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Abstract**

Management decisions of a mutual life company involving the amounts and relative proportions of participating (with profits) and nonparticipating (without profits) business and the level of expenses are examined in relation to their effect on participating policyholders' returns. A particular expense ratio is defined that plays a key role in a framework for making such decisions. The sensitivity of participating policy returns to changes in each factor are analyzed. Companies with expense ratios (as defined) of less than 2 are shown to prefer a different strategy from companies with higher ratios. There is an incomplete tendency for the ratio to stabilize either at unity or to tend to infinity. The practical implications and limitations of the approach are considered.

Key words: *decision making; expenses; new business*

1 Introduction

This paper concerns certain management decisions relating to mutual life companies (offices); the position regarding stock (proprietary) companies is different and is only briefly discussed.

A United Kingdom (U.K.) environment is assumed, although the circumstances are general enough to make the conclusions appropriate to other countries, including the United States. Some of the comments made and procedures adopted in the paper, however, reflect peculiarities of the U.K. (including methods of dividend distribution,

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product design, and statutory regulation). Brief descriptions of these features will be given to assist non-U.K. readers.

In the U.K., participating (with profits) policyholders' dividends are paid in two forms, referred to as *reversionary* and *terminal bonuses*. Reversionary bonuses are additions to the contractual policy benefit; they usually are made annually, at the discretion of the company's actuary, to reflect a proportion of the surplus earned during the previous year. Terminal bonuses are added at the claim date of the policy, again at the discretion of the actuary, so that the total policy benefit on maturity of a policy will be equal to the policy's asset share plus an element of smoothing. In a mutual company the return to the participating policyholder also will include a share in the company's profits or losses from other sources, such as those generated by nonparticipating business, plus any contribution made to or from the estate.

The nature of the statutory regulations regarding the valuation of assets and liabilities combined with the particular features of the participating business described above result in different patterns of emergence of statutory surplus. Nonparticipating (without profit) business generally produces large initial surplus strains, followed by small regular profits emerging in subsequent years. The large strains, however, can be reduced by modern product designs. Participating business, for which reserves only are required for the contractual benefit plus declared bonuses, lead to reduced or even nonzero initial strains, followed by relatively large contributions to statutory surplus for a considerable period of the policy's duration. A large strain then is produced at the claim date when the terminal bonus becomes payable. As a result, the issue of new participating business will tend to improve the statutory surplus position, while the issue of nonparticipating business will tend to have the opposite effect. This is a factor that will bear on later discussion.

In the U.K., traditional nonparticipating business such as term and whole life insurances do not constitute much of a mutual company's portfolio. A considerable and possibly increasing volume of business consists of unit-linked contracts.[1] Later in the paper situations are hypothesized in which 35 percent or more of a mutual company's portfolio consists of nonparticipating business. While such a proportion may have been unlikely historically, more recently this would not be an incredible figure for some firms.

[1] In a unit-linked insurance contract, premiums (after deductions for expense and claim charges) are allocated to units, the value of which directly reflect the returns obtained from a specific pool of assets. The charges represent the nonparticipating premium to the company for these contracts

The decisions considered in this paper are those that ultimately have an effect upon the volumes of new participating and nonparticipating business issued by a mutual company and in the management of expense levels.

According to a basic principle of economics, the more units of product that are sold at the same price for a fixed level of expense, the greater will be the profit per unit sold. Furthermore, an increase in expense levels if accompanied by a greater proportionate increase in units sold will increase unit-profit. This is referred to as *economies of scale*.

In the life insurance business, units of product (policies) are sold, at least partly, with the aim of making a profit and with the knowledge that the activities of selling and managing the business involve expenses that offset profit. A stock company issuing nonparticipating policies will conform ultimately to the basic economic principles stated above, as will a nonparticipating portfolio within a mutual company.

A mutual company, which must have a significant portfolio of participating policyholders on its books, is in an unusual position. As a mutual, all profits earned by both the participating and nonparticipating portfolios must be distributed (ultimately) to the participating policyholders. This means that while increasing the number of participating units sold for a given level of expense will reduce the average cost for each unit sold (thereby increasing unit profit), it also will reduce each unit's share of the profits earned by the nonparticipating portfolio (thereby decreasing unit profit). The position of the mutual company is therefore more complex than the position of a nonparticipating stock company case. The overall profitability of a mutual company depends on the relative levels of profit from the nonparticipating portfolio compared with the level of expenses. It is this position that will be explored in section 3 of this paper.

Profit is not the only consideration of importance to management when arriving at decisions that may affect business volume. For example, the mutual company at all times must maintain a sufficient statutory surplus both to satisfy the regulators and to make investments that are in the best long-term interests of the policyholders, including investment in the issue of new nonparticipating contracts. This surplus is provided by the existence of a participating portfolio, as well as from profits retained from earlier generations of policyholders. A certain relative level of participating business is necessary; without it, a mutual company could not exist.

There are also factors at work in the market that may affect business volume irrespective of any other ambitions the management may have. For example, sales of nonparticipating contracts may be affected by premium rate, while sales of participating policies may be influenced by historical and current profitability. Customer preferences for products may change over time, and changes to tax legislation (e.g., removal of tax reliefs on insurance premiums) dramatically can influence sales volume. These factors must be borne in mind when considering the implications of the results described in this paper.

The present analysis will need to distinguish between two types of expenses: proportionate and nonproportionate expenses:[2]

a) *Proportionate expenses* are variable expenses associated with participating and nonparticipating portfolios, and these expenses are proportionate to the volumes of business sold.

b) *Nonproportionate expenses* are the remaining expenses, consisting of other variable expenses and fixed expenses. Nonproportionate expenses can be considered as expenses that collectively vary with the decision made, but not necessarily in proportion to any change in volume of business resulting from the decision.

For example, a particular management decision may lead to an increase in nonproportionate expenses of X percent, coupled with an increase in nonparticipating sales of Y percent; X and Y are not linked to each other in any way other than that they are both dependent upon the decision made. A mutual company attempting to expand its operations to produce economies of scale may be faced with such a decision set. As will be seen in section 3 below, it is always best to choose the decision that produces the greatest increase in sales for the smallest increase in nonproportionate expenses, everything else being equal.

2 Construction of Total Profit

All references to present values refer to a time origin (time 0) unless otherwise stated. For the sake of simplicity, it is assumed that the discount rate used to calculate present values is equal to the rate of investment return earned over the lifetime of the portfolio. Further, without loss of generality, it is assumed that the mutual

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[2] Chalke (1991) considers expenses at any decision point to be "nonmarginal" if they are invariable by any of the possible decisions made. Expenses that vary according to the decision made are described as "marginal expenses." Ramsay (1991), in his comment on Chalke's paper, points out that these expenses more appropriately are described as "fixed" and "variable" respectively, in accordance with more traditional parlance. Chalke notes that fixed expenses at one decision point may become the variable expenses of the next decision point.

company's business consists of one tranche[3] of nonparticipating business and one tranche of participating business, all issued at time 0.[4] The policies within each tranche are assumed to be identical. The company is assumed to incur three distinct types of expenses:

a) Proportionate expenses of the nonparticipating business;
b) Proportionate expenses of the participating business;
c) Nonproportionate expenses.

The management also has ultimate control of business volume, separately for each tranche.

Three types of profit, P_n, P_w, and P_w', need to be defined.

P_n = Actuarial present value of future marginal profits (net of proportionate expenses) earned by a single nonparticipating policy issued at time 0;

P_w = Actuarial present value of future marginal profits (net of proportionate expenses) earned by a single participating policy issued at time 0; and

P_w' = Actuarial present value of the marginal profits earned by a single participating policy including the value of the benefit payments.

Appendix 1 contains a detailed description of the method used to calculate P_n and P_w.

While in reality individual policies, even of the same size and type, earn different profits (e.g., due to different dates of claim), it is assumed that each policy earns the same average (or expected) profit. The effect of changes in business volume on profit variability is not considered in this paper.

It is assumed that marginal profits are fixed and independent of sales volume. In practice this is not entirely true: cheaper products are easier to sell, but will have lower marginal profit. In the present context it is helpful to think of the nonparticipating business as a body of unit-linked policies with premium rates that are effectively the charges deducted from the policy benefits. In these cases, policy sales depend more on expected investment returns obtained from the policyholder's unit-holding than upon the rates of charge levied to

[3] Here *tranche* refers to business issued within a specific and limited time period.

[4] Similar conclusions could be drawn assuming the company is in a stationary position, in real terms, issuing constant volumes of new business each year. A single tranche model, however, is much easier to visualize.

cover expenses and other costs, at least up to a point. Hence, an assumption of invariant marginal profit per policy can be justified for the purpose of illustrating the point of interest in this paper. The effect of introducing a price/volume relationship for the nonparticipating business in the model is an aspect worthy of further investigation.

Let

N_n = Number of nonparticipating policies issued at time 0;
N_w = Number of participating policies issued at time 0;
$E^{(n)}$ = Actuarial present value of all future nonproportionate expenses (with respect to these two tranches of business).

The present value of the company's future retained profits from the two tranches, TP, then is given by:

$$TP = N_n P_n + N_w P_w - E^{(n)}.$$

Because, over the lifetime of the business, all the profits earned by the two tranches are paid to the participating policyholders in policy benefits,[5] it follows that $TP = 0$.

Let c be the present value of future benefits paid to a single participating policy (assumed to be the same for all participating policies), then P'_w is given by

$$P'_w = P_w + c\,.$$

P'_w can be considered as the value of future premiums, less proportionate expenses, plus the policy's returns on investment. Hence:

$$0 = N_n P_n + N_w P'_w - N_w c - E^{(n)}$$

or

$$c = P'_w + \frac{N_n P_n - E^{(n)}}{N_w}\,. \tag{1}$$

[5] This may not always be the case. Smoothing participating policy returns may result in more or less than asset shares being paid, while there may be a strategy to expand or contract the estate for good management reasons. Because policy benefits are designed to follow asset shares and the estate is ultimately a policyholder asset, then it seems appropriate to assume that, on average, $TP = 0$.

In other words, the present value of the benefits under a single participating policy is equal to the value of its premiums, including investment income and net of proportionate expenses, plus that policy's share of the profits from the tranche of nonparticipating policies, less that policy's share of the nonproportionate expenses of the company.

From equation (1) it easily can be seen that increasing the volume of nonparticipating business N_n, or reducing the amount of nonproportionate expenses $E^{(n)}$, will increase the return to the individual participating policyholder c. Increasing the volume of participating business only will increase returns, however, if $(N_n P_n - E^{(n)})$ is negative. That is, the ratio $E^{(n)}/(N_n P_n)$ is greater than unity. This ratio will be referred as R, or as the *expense ratio,*

$$R = \frac{E^{(n)}}{N_n P_n}$$

and it represents the extent to which the nonproportionate expenses of the portfolio are covered by the nonparticipating business.

The rest of this paper is concerned with identifying the relative effects of varying N_n, N_w, and $E^{(n)}$ on participating policy returns for different values of R. In addition, the paper establishes a framework for the construction of management decisions for companies with particular expense ratios subject to different business prospects.

3 Controlling the Variables to Maintain or Improve Per Policy Profit

3.1 The Variables

It will be assumed that at time 0 management can make decisions that affect N_n, N_w, $E^{(n)}$, or any combination of these quantities. Equation (2) below represents the value of the participating per policy returns (subsequently referred to as *per policy* returns) after changes in each of these variables,

$$(1 + \alpha_c)c = P_w' + \frac{(1 + \alpha_n)N_n P_n - (1 + \alpha_e^{(n)})E^{(n)}}{(1 + \alpha_w)N_w} \tag{2}$$

$$-1 \leq \alpha_e^{(n)}, \alpha_n < \infty \text{ and } \alpha_w > -1,$$

where α_n, α_w, $\alpha_e^{(n)}$, and α_c are parameters indicating the proportional changes in the number of nonparticipating policies, number of participating policies, nonparticipating expenses, and per policy returns respectively.

3.2 Maintaining Returns

Whenever conditions change, it is reasonable to assume that the aim of management will be to ensure that per policy returns do not fall, (i.e., to ensure that α_c is never negative). Subtracting equation (1) from equation (2) yields:

$$\alpha_c c = \frac{(1+\alpha_n)N_n P_n - (1+\alpha_e^{(n)})E^{(n)}}{(1+\alpha_w)N_w} - \frac{N_n P_n - E^{(n)}}{N_w}$$

$$= \frac{N_n P_n}{N_w}\left[\frac{(1+\alpha_n) - (1+\alpha_e^{(n)})R}{(1+\alpha_w)} - (1-R)\right]$$

which implies:

$$\alpha_c = \frac{N_n P_n}{c N_w}\left[\frac{\alpha_n - (1-R)\alpha_w - R\alpha_e^{(n)}}{(1+\alpha_w)}\right]. \tag{3}$$

It is instructive to examine the behavior of α_c with respect to the other parameters. From equation (3),

$$\frac{\partial \alpha_c}{\partial \alpha_n} = \frac{1}{(1+\alpha_w)}\left(\frac{N_n P_n}{c N_w}\right) \geq 0 \tag{4}$$

because the constants c and N_w are positive and N_n and P_n are non-negative. Notice that the right side of equation (4) is independent of α_n, $\alpha_e^{(n)}$, and R. Thus, returns increase at a constant rate for any change in these quantities.

Similarly,

$$\frac{\partial \alpha_c}{\partial \alpha_w} = \frac{-(1-R)}{(1+\alpha_w)^2}\left(\frac{N_n P_n}{c N_w}\right) \tag{5}$$

and

$$\frac{\partial \alpha_c}{\partial \alpha_e^{(n)}} = \frac{-R}{(1+\alpha_w)}\left(\frac{N_n P_n}{c N_w}\right) \leq 0. \tag{6}$$

From equation (5), returns (as a function of α_w) either are decreasing, zero, or increasing if $R < 1$, $R = 1$, or $R > 1$, respectively. Finally, from equation (6) we see that for a given α_w, returns decrease at a constant rate regardless of the level of $\alpha_e^{(n)}$.

Let us now investigate the behavior of α_n, α_w, and $\alpha_e^{(n)}$ when there is *no* change in the level of returns; that is, when $\alpha_c = 0$. First, setting $\alpha_c = 0$ in equation (3) yields :

$$\alpha_n = (1-R)\alpha_w + R\alpha_e^{(n)} . \tag{7}$$

That is, to maintain returns, the proportional change in the number of nonparticipating policies (α_n) must be a weighted average of the proportional change in the number of participating policies (α_w) and the change in nonproportional expenses ($\alpha_e^{(n)}$). Here the weights can be negative (if $R > 1$). When $0 < R < 1$, in order for returns to be maintained, nonparticipating business has to be increased in response to increases in both expenses and participating business. Decreases in $E^{(n)}$ and N_w would allow nonparticipating volume to fall while maintaining returns.

Consider the following pairs of parameters: (α_n, α_w), $(\alpha_n, \alpha_e^{(n)})$, and $(\alpha_w, \alpha_e^{(n)})$ in equation (7), subject to the third parameter being set equal to zero. Define $f_{x/y}$ as:

$$f_{x/y} = \frac{\alpha_x}{\alpha_y} \tag{8}$$

where (α_x, α_y) is one of the pairs of parameters listed above and subject to the constraints of equation (7). In other words, α_x is the change in the factor identified by x which is exactly sufficient to maintain returns (i.e., $\alpha_c = 0$) following a change of α_y in the factor identified by y and no change in the third factor in equation (7).

Definition 1
When $|f| < 1$, the response is termed efficient; *when $|f| \geq 1$, the response is termed* inefficient.

Definition 2
If $|f_{x/z}| < |f_{y/z}|$, then a change in z is compensated for more efficiently *(or* less inefficiently*) by changing x rather than y.*

Consider the pair (α_n, α_w). By setting $\alpha_e^{(n)} = 0$ in equation (7), we have $\alpha_n = (1-R)\alpha_w$ which implies that:

$$\frac{\alpha_n}{\alpha_w} = f_{n/w} = (1-R) \tag{9}$$

Similarly, setting $\alpha_w = 0$ gives $\alpha_n = R\alpha_e^{(n)}$ and

$$\frac{\alpha_n}{\alpha_e^{(n)}} = f_{n/e} = R \tag{10}$$

while setting $\alpha_n = 0$ gives $(1-R)\alpha_w + R\alpha_e^{(n)} = 0$ and

$$\frac{\alpha_w}{\alpha_e^{(n)}} = f_{w/e} = \frac{R}{R-1}. \tag{11}$$

The following results are derived easily from Definition 1:

	Efficient Region	Inefficient Region
$f_{n/w} = (1-R)$	$0 < R < 2$	$R \geq 2$
$f_{n/e} = R$	$0 < R < 1$	$R \geq 1$
$f_{w/e} = \frac{R}{R-1}$	$0 < R < 1/2$	$R \geq 1/2$.

Tables 1 through 3 display summary information on the effects of controlling various parameters to maintain per policy returns.

TABLE 1
Summary of the Nonparticipating Business Response With Respect to Changes in Expenses and Volume of Participating Business in Order to Maintain per Policy Returns

R	Nonparticipating Response	Due to Nonparticipating:		Due to Expenses:		Notes
(0, 1/2)	INC	INC	E	INC	E	$f_{n/w} > f_{n/e}$
(1/2, 1)	INC	INC	E	INC	E	$f_{n/e} > f_{n/w}$
(1, 2)	INC	DEC	E	INC	I	
(2, ∞)	INC	DEC	I	INC	I	

INC = Increase
DEC = Decrease
E = Efficient
I = Inefficient

TABLE 2
Summary of the Participating Business Response With Respect to Changes in Expenses and Volume of Nonparticipating Business in Order to Maintain Per Policy Returns

R	Participating Response	Due to Nonparticipating:		Due to Expenses:		Notes
(0, 1/2)	DEC	DEC	I	INC	E	No solution where $R(1+\alpha_e) > (1+\alpha_n)$
(1/2, 1)	DEC	DEC	I	INC	I	
(1, 2)	INC	DEC	I	INC	I	
(2, ∞)	INC	DEC	E	INC	I	

For key, see bottom of Table 1

TABLE 3
Summary of the Expenses Response With Respect to Changes in the Volumes of Nonparticipating and Participating Business in Order to Maintain Per Policy Returns

R	Expenses Response	Due to Participating:		Due to Nonparticipating:		Notes
(0, 1/2)	DEC	INC	I	DEC	I	No solution where $\alpha_n < \alpha_w - R(1+\alpha_w)$
(1/2, 1)	DEC	INC	E	DEC	I	
(1, 2)	DEC	DEC	E	DEC	E	
(2, ∞)	DEC	DEC	E	DEC	E	

For key, see bottom of Table 1

4 Sensitivity Analysis

The extent to which changes in the three factors affect the per policy returns now will be analyzed using a hypothetical model company.

The model company is composed entirely of 10 year annual premium pure endowments, with one tranche in unit-linked (nonparticipating) form, the other as participating. The methodology used to calculate P_n, P_w', and $E^{(n)}$ are described fully in Appendix 1. The assumptions used to calculate P_n and P_w' are given in Appendix 2. These assumptions lead to P_n = £255.69; P_w' = £3517.45

The present value of future nonproportionate expenses $E^{(n)}$ is calculated such that P_n less one policy's share of these expenses is equal to 50 percent of the initial commission (IC), i.e.,

$$0.5 \times IC = P_n - \frac{E^{(n)}}{N_n + N_w}.$$

This implies that:

$$E^{(n)} = (N_n + N_w)(P_n - 0.5 \times IC)$$

$$= 105.69\,(N_n + N_w)$$

(according to these assumptions).

The present value of the participating maturity benefit c is calculated according to equation (1). The participating policy is assumed to have a sum assured S such that a compound reversionary bonus of 5 percent per annum (with no terminal bonus) will lead to the implied maturity value of $c \times (1.1)^{10}$, i.e.,

$$S = c \times \left(\frac{1.1}{1.05}\right)^{10}.$$

The analysis involves calculating $c(1+\alpha_c)$ using equation (2), produced for values of α_c of +0.5 and -0.5, for each of the three factors in turn for R = 0.5, 0.75, 1.0, 1.5, 2.0, 3.0. Note that α_c can be expressed in terms of the implied revised reversionary bonus rate (r), which satisfies:

$$S(1+r)^{10} = c \times (1+\alpha_c)(1.1)^{10}.$$

The results are given in Table 4, and the changed values of R which correspond to these revised bonus rates are given in Table 5.

TABLE 4
Implied Reversionary Bonus Yields Percent for 50 Percent Variations in Fixed Expenses and in the Volumes of Nonparticipating and Participating Business According to the Model Described in Section 4 and Appendix 1
N_w = 1000 throughout (A value of 5 percent indicates no change in yield)

R	N_n	$\alpha_n =$ 0.5	$\alpha_n =$ -0.5	$\alpha_w =$ 0.5	$\alpha_w =$ -0.5	$\alpha_e^{(n)} =$ 0.5	$\alpha_e^{(n)} =$ -0.5
0.50	4771.00	6.46	3.33	4.47	6.46	4.20	5.75
0.75	1228.00	5.45	4.53	4.92	5.23	4.65	5.34
1.00	704.62	5.27	4.73	5.00	5.00	4.73	5.27
1.50	380.40	5.15	4.85	5.05	4.85	4.78	5.22
2.00	260.50	5.10	4.90	5.07	4.80	4.80	5.20
3.00	159.80	5.06	4.94	5.08	4.75	4.81	5.19

TABLE 5
Changes to the Expense Ratio *R* After 50 Percent Variations in Fixed Expenses and in the Volumes of Nonparticipating and Participating Business, Where These Values Correspond to the Same Changed Situations That Produce the Yields Shown in Table 4 at any Given Value of *R*

$\alpha_n =$ 0.5	$\alpha_n =$ -0.5	$\alpha_w =$ 0.5	$\alpha_w =$ -0.5	$\alpha_e^{(n)} =$ 0.5	$\alpha_e^{(n)} =$ -0.5
0.67R	2*R*	*R*	*R*	1.5*R*	0.5*R*

5 Discussion

5.1 Interpretation of the Results

In this section reference will be made particularly to Tables 1 to 4 and to equations (9) to (11).

Consider first Table 4. Sensitivity to changes varies both according to the company' expense ratio, R, and according to the factor involved. Returns become extremely sensitive at expense ratios below 0.5. But as these values imply high nonparticipating volumes coupled with low expenses, ratios in this region are unlikely in mutual life companies, which need a substantial volume of participating business to be viable.

As a general observation, yield becomes less sensitive to changes the higher is the expense ratio. At values of R above about 1.5 the improvements in yield due to increasing the volume of either types of business are barely appreciable. For these values of R, the greatest improvements are achieved by reducing nonproportionate expense levels.

At values of R above about 2, the most significant adverse effect is due to a decrease in the volume of participating business; hence, maintaining the volume of this business should be of most concern to a company with such a ratio. From Definition 2, $|f_{e/w}| < |f_{n/w}|$ indicates that an unavoidable fall in participating volume is much more efficiently dealt with by decreasing expenses than by increasing nonparticipating volume. This difference in efficiency becomes more marked for increasingly large values of R. Similarly, an increase in expenses is compensated for more efficiently by increasing participating rather than nonparticipating volume ($|f_{w/e}| < |f_{n/e}|$).

Offices with ratios between 1 and 2 should become more concerned with falls in nonparticipating volume and increases in expense levels. Reducing expense levels is a much more efficient way of compensating for a fall in nonparticipating volume than increasing participating volume ($|f_{e/n}| < |f_{w/n}|$). At ratios close to unity, varying the participating volume will have almost no effect on yield. There is no efficient way to deal with increasing expenses at these ratios; hence, this would appear to be the most significant problem. If increasing expenses is unavoidable, then increasing the nonparticipating volume is the least inefficient way of compensating ($|f_{n/e}| < |f_{w/e}|$). The greatest improvements at these ratios can be achieved by increasing nonparticipating volume or by decreasing the nonproportionate expenses.

At ratios below unity a rather peculiar and apparently unstable situation exists, as per policy returns increase with a fall in participating volume, reflecting the increased share of the (positive) value of $(N_n P_n - E^{(n)})$ per participating policy. Returns become increasingly sensitive to changes in all factors, but particularly to changes in the nonparticipating volume. There is no efficient way of compensating for a fall in nonparticipating volume at these levels—it is of particular concern to management to maintain nonparticipating volume here. Between ratios of 0.5 and 1, $|f_{e/n}| < |f_{w/n}|$, i.e., it is less inefficient to compensate for falling nonparticipating business by reducing expenses than by decreasing participating sales; however, the opposite is the case for the (rather unlikely) situation where the expense ratio is below 0.5.

At ratios below unity, $|f_{n/e}| < 1$, so that an increase in expenses can be compensated for efficiently by increasing nonparticipating volume. Reducing the participating business is also an efficient way of dealing with increased expenses at ratios below 0.5, although the nonparticipating route is always the most efficient method.

5.2 Consequences of Management Decisions

A company with an expense ratio exceeding 2 would be most concerned with maintaining and increasing participating sales and controlling expenses. Economies of scale are easier to achieve using participating sales the larger the value of the expense ratio. But in all cases, a greater proportionate increase in sales than in expenses is needed to secure these economies. These actions would tend to increase the expense ratio, making it proportionately easier to achieve further economies of scale. The high ratio position persists and tends to become increasingly stable as $R \to \infty$.

At expense ratios in the region $1 < R < 2$, it becomes increasingly easier (in proportionate terms) to maintain or to improve returns by increasing nonparticipating business or by reducing expenses. Economies of scale best would be achieved by increasing nonparticipating sales, although the proportionate increase in sales has to be larger than that of the expenses. All these actions would result in yet lower expense ratios, making economies of scale easier to achieve and hence continuing the reduction in expense ratio to unity. Any attempt to obtain economies of scale by increasing the participating portfolio becomes increasingly difficult and inefficient, the closer the expense ratio is to 1 from 2. If successful, though, such an action would tend to increase the ratio.

At expense ratios below unity, economies of scale can be achieved efficiently by increasing the nonparticipating business (i.e., if the result of the decision is for $1 > \alpha_n > f_{n/e} \times \alpha_e^{(n)}$). This action (i.e., efficiently producing economies of scale) would tend to increase the expense ratio toward unity. Even greater returns could be achieved if $\alpha_n > 1$, in which case the ratio will reduce. Participating sales, however, cannot be increased without lowering per policy profit (or at least without increasing the nonparticipating portfolio sufficiently to compensate for the losses). On the other hand, a company in such a position may be providing higher returns than its market competitors, other things being equal. Such returns would make the company attractive to new participating policyholders, who would accept a fall in per policy profit just to obtain a share of some of it; alternatively, the company could be a potential candidate for demutualization. Hence, market forces could act to increase the participating portfolio—if this ultimately leads to increases in non-proportionate expenses, then this also will increase the expense ratio. Another alternative is for the company to reduce its nonparticipating premium rates (or charges), which would tend to increase the expense ratio as P_n would be reduced. This effectively transfers some of the superprofits to the nonparticipating policyholders, an action that might be required on the grounds of equity. The need to maintain the participating portfolio in order to provide an adequate statutory surplus also should be borne in mind.

If the only consideration of management is to increase per policy returns, then once $R < 1$ the optimum decision would be to reduce the participating portfolio down to one policy. Market forces, coupled with the company's need to provide capital, would tend to reverse the trend. The ultimate position (i.e., value of R) at which a company would tend to maintain itself would be largely dependent upon the market level of per policy profits expected from participating policies, although there is a partially stable point at $R = 1$ caused by attempts to produce economies of scale through efficient increases to the nonparticipating portfolio.

There are two distinct strategies that a company can adopt to maintain a required level of profit, associated respectively with low and high expense ratios.

a) **Low Ratio Strategy**—A company with an expense ratio in the region of unity would be in a highly manageable position. With all nonproportionate expenses covered by nonparticipating business, participating volume can be increased or decreased with no change to returns, provided the statutory solvency position is not compromised by any decrease in volume. Control of per policy profit would rest entirely with controlling the volume of nonparticipating business and level of nonproportionate expenses (and in controlling the expense ratio). Market demand for profit levels would tend to dictate where the ratio ultimately would lie, although pursuit of economies of scale introduces a partial optimum expense ratio at unity itself.

b) **High Ratio Strategy**—A company with a high expense ratio implies that nonparticipating business is essentially an insignificant proportion of the portfolio. Control of per policy profit would rest almost entirely with controlling the volume of participating business and the level of nonproportionate expenses, while pursuit of economies of scale would tend to increase the expense ratio still further.

5.3 Practical Implications

The main implications from the above are for mutual life companies that maintain significant volumes of nonparticipating (including unit-linked) business, implying low expense ratios and hence requiring a low ratio strategy. The lower the ratio, the more sensitive per policy profits are to changes in the constituents of the expense ratio. At ratios less than unity, the fact that increasing participating business reduces profit should be borne in mind. At ratios near unity, management should bear in mind that no increase (or decrease) in the participating portfolio will affect returns; a policy of expansion involving increasing expenses matched by increasing the sales of participating contracts would have only adverse effects on returns. Such companies also need to consider the need to meet statutory solvency levels, always an important consideration where significant levels of nonparticipating business are involved.

The actual value of the ratio for any particular company will determine the required response to adopt for any particular situation: for example, when pursuing economies of scale, in determining the minimum increase required to the nonparticipating portfolio to cover an increase in expense levels. Other management decisions that can be assisted by the response relationships described in this paper include:

a) How can market share be increased most efficiently in order to make minimum losses/maximum profits for the participating policyholders?
b) When business is falling, to what extent should expenses be reduced and which type of business is it most important to retain?

6 Summary

The ratio of nonproportionate expenses to total marginal profits from nonparticipating business (the *expense ratio*) is a key factor in determining management policy regarding business volume and expenses.

Relationships presented in this paper can be used to determine minimum responses required to compensate for changes in any of the factors in order to maintain per policy returns and also to assist management in choosing appropriate strategies for achieving such aims as economies of scale, increasing market share, or cost-cutting.

Decision choices should vary depending on whether the company has a low expense ratio (less than 2) or a high expense ratio (greater than 2). There are two partially optimum ratios, at $R = 1$ and $R \rightarrow \infty$, both resulting from companies choosing the most efficient methods to produce economies of scale at $R < 2$ and $R > 2$, respectively.

It is not sufficient to assume that increasing sales of participating contracts will improve per policy returns, as the greater coverage of expenses is offset to a greater or lesser extent by the dilution of profits from the nonparticipating portfolio.

Particularly at low expense ratios, decision choices identified by the relationships described in this paper will be constrained by the need to meet statutory solvency levels and to maintain adequate investment flexibility. Other factors, such as market forces, also can affect levels of business. All relevant factors should be considered together.

References

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Appendix 1

Let

$^{w}q_x$ = Probability of a participating policyholder at age x dying before age $x + 1$;

$^{n}q_x$ = Probability of a nonparticipating policyholder at age x dying before age $x + 1$;

$^{w}p_x$ = $1 - {}^{w}q_x$;

$^{n}p_x$ = $1 - {}^{n}q_x$.

Assume that all participating policies are t year pure endowments issued to a life age x. If a participating policyholder dies before the policy matures at age $x + t$, there is a return of the accumulated fund at the end of the year of death. The fund is set equal to the participating policy's asset share on death (including its share of nonparticipating policy profits and its share of nonproportionate expenses). Premiums are level and are paid for t years.

Define F_k to be the expected fund at time k immediately *before* the payment of the death benefit:

$$F_k = {}^{w}F_k + {}^{n}F_k - {}^{e}F_k \tag{A.1}$$

where ${}^{w}F_k$, ${}^{n}F_k$, and ${}^{e}F_k$ are defined below.

$$^{w}F_k = N_w \times {}_{k-1}^{\;\;w}p_x \sum_{r=0}^{k-1} (G - {}^{w}E_r^{(p)})(1 + i)^{k-r} \tag{A.2}$$

$$^{n}F_k = N_n \sum_{r=0}^{k-1} {}_{k-1-r}^{\;\;\;\;\;w}p_{x+r} \times {}_{r}^{n}p_x \times (H_r - {}^{n}E_r^{(p)})(1 + i)^{k-r} \tag{A.3}$$

$$^{e}F_k = \sum_{r=0}^{k-1} {}_{k-1-r}^{\;\;\;\;\;w}p_{x+r} \times E_r^{(n)} (1 + i)^{k-r} \tag{A.4}$$

where:

v = $(1+i)^{-1}$;

G = Annual gross premium;

$^{w}E_r^{(p)}$ = Per policy proportionate expenses for a single participating policy paid at time r;

$^{n}E_r^{(p)}$ = Per policy proportionate expenses for a single nonparticipating policy paid at time r;

$^{n}H_r$ = Charges paid at time r per nonparticipating policy; and

$E_r^{(n)}$ = Total nonproportional expenses at time r.

Note that the payment $({}^{n}H_r - {}^{n}E_r^{(p)})$ at time r depends on the survival of nonparticipating policyholders to time r. What remains of these payments by time $k-1$ depends on how many *participating* policyholders survive to time $k-1$. Clearly ${}^{w}F_k$, ${}^{n}F_k$, and ${}^{e}F_k$ are actuarial "accumulated" values up to time $k-1$ (including both interest and mortality) and from time $k-1$ to time k using interest only.

The expected actuarial present value of future claims per participant policy is c where

$$c = \frac{\sum_{k=0}^{t-1} v^{k+1} \times {}^{w}q_{x+k} \times F_{k+1} + v^t \times {}^{w}p_{x+t-1} \times F_t}{N_w}. \tag{A.5}$$

Next, define P'_w, P_n, and $E^{(n)}$ as follows:

$$\begin{aligned} P'_w &= \sum_{k=0}^{t-1} v^{k+1} \times {}^{w}q_{x+k} \times \frac{{}^{w}F_{k+1}}{N_w} + v^t \times {}^{w}p_{x+t-1} \times \frac{{}^{w}F_t}{N_w} \\ &= \sum_{k=0}^{t-1} {}^{w}_{k}p_x \times {}^{w}q_{x+k} \sum_{r=0}^{k} (G - {}^{w}E_r^{(p)})\, v^r \\ &\quad + {}^{w}_{t}p_x \sum_{r=0}^{t-1} (G - {}^{w}E_r^{(p)})\, v^r \end{aligned} \tag{A.6}$$

$$\begin{aligned} P_n &= \sum_{k=0}^{t-1} v^{k+1} \times {}^{w}q_{x+k} \times \frac{{}^{n}F_{k+1}}{N_n} + v^t \times {}^{w}p_{x+t-1} \times \frac{{}^{n}F_t}{N_n} \\ &= \sum_{k=0}^{t-1} {}^{w}q_{x+k} \sum_{r=0}^{k} {}^{w}_{k-r}p_{x+r} \times {}^{n}_{r}p_x \times (H_r - {}^{n}E_r^{(p)})\, v^r \\ &\quad + \sum_{r=0}^{t-1} {}^{w}_{t-r}p_{x+r} \times {}^{n}_{r}p_x \times (H_r - {}^{n}E_r^{(p)}) v^r \end{aligned} \tag{A.7}$$

and

$$E^{(n)} = \sum_{k=0}^{t-1} v^{k+1} \times {}^{w}q_{x+k} \times {}^{e}F_{k+1}$$

$$= \sum_{k=0}^{t-1} {}^{w}q_{x+k} \sum_{r=0}^{k} {}_{k-r}^{\;\;w}p_{x+r} \times E_r^{(n)} v^r + \sum_{r=0}^{t-1} {}_{k-r}^{\;\;w}p_{x+r} \times E_r^{(n)} v^r \,. \quad \text{(A.8)}$$

From the definition of c in equation (A.5),

$$c = \sum_{k=0}^{t-1} v^{k+1} \times {}^{w}q_{x+k} \left[\frac{{}^{w}F_{k+1}}{N_w} + \frac{N_n}{N_w} \times \frac{{}^{n}F_{k+1}}{N_n} - \frac{{}^{e}F_{k+1}}{N_w} \right]$$

$$+ v^t \times {}^{w}p_{x+t-1} \left[\frac{{}^{w}F_t}{N_w} + \frac{N_n}{N_w} \times \frac{{}^{n}F_t}{N_n} - \frac{{}^{e}F_t}{N_w} \right]$$

$$= P_w' + \frac{N_n}{N_w} P_n - \frac{E^{(n)}}{N_w}$$

$$= P_w' + \frac{N_n P_n - E^{(n)}}{N_w} \,. \quad \text{(A.9)}$$

Appendix 2—Model Office Assumptions

Annual premium = £600

Proportionate Expenses

Initial commission	=	50 percent of annual premium
Renewal commission	=	2.5 percent of annual premium
Other initial expenses	=	£60
Investment expenses	=	0.25 percent of accumulated asset share at end of each year
Other renewal expenses	=	£6 per annum, inflating at 7.5 percent per annum

Charges for Unit-Linked Policy

Initial	=	£500
Renewal for commission	=	2.5 percent of annual premium
Renewal for fund management charge	=	0.5 percent of unit fund at end of each year
Renewal for other	=	£15 inflating at 7.5 percent per annum

Other Assumptions

Asset accumulation rate	=	10 percent per annum
Rates of mortality and withdrawal	=	nil
Tax rates	=	nil
Discount rate for calculating present values	=	10 percent per annum

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