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mental rotation effect. Other picture-plane correlations may be used in other cases, to describe response times and other data given as evidence for a mental process of rotation.

Where an operation of rotation was thought necessary to provide a criterion for judging the identity of solid forms, these correlations offer only a measure of similarity. Yet, such correlations might help explain why response times to these picture pairs are unique rather than bivalent (as an operation of rotation can proceed either the shorter way around, or the long way). They might explain why response times to views of solids paired with views of enantiomorphic (left- and right-handed) solids increase linearly with depicted angular difference. They might also explain why identical picture pairs are associated with substantially lower response times than picture pairs that represent a small rotation in space. These changes in correlation may seem a nuisance, a confounding variable in the search for a more complete characterization of the new mental kinematics. On the other hand, such measures on the surfaces of picture pairs could account for most of the story. Correlational measures (in contradistinction to invariants) may account for the mental rotation effect in depth without recourse to interpretation of the pictures as representations of depth (see also Niall 1997). Such an approach promises a simple, concrete account of some evidence in support of a kinematics of the mind.

The beauty of **SHEPARD**'s proposal for a kinematics of the mind is the dimly-reflected beauty of geometry, of invariants (i.e., the beauty of group structure and the invariant theory of classical kinematics). Yet we may not require that geometry of three dimensions to explain the experimental phenomena at hand. **HECHT** makes the astute claim that it is an *empirical* matter if invariants or other candidate regularities of the environment provide a model of some aspect of vision. HECHT makes another point that such invariants ought to be "non-trivial" – yet his and SHEPARD's examples are trivial ones which confuse invariants with recurrent environmental characteristics. Invariants are nothing like the direction of illumination for a standing observer, the conservation of water level, or else statistics over *unspecified* geometric properties. For a better account of invariants in the study of vision, see Mundy et al. 1994.

TODOROVIČ makes another strong point that our real knowledge of kinematics is based on a capacity for idealization, different from our ability to see. In contrast to Proffitt's suggestion to **KUBOVY & EPSTEIN**, one can say that motion *often* violates pure kinematics. The friction of rough surfaces, the surface tension of fluids, and many other physical effects underlie ordinary visible phenomena, but do not enter into the idealizations of kinematics. Also, the development of knowledge about kinematics itself is not a story of internalization: such a claim would fictionalize the history of science. The development of kinematics is not a chapter in a psychology of the individual, since the development of physics has supposed an epistemological division of labor. It is not a chapter in the evolutionary psychology of the species either, since organisms adapt to existing local conditions, and not to counterfactual or universal conditions. A psychology which fails to acknowledge the place of ideals in its description of intellectual competence – including competence in kinematics – is a psychology which fails to draw a cogent distinction between perception and thought.

The notion of an internalized kinematics addresses a fundamental problem in psychology – better said, the notion scratches a deep conceptual itch. **HECHT** claims that the notion solves one of the hardest problems in the study of perception, the underspecification problem. **KUBOVY & EPSTEIN** describe the inverse projection problem as fundamental to the problem of vision; the inverse projection problem revisits the underspecification problem for vision. This problem is neither deep nor hard nor fundamental; its conceptual itch is illusory – if anything, the problem is the result of a deep confusion. (Kubovy & Epstein cite James Gibson (1979) as calling it a "pseudoproblem.") No solution involving an internalized kinematics is required where there exists no problem.

Some psychological phenomena like "mental rotation" may arise as a consequence of the characteristics of illumination, or the perspective geometry of pictures. **SHEPARD** remarks that the evolutionary significance of the invariant characteristics of light-reflecting objects is primary to that of the characteristics of light or light sources. Yet for vision, the invariants preserved and the variants generated in the propagation of light are primary to other "invariants" of light-reflecting objects – those which are not preserved when reflected light reaches our eyes. Of course we might prefer to expound the psychology of representation without any detour of discourse about the senses, including the sense of sight. But "in the acutal use of expressions we make detours, we go by sideroads. We see the straight highway before us, but of course we cannot use it, because it is permanently closed" (Wittgenstein 1953/1967, p. 127e).

Functional resemblance and the internalization of rules

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Abstract: Kubovy and Epstein distinguish between systems that *follow* rules, and those that merely *instantiate* them. They regard compliance with the principles of kinematic geometry in apparent motion as a case of instantiation. There is, however, some reason to believe that the human visual system internalizes the principles of kinematic geometry, even if it does not explicitly represent them. We offer functional resemblance as a criterion for internal representation.
[KUBOVY & EPSTEIN]

According to **KUBOVY & EPSTEIN** (K&E), there are two ways of construing the fact that the perceived paths of apparently moving objects conform to the principles of kinematic geometry (Shepard 1994, pp. 4–6). One might suppose, with **SHEPARD**, that our visual system proceeds by applying internal knowledge of kinematic geometry. Alternatively, one might suppose, as K&E urge, that our visual system proceeds *as if* it possessed knowledge of kinematic geometry. The latter is always an option, say K&E, because of the difference between physical devices that *follow* rules and those that merely *instantiate* them (see target article, p. 619). Although K&E don't elaborate, their supporting discussion suggests they have in mind the well known distinction between physical systems whose behaviour is driven by internally represented rules (such as stored program digital computers) and those whose behaviour merely conforms to rules/laws, without internally representing them (the approximate conformity of the planets to Newton's universal law of gravitation is the standard example). There is, however, some reason to believe that the human visual system does not merely instantiate the principles of kinematic geometry. Consequently, if the visual system does behave in accordance with these principles, as K&E concede, it must internally represent them in some way. We will argue for this view by briefly re-examining the distinction between rule-following and rule instantiation.

The paradigm case of a device whose behaviour is driven by represented rules – of rule-following – is the Turing machine. The causal operation of a Turing machine is entirely determined by the tokens written on the machine's tape together with the configuration of the machine's read/write head. One of the startling features of a Turing machine is that the machine's tape can be used not only to store data to be manipulated, but also to *explicitly* represent the computational rules according to which this manipulation occurs. This is the basis of stored program digital computers and the possibility of a Universal Turing machine (one which can emulate the behaviour of any other Turing machine).

This neat picture gets a little messy, however, when we consider that not all of the computational rules that drive the behaviour of a Turing machine can be explicitly represented in the form of tokens written on the machine's tape. At the very least, there must be some primitive rules or instructions built into the system in a nonexplicit fashion, these residing in the machine's read/write head. Since these "hardwired" rules are not encoded in the form of discrete tokens written on the machine's tape, many theorists claim that they are *tacitly* represented (see, e.g., Cummins 1986; Dennett 1982; O'Brien & Opie 1999; Pylyshyn 1984). But what licenses this terminology? Is there any real difference between the behaviour of a Turing machine driven by "tacitly represented" rules and a planet obeying Newton's laws?

We think there is. Consider a Turing machine that adds integers. Such a machine receives as input a set of tokens representing the numbers to be added, and eventually produces further tokens representing their sum. Since the sequence of tokens on the machine's tape (representing both summands and sums) is a set of discrete physical objects, the Turing machine's operation can be characterised in terms of a pattern of causal relations among its tokens. From this perspective, the Turing machine succeeds in adding numbers because the causal relations among its inputs and outputs, considered as physical objects, mirror the numerical relations among sums and summands. The computational power of the Turing machine thus depends on the existence of a homomorphism between an empirical relational structure (in this case a causal one) and a mathematical relational structure, as **K&E** would put it (sect. 2.1, p. 621). We will characterise the relationship between the system of tape tokens and the integers as one of *functional resemblance.* One system functionally resembles another when the pattern of causal relations among the objects in the first system preserves or mirrors at least some of the relations among the objects in the second (for further discussion see O'Brien & Opie, forthcoming).

If, by virtue of the causal relations among its internal states, a physical system functionally resembles some domain D, then in our view it is appropriate to interpret the mechanisms that drive the system as *internal representations* of the relations between the objects in D. In the case of our imagined Turing machine, D is the (abstract) domain of integers, which are subject to various arithmetic relations, including those codified in the rules of addition. Consequently, it's appropriate to interpret the Turing machine as embodying internally represented rules of numerical addition. It does not matter whether the Turing machine explicitly represents these rules in the form of tokens written on its tape, or tacitly represents them courtesy of the configuration of its read/write head. What matters is that the casual relations among some of its internal states mirror specific mathematical relations among the integers.

Functional resemblance serves to distinguish devices like the Turing machine, which represent rules, from other physical systems that merely conform to rules. In the case of the solar system, for example, while the motions of the planets respect Kepler's laws, which can in turn be derived from Newton's universal law of gravitation, there is little sense to be made of the idea that these laws are internally represented by the system. Such laws are actually *our* attempt to represent (in mathematically tractable form) the regularities inherent in the causal dynamics of the system. Thus, when we simulate the planetary motions on a digital computer, we arrange things so that the causal relations among some of the internal states of the computer mirror the geometric and dynamical relations among the planets. The functional resemblance runs from simulation to planetary system, not the other way around. We are thereby warranted in saying that the inherent gravitational constraints of the solar system are *represented* in the computer, but *not* that the solar system represents the laws of motion – it merely instantiates them (to use **K&E**'s language).

What, then, of the human visual system's conformity to the principles of kinematic geometry, at least where the behaviour of apparently moving objects is concerned? Here it would seem that a relationship of functional resemblance does obtain between internal states of the human visual system and the motions of objects in the world. Of course, we don't yet know which brain processes are responsible for producing experiences of apparent motion. But it is reasonable to infer that the causal processes involved are systematically related to the structure of the experiences themselves. Such experiences portray objects that are subject to the kinds of constraints identified by Shepard, namely, they are conserved, are restricted to movements in two or three dimensions, and traverse kinematically simple paths (Shepard 1994, pp. 4–6). By assumption, these constraints are mirrored in the causal relations among the neural vehicles of apparent motion: there is a functional resemblance (if not an isomorphism) between brain states and perceived paths. Although real objects do not invariably move in accordance with kinematic constraints, the motions delimited by those constraints certainly constitute a class of *possible* object motions. Indeed, motions defined with respect to axes of symmetry are common in the context of manual object manipulation. By the transitivity of resemblance, we thus establish that there is a functional resemblance between the system of internal vehicles responsible for experiences of apparent motion and the motions of real objects acting under kinematic constraints. In light of our earlier discussion, this suggests that we may regard the visual system as representing the principles of kinematic geometry, not merely instantiating them.

Even if the principles of kinematic geometry are not explicitly encoded by the visual system, it therefore appears that kinematic principles *are* "lodged in the mind" (**K&E**, p. 619). Kinematic constraints are built into the very fabric of the visual system. They are not merely "passive guarantors or underwriters that are external to the perceptual process," but "active constituents in the perceptual process" (ibid.) (at least under the stimulus conditions that give rise to apparent motion). In other words, we must reject **K&E**'s modest interpretation of **SHEPARD**'s observations, leaving Shepard's own conclusion: kinematic constraints are internally represented, because they have been "internalized" by the brain.

The mathematics of symmetry does not provide an appropriate model for the human understanding of elementary motions

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Abstract: Shepard's article presents an impressive application of the mathematics of symmetry to the understanding of motion. However, there are basic psychological phenomena that the model does not handle well. These include the importance of the orientations of rotational motions to salient reference systems for the understanding of the motions. An alternative model of the understanding of rotations is sketched.
[**SHEPARD**]

In even the most elementary domains of physical understanding, there are clear distinctions between problems that are natural and intuitive for people, and ones that are challenging. These phenomena extend into many areas of cognition, including spatial organization, object recognition, and event knowledge; and explanation of this variation in physical understanding is an important undertaking for cognitive theory.

In the first part of his article, **SHEPARD** constructs an explanation of variation in our understanding of elementary motion from the modern mathematics of symmetry. In this view, our understanding of motion is embodied in a six-dimensional manifold, and those motions that are natural for us to perceive or imagine are the geodesics in the manifold; the structure of the manifold, and the lengths of the geodesics in it, are reduced when the objects that move are rotationally symmetric. This geometric model is an impressive achievement, and it appears to represent a thorough exploration of the application of this mathematics to spatial cognition. Despite containing important elements of truth, however, I