Probabilistic Analysis of Multi-layered Soil Effects on Shallow Foundation Settlement

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Summary: The results of a preliminary investigation into the effects of multi-layered soil on foundation settlement are reported in this paper. In this study, the settlement of a square pad footing placed on two-layered soil profile is examined. Using a combination of Finite Element Method (FEM) analyses and random fields simulation, probabilistic analyses of the settlement of footing founded on two-layered soil profile is established through Monte Carlo simulation. The Young's modulus field has been simulated via Local Average Subdivision method (LAS) with a fixed mean, various coefficients of variation and spatial correlation structures. The coefficients of variation and spatial correlation structures in each layer are set to be different. It is assumed that the boundary between two soil layers is abrupt as may be represented by faults and geological unconformities in the soil mass. The results of the analyses indicate a modest decrease in the coefficient of variation of settlement of the 2-layered soil profile compared to the settlement of the uniform single layer soil mass.

INTRODUCTION

Foundations of engineering structures transfer and distribute their loading to the underlying soil and rock. Foundations are mostly supported by multi-layered soil profiles. It is widely known that the spatial variability in soil properties has significant effects on foundation performance (i.e. bearing capacity and settlement).

Due to the variety of soil types and multi-layered soil profiles that exist in nature, establishing a probabilistic analysis experimentally without the aid of modern high-speed computers would be tedious, if not impossible. Therefore, stochastic numerical modelling combined with Monte Carlo simulation has been adopted in this study. Random field simulation is incorporated into the finite element modelling to simulate the soil medium as spatially random fields, taking into consideration the correlation structures. In the present study, the soil medium is divided into two layers, each with different spatial statistics. A probabilistic study is carried out on the settlement of a uniformly loaded rigid footing placed at the surface of the two-layered soil medium.

PROBABILISTIC ANALYSIS OF MULTILAYER-SOIL PROFILES

Previous research on investigating and establishing probabilistic performance of foundations founded on a single layer spatially random field has been reported by Baecher and Ingra (1981); Righetti and Harrop-Williams (1988); Zeitoun and Baker (1992); Paice et al. (1994, 1996) and Fenton and Griffiths (2002). These research works represent the pioneering works of developing probabilistic analysis on foundation settlements found on a randomly distributed soil using a combination of stochastic finite element analysis and Monte Carlo simulation. The random soil medium is modelled based on random filed theory (Vanmarcke 1983), which assumes stationarity (i.e. mean and variance are constant throughout the entire soil mass). Two-dimensional finite element analysis has been employed and the footing is assumed to extend large distances perpendicular to the plane of the random field. It is also assumed that the 2D elastic modulus field has an infinite correlation length in this out-of-plane direction. In random field modelling, since any abrupt change in the soil profile violates the basic assumption of stationarity, the soil profiles examined by previous researchers were limited to single layers.

In the present study, the Local Average Subdivision (LAS) method (Fenton 1990; Fenton and Vanmarcke 1990, Fenton 1994) has been employed due to its simplicity to generate realizations of possible elastic modulus random fields. A Gaussian random field with zero mean, unit variance, and spatial correlation length, $\theta_{\ln E}$, is first simulated and then transformed into a log-normal random field using standard statistics techniques. A log-normal distribution is used to avoid negative values and because such distributions have been observed in practice. Similar to other random field generators, LAS is also based on the stationarity assumption. Any abrupt change in the random process will contravene this basic assumption.

In elastic settlement analyses, two constitutive parameters are required, for example, the Young's modulus, E, and the Poisson's ratio, v. For the present study, the Poisson's ratio is fixed at 0.3 and the Young's modulus is modelled as a random field with a constant mean, μ_E, of 20,000 kPa over the entire soil mass for all analyses. The coefficient of variation (COV_E) of the Young's modulus field has been reported to vary between 2% and 42% (Lee et al. 1983). The statistical distribution of the Young's modulus field is assumed to follow a lognormal distribution. The correlation length, $\theta_{\ln E}$, describes the spatial continuity of the random field; that is, values at adjacent locations are more correlated than those separated by larger distances. The spatial dependence is assumed to follow an isotropic Markovian correlation function:

$$\rho_{\ln E}(\tau) = \exp\left\{-\frac{2|\tau|}{\theta_{\ln E}}\right\} \tag{1}$$

where $\rho_{\ln E}$ is the coefficient of correlation between two points separated by a distance τ. For simplicity, the spatial correlation length in the x, y and z directions are assumed to be the same, that is, isotropic.

In the 3-dimensional finite element modelling, a two layered soil medium with a fixed total thickness, H_T, of 30 m is assumed to overlay a rigid stratum. A rigid footing carrying a uniform load of, $q_0 = 250$ kPa, is placed at the centre of soil mass. Four sides of the finite element model are restrained against horizontal displacement but free to deform vertically. The base of the model is fully fixed. The soil mass is discretized into 1 m × 1 m × 1 m eight-noded quadrilateral elements. The size of the finite element mesh in the horizontal direction is adjusted according to the width of the footing. It is generally recognized that the mesh boundaries should be set at a distance at least five times the loaded area to ensure that boundary effects do not influence the results (Desai and Abel 1972). The finite element program used in this study is 3-dimensional and is identical to that given by Smith and Griffiths (1998).

Figure 1 shows grey scale representations of a 3D random field, each simulated using different correlation lengths by the LAS method. Darker areas in the simulated soil mass designate higher stiffness, whilst lighter areas denote low values of elastic modulus. It can be seen that as the correlation length increases, the randomness decreases (there is transition from darker areas to lighter areas).

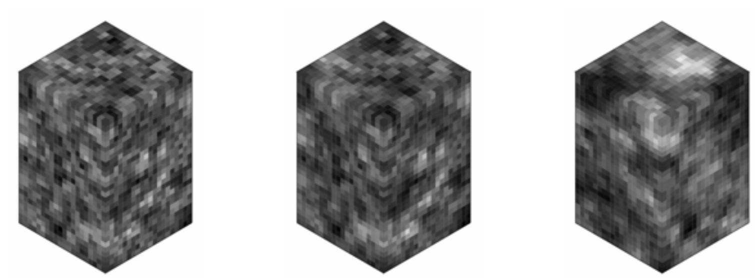
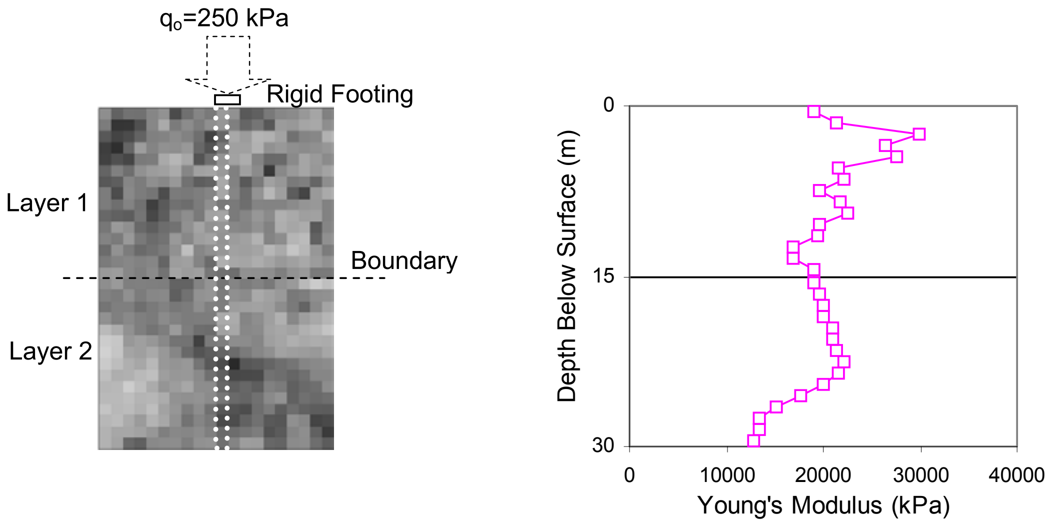


Figure 1. Simulated Random Soil Profile Via LAS: (i) $\theta_E = 2$ m ; (ii) $\theta_E = 4$ m ; (iii) $\theta_E = 8$ m .

Figure 2 shows a 2D x - z plane in grey-scale of a potential realization of the 3D Young's modulus field. The values of Young's modulus, E, taken vertically below the footing are shown in the plot illustrating an abrupt change across the boundary. To simulate a two-layered random soil medium without defying the basic stationarity assumption, a simple approach is adopted. The soil medium is divided into two layers; each with uniform thickness, lying horizontally and separated by an abrupt boundary. Each layer is assigned different spatial statistics. It is assumed that each soil layer is unrelated to the other and any spatial continuity across the boundary is ignored. This boundary may represent faults or geological unconformities present in the soil mass. Each layer is then simulated independently and the two merged based on the location of the boundary and thickness of the layers. The simulated Young's modulus fields are assigned to the finite element mesh after which an analysis of settlement is undertaken.



(a) (b)

Figure 2. (a) Two-dimensional representation of a single (2 m × 2 m) pad footing founded on a two-layered soil medium; (b) The variation of elastic modulus with depth under the pad footing (shown with white dots in (a)).

For the parametric study undertaken in the present study, the ranges of coefficient of variation, COV_E, the scale of fluctuation, $\theta_{\ln E}$, the width of footing, W_f, and the ratio of the soil layer thickness, H_1 / H_2, are summarised in Table 1. It has been reported that equilibrium solutions can be obtained by analysing the results of Monte Carlo simulations consisting of 2,000 realisations (Paice et al. 1996; Fenton and Griffiths 2002).

Table 1. Parametric Study: Varying Input Parameters in the Study.

Parameter	**Values Considered**
COV_E	10%, 20%
$\theta_{\ln E}$ (m)	1.0, 2.0, 4.0, 8.0
W_f (m)	2.0, 4.0
H_1 / H_2	0.2, 0.5, 1.0, 2.0, 5.0

RESULTS AND DISCUSSIONS

In order to assess the effect of the abrupt boundary in the soil mass on the probabilistic settlement of a pad footing, first, a single soil layer was analysed and the results are presented in Figures 3 and 4. Two footing widths, W_f = {2.0 m, 4.0 m}, soil correlation lengths, $\theta_{\ln E}$ = {1.0 m, 2.0 m, 4.0 m, 8.0 m}, and $COV_E = \{10\%, 20\%\}$, were considered in this case.

Figure 3 shows the variation of COV of footing settlement, $\mu_\delta / \sigma_\delta$ or COV_δ, with the variation of $\theta_{\ln E} / W_f$ whilst Figure 4 shows the variation of the COV_δ with the variation of $\theta_{\ln E} / H_T$. These findings are validated by the results of previous analytical evaluations by other researchers (i.e. Paice et al. 1996; Fenton and Griffiths 2002). Figures 3 and 4 show that, as the correlation length increases, the COV_δ also increases for all footing sizes. Also, as W_f increases, COV_δ decreases.

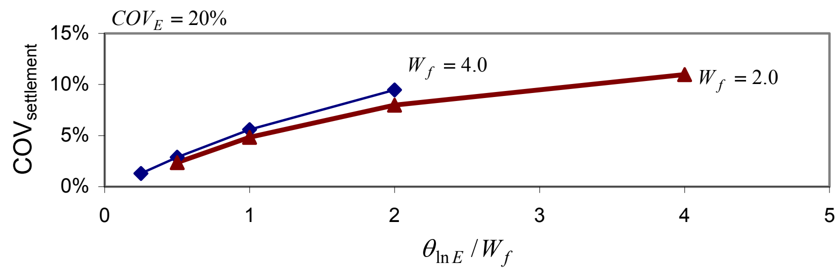


Figure 3. Coefficient of Variation of Settlement with Varying $\theta_{\ln E} / W_f$ for a Single Soil Layer Profile.

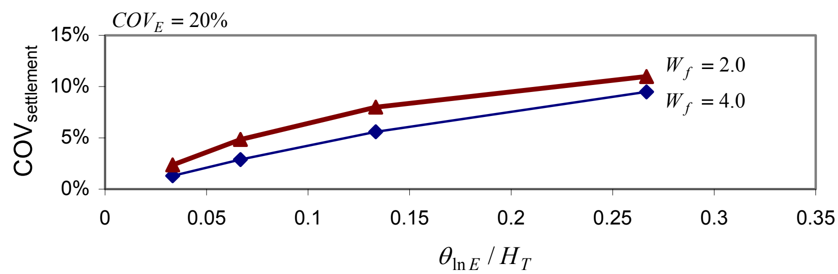


Figure 4. Coefficient of Variation of Settlement with Varying $\theta_{\ln E} / H_T$ for a Single Soil Layer Profile.

Figure 5 shows the variation of COV of footing settlement in a soil profile with a discontinuity located 5 m below surface compared with the variation of COV of footing settlement in a uniform layer. The results indicate that COV_δ decreases in a 2-layered soil profile compared to a single uniform soil profile. It also shows the decrement is significant if the location of the discontinuity, H_1 / W_f, is near to the footing.

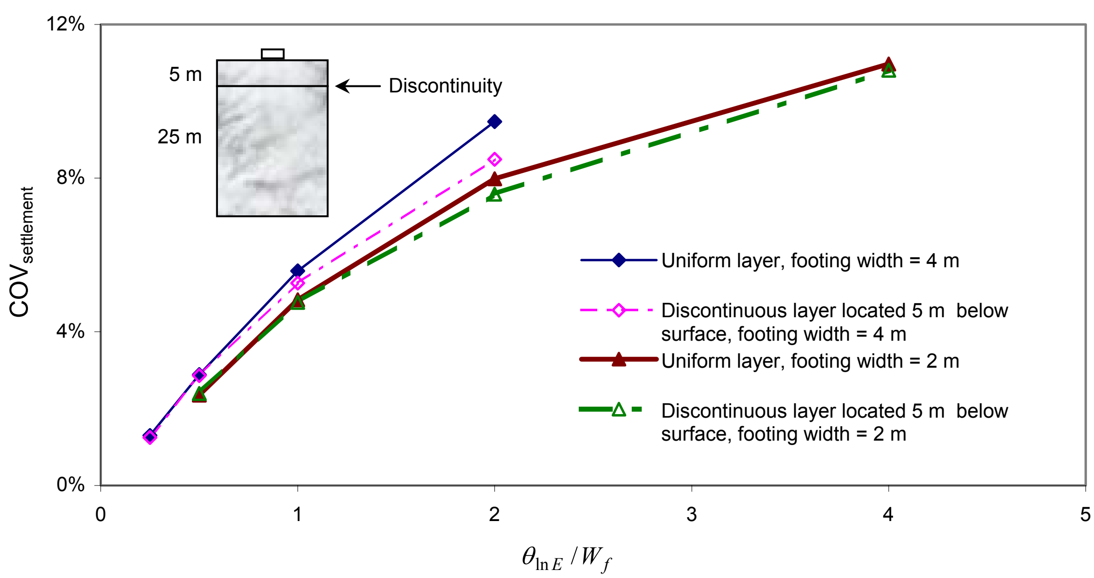


Figure 5. Variation of COV of Settlement in Soil Layer with Discontinuity Located 5 m Below Surface.

Figures 6 to 9 show some of the results of the parametric analyses performed. The results are grouped by the ratio of the correlation length of the top layer over the correlation length of the bottom layer, $\theta_{LnE_1} / \theta_{LnE_2}$, and the width of the footing, W_f. It can be seen that the top layer (COV_{E_1}) has the biggest influence on COV_δ for all the cases considered. This is to be expected as the footing is founded on this layer resulting in larger stress and strain increments within the upper layer.

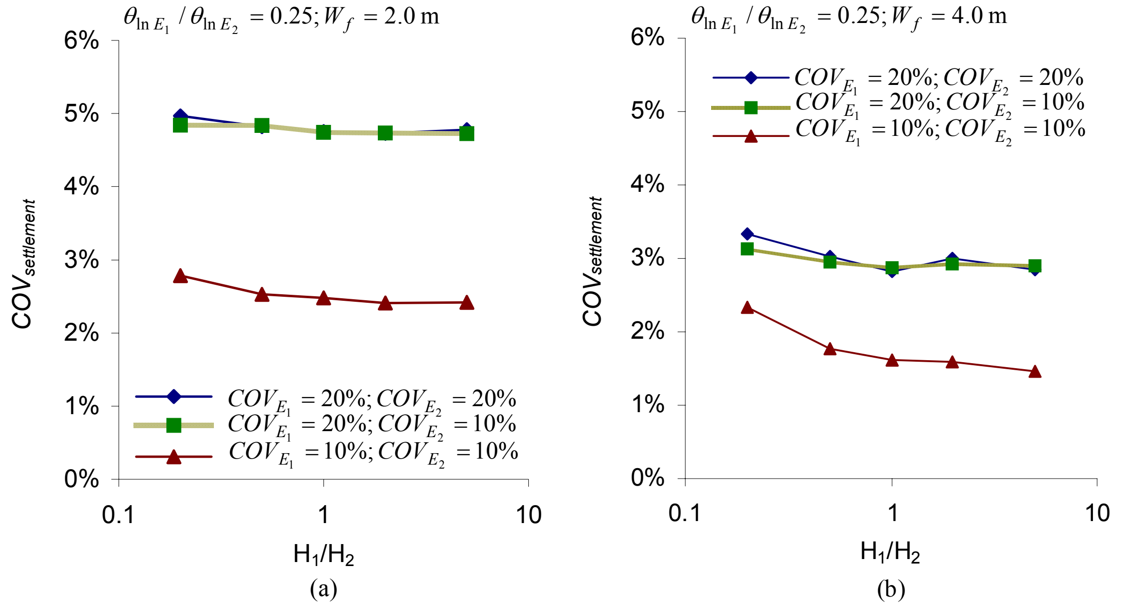


Figure 6. Variation of the COV of Settlement for Various Layer Thickness ($\theta_{\ln E_1} = 2.0$, $\theta_{\ln E_2} = 8.0$ and $\theta_{\ln E_1}/\theta_{\ln E_2} = 0.25$).

These results show the variation of COV_δ of footing settlement with the variation of the ratio of the thickness of the two soil layers, H_1/H_2, which indicates the location of discontinuity below the surface. Whilst the results show that the COV_δ of a 2 m wide footing is higher than that for a 4 m wide footing, the results also suggest that the variation of COV_δ of a 4m wide footing, which has a greater influence zone with respect to H_1/H_2, is greatly influenced by the location of discontinuity in the soil mass compare to a 2m wide footing. The findings also show that as H_1/H_2 increases, COV_δ approaches an asymptotic value, suggesting that there is little influence on COV_δ as the discontinuity is located further from the surface of the soil mass and beyond the influence zone of the footing.

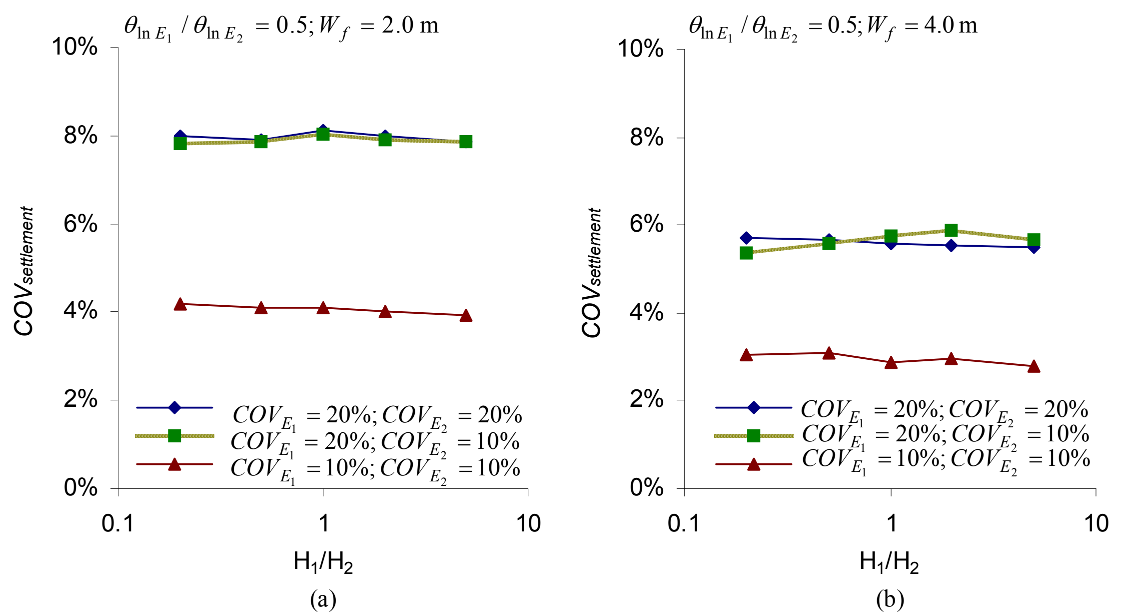


Figure 7. Variation of the COV of Settlement for Various Layer Thickness ($\theta_{\ln E_1} = 4.0$, $\theta_{\ln E_2} = 8.0$ and $\theta_{\ln E_1}/\theta_{\ln E_2} = 0.5$).

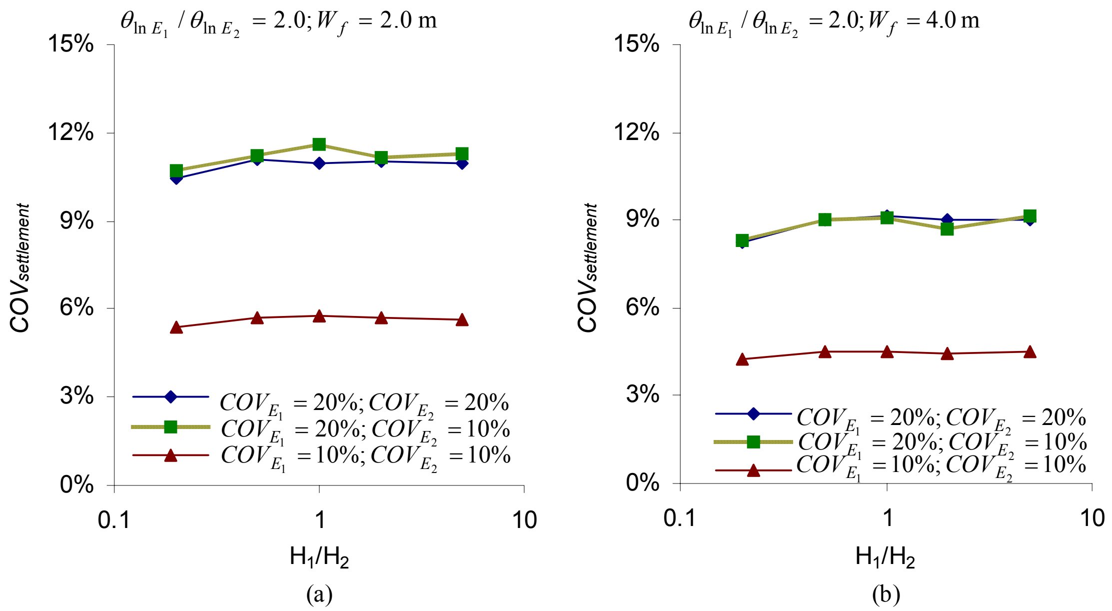


Figure 8. Variation of the COV of Settlement for Various Layer Thickness ($\theta_{\ln E_1} = 8.0$, $\theta_{\ln E_2} = 4.0$ and $\theta_{\ln E_1} / \theta_{\ln E_2} = 2.0$).

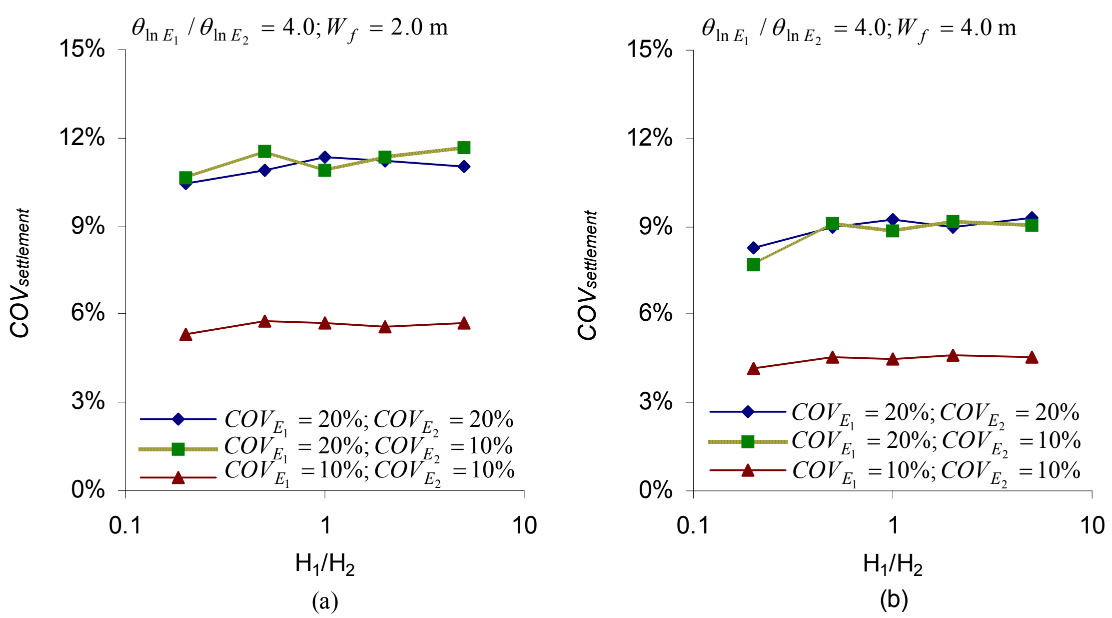


Figure 9. Variation of the COV of Settlement for Various Layer Thickness ($\theta_{\ln E_1} = 8.0$, $\theta_{\ln E_2} = 2.0$ and $\theta_{\ln E_1} / \theta_{\ln E_2} = 4.0$).

CONCLUSIONS

In the present study, the response of a rigid footing founded on a two-layered spatially random soil profile has been investigated using combined stochastic finite element analysis with Monte Carlo simulation. Parametric studies have been carried out for a number of COV of elastic modulus, correlation length, width of the footing and thickness of top and bottom layers, while the mean of elastic modulus was kept constant. The results indicate that the COV of footing settlement is lower in soil layer with a discontinuity compared to a footing founded on a single uniform soil layer. The decrement is more significant if the discontinuity is located near the surface of the soil profile and within the influence zone of the footing. It is suggested that the variation in the soil layer that is nearest the foundation has the most dominating effect on foundation settlement. The results also suggest that COV_δ of a wider footing, which has a deeper zone of influence, is found to be more sensitive to the location of the discontinuity in the soil mass.

ACKNOWLEDGEMENTS

The research presented in this paper is part of ongoing work at the University of Adelaide funded by the Australia Research Council. Their support is gratefully acknowledged.

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