Spatio-temporal log-Gaussian Cox processes on earthquake events

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Abstract. *This work presents an application of spatio-temporal log-Gaussian Cox processes for the description of earthquakes events. To explain the overall spatial trend, spatial geological information in the study area such as faults and volcanoes are introduced in the model. Moreover, an anisotropic specification of the covariance matrix of the Gaussian process is used to improve the explanation of the phenomenon. We apply and compare different models to explain the seismic events occurred in Alaska over the last decades.*

Keywords. *Anisotropy; Earthquake data; log-Gaussian Cox processes*

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1 Introduction

Modelling the spatio-temporal dynamics of earthquake events provide useful information about the evolution of the phenomenon. For instance, the distribution of earthquakes exhibits spatial inhomogeneity due to a different geological characteristics in the area and anisotropy due to the directional displacement of events. Self-exciting point processes are largely used to describe earthquakes characteristics, assuming that the occurrence of an event increases the probability of occurrence of other events in time and space (Hawkes [4] or ETAS model [8]). According to the classification of models for continuous-time data in [3], the previous models can be called mechanistic, since they explain the process defining a conditional intensity function that properly describes the underlying scientific mechanism.

Although the log-Gaussian Cox process does not model directly interactions among events, it can describe clustered patterns empirically, accounting for the overall spatial and temporal trends. The stochastic process characterizes local spatio-temporal interactions. In our work we apply an empirical model framework using the log-Gaussian Cox process because we want to focus the attention on the dependence of events on the underline environmental information (spatial geological covariates), capturing and describing in a proper way the anisotropy that intrinsically characterizes earthquake events. We describe the spatial and temporal evolution of Alaska earthquakes with magnitude greater than 3 considering a 30 days time resolution.

2 Spatio-temporal log-Gaussian Cox process

A Cox process is a doubly stochastic point process formed as an inhomogeneous Poisson process with a stochastic intensity [2]. The intensity of the spatio-temporal log Gaussian Cox process (LGCP) is given by [6, 3]

$$\Lambda(\boldsymbol{x},t) \quad = \quad \lambda(\boldsymbol{x},t)exp(S(\boldsymbol{x},t)) \quad x \in W \subset R^2, t \in R^+$$

In an epidemiological context, the deterministic part ($\lambda(\boldsymbol{x},t)$) is the endemic part of the process, whilst the Gaussian process ($S(\boldsymbol{x},t)$) captures the epidemic part that can be seen as the residual variation. Basically, the properties of the model depend on the specification of the mean and the covariance structure of the Gaussian process, and under separability assumption we have that

$$Cov(S(\boldsymbol{x}_1,t_1),S(\boldsymbol{x}_2,t_2)) = \gamma(u,v) = \sigma^2 r_s(||x_2-x_1||;\theta_s)r_t(|t_2-t_1|;\theta_t)$$

where $u = ||x_2 - x_1||$ and $v = |t_2 - t_1|$. With an anisotropy specification [7], the spatial component is given by the product

$$r_s(||s_2-s_1||;\theta_s)\sqrt{u \ \Sigma_{\theta,\varsigma}^{-1} \ u^t}$$

where $u \in R^2$ and Σ is a symmetric positive definite matrix with positive determinant containing the angle radians parameter (θ) and the anisotropic factor (ς). In our work, we consider the covariance matrix with an isotropic and anisotropic specification for the spatial part. The parameters of the covariance matrix are estimated by minimum contrast estimation, comparing the theoretical form of the covariance function that involves the model parameters, with its non-parametric empirical form ([1, 7]).

3 Dataset

Alaska is the most seismically active region of the United States. Figure 1 shows locations of Alaska's faults [5] and volcanoes in the area.

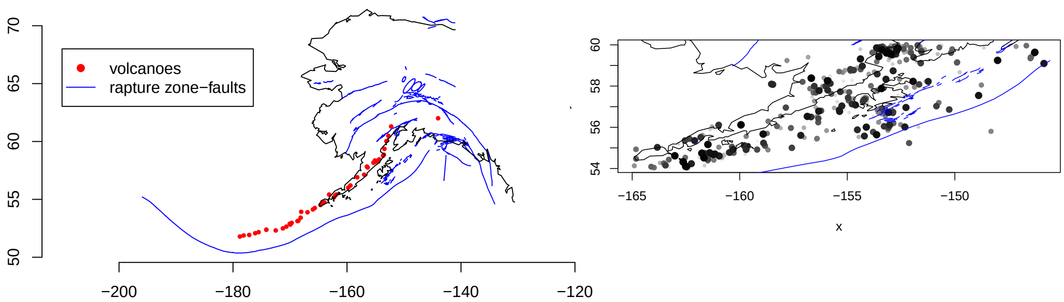

Figure 1: Volcanoes and faults in the stusy area. Events with a magnitude greater than 3 in the Alaska peninsula in 2000 and 2001, grey and small dots correspond to the oldest events

We consider the earthquakes catalogue provided by the Alaska Earthquake Center, all seismic events from January 1970 up to nowadays. For each event, the spatial coordinates, the hypocentre depth and the magnitude are reported. We want to describe using the log-Gaussian Con process the spatio-temporal variation of earthquakes considering a discrete time resolution of 30 days.

In Figure 1, the events with magnitude greater than 3 in the Alaska peninsula for 24 period of time starting from 1^{st} January 2000 are shown. It is clear both inhomogeneity and a prevailing direction in this point pattern. Moreover, looking the size and the colour of the points that depend on the time, it is clear the spatio-temporal dependence. A discretization in space and time is used to estimate the model. We consider as time unit 30 days of events and we estimate the deterministic part using generalized additive models for count data [9]. For the spatial part, we introduce geological spatial variables: distance to the nearest fault and distance to the nearest volcano.

These models can be useful in estimating the intensity surface of a spatio-temporal point process, in constructing spatially maps of earthquake risk, and in predicting seismic activity.

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