frac{\alpha}{2} \langle c_y^b i(\partial D_r \bar{c})_x^a \rangle \right] . \end{aligned} \tag{27}$$

For $\alpha = 0$ (or 2) the left-hand side reduces to unity and one obtains the ghost DSE of standard Faddeev-Popov theory. For $\alpha = 1$ both terms on the r.h.s. are identical and add up to that of Eq. (23).

The main difference, as compared to the ordinary Faddeev-Popov gauge, in an explicit representation of the ghost DSE will be a new type of diagrams generated by the four-ghost interaction. The formal structure of the gluon DSE, on the other hand, remains unchanged.

For completeness we have provided a derivation of the ghost DSE starting directly from the Lagrangian (1) in Appendix A. For all details, the interested reader is referred to this appendix as well as Appendix B, which contains the definitions of Green's functions and the decompositions of full into connected and one-particle irreducible Green's functions. Employing the definitions of the bare ghost-gluon and the bare four-ghost vertex, see Appendix B, the Dyson-Schwinger equation for the ghost propagator in coordinate space reads

$$[D_G^{ab}(x-y)]^{-1}=\tilde{Z}_3[D_G^{(0)ab}(x-y)]^{-1}-\tilde{Z}_1\int d^4z d^4u d^4v d^4z_1 d^4z_2 d^4z_3 \Gamma_\mu^{(0)bde}(y,u,v)D_{\mu\nu}^{ef}(v-z_1)$$
$$\times\Gamma_\nu^{fha}(z_1,z_3,x)D_G^{hd}(u-z_3).-\tilde{Z}_4\int d^4u d^4v \Gamma_{4gh}^{(0)bdfa}(x,u,v,y)D_G^{fd}(v-u)$$
$$-\tilde{Z}_4\frac{1}{2}\int d^4z d^4u d^4v d^4u_1 d^4u_2 d^4u_3 d^4u_4 \Gamma_{4gh}^{(0)bdgf}(y,z,v,u)D_G^{fe}(u-u_4)D_G^{gi}(v-u_2)$$
$$\times\Gamma_{4gh}^{jaei}(u_3,x,u_4,u_2)D_G^{jd}(u_3-z)-\tilde{Z}_4\frac{1}{2}\int d^4z d^4u d^4v d^4u_1 d^4u_2 d^4u_3 d^4u_4 d^4u_5$$
$$\times\Gamma_{4gh}^{(0)bdgf}(y,z,v,u)D_{\mu\nu}^{ek}(u_1-u_4)D_G^{fl}(u-u_5)\Gamma_\nu^{kal}(u_4,x,u_5)D_G^{gi}(v-u_2)\Gamma_\mu^{eij}(u_1,u_3,u_2)D_G^{jd}(u_3-z). \tag{28}$$

Fourier transformation to momentum space yields

$$[D_G(p)]^{-1}=\tilde{Z}_3[D_G^{(0)}(p)]^{-1}-\tilde{Z}_1\frac{g^2N_c}{(2\pi)^4}\int d^4q\Gamma_\mu^{(0)}(p,q)D_{\mu\nu}(p-q)\Gamma_\nu(q,p)D_G(q)-\tilde{Z}_4\frac{g^2N_c}{(2\pi)^4}\int d^4q\Gamma_{4gh}^{(0)}D_G(q)$$
$$+\tilde{Z}_4\frac{1}{2}\frac{g^4N_c^2}{(2\pi)^8}\int d^4q_1q_2\Gamma_{4gh}^{(0)}D_G(q_1)D_G(p-q_1-q_2)\Gamma_{4gh}(p,q_1,q_2)D_G(q_2)-\tilde{Z}_4\frac{1}{4}\frac{g^4N_c^2}{(2\pi)^8}$$
$$\times\int d^4q_1q_2\Gamma_{4gh}^{(0)}D_{\mu\nu}(p-q_1)\ D_G(q1)\Gamma_\nu(p,q_1)\ D_G(q_2)\Gamma_\mu(-p+q_1+q_2,q_2)D_G(p-q_1-q_2). \tag{29}$$

The color traces have already been carried out and the reduced vertices defined in Appendix B have been used. The four-ghost interaction generates three new diagrams in the ghost equation, a tadpole contribution and two two-loop diagrams. Furthermore, the bare ghost-gluon vertex depends on the gauge parameter α,

$$\Gamma_\mu^{(0)abc}(k,p,q)=gf^{abc}(2\pi)^4\delta^4(k+q-p)\Gamma_\mu^{(0)}(p,q)$$
$$\Gamma_\mu^{(0)}(p,q)=\left[\left(1-\frac{\alpha}{2}\right)q_\mu+\frac{\alpha}{2}p_\mu\right]. \tag{30}$$

Note the symmetry between the the ghost momentum p_μ and the antighost momentum q_μ, when the gauge parameter α is set to 1.

C. Projection of the gluon equation

The respective equation for the gluon propagator is formally the same as in the Faddeev-Popov case. Differences occur in the explicit form of the bare ghost-gluon vertex and the dressed vertices in general depend on the gauge parameters. The gluon DSE reads

$$[D(p)]_{\mu\nu}^{-1}=Z_3[D^{(0)}(p)]_{\mu\nu}^{-1}+\tilde{Z}_1\frac{g^2N_c}{(2\pi)^4}\int d^4q\Gamma_\mu^{(0)}(p,q)\ D_G(p-q)\Gamma_\nu(q,p)D_G(q)$$
$$-Z_1\frac{1}{2}\frac{g^2N_c}{(2\pi)^4}\int d^4q\Gamma_{\mu\rho\sigma}^{(0)}(p,q)\ D_{\rho\rho'}(p-q)\Gamma_{\rho'\nu\sigma'}(q,p)D_{\sigma\sigma'}(q)-Z_4\frac{1}{2}\frac{g^2N_c}{(2\pi)^4}\int d^4q\Gamma_{\mu\nu\rho\sigma}^{(0)}D_{\rho\sigma}(q)$$
$$-Z_4\frac{1}{6}\frac{g^4N_c^2}{(2\pi)^8}\int d^4q_1q_2\Gamma_{\mu\rho\sigma\lambda}^{(0)}D_{\rho\rho'}(q_2)\ D_{\sigma\sigma'}(p-q_2-q_1)\Gamma_{\rho'\nu\lambda'\sigma'}(p,q_1,q_2)D_{\lambda\lambda'}(q_1)$$
$$-Z_4\frac{1}{2}\frac{g^4N_c^2}{(2\pi)^8}\int d^4q_1q_2\Gamma_{\mu\rho\sigma\lambda}^{(0)}D_{\rho\rho'}(p-q_1-q_2)D_{\sigma\sigma'}(q_2)\Gamma_{\rho'\zeta\sigma'}(p-q_1-q_2,q_2)$$
$$\times D_{\zeta\zeta'}(p-q_1)\Gamma_{\zeta'\nu\lambda'}(p-q_1,q_1)\ D_{\lambda\lambda'}(q_1). \tag{31}$$

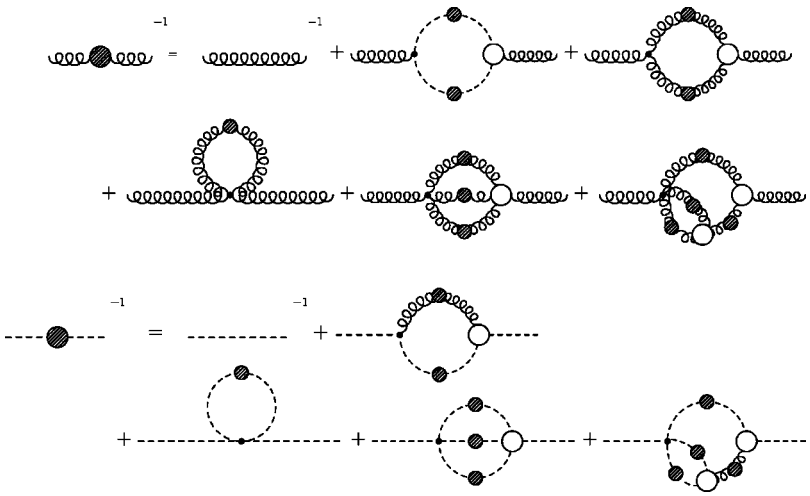
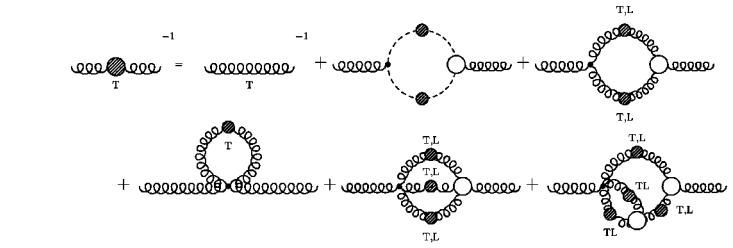


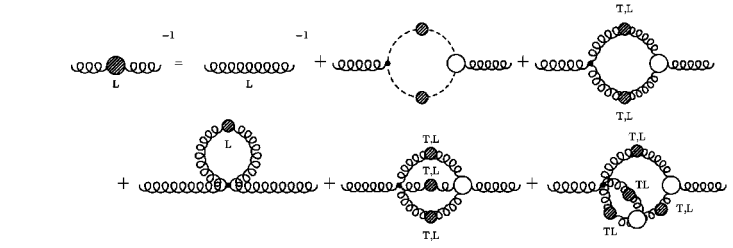
FIG. 1. The coupled gluon and ghost Dyson-Schwinger equations from a BRS and anti-BRS symmetric Lagrangian. Each equation contains one-loop diagrams, a tadpole contribution, and a sunset and a squint diagram.

Both equations are shown diagrammatically in Fig. 1. One clearly sees the striking similarity between the ghost and the gluon equation once a four-ghost interaction has been introduced. Both equations have bare and one-loop parts, a tadpole contribution, a sunset, and a squint diagram.

In order to sort the various contributions of the gluon equation to the inverse of the gluon propagator on the left-hand side we project the equation on its longitudinal and transverse parts. It is well known that for linear covariant gauges, $\alpha=0$, the longitudinal part of the gluon propagator remains undressed [3]. However, away from linear covariant gauges this is not the case, as can be seen from the corresponding Slavnov-Taylor identity derived in [18]. We then have three dressing functions in the general case and the

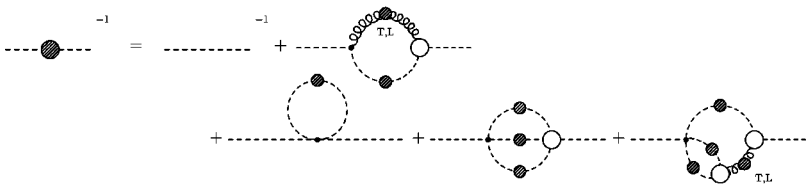


$$\frac{1}{Z(p^2)} = Z_3 + V^{ghost} + V_{TT}^{glue} + V_{TL}^{glue} + V_{LL}^{glue} + V_T^{tad} + V_{TTT}^{sun} + V_{TTL}^{sun} + V_{TLL}^{sun} + V_{LLL}^{sun} + V_{TTTT}^{squint} + V_{TTTL}^{squint} + V_{TTLL}^{squint} + V_{TLLL}^{squint} + V_{LLLL}^{squint}$$



$$\frac{1}{\lambda L(p^2)} = \frac{Z_3}{Z_\lambda \lambda} + W^{ghost} + W_{TT}^{glue} + W_{TL}^{glue} + W_{LL}^{glue} + W_L^{tad} + W_{TTT}^{sun} + W_{TTL}^{sun} + W_{TLL}^{sun} + W_{LLL}^{sun} + W_{TTTT}^{squint} + W_{TTTL}^{squint} + W_{TTLL}^{squint} + W_{TLLL}^{squint} + W_{LLLL}^{squint}$$

FIG. 2. Various contributions from the respective diagrams in the transverse and longitudinal gluon equation and the equation for the ghost dressing function.



$$\frac{1}{G(p^2)} = \tilde{Z}_3 + U_T^{dress} + U_L^{dress} + U^{tad} + U^{sun} + U_T^{squint} + U_L^{squint}$$

propagators are given by

$$D_{\mu\nu}(p)=[D_{\mu\nu}(p)]_T+[D_{\mu\nu}(p)]_L$$
$$=\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}+\lambda L(p^2)\frac{p_\mu p_\nu}{p^4}, \tag{32}$$

$$D_G(p)=-\frac{G(p)}{p^2}. \tag{33}$$

The transversal and longitudinal gluon dressing functions $Z(p^2)$ and $L(p^2)$ can be extracted by contracting the gluon equation with the transversal and longitudinal projector, respectively. The results are given graphically in Fig. 2, where we also specify our notation for the different contributions being analyzed in the next section. Contributions in the transversal part of the gluon equation are denoted by the symbol *V*, contributions in the longitudinal part by *W*, and the ones in the ghost equation by *U*. The subscripts *T* and *L* indicate the respective parts of the gluon propagator running around in the loops of the diagrams and abbreviations for the diagrams are used. For example, the symbol W^{sun}_{LLT} denotes a contribution from the sunset diagram to the longitudinal gluon equation with two longitudinal and one transverse part of the gluon propagator running in the loop. To isolate the dressing functions, the left-hand sides of the equations have already been divided by factors of $3p^2$ and p^2, respectively.

D. Generalized Slavnov-Taylor identities

To derive the generalization of the Slavnov-Taylor identity for the longitudinal gluon propagator, we start from the following BRS variations:

$$s_r(\partial A^a_x \bar{c}^b_y)=-\tilde{Z}_3(\partial D_r c)^a_x \bar{c}^b_y+\partial A^a_x\left(B-\tilde{Z}_1\frac{\alpha}{2}(\bar{c}\times c)\right)^b_y, \tag{34}$$

$$\overline{s_r}(\partial A^a_x c^b_y)=-\tilde{Z}_3(\partial D_r \bar{c})^a_x c^b_y-\partial A^a_x\left[B+\tilde{Z}_1\left(1-\frac{\alpha}{2}\right)(\bar{c}\times c)\right]^b_y. \tag{35}$$

The corresponding vacuum expectation values vanish, and taking combinations of the expectation values of these equations we obtain

$$0=\left(1-\frac{\alpha}{2}\right)\langle s_r(\partial A^a_x \bar{c}^b_y)\rangle-\frac{\alpha}{2}\langle \overline{s_r}(\partial A^a_x c^b_y)\rangle$$
$$=-\tilde{Z}_3\left(1-\frac{\alpha}{2}\right)\langle(\partial D_r c)^a_x \bar{c}^b_y\rangle$$
$$+\tilde{Z}_3\frac{\alpha}{2}\langle(\partial D_r \bar{c})^a_x c^b_y\rangle+\langle \partial A^a_x B^b_y\rangle. \tag{36}$$

Upon insertion of the e.o.m. of the *B*-field, $Z_\lambda \lambda\ B=iZ_3\partial A$, this directly leads to Eq. (27). On the other hand, the ghost DSEs from Eqs. (13) and (14) allow us to eliminate the first two terms on the r.h.s., multiplying to them appropriate factors of $\alpha/2$ and $1-\alpha/2$ and inserting these expression in Eq. (36), yielding

$$Z_3\langle \partial A^a_x \partial A^b_y\rangle=Z_\lambda\lambda\left\{\delta^{ab}\delta_{xy}-i\tilde{Z}_1\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\langle(\partial A\times c)^a_x \bar{c}^b_y\rangle+\frac{Z_\lambda\lambda}{Z_3}\tilde{Z}_1^2\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)^2\langle[(\bar{c}\times c)\times c]^a_x \bar{c}^b_y\rangle\right.$$
$$\left.+i\tilde{Z}_1\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\langle(\partial A\times \bar{c})^a_x c^b_y\rangle+\frac{Z_\lambda\lambda}{Z_3}\tilde{Z}_1^2\frac{\alpha^2}{4}\left(1-\frac{\alpha}{2}\right)\langle[(\bar{c}\times c)\times \bar{c}]^a_x c^b_y\rangle\right\}. \tag{37}$$

This generalizes the Slavnov-Taylor identity for the longitudinal part of the gluon propagator which, contrary to the standard Faddeev-Popov gauges, does in general acquire renormalization by the interactions, cf. Eq. (7). On the r.h.s. of the Slavnov-Taylor identity, the terms on the third line are the Faddeev-Popov conjugate of those on the second. In the ghost antighost symmetric case for $\alpha=1$ they are identical. In this case the Slavnov-Taylor identity simplifies,

$$Z_3\langle \partial A^a_x \partial A^b_y\rangle=Z_\lambda\lambda\left\{\delta^{ab}\delta_{xy}-i\tilde{Z}_1\frac{1}{2}\langle(\partial A\times c)^a_x \bar{c}^b_y\rangle\right.$$
$$\left.+\frac{Z_\lambda\lambda}{Z_3}\frac{\tilde{Z}_1^2}{4}\langle[(\bar{c}\times c)\times c]^a_x \bar{c}^b_y\rangle\right\}. \tag{38}$$

Note that close to the Landau gauge the corrections to the unity of the standard Faddeev-Popov gauges are suppressed by one order in the gauge parameter λ.

A further Slavnov-Taylor identity is obtained by adding the expectation values of the BRS variations in Eqs. (34) and (35):

$$0=\langle s_r(\partial A^a_x \bar{c}^b_y)\rangle+\langle \overline{s_r}(\partial A^a_x c^b_y)\rangle$$
$$=-\tilde{Z}_3\langle(\partial D_r c)^a_x \bar{c}^b_y\rangle-\tilde{Z}_3\langle(\partial D_r \bar{c})^a_x c^b_y\rangle$$
$$-\tilde{Z}_1\langle \partial A^a_x(\bar{c}\times c)^b_y\rangle. \tag{39}$$

This leads to Eq. (26) as promised in the previous subsection.

These Slavnov-Taylor identities indicate that the Landau gauge limit $\lambda \to 0$ is smooth. Based on Eqs. (37) and (39), one may anticipate that an infrared masslesslike longitudinal part of the gluon propagator leads for sufficiently small values of the gauge parameter λ to the same infrared enhancement of ghosts as observed in the Landau gauge.

III. INFRARED ANALYSIS WITH BARE VERTICES FOR ARBITRARY GAUGE PARAMETERS

In this section, we will analyze the behavior of the two-point functions at small momenta p^2. We will employ a truncation scheme that successfully has been applied in the case of the Landau gauge [7–9] and explore its applicability to general gauges.

An interesting result of the investigations in the Landau gauge is the observation that there is no qualitative difference of the solutions found with bare vertices or with vertices dressed by the use of Slavnov-Taylor identities. This has not only been found in truncations using angular approximations [4,5] for the integrals, but has been confirmed recently for a range of possible vertex dressings in a truncation scheme without any angular approximations [8]. The reason for this somewhat surprising result has been attributed to the nonrenormalization of the ghost-gluon vertex in the Landau gauge, that is, $\tilde{Z}_1 = 1$. It seems as if the violation of gauge invariance using a bare vertex is not that severe in the Landau gauge such that the resulting equations still provide meaningful results. In the following, we will explore to what extent such a simple truncation idea is applicable in other gauges where $\tilde{Z}_1 \neq 1$.

In the Landau gauge, the coupled set of Dyson-Schwinger equations is solved by pure power laws for the ghost and gluon dressing functions. Such solutions are determined analytically by plugging a power-law *Ansatz* in the equations and matching appropriate powers on the left- and right-hand sides. Once several power solutions have been found, the remaining task is to single out the one matching the numerical solution of the renormalized equation. In the Landau gauge, it has been shown that indeed only one of the power solutions found in Refs. [7,8] is the correct infrared limit of the renormalized solution [9] by solving the equations numerically for all momenta. In the following, we will investigate whether there are power solutions at all using bare vertices for general gauge parameter α and λ.

Now we employ the power-law *Ansatz* for the dressing functions,

$$G(x) = Bx^{\beta}, \quad Z(x) = Ax^{\sigma}, \quad L(x) = Cx^{\rho}, \tag{40}$$

where $x = p^2$ has been used. Together with the expressions for the bare vertices given in Appendix B, we plug the power laws into the ghost and the gluon equation. The formulas for the various integrals are given in Appendix C. The straightforward but tedious algebra is done with the help of the algebraic manipulation program FORM [40]. In Ref. [8] it has been shown that the renormalization functions Z_3 and $\tilde{Z}_3$ do not play a role in the determination of possible power solutions of the equations in the infrared region of momentum. Furthermore, the tadpoles just give constant contributions to the respective propagators which vanish in the process of renormalization. Thus we safely omit them in the present investigation.

For the most general gauges, $\alpha \neq 0$ and $\lambda \neq 0$, we obtain the following structure:

$$B^{-1}x^{-\beta} = x^{\sigma+\beta}(U')^{dress}_{T} + x^{\rho+\beta}(U')^{dress}_{L} + x^{3\beta}(U')^{sun} + x^{\sigma+3\beta}(U')^{squint}_{T} + x^{\rho+3\beta}(U')^{squint}_{L}, \tag{41}$$

$$\begin{aligned} A^{-1}x^{-\sigma} = {} & x^{2\beta}(V')^{ghost} + x^{2\sigma}(V')^{glue}_{TT} + x^{\sigma+\rho}(V')^{glue}_{TL} + x^{2\rho}(V')^{glue}_{LL} + x^{3\sigma}(V')^{sun}_{TTT} + x^{2\sigma+\rho}(V')^{sun}_{TTL} + x^{\sigma+2\rho}(V')^{sun}_{TLL} \\ & + x^{3\rho}(V')^{sun}_{LLL} + x^{4\sigma}(V')^{squint}_{TTTT} + x^{3\sigma+\rho}(V')^{squint}_{TTTL} + x^{2\sigma+2\rho}(V')^{squint}_{TTLL} + x^{\sigma+3\rho}(V')^{squint}_{TLLL} + x^{4\rho}(V')^{squint}_{LLLL}, \end{aligned} \tag{42}$$

$$\begin{aligned} (C\lambda)^{-1}x^{-\rho} = {} & + x^{2\beta}(W')^{ghost} + x^{2\sigma}(W')^{glue}_{TT} + x^{\sigma+\rho}(W')^{glue}_{TL} + x^{3\sigma}(W')^{sun}_{TTT} + x^{2\sigma+\rho}(W')^{sun}_{TTL} + x^{\sigma+2\rho}(W')^{sun}_{TLL} \\ & + x^{3\rho}(W')^{sun}_{LLL} + x^{4\sigma}(W')^{squint}_{TTTT} + x^{3\sigma+\rho}(W')^{squint}_{TTTL} + x^{2\sigma+2\rho}(W')^{squint}_{TTLL} + x^{\sigma+3\rho}(W')^{squint}_{TLLL}. \end{aligned} \tag{43}$$

Here the primed quantities are momentum-independent functions of β, σ, and ρ; cf. Fig. 2, where the corresponding unprimed, momentum-dependent quantities have been introduced. The pattern of the equation is such that each primed factor on the right-hand side is accompanied by the squared momentum x to the power of the dressing function content of the respective diagram. In Appendix D, we demonstrate how such a pattern emerges, for example, from the sunset diagram in the ghost equation, $(U)^{sun}$. Note that the contributions $(W')^{glue}_{LL}$ and $(W')^{squint}_{LLLL}$ are zero and therefore missing in the longitudinal gluon equation (43) as momentum conservation cannot hold with three longitudinal gluons in the three-gluon vertex.

For the following argument, we focus on one particular contribution on each right-hand side of the equations:

$$B^{-1}x^{-\beta} = x^{3\beta}(U')^{sun} + \cdots, \tag{44}$$

$$A^{-1}x^{-\sigma} = x^{4\sigma}(V')^{squint}_{TTTT} + \cdots, \tag{45}$$

$$(C\lambda)^{-1}x^{-\rho} = x^{3\rho}(W')^{sun}_{LLL} + \cdots. \tag{46}$$

The coefficients $(U')^{sun}$, $(V')^{squint}_{TTTT}$, and $(W')^{sun}_{LLL}$ are nonzero and explicitly given in Appendix D. *First*, it is now easy to see from Eqs. (44), (45), and (46) that neither β nor σ nor ρ can be negative. If one of these powers would be negative, the limit $x\to 0$ would lead to a vanishing left-hand side of the respective equation whereas the right-hand side is singular in this limit. This is a contradiction as the power on the left-hand side of the equation should match the leading power on the right-hand side. *Second*, if one of β, σ, or ρ would be positive, then the diverging left-hand side of the respective equation would require a diverging counterpart on the right-hand side. However, all powers on the right-hand side are positive as we already concluded that β, σ or ρ are not negative and there are no minus signs in any powers on the right- hand sides, cf. Eqs. (41), (42), and (43). Therefore, for positive powers all terms on the right-hand side vanish in the limit $x\to 0$, which leads again to a contradiction. The last possibility is then $\beta=\sigma=\rho=0$, but then one gets perturbative logarithms on the right-hand side of the equation which do not match the constant on the left-hand side. Thus in the all-bare-vertex truncation there is no power solution for general values of the gauge parameters $\lambda\neq 0$ and $\alpha\neq 0$. Based on the considerations on the Slavnov-Taylor identities given in the previous section, we therefore arrive at the conclusion that this truncation is insufficient to determine the infrared behavior of the propagators even qualitatively.

There are two limits for the gauge parameters α and λ in which the situation changes. The first one is $\alpha=0$, that is, ordinary linear covariant gauges. Due to the corresponding Slavnov-Taylor identity, the longitudinal part of the gluon propagator remains undressed, $L(p^2)=1$ [3]. However, replacing dressed vertices by bare ones in the infrared, this identity might be violated (which does not happen in perturbation theory, of course). We therefore employ the general expression $L(p^2)=C(p^2)^\rho$ for the longitudinal gluon dressing function and explore whether the limit $\rho\to 0$ can be taken with bare vertices. In the ghost equation, the squint as well as the sunset diagram disappear and we are left with the one-loop contributions U_T^{dress} and U_L^{dress}. The explicit expression for the ghost equation is given by (cf. Appendix D)

$$B^{-1}x^{-\beta}=U_T^{dress}+U_L^{dress} \tag{47}$$

$$\begin{aligned}
&=x^{(\beta+\sigma)}\frac{g^2N_c\tilde{Z}_1AB}{16\pi^2}\,\frac{-3}{2(\beta+\sigma)(-1+\beta+\sigma)}\\
&\quad\times\frac{\Gamma(2+\beta)\Gamma(1+\sigma)\Gamma(2-\beta-\sigma)}{\Gamma(1-\beta)\Gamma(2-\sigma)\Gamma(3+\beta+\sigma)}\\
&\quad-x^{(\beta+\rho)}\frac{g^2N_c\tilde{Z}_1\lambda BC}{16\pi^2}\,\frac{\rho+1/2}{\beta}\\
&\quad\times\frac{\Gamma(2+\beta)\Gamma(1+\rho)\Gamma(-\beta-\rho)}{\Gamma(-\beta)\Gamma(2-\rho)\Gamma(3+\rho+\beta)}.
\end{aligned} \tag{48}$$

For $\rho\to 0$, we run into the same contradiction as explained above for general values of the gauge parameters α and λ. However, admitting the generation of a (spurious) longitudinal gluon dressing, this contradiction can be resolved in the following way: Equation (45) for the transversal gluon dressing function does not change in structure, therefore $\sigma>0$. Furthermore, we have $\rho>0$ from Eq. (46). Then we have $-2\beta=\sigma$ and/or $-2\beta=\rho$ in the ghost equation (48) and $\beta<0$, i.e., a diverging ghost dressing function in the infrared. From this it follows immediately that the ghost loop is the dominant contribution in both the equations for the transversal and longitudinal gluon dressing function. From these two equations we therefore infer

$$-\beta=\sigma/2=\rho/2=:\kappa, \tag{49}$$

which is consistent with the ghost equation. We thus find an infrared vanishing gluon dressing and a singular ghost dressing function for all values of the gauge parameter λ. This result is identical to the one in the Landau gauge [7–9]. However, a word of caution is in order. In the Landau gauge there are indications [4,41] that the general result (49) does not change when the vertices are dressed. This has been confirmed recently for a range of possible vertex dressings [8]. It is an as yet open question whether this is true for $\lambda\neq 0$ in the same way.

Having addressed the case of linear covariant gauges with $\alpha=0$, we now turn to the other interesting limit, that is, $\lambda=0$, while $\alpha\neq 0$. It is easy to see that the α-dependence of the Lagrangian (1) can be eliminated in this case by partial integration using the constraint $\partial A=0$. However, on the level of the DSEs with bare vertices there remain spurious α-dependent terms on the right-hand side of the gluon equation. In the next section we will investigate the dependence of the Landau gauge solution on these spurious α terms.

IV. SOLUTIONS IN THE LANDAU GAUGE

To assess the influence of the spurious α terms in the Landau gauge, we use the truncation scheme developed in [9]. There the two-loop diagrams in the gluon equation have been neglected as they are subleading in the perturbative regime and ghost loop dominance has been assumed in the infrared. In order to obtain the correct one-loop behavior of the ghost and gluon dressing functions, the gluon loop has been modified by replacing the renormalization constant Z_1 by a momentum-dependent function $\mathcal{Z}_1$,

$$\begin{aligned}
Z_1(L,s)&\to\mathcal{Z}_1(x,y,z;s,L)\\
&=\frac{G(y)^{(-2-6\delta)}}{Z(y)^{(1+3\delta)}}\,\frac{G(z)^{(-2-6\delta)}}{Z(z)^{(1+3\delta)}}.
\end{aligned} \tag{50}$$

Here $L=\Lambda^2$ denotes a cutoff and $s=\mu^2$ a renormalization scale in units of squared momenta. The momentum $x=p^2$ is the one flowing into the loop, $y=q^2$ is the loop momentum over which it is integrated, and $z:=k^2=(p-q)^2$. Furthermore, the anomalous dimension δ of the ghost dressing function has been used. The gluon equation is contracted with the general tensor

$$\mathcal{P}^{(\zeta)}_{\mu\nu}(p)=\delta_{\mu\nu}-\zeta\frac{p_\mu p_\nu}{p^2}. \tag{51}$$

As a completely transversal gluon equation would be independent of the parameter ζ, the use of the general projector provides an opportunity to test for violations of transversality due to the truncation. For $\zeta\neq 4$, one has to take care of spurious quadratic divergencies that have to be subtracted in the kernel of the gluon equation.

The coupled set of equations for the ghost and gluon dressing functions then read as follows:

$$\frac{1}{G(x)}=Z_3-g^2N_c\int\frac{d^4q}{(2\pi)^4}\frac{K(x,y,z)}{xy}G(y)Z(z), \tag{52}$$

$$\frac{1}{Z(x)}=\tilde{Z}_3+g^2\frac{N_c}{3}\int\frac{d^4q}{(2\pi)^4}\frac{M(x,y,z)}{xy}G(y)G(z)+g^2\frac{N_c}{3}\int\frac{d^4q}{(2\pi)^4}\frac{Q(x,y,z)}{xy}Z(y)Z(z)\mathcal{Z}_1(y,z). \tag{53}$$

The kernels ordered with respect to powers of $z:=p^2=(k-q)^2$ have the form

$$K(x,y,z)=\frac{1}{z^2}\left(-\frac{(x-y)^2}{4}\right)+\frac{1}{z}\left(\frac{x+y}{2}\right)-\frac{1}{4}, \tag{54}$$

$$M(x,y,z)=\frac{1}{z}\left(\frac{(\zeta-1)\alpha^2-(\zeta-1)2\alpha+\zeta-2}{4}x+\frac{y}{2}-\frac{\zeta}{4}\frac{y^2}{x}\right)+\frac{1}{2}+\frac{\zeta}{2}\frac{y}{x}-\frac{\zeta}{4}\frac{z}{x}, \tag{55}$$

$$Q(x,y,z)=\frac{1}{z^2}\left(\frac{1}{8}\frac{x^3}{y}+x^2-\frac{19-\zeta}{8}xy+\frac{5-\zeta}{4}y^2+\frac{\zeta}{8}\frac{y^3}{x}\right)+\frac{1}{z}\left(\frac{x^2}{y}-\frac{15+\zeta}{4}x-\frac{17-\zeta}{4}y+\zeta\frac{y^2}{x}\right)-\left(\frac{19-\zeta}{8}\frac{x}{y}+\frac{17-\zeta}{4}+\frac{9\zeta}{4}\frac{y}{x}\right)+z\left(\frac{\zeta}{x}+\frac{5-\zeta}{4y}\right)+z^2\frac{\zeta}{8xy}+\frac{5}{4}(4-\zeta). \tag{56}$$

First we accomplish the infrared analysis. With Eq. (49) we employ the *Ansatz*

$$Z(x)=Ax^{2\kappa},\quad G(x)=Bx^{-\kappa} \tag{57}$$

in Eqs. (52) and (53). After integration we match coefficients of equal powers on both side of the equations and obtain

$$\frac{1}{18}\frac{(2+\kappa)(1+\kappa)}{(3-2\kappa)}=\frac{(4\kappa-2)(-1+\kappa)}{(\zeta-1)[4\kappa^2(\alpha^2-2\alpha+1)+8\kappa\alpha(2-\alpha)+3\alpha(\alpha-2)]+\kappa(10-7\zeta)-6+3\zeta}. \tag{58}$$

The values of κ for different projectors $\mathcal{P}^{(\zeta)}$ can be read off Fig. 3. The curve given by the fully drawn line represents the term on the left-hand side of Eq. (58), whereas the other lines depict the right-hand side for several values of the parameter ζ. Only the two $\zeta=1$ solutions are manifestly independent of α, as pointed out in [8]. The spurious α dependence of the $\zeta=4$ values reported therein here implies that general ζ solutions must necessarily show such an α dependence also, whenever $\zeta\neq 1$. However, the bulk of solutions between $\kappa=0.5$ and $\kappa=0.6$ remains nearly unchanged when α is varied, whereas most of the solutions for $\kappa\geqslant 1$ disappear. For the Brown-Pennington projector $\zeta=4$, no solution can be found for the symmetric case, $\alpha=1$, in complete agreement with the findings of Ref. [8]. Indeed it has been shown [9] that only the smaller solutions are those that connect to numerical results for finite momenta.

We now explore the impact of the spurious α term on the behavior of the solutions for all momenta x. We have solved Eqs. (52) and (53) numerically using the same technique as described in [9]. The results can be seen in Fig. 4. As the dependence of the kernel of the ghost loop on α vanishes in the case of the transverse projector, $\zeta=1$, this solution is the same as the one already calculated in [9]. For the other cases, the power κ changes from 0.5953 for $\zeta=1$ to 0.5020 for $\zeta=3.9$ in accordance with the infrared analysis. The ultraviolet properties of the solutions are slightly disturbed compared to the cases $\alpha=0$ and $\alpha=2$. An analysis of the ultraviolet behavior done similarly to the one in Ref. [9] reveals that the α term in the ghost loop induces a spurious dependence of the anomalous dimensions on the parameter ζ,

$$\gamma=\frac{-26-(\zeta-1)\alpha(2-\alpha)}{44+(\zeta-1)\alpha(2-\alpha)},$$

$$\delta=\frac{-9}{44+(\zeta-1)\alpha(2-\alpha)}. \tag{59}$$

For general α only the transverse projector removes the spurious term in the ghost equation and leads to the correct one-loop scaling of the equations, that is, $\delta=-9/44$ for the ghost and $\gamma=-13/22$ for the gluon dressing function for an arbitrary number of colors and zero flavors.

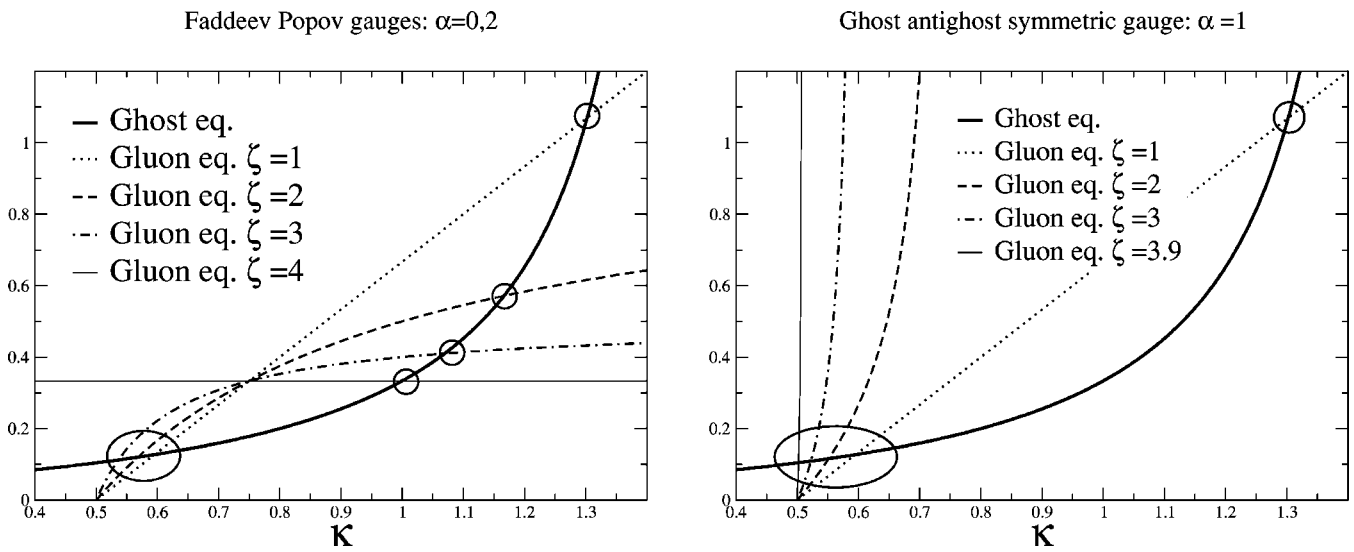


FIG. 3. Here the graphical solution to Eq. (58) is shown. The thick line represents the left hand side of Eq. (58), whereas the other curves depict the right-hand side for different values of the parameters ζ. The left figure shows results for $\alpha=0$ and $\alpha=2$, whereas in the figure on the right $\alpha=1$. The ellipse marks the bulk of solutions between $\kappa=0.5$ and $\kappa=0.6$ for $\zeta=1$, whereas the circles in the left figure show the movement of the solution for the Brown-Pennington case $\zeta=4$ from $\kappa=1$ to $\kappa=1.3$.

V. CONCLUSION

We have studied the infrared behavior of the ghost and gluon propagators in general covariant gauges. These gauges allow us to interpolate via a second gauge parameter between the linear-covariant ones of standard Faddeev-Popov theory and the ghost-antighost symmetric gauges. We derived the corresponding generalized Dyson-Schwinger equations for the propagators which include the ones of linear-covariant gauges as the limit where the second gauge parameter vanishes. Note that ghost-antighost symmetric gauges are particularly interesting as they allow an interpretation of the antighost field being the antiparticle of the ghost which includes also the possibility of a ghost-antighost condensate. Due to the emergence of a four-ghost interaction term in the Lagrangian for general values of gauge parameters, the Dyson-Schwinger equation of the ghost propagator displays a rich structure very similar to the one of the gluon equation. On the other hand, in the gluon equation we obtain the same structure as in linear covariant gauges apart from the fact that the gluon propagator acquires a nontrivial longitudinal part which appears in turn in all diagrams. The gluon and ghost equations depend therefore on three dressing functions, one for the ghost, one for the transverse part of the gluon propagator, and one for the longitudinal one, which are constrained, however, by Slavnov-Taylor identities in an intricate way.

We then employed a truncation scheme for the Dyson-Schwinger equations that uses bare vertices in place of the dressed ones. The success of this particular truncation scheme in the Landau gauge has been attributed to the nonrenormalization of the ghost-gluon vertex, that is, $\tilde{Z}_1=1$. We addressed the infrared behavior of the ghost and gluon propagators for general gauges by employing power-law *Ansätze* for the respective dressing functions. We then have been able to evaluate the infrared behavior of the gluon and ghost equations analytically.

For all linear covariant gauges we find a similar result as compared to the one in the Landau gauge: an infrared-suppressed gluon propagator and an infrared-enhanced ghost. Whereas in the Landau gauge there are indications that this generic result is not changed when the vertices are dressed [8], it remains an open question whether this is the case in linear covariant gauges in general. Away from linear covariant gauges, that is in the general case $\alpha\neq0$ and $\lambda\neq0$, we do not find power solutions for the dressing functions. However, we expect this to change with appropriate vertex dressings. Nevertheless, it remains to be emphasized that therefore also the occurrence of a ghost and/or gluon mass is excluded in this specific truncation scheme within this class of gauges. A Dyson-Schwinger equation-based investigation of the related question of a ghost-antighost vacuum condensate, or more generally, of an ''on-shell''-BRS-invariant dimension 2 condensate, needs to take into account the generalized Slavnov-Taylor identities (37) and (39). The question arises whether an infrared masslesslike longitudinal part of the gluon propagator leads for all values of the gauge parameters to the same infrared enhancement of ghosts as observed in the Landau gauge. Work in this direction is in progress.

A special case among all gauges considered here is the Landau gauge. In the limit $\lambda=0$, the general Lagrangian (1) becomes independent of the second gauge parameter α, thus the Landau gauge is also a special case of ghost-antighost symmetric gauges. Although the Lagrangian of the theory is independent of the gauge parameter α, our simple truncation scheme breaks this invariance and spurious α-dependent terms arise in the ghost loop of the gluon Dyson-Schwinger equation. Examining the case $\alpha=1$, we showed that the influence of these spurious terms is very small. We determined

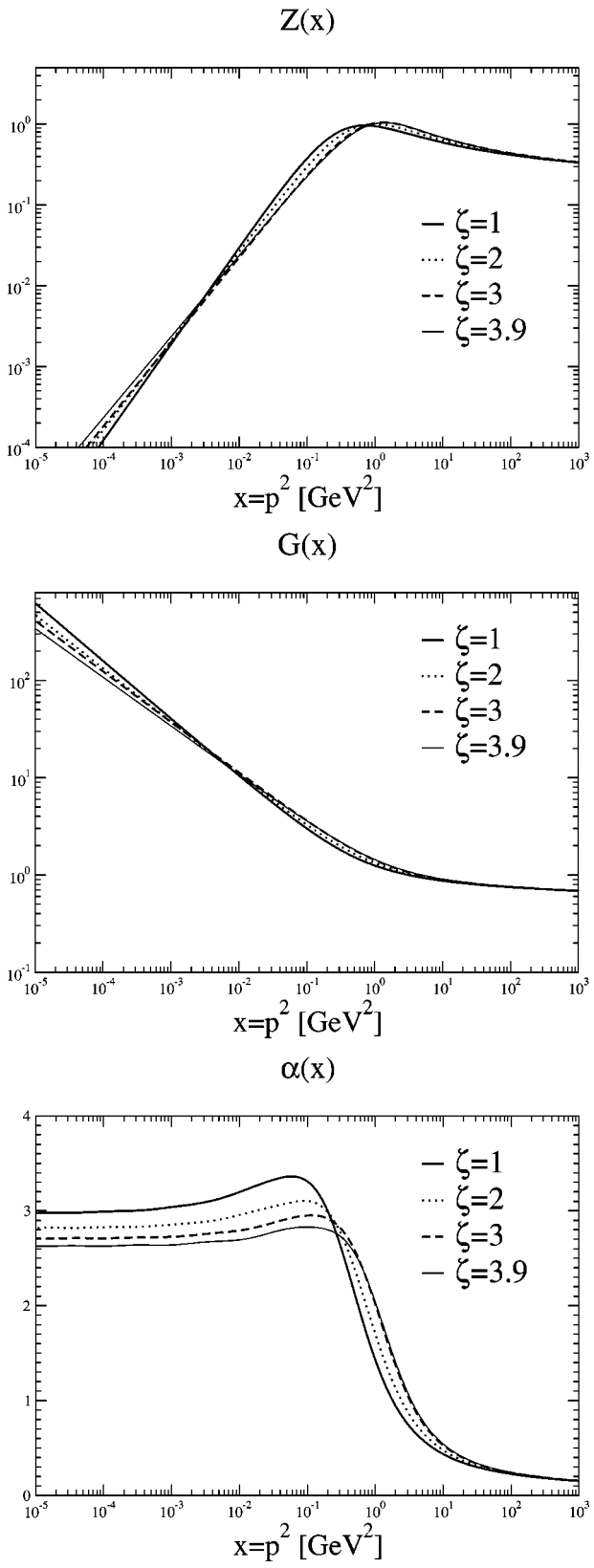


FIG. 4. Shown are the gluon dressing function, the ghost dressing function, and the running coupling in the truncation scheme [9] for the gauge parameters $\alpha=1$ and $\lambda=0$ and different projectors $\mathcal{P}^{(\zeta)}$.

solutions for the ghost and gluon dressing functions both analytically in the infrared and numerically for finite momenta and found solutions identical to the ones of Ref. [9] provided the gluon equation is projected onto its physical, transversal components. We thus recovered the results of the Landau gauge from a different direction in the two-dimensional space of gauge parameters.

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APPENDIX A: DERIVATION OF THE DYSON-SCHWINGER EQUATION FOR THE GHOST PROPAGATOR

We start by transforming the Lagrangian (1) into a more suitable form by partial integration, assuming the usual boundary conditions of vanishing fields at infinity. In order to keep notation on a readable level, we will suppress renormalization constants in this appendix: The derivation of the Dyson-Schwinger equation for the ghost propagator remains formally unchanged by the rescaling (5) and thus the appropriate renormalization constants can be regained straightforwardly. We obtain

$$\begin{aligned}\mathcal{L}=&\frac{1}{2}A^a_\mu\left[-\partial^2\delta_{\mu\nu}+\left(1-\frac{1}{\lambda}\right)\partial_\mu\partial_\nu\right]A^a_\nu-gf^{abc}(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu\\&+\frac{g^2}{4}f^{abe}f^{cde}A^a_\mu A^b_\nu A^c_\mu A^d_\nu+\bar{c}^a\partial^2 c^a\\&+\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{\lambda}{2}g^2f^{ace}f^{bde}\bar{c}^a\bar{c}^b c^c c^d\\&+i\left(1-\frac{\alpha}{2}\right)gf^{abc}\bar{c}^a\partial_\mu(A^c_\mu c^b)+i\frac{\alpha}{2}gf^{abc}\bar{c}^a A^c_\mu\partial_\mu c^b.\end{aligned}\tag{A1}$$

The partition function of the theory is given by

$$\begin{aligned}Z[J,\sigma,\bar{\sigma}]=&\int\mathcal{D}[A\bar{c}c]\\&\times\exp\left\{-\int d^4z\,\mathcal{L}+\int d^4z(A^aJ^a+\bar{\sigma}c+\bar{c}\sigma)\right\}\end{aligned}\tag{A2}$$

with the sources J, σ and $\bar{\sigma}$ of the gluon, antighost and ghost fields, respectively. The action is given by $\mathcal{S}[J,c,\bar{c}]=\int d^4z\,\mathcal{L}$. The generating functional of connected Green's functions, $W[J,\sigma,\bar{\sigma}]$, is defined as the logarithm of the partition function. The functional Legendre transform of W is the effective action

$$\Gamma[A,\bar{c},c] = -W[J,\sigma,\bar{\sigma}] + \int d^4z (A^a J^a + \bar{\sigma} c + \bar{c}\sigma), \tag{A3}$$

which is the generating functional of one-particle irreducible vertex functions. The fields and sources can be written as functional derivatives of the respective generating functionals in the following way:

$$\frac{\delta W}{\delta \sigma} = -\bar{c}, \quad \frac{\delta W}{\delta \bar{\sigma}} = c, \quad \frac{\delta W}{\delta J_\mu} = A_\mu,$$

$$\frac{\delta \Gamma}{\delta c} = -\bar{\sigma}, \quad \frac{\delta \Gamma}{\delta \bar{c}} = -\sigma, \quad \frac{\delta \Gamma}{\delta A_\mu} = J_\mu. \tag{A4}$$

The sign conventions have been chosen such that derivatives with respect to $\bar{c}$ and $\bar{\sigma}$ are left derivatives whereas the ones with respect to c and σ are right derivatives,

$$\frac{\delta}{\delta(\bar{\sigma},\bar{c})} := \text{left derivative}, \quad \frac{\delta}{\delta(\sigma,c)} := \text{right derivative}. \tag{A5}$$

Given that the functional integral is well-defined, the Dyson-Schwinger equation for the ghost propagator is derived from the observation that the integral of a total derivative vanishes provided the measure is invariant under field translations. We take the derivative with respect to the antighost field and obtain

$$\begin{aligned} 0 &= \int \mathcal{D}[A\bar{c}c] \frac{\delta}{\delta \bar{c}} \exp\left\{ -\int d^4z \mathcal{L} + \int d^4z (A^a J^a + \bar{\sigma} c + \bar{c}\sigma) \right\} \\ &= \int \mathcal{D}[A\bar{c}c] \left(-\frac{\delta S[A,c,\bar{c}]}{\delta \bar{c}} + \sigma \right) \\ &\quad \times \exp\left\{ -\int d^4z \mathcal{L} + \int d^4z (A^a J^a + \bar{\sigma} c + \bar{c}\sigma) \right\} \\ &= \left(-\frac{\delta S\left[\frac{\delta}{\delta J}, \frac{\delta}{\delta \bar{\sigma}} \frac{\delta}{\delta \sigma} \right]}{\delta \bar{c}} + \sigma \right) Z[J,\sigma,\bar{\sigma}]. \end{aligned} \tag{A6}$$

Now we use the relations (A4) and apply a further functional derivative with respect to the source $\sigma^b(y)$. We arrive at

$$0 = \left(-\frac{\delta S}{\delta \bar{c}^c(z)} \bar{c}^b(y) + \sigma^c(z) \bar{c}^b(y) + \delta(z-y)\delta_{cb} \right) Z[J,\sigma,\bar{\sigma}] \tag{A7}$$

with explicit color indices and space-time arguments. Setting the sources equal to zero we obtain the ghost Dyson-Schwinger equation

$$\left\langle \frac{\delta S}{\delta \bar{c}^c(z)} \bar{c}^b(y) \right\rangle = \delta(z-y)\delta_{cb}. \tag{A8}$$

The derivative is easily calculated

$$\begin{aligned} \frac{\delta S}{\delta \bar{c}^c(z)} &= \partial^2 c^c(z) + \frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right) \frac{\lambda}{2} g^2 f^{cde} f^{fge} \bar{c}^d(z) c^f(z) c^g(z) \\ &\quad + i\left(1-\frac{\alpha}{2}\right) g f^{cde} \partial_\mu [A^e_\mu(z) c^d(z)] \\ &\quad + i\frac{\alpha}{2} g f^{cde} A^e_\mu(z) \partial_\mu c^d(z). \end{aligned} \tag{A9}$$

Whereas in the covariant formalism full and connected *three-point functions* are the same, the *four-point correlations* have to be decomposed into disconnected and connected parts. For the four-ghost correlation function this results in

$$\begin{aligned} &\langle \bar{c}^b(y) \bar{c}^d(z) c^f(z) c^g(z) \rangle \\ &\quad = \langle \bar{c}^b(y) c^g(z) \rangle \langle \bar{c}^d(z) c^f(z) \rangle - \langle \bar{c}^b(y) c^f(z) \rangle \\ &\quad \times \langle \bar{c}^d(z) c^g(z) \rangle + \langle \bar{c}^b(y) \bar{c}^d(z) c^f(z) c^g(z) \rangle_{\text{conn}}. \end{aligned} \tag{A10}$$

Keeping in mind the Grassmann nature of the ghost and antighost fields we then obtain

$$\begin{aligned} -\delta(z-y)\delta_{cb} &= \partial^2 \langle \bar{c}^b(y) c^c(z) \rangle + \frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right) \frac{\lambda}{2} g^2 f^{cde} f^{fge} \{ \langle \bar{c}^b(y) \bar{c}^d(z) c^f(z) c^g(z) \rangle + (\langle \bar{c}^b(y) c^g(z) \rangle \langle \bar{c}^d(z) c^f(z) \rangle \\ &\quad - \langle \bar{c}^b(y) c^f(z) \rangle \langle \bar{c}^d(z) c^g(z) \rangle) \} + \left(1-\frac{\alpha}{2}\right) g f^{cde} \langle \bar{c}^b(y) \partial_\mu [A^e_\mu(z) c^d(z)] \rangle + \frac{\alpha}{2} g f^{cde} \langle \bar{c}^b(y) A^e_\mu(z) \partial_\mu c^d(z) \rangle, \end{aligned} \tag{A11}$$

where all correlations are connected Green's functions. We now use the relation

$$\delta(y-x)\,\delta^{ab}=\frac{\delta\overline{\sigma}^{b}(y)}{\delta\overline{\sigma}^{a}(x)}=\int d^4z\frac{\delta\overline{\sigma}^{b}(y)}{\delta\overline{c}^{d}(z)}\,\frac{\delta\overline{c}^{d}(z)}{\delta\overline{\sigma}^{a}(x)}$$

$$=\int d^4z\frac{\delta^2\Gamma}{\delta\overline{c}^{d}(z)\,\delta c^{b}(y)}\,\frac{\delta^2 W}{\delta\overline{\sigma}^{a}(x)\,\delta\sigma^{d}(z)}$$

$$=:\int d^4z[D_G^{db}(z-y)]^{-1}D_G^{ad}(x-z) \tag{A12}$$

and multiply Eq. (A11) with $-[D_G^{ac}(x-z)]^{-1}=[\langle\overline{c}^{c}(z)c^{a}(x)\rangle]^{-1}$. We arrive at

$$[D_G^{ab}(x-y)]^{-1}=\partial^2\delta(x-y)\,\delta^{ab}-\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{\lambda}{2}g^2f^{cde}f^{fge}\int d^4z[D_G^{ac}(x-z)]^{-1}\{\langle\overline{c}^{b}(y)\overline{c}^{d}(z)c^{f}(z)c^{g}(z)\rangle$$

$$+\langle\overline{c}^{b}(y)c^{g}(z)\rangle\,\langle\overline{c}^{d}(z)c^{f}(z)\rangle-\langle\overline{c}^{b}(y)c^{f}(z)\rangle\,\langle\overline{c}^{d}(z)c^{g}(z)\rangle\}-i\left(1-\frac{\alpha}{2}\right)gf^{cde}$$

$$\times\int d^4z[D_G^{ac}(x-z)]^{-1}\langle\overline{c}^{b}(y)\partial_\mu[A_\mu^{e}(z)c^{d}(z)]\rangle-i\frac{\alpha}{2}gf^{cde}\int d^4z[D_G^{ac}(x-z)]^{-1}\langle\overline{c}^{b}(y)A_\mu^{e}(z)\partial_\mu c^{d}(z)\rangle. \tag{A13}$$

Before we decompose the connected Green's functions into one-particle irreducible ones, we have to take care of the space-time derivatives. Noting that

$$\partial_\mu^{z}\frac{\delta^2 W}{\delta J_\mu^{c}(z)\sigma^{d}(z)}=-\int d^4u\,\partial_\mu^{u}[\,\delta(u-z)]\frac{\delta^2 W}{\delta J_\mu^{c}(u)\sigma^{d}(u)}$$

$$=-\int d^4[uv]\,\partial_\mu^{u}[\,\delta(u-z)]\,\delta(u-v)\,\frac{\delta^2 W}{\delta J_\mu^{c}(v)\sigma^{d}(u)} \tag{A14}$$

with the abbreviation $d^4u\,d^4v=:d^4[uv]$, and

$$\frac{\delta}{\delta J_\mu^{c}(z)}\partial_\mu^{z}\frac{\delta W}{\sigma^{d}(z)}=\int d^4[uv]\,\delta(u-z)\,\delta(u-v)\,\frac{\delta}{\delta J_\mu^{c}(v)}\partial_\mu^{u}\frac{\delta W}{\sigma^{d}(u)}$$

$$=-\int d^4[uv]\,\partial_\mu^{u}[\,\delta(u-z)\,\delta(u-v)]\frac{\delta^2 W}{\delta J_\mu^{c}(v)\sigma^{d}(u)} \tag{A15}$$

we can replace the derivative terms by the bare ghost-gluon vertex defined in Appendix B. The tadpole term can be treated in the following way:

$$\int d^4z[D_G^{ac}(x-z)]^{-1}f^{cde}f^{fge}\{\langle\overline{c}^{b}(y)c^{g}(z)\rangle\langle\overline{c}^{d}(z)c^{f}(z)\rangle-\langle\overline{c}^{b}(y)c^{f}(z)\rangle\langle\overline{c}^{d}(z)c^{g}(z)\rangle\}$$

$$=2\int d^4z[D_G^{ac}(x-z)]^{-1}f^{cde}f^{fge}\{\langle\overline{c}^{b}(y)c^{g}(z)\rangle\langle\overline{c}^{d}(z)c^{f}(z)\rangle\}$$

$$=2\int d^4[zuv][D_G^{ac}(x-z)]^{-1}\,\delta(z-u)\,\delta(u-v)f^{cde}f^{fge}D_G^{gb}(z-y)D_G^{fd}(v-u)$$

$$=2\int d^4[uv]\,\delta(x-y)\,\delta(z-u)\,\delta(u-v)f^{bde}f^{fae}D_G^{fd}(v-u). \tag{A16}$$

Plugging the expressions for the ghost-gluon loop and the one for the tadpole into Eq. (A13) and using the expression for the bare four-ghost vertex given in Appendix B, we obtain

$$[D_G^{ab}(x-y)]^{-1}=\partial^2\delta(x-y)\,\delta^{ab}-\int d^4[uv]\Gamma_{4gh}^{(0)bdfa}(x,u,v,y)D_G^{fd}(v-u)+\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{\lambda}{2}g^2f^{cde}f^{fge}$$
$$\times\int d^4[zuv]\delta(z-u)\,\delta(u-v)[D_G^{ac}(x-z)]^{-1}\langle\overline{c}^b(y)\overline{c}^d(z)c^f(u)c^g(v)\rangle$$
$$-\int d^4[zuv]\Gamma_\mu^{(0)cde}(z,u,v)[D_G^{ac}(x-z)]^{-1}\langle\overline{c}^b(y)A_\mu^e(v)c^d(u)\rangle. \tag{A17}$$

To decompose the connected Green's functions into one-particle irreducible ones, we use the relations

$$\langle A_\mu^e(v)\overline{c}^b(y)c^d(u)\rangle=\int d^4[z_1z_2z_3]D_{\mu\nu}^{ef}(v-z_1)D_G^{bg}(y-z_2)\Gamma_\nu^{fhg}(z_1,z_3,z_2)D_G^{hd}(u-z_3), \tag{A18}$$

$$\langle\overline{c}^b(y)\overline{c}^d(z)c^f(u)c^g(v)\rangle=\int d^4[u_1u_2u_3u_4u_5u_6]\{D_{\mu\nu}^{ek}(u_1-u_4)D_G^{fl}(u-u_5)\Gamma_\nu^{khl}(u_4,u_6,u_5)D_G^{hb}(u_6-y)$$
$$\times D_G^{gi}(v-u_2)\Gamma_\mu^{eji}(u_1,u_3,u_2)D_G^{jd}(u_3-z)\}-\int d^4[u_1u_2u_3u_4u_5]$$
$$\times\{D_G^{fe}(u-u_4)D_G^{hb}(u_5-y)\times D_G^{gi}(v-u_2)\Gamma_{4gh}^{jhei}(u_3,u_5,u_4,u_2)D_G^{jd}(u_3-z)\}, \tag{A19}$$

which have been derived in Appendix B.

Substituting these expressions into Eq. (A17), we arrive at the final expression for the ghost Dyson-Schwinger equation in coordinate space:

$$[D_G^{ab}(x-y)]^{-1}=[D_G^{(0)ab}(x-y)]^{-1}-\int d^4[uv]\Gamma_{4gh}^{(0)bdfa}(x,u,v,y)D_G^{fd}(v-u)$$
$$-\frac{1}{2}\int d^4[zuvu_1u_2u_3u_4u_5]\Gamma_{4gh}^{(0)bdgf}(y,z,v,u)D_{\mu\nu}^{ek}(u_1-u_4)D_G^{fl}(u-u_5)\Gamma_\nu^{kal}(u_4,x,u_5)D_G^{gi}(v-u_2)$$
$$\times\Gamma_\mu^{eij}(u_1,u_3,u_2)D_G^{jd}(u_3-z)-\frac{1}{2}\int d^4[zuvu_1u_2u_3u_4]\Gamma_{4gh}^{(0)bdgf}(y,z,v,u)D_G^{fe}(u-u_4)D_G^{gi}(v-u_2)$$
$$\times\Gamma_{4gh}^{jaei}(u_3,x,u_4,u_2)D_G^{jd}(u_3-z)-\int d^4[zuvz_1z_2z_3]\Gamma_\mu^{(0)bde}(y,u,v)D_{\mu\nu}^{ef}(v-z_1)\Gamma_\nu^{fha}(z_1,z_3,x)D_G^{hd}(u-z_3), \tag{A20}$$

where an additional minus sign arises from the interchange of the color indices f and g in the bare four-ghost vertices and from the interchange of j and i in the ghost-gluon vertex.

After performing a Fourier transformation we obtain the respective expression in momentum space

$$[D_G(p)]^{-1}=[D_G^{(0)}(p)]^{-1}+(-N_c)\frac{g^2}{(2\pi)^4}\int d^4q\Gamma_{4gh}^{(0)}D_G(q)+\left(\frac{-N_c^2}{2}\right)\frac{1}{2}\frac{g^4}{(2\pi)^8}\int d^4[q_1q_2]\Gamma_{4gh}^{(0)}D_{\mu\nu}(p-q_1)D_G(q1)$$
$$\times\Gamma_\nu(p,q_1)D_G(q_2)\Gamma_\mu(-p+q_1+q_2,q_2)D_G(p-q_1-q_2)-(-N_c^2)\frac{1}{2}\frac{g^4}{(2\pi)^8}\int d^4[q_1q_2]\Gamma_{4gh}^{(0)}D_G(q_1)$$
$$\times D_G(p-q_1-q_2)\Gamma_{4gh}(p,q_1,q_2)D_G(q_2)+(-N_c)\frac{g^2}{(2\pi)^4}\int d^4q\Gamma_\mu^{(0)}(p,q)D_{\mu\nu}(p-q)\;\Gamma_\nu(q,p)D_G(q), \tag{A21}$$

where the color traces have been carried out and the reduced vertices defined in Appendix B have been used.

APPENDIX B: DEFINITIONS AND DECOMPOSITIONS

1. Ghost and gluon propagators

The full ghost and gluon propagators in coordinate space are defined to be

$$\langle c^a(x)\overline{c^b}(y)\rangle = \frac{\delta W}{\delta\sigma^b(y)\,\delta\overline{\sigma^a}(x)} = D_G^{ab}(x-y), \tag{B1}$$

$$\langle A_\mu^a(x)A_\nu^b(y)\rangle = \frac{\delta W}{\delta J_\nu^b(y)\,\delta J_\nu^a(x)} = D_{\mu\nu}^{ab}(x-y). \tag{B2}$$

The bare propagators in coordinate space can be easily derived from the quadratic part of the action,

$$S_{quad} = \int d^4x' \left\{ \frac{1}{2}A_\mu^a \left[-\partial^2\delta_{\mu\nu} + \left(1-\frac{1}{\lambda}\right)\partial_\mu\partial_\nu \right] A_\nu^a + \overline{c^a}\partial^2 c^a \right\} \tag{B3}$$

and are given by

$$[D_G^{(0)ab}(x-y)]^{-1} = \frac{\delta^2 S_{quad}}{\delta\overline{c^a}(x)c^b(y)} = \delta^{ab}\partial^2\delta(x-y), \tag{B4}$$

$$[D_{\mu\nu}^{(0)ab}(x-y)]^{-1} = \frac{\delta^2 S_{quad}}{\delta A_\mu^a(x)A_\nu^b(y)} = \delta^{ab}\left(-\partial^2\delta_{\mu\nu} + \left(1-\frac{1}{\lambda}\right)\partial_\mu\partial_\nu \right)\delta(x-y), \tag{B5}$$

with the gauge parameter λ. After Fourier transformation one obtains the corresponding expressions in momentum space:

$$[D_G^{(0)ab}(p)]^{-1} = -\delta^{ab}p^2 \tag{B6}$$

$$[D_{\mu\nu}^{(0)ab}(p)]^{-1} = \delta^{ab}\left(\delta_{\mu\nu} - \left(1-\frac{1}{\lambda}\right)\frac{p_\mu p_\nu}{p^2} \right)p^2. \tag{B7}$$

2. Ghost-gluon vertex

From the ghost gluon part of the action

$$S_{ghgl} = \int d^4x' \left\{ -i\left(1-\frac{\alpha}{2}\right) g f^{abc}(\partial^\mu\overline{c})A_\mu^c c^b + i\frac{\alpha}{2} g f^{abc}\overline{c^a}A_\mu^c\partial^\mu c^b \right\} \tag{B8}$$

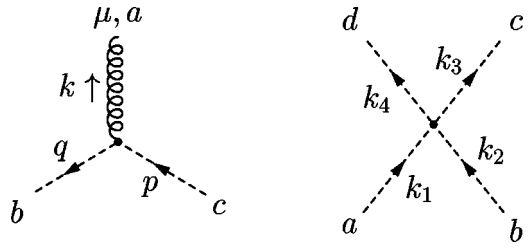

FIG. 5. Momentum routing for the tree level ghost-gluon and four-ghost vertices.

the tree level ghost gluon vertex Γ_μ^{abc} is easily derived,

$$\Gamma_\mu^{(0)abc}(x,y,z) = \frac{\delta^3 S_{ghgl}}{\delta A_\mu^a(x)\,\delta\overline{c^b}(y)\,\delta c^c(z)} = -gf^{abc}\left[i\left(1-\frac{\alpha}{2}\right)[\partial_\mu^z\delta^4(z-y)]\delta^4(z-x) + i\frac{\alpha}{2}\partial_\mu^z[\delta^4(z-y)\,\delta^4(z-x)] \right]. \tag{B9}$$

Using the momentum conventions of Fig. 5, the Fourier-transformed bare ghost-gluon vertex reads

$$\Gamma_\mu^{(0)abc}(k,p,q) = \int d^4[xyz]\Gamma_\mu^{abc}(x,y,z)e^{i(k\cdot x+q\cdot y-p\cdot z)} = gf^{abc}(2\pi)^4\delta^4(k+q-p) \times\left[\left(1-\frac{\alpha}{2}\right)q_\mu + \frac{\alpha}{2}p_\mu \right], \tag{B10}$$

where the abbreviation $d^4x d^4y d^4z =: d^4[xyz]$ has been introduced. Note the symmetry of the vertex in the ghost momenta p_μ and q_μ if $\alpha=1$. For convenience we define a reduced vertex function $\Gamma_\mu^{(0)}(p,q)$ by

$$\Gamma_\mu^{(0)abc}(k,p,q) = gf^{abc}(2\pi)^4\delta^4(k+q-p)\Gamma_\mu^{(0)}(p,q)$$

$$\Gamma_\mu^{(0)}(p,q) = \left[\left(1-\frac{\alpha}{2}\right)q_\mu + \frac{\alpha}{2}p_\mu \right]. \tag{B11}$$

The full one-particle irreducible ghost gluon vertex in coordinate space is given by

$$\Gamma_\mu^{abc}(x,y,z) = \frac{\delta\Gamma}{\delta J_\mu^a(x)\,\delta\overline{c^b}(y)\,\delta c^c(z)}. \tag{B12}$$

3. Four-ghost vertex

The four-ghost vertex Γ_{4g}^{abcd} is derived from the four-ghost part of the action,

$$S_{4gh}=\int d^4x'\left\{\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\frac{\lambda}{2}g^2 f^{ace}f^{bde}\bar{c}^a\bar{c}^b c^c c^d\right\}, \tag{B13}$$

which leads to

$$\begin{aligned}\Gamma_{4g}^{(0)abcd}(x,y,z,w)&=\frac{\delta^4 S_{4gh}}{\delta\bar{c}^a(x)\,\delta\bar{c}^b(y)\,\delta c^c(z)\,\delta c^d(w)}\\&=\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\lambda g^2 f^{abe}f^{cde}\delta^4(x-y)\\&\quad\times\delta^4(y-z)\,\delta^4(z-w).\end{aligned} \tag{B14}$$

Again using the momentum conventions of Fig. 5, one obtains for the Fourier-transformed bare four-ghost vertex

$$\begin{aligned}\Gamma_{4g}^{(0)abcd}(k_1,k_2,k_3,k_4)&=\frac{\delta^4 S_{4gh}}{\delta\bar{c}^a(x)\,\delta\bar{c}^b(y)\,\delta c^c(z)\,\delta c^d(w)}\\&=\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\lambda g^2 f^{abe}f^{cde}(2\pi)^4\\&\quad\times\delta^4(k_1+k_2-k_3-k_4).\end{aligned} \tag{B15}$$

We define a reduced vertex function $\Gamma_{4g}^{(0)}$ by

$$\begin{aligned}\Gamma_{4g}^{(0)abcd}(k_1,k_2,k_3,k_4)&=g^2 f^{abe}f^{cde}(2\pi)^4\\&\quad\times\delta^4(k_1+k_2-k_3-k_4)\Gamma_{4g}^{(0)}\end{aligned}$$

$$\Gamma_{4g}^{(0)}=\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\lambda. \tag{B16}$$

The full four-ghost vertex in coordinate space is formally given by

$$\Gamma^{abcd}(x,y,z)=\frac{\delta\Gamma}{\delta\bar{c}^a(x)\,\delta\bar{c}^b(y)\,\delta c^c(z)\,\delta c^d(y)}. \tag{B17}$$

4. Decomposition of connected ghost-gluon Green's function

With the help of the matrix relation

$$\frac{\delta\chi^{-1}}{\delta\phi}=-\chi^{-1}\frac{\delta\chi}{\delta\phi}\chi^{-1}, \tag{B18}$$

and the identity

$$\begin{aligned}\delta(y-x)\,\delta^{ab}&=\int d^4z\frac{\delta\bar{c}^b(y)}{\delta\bar{c}^d(z)}\frac{\delta\bar{c}^d(z)}{\delta\bar{\sigma}^a(x)}\\&=\int d^4z\frac{\delta^2\Gamma}{\delta\bar{c}^d(z)\,\delta c^b(y)}\frac{\delta^2 W}{\delta\bar{\sigma}^a(x)\,\delta\sigma^d(z)},\end{aligned} \tag{B19}$$

we decompose the connected ghost-gluon correlation function, $\langle A_\mu^a(x)\bar{c}^b(y)c^c(z)\rangle$, in the following way:

$$\begin{aligned}\langle A_\mu^a(x)\bar{c}^b(y)c^c(z)\rangle&=\frac{\delta^3 W}{\delta J_\mu^a(x)\,\delta\bar{\sigma}^b(y)\,\delta\sigma^c(z)}\\&=\frac{\delta}{\delta J_\mu^a(x)}\left[\frac{\delta^2\Gamma}{\delta\bar{c}^b(y)\,\delta c^c(z)}\right]^{-1}\\&=\int d^4u_1\frac{\delta A_\nu^d(u_1)}{\delta J_\mu^a(x)}\frac{\delta}{\delta A_\nu^d(u_1)}\left[\frac{\delta^2\Gamma}{\delta\bar{c}^b(y)\,\delta c^c(z)}\right]^{-1}\\&=\int d^4[u_1u_2u_3]\frac{\delta^2 W}{\delta J_\mu^a(x)\,\delta J_\nu^d(u_1)}\frac{\delta^2 W}{\delta\bar{\sigma}^b(y)\,\delta\sigma^e(u_2)}\frac{\delta^3\Gamma}{\delta A_\nu^d(u_1)\,\delta\bar{c}^e(u_2)\,\delta c^f(u_3)}\frac{\delta^2 W}{\delta\bar{\sigma}^f(u_3)\,\delta\sigma^c(z)}\\&=\int d^4[u_1u_2u_3]D_{\mu\nu}^{ad}(x-u_1)D_G^{eb}(u_2-y)\Gamma_\nu^{def}(u_1,u_2,u_3)D_G^{cf}(u_3-z).\end{aligned} \tag{B20}$$

Here we used the abbreviation $d^4[u_1u_2u_3]:=d^4u_1\,d^4u_2\,d^4u_3$ and the definitions of the gluon propagator $D_{\mu\nu}$, the ghost propagator D_G, and the ghost-gluon vertex Γ_ν given in previous subsections.

5. Decomposition of connected four-ghost Green's function

Furthermore, we need the decomposition of the four-ghost correlation function into one-particle irreducible parts. We start at a stage where the sources are still present and set them to zero at the end of the derivation. We first give the decomposition of the connected ghost-antighost-ghost three-point function

$$\langle \overline{c^b}(y) c^c(z) \overline{c^d}(w) \rangle = \frac{\delta^3 W}{\delta \overline{\sigma^b}(y)\, \delta \sigma^c(z)\, \delta \overline{\sigma^d}(w)}$$
$$= \frac{\delta}{\delta \overline{\sigma^b}(y)} \left[\frac{\delta^2 \Gamma}{\delta \sigma^c(z)\, \delta \overline{\sigma^d}(w)} \right]^{-1}$$
$$= \int d^4 u_1 \frac{\delta A^e_\nu(u_1)}{\delta \overline{\sigma^b}(y)} \frac{\delta}{\delta A^e_\nu(u_1)} \left[\frac{\delta^2 \Gamma}{\delta \sigma^c(z)\, \delta \overline{\sigma^d}(w)} \right]^{-1}$$
$$= \int d^4[u_1 u_2 u_3] \frac{\delta^2 W}{\delta \overline{\sigma^b}(y)\, \delta J^e_\nu(u_1)} \frac{\delta^2 W}{\delta \sigma^c(z)\, \delta \overline{\sigma^f}(u_2)} \frac{\delta^3 \Gamma}{\delta A^e_\nu(u_1)\, \delta c^f(u_2)\, \delta \overline{c^g}(u_3)} \frac{\delta^2 W}{\delta \sigma^g(u_3)\, \delta \overline{\sigma^d}(w)}. \tag{B21}$$

Then we decompose the connected four-ghost Green's function:

$$\langle c^a(x) \overline{c^b}(y) c^c(z) \overline{c^d}(w) \rangle = \frac{\delta^4 W}{\delta \sigma^a(x)\, \delta \overline{\sigma^b}(y)\, \delta \sigma^c(z)\, \delta \overline{\sigma^d}(w)}$$
$$= \frac{\delta}{\delta \sigma^a(x)} \int d^4[u_1 u_2 u_3] \frac{\delta^2 W}{\delta \overline{\sigma^b}(y)\, \delta J^e_\mu(u_1)} \frac{\delta^2 W}{\delta \sigma^c(z)\, \delta \overline{\sigma^f}(u_2)} \frac{\delta^3 \Gamma}{\delta A^e_\mu(u_1)\, \delta c^f(u_2)\, \delta \overline{c^g}(u_3)} \frac{\delta^2 W}{\delta \sigma^g(u_3)\, \delta \overline{\sigma^d}(w)}. \tag{B22}$$

Carrying out the remaining derivative gives four terms. The two terms where the derivative acts on the second and on the last propagator vanish, because the term $\delta W / [\, \delta \overline{\sigma^b}(y)\, \delta J^e_\nu(u_1)]$ vanishes when the sources are set to zero. The contribution where the derivative acts on the first propagator can be treated using Eq. (B20). In the expression with the derivative acting on the vertex we use

$$- \frac{\delta^2 W}{\delta \overline{\sigma^b}(y)\, \delta J^e_\mu(u_1)} \frac{\delta^4 \Gamma}{\delta \sigma^a(x)\, \delta A^e_\mu(u_1)\, \delta c^f(u_2)\, \delta \overline{c^g}(u_3)}$$
$$= \frac{\delta^4 \Gamma}{\delta \sigma^a(x)\, \delta \overline{\sigma^b}(y)\, \delta c^f(u_2)\, \delta \overline{c^g}(u_3)}$$
$$= \int d^4 u_4 \frac{\delta^2 W}{\delta \sigma^a(x)\, \overline{\sigma^e}(u_4)} \frac{\delta^4 \Gamma}{\delta c^e(u_4)\, \delta \overline{\sigma^b}(y)\, \delta c^f(u_2)\, \delta \overline{c^g}(u_3)}$$
$$= \int d^4[u_4 u_5] \frac{\delta^2 W}{\delta \sigma^a(x)\, \overline{\sigma^e}(u_4)} \frac{\delta^2 W}{\delta \overline{\sigma^b}(y)\, \delta \sigma^h(u_5)} \frac{\delta^4 \Gamma}{\delta c^e(u_4)\, \delta \overline{c^h}(u_5)\, \delta c^f(u_2)\, \delta \overline{c^g}(u_3)}$$
$$= - \int d^4[u_4 u_5] \frac{\delta^2 W}{\delta \sigma^a(x)\, \overline{\sigma^e}(u_4)} \frac{\delta^2 W}{\delta \sigma^h(u_5)\, \delta \overline{\sigma^b}(y)} \frac{\delta^4 \Gamma}{\delta \overline{c^g}(u_3)\, \delta \overline{c^h}(u_5)\, \delta c^e(u_4)\, \delta c^f(u_2)}. \tag{B23}$$

Collecting all this together we arrive at

$$\langle c^a(x) \overline{c^b}(y) c^c(z) \overline{c^d}(w) \rangle = \int d^4[u_1 u_2 u_3 u_4 u_5 u_6] \left\{ \frac{\delta^2 W}{\delta J^e_\mu(u_1)\, \delta J^f_\nu(u_4)} \frac{\delta^2 W}{\delta \sigma^a(x)\, \delta \overline{\sigma^g}(u_5)} \frac{\delta^3 \Gamma}{\delta A^f_\nu(u_4)\, \delta c^g(u_5)\, \delta \overline{c^h}(u_6)} \right.$$
$$\left. \times \frac{\delta^2 W}{\delta \sigma^h(u_6)\, \delta \overline{\sigma^b}(y)} \frac{\delta^2 W}{\delta \sigma^c(z)\, \delta \overline{\sigma^i}(u_2)} \frac{\delta^3 \Gamma}{\delta A^e_\mu(u_1)\, \delta c^i(u_2)\, \delta \overline{c^j}(u_3)} \frac{\delta^2 W}{\delta \sigma^j(u_3)\, \delta \overline{\sigma^d}(w)} \right\}$$
$$- \int d^4[u_1 u_2 u_3 u_4 u_5] \left\{ \frac{\delta^2 W}{\delta \sigma^a(x)\, \overline{\sigma^e}(u_4)} \frac{\delta^2 W}{\delta \sigma^h(u_5)\, \delta \overline{\sigma^b}(y)} \frac{\delta^2 W}{\delta \sigma^c(z)\, \delta \overline{\sigma^f}(u_2)} \right.$$
$$\left. \times \frac{\delta^4 \Gamma}{\delta \overline{c^g}(u_3)\, \delta \overline{c^h}(u_5)\, \delta c^e(u_4)\, \delta c^f(u_2)} \frac{\delta^2 W}{\delta \sigma^g(u_3)\, \delta \overline{\sigma^d}(w)} \right\}. \tag{B24}$$

Interchanging some Grassmann fields in the correlations and using the definitions for the propagators and vertices given in the previous subsections, we arrive at

$$\begin{aligned}\langle \overline{c^b}(y)\overline{c^d}(w)c^a(x)c^c(z)\rangle = &\int d^4[u_1u_2u_3u_4u_5u_6]\{D^{ef}_{\mu\nu}(u_1-u_4)D^{ag}_G(x-u_5)\Gamma^{fhg}_{\nu}(u_4,u_6,u_5)D^{hb}_G(u_6-y)D^{ci}_G(z-u_2)\\ &\times\Gamma^{eji}_{\mu}(u_1,u_3,u_2)D^{jd}_G(u_3-w)\}+\int d^4[u_1u_2u_3u_4u_5]\{D^{ae}_G(x-u_4)D^{hb}_G(u_5-y)\\ &\times D^{cf}_G(z-u_2)\Gamma^{hgef}_{4gh}(u_5,u_3,u_4,u_2)D^{gd}_G(u_3-w)\},\end{aligned} \tag{B25}$$

which is the decomposition of the four-ghost correlation used in Appendix A.

APPENDIX C: TENSOR INTEGRALS

The explicit expression for the scalar bubble integral I, defined in Eq. (C1), can be easily evaluated in Euclidean space-time using the Feynman parametrization. With the squared momenta $x=p^2$, $y=q^2$, and $z=(p-q)^2$, the result is given by

$$I(a,b,p) := \int d^4q \frac{1}{y^a z^b} \tag{C1}$$

$$= \pi^2 x^{2-a-b}\frac{\Gamma(2-a)\Gamma(2-b)\Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(4-a-b)}. \tag{C2}$$

The corresponding tensor integrals can be reduced to scalar integrals by extracting combinations of momenta p_μ and the symmetric tensor $\delta_{\mu\nu}$ according to the symmetry properties of the integrand:

$$J_\mu(a,b,p) := \int d^4q \frac{q_\mu}{y^a z^b} = J_1(a,b,p)p_\mu, \tag{C3}$$

$$K_{\mu\nu}(a,b,p) := \int d^4q \frac{q_\mu q_\nu}{y^a z^b} = K_1(a,b,p)p_\mu p_\nu + K_2(a,b,p)x\delta_{\mu\nu}, \tag{C4}$$

$$L_{\mu\nu\rho}(a,b,p) := \int d^4q \frac{q_\mu q_\nu q_\rho}{y^a z^b} = L_1(a,b,p)p_\mu p_\nu p_\rho + L_2(a,b,p)x(p_\mu\delta_{\nu\rho}+p_\nu\delta_{\rho\mu}+p_\rho\delta_{\mu\nu}), \tag{C5}$$

$$\begin{aligned}M_{\mu\nu\rho\sigma}(a,b,p) := &\int d^4q \frac{q_\mu q_\nu q_\rho q_\sigma}{y^a z^b}\\ &= M_1(a,b,p)p_\mu p_\nu p_\rho p_\sigma + M_2(a,b,p)x(\delta_{\mu\nu}p_\rho p_\sigma+\delta_{\mu\rho}p_\nu p_\sigma+\delta_{\mu\sigma}p_\rho p_\mu+\delta_{\nu\rho}p_\mu p_\sigma+\delta_{\nu\sigma}p_\rho p_\mu+\delta_{\rho\sigma}p_\mu p_\nu)\\ &\quad + M_3(a,b,p)x^2(\delta_{\mu\nu}\delta_{\rho\sigma}+\delta_{\mu\rho}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\rho\nu}).\end{aligned} \tag{C6}$$

The scalar integrals in these expressions are calculated by contracting them with appropriate tensors, writing all scalar products in terms of squared momenta x, y, and z and applying Eq. (C2). One arrives at

$$J_1=\pi^2\frac{\Gamma(3-a)\Gamma(2-b)\ \Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(5-a-b)}x^{2-a-b}, \tag{C7}$$

$$K_1=\pi^2\frac{\Gamma(4-a)\Gamma(2-b)\Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(6-a-b)}x^{2-a-b}, \tag{C8}$$

$$K_2=\pi^2\frac{\Gamma(3-a)\Gamma(3-b)\Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(6-a-b)}\times\frac{1}{2(-3+a+b)}x^{2-a-b}, \tag{C9}$$

$$L_1=\pi^2\frac{\Gamma(5-a)\Gamma(2-b)\Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(7-a-b)}x^{2-a-b}, \tag{C10}$$

$$L_2=\pi^2\frac{\Gamma(4-a)\Gamma(3-b)\Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(7-a-b)}\times\frac{1}{2(-3+a+b)}x^{2-a-b}, \tag{C11}$$

$$M_1=\pi^2\frac{\Gamma(6-a)\Gamma(2-b)\Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(8-a-b)}x^{2-a-b}, \tag{C12}$$

$$M_2=\pi^2\frac{\Gamma(5-a)\Gamma(3-b)\Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(8-a-b)}\times\frac{1}{2(-3+a+b)}x^{2-a-b}, \tag{C13}$$

$$M_3=\pi^2\frac{\Gamma(4-a)\Gamma(4-b)\Gamma(a+b-2)}{\Gamma(a)\Gamma(b)\Gamma(8-a-b)}\times\frac{1}{4(-3+a+b)(-4+a+b)}x^{2-a-b}. \tag{C14}$$

APPENDIX D: EXPRESSIONS FOR SOME DIAGRAMS IN THE BARE VERTEX APPROXIMATION

In this appendix we give explicitly the expressions for some diagrams needed for our investigation in the main body of the paper. All algebraic manipulations have been done using the program FORM [40]. Our *Ansätze* for the small momentum behavior of the ghost dressing function G, the transversal gluon dressing function Z, and the longitudinal gluon dressing function L are the power laws

$$G(x)=Bx^{\beta},Z(x)=Ax^{\sigma},L(x)=Cx^{\rho}, \tag{D1}$$

where we have used the abbreviation $x=p^2$.

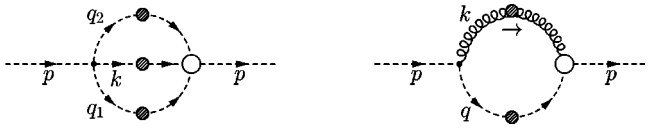

FIG. 6. Momentum routing for the sunset and for the dressing diagram in the ghost equation.

We first evaluate the sunset diagram in the ghost equation given diagrammatically in Fig. 6. With the bare four-ghost vertex given in Eq. (B16) and the abbreviations for the squared momenta $x=p^2$, $y_1=(q_1)^2$, $y_2=(q_2)^2$, $z_1=(p-q_1)^2$, and $z_2=(p-q_1-q_2)^2$ the sunset diagram reads

$$U^{sun}=\frac{N_c^2g^4\tilde{Z}_4}{2(2\pi)^8}\left[\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\lambda\right]^2\int d^4q_1\frac{B(y_1)^{\beta}}{xy_1}\times\int d^4q_2\frac{B^2(y_2)^{\beta}(z_2)^{\beta}}{y_2z_2}. \tag{D2}$$

The factor $1/x$ in the first integral stems from the left-hand side of the ghost equation. We now integrate the inner loop with the help of formula (C2) and obtain

$$U^{sun}=\frac{N_c^2g^4\tilde{Z}_4B^3}{512\pi^6}\left[\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\lambda\right]^2\frac{\Gamma^2(1+\beta)\Gamma(-2\beta)}{\Gamma^2(1-\beta)\Gamma(2+2\beta)}\times\int d^4q_1\frac{(y_1)^{\beta}}{xy_1}(z_1)^{2\beta}, \tag{D3}$$

where z_1 is the total squared momentum flowing through the integrated loop. The second integration is done in the same way. We arrive at

$$U^{sun}=x^{3\beta}\frac{N_c^2g^4\tilde{Z}_4B^3}{512\pi^4}\left[\frac{\alpha}{2}\left(1-\frac{\alpha}{2}\right)\lambda\right]^2\frac{\Gamma^3(1+\beta)\Gamma(-3\beta-1)}{\Gamma^3(1-\beta)\Gamma(3+3\beta)}$$

$$=:x^{3\beta}(U')^{sun}. \tag{D4}$$

As each integration step eats up the two squared momenta in the denominators of the integral kernels, only powers of x to the anomalous dimensions of the dressing functions in the loop (here 3β from three ghost propagators) survive. This mechanism works in the same way for all diagrams and explains the pattern in Eqs. (41), (42), and (43) in the main body of the paper.

Next we evaluate the two contributions in the gluon equation needed for the argument below Eq. (46). The explicit expressions for the kernels of two-loop gluon diagrams are rather lengthy but the calculation is done along the same lines as in the ghost sunset diagram above. Therefore, we just give the final results,

$$V_{TTTT}^{squint} = x^{4\sigma}\frac{-27g^4N_c^2Z_4A^4}{4096\pi^4} \times\frac{\Gamma(-1-4\sigma)\Gamma(1/2-\sigma)\Gamma(3\sigma)\Gamma^2(1+\sigma)}{\Gamma(4-3\sigma)\Gamma^2(2-\sigma)\Gamma(3/2-\sigma)\Gamma(4+4\sigma)} \times 2^{-4\sigma}(-1+3\sigma)(10+\sigma-66\sigma^2+63\sigma^3) \times(5+43\sigma+47\sigma^2) := x^{4\sigma}(V')_{TTTT}^{squint}, \tag{D5}$$

$$W_{LLL}^{sun} = x^{3\delta}\frac{g^4N_c^2Z_4C^3}{1536\pi^4}\frac{1}{(1+3\delta)}\frac{\Gamma^3(1+\delta)\Gamma(1-3\delta)}{\Gamma^3(2-\delta)\Gamma(3+3\delta)}\lambda^3 = x^{3\delta}(W')_{LLL}^{sun}. \tag{D6}$$

Finally we calculate that part in the dressing diagram of the ghost equation which contains the longitudinal part of the gluon propagator for the special case $\alpha=0,2$. These are the linear covariant gauges where $L(x)=1$ by virtue of the Slavnov-Taylor identity. Replacing dressed vertices with bare ones, however, violates this identity. We therefore start with the general expression $L(x)=Cx^\rho$ and investigate whether the limit $\rho\to 0$ can be performed consistently. With the momentum assignments $x=p^2$, $y=q^2$, and $z=k^2=(p-q)^2$ the longitudinal part of the diagram is given by

$$U_L^{dress} = -\frac{N_c g^2\tilde{Z}_1\lambda}{(2\pi)^4}\int d^4q q_\mu\frac{k_\mu k_\nu}{z^2}p_\nu\frac{By^\beta Cz^\rho}{xy} \tag{D7}$$

$$= -\frac{N_c g^2\tilde{Z}_1\lambda BC}{(2\pi)^4}\int d^4q\left\{p_\mu\frac{q_\mu}{y^{1-\beta}z^{2-\rho}} - \frac{p_\mu p_\nu}{x}\frac{q_\nu q_\mu}{y^{1-\beta}z^{2-\rho}} - \frac{1}{y^{-\beta}z^{2-\rho}} + \frac{p_\mu}{x}\frac{q_\mu}{y^{-\beta}z^{2-\rho}}\right\}, \tag{D8}$$

where again the extra factor $1/x$ stems from the left-hand side of the ghost DSE. At this stage of the calculation it is not clear whether there are infrared singularities in the limit $\rho\to 0$. We employ the tensor integrals given in Appendix C, use $x\Gamma(x)=\Gamma(1+x)$, and obtain

$$U_L^{dress} = -\frac{N_c g^2\tilde{Z}_1\lambda BC}{16\pi^2}x^{\beta+\rho}\frac{\Gamma(2+\beta)\Gamma(\rho)\Gamma(-\beta-\rho)}{\Gamma(-\beta)\Gamma(2-\rho)\Gamma(2+\rho+\beta)} \times\frac{\rho^2+\rho/2}{\beta(2+\beta+\rho)}. \tag{D9}$$

In the limit $\rho\to 0$, this expression is infrared-finite, as $\lim_{\rho\to 0}\Gamma(\rho)\rho=1$. We then obtain

$$U_L^{dress} = -\frac{N_c g^2\tilde{Z}_1\lambda BC}{16\pi^2}x^\beta\frac{1}{2\beta(2+\beta)}. \tag{D10}$$

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[42] This has been demonstrated in Refs. [7–9]. On the other hand, self-consistent solutions can also be obtained once the two-loop diagrams are assumed to contribute in the infrared [6].
[43] Note the typos in [18]. For explicit calculations of the renormalization constants for the ghost-antighost symmetric case $\alpha=1$ with Curci-Ferrari mass term and quarks, up to three loops, see [38,39].
[44] See the appendix of Ref. [3].