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Jerome J. Siewert
Wausau Insurance Companies, Siewert@e-mail.com

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Jerome J. Siewert*

Abstract†

In this paper I describe several approaches for estimating liabilities under a high deductible program, including a proposal for a more sophisticated approach relying upon a loss distribution model. The discussion addresses several related issues dealing with deductible size and mix, absence of long-term histories, and the determination of consistent loss development factors among deductible limits. In addition, I propose several approaches for estimating aggregate loss limit charges, if any, and the asset value for associated servicing revenue.

Key words and phrases: *loss ratio, excess loss, ultimate loss, IBNR, development factors, inverse power curve*

1 Introduction

With the advent of the workers compensation high deductible program in the early 1990s, actuarial efforts focused principally on pricing issues. Insurers initially developed this program to:

*Jerome J. Siewert, F.C.A.S., M.A.A.A., is an assistant vice president and actuary at the Wausau Insurance Companies. He manages the reserving unit of the actuarial department. Mr. Siewert's previous experience includes managing several pricing units and serving on various Casualty Actuarial Society ratemaking committees. Prior to joining Wausau, he spent four years as a secondary school mathematics teacher.

Mr. Siewert's address is: Wausau Insurance Companies, PO Box 8017, Wausau WI 54402, USA. Internet address: *jsiewert@e-mail.com*

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- Achieve price flexibility while passing additional risk to larger insureds in what was considered at that time an unprofitable line of business;
- Ameliorate onerous residual market charges and premium taxes in some states;
- Realize cash flow advantages similar to those of the closely related product, the paid loss retro;
- Provide insureds with another vehicle to control losses while protecting them against random, large losses; and
- Allow self-insurance without submitting insureds to sometimes demanding state requirements.

As the program matured, insurers' focus shifted to the liability side. Questions now are being asked about what losses are emerging and what they ultimately will cost insurers. For the actuary, the question is how best to estimate these liabilities when losses are not expected to emerge above deductible limits for many years.

Many questions still need to be addressed, for example:

- In the absence of long-term development histories under a deductible program, how can the actuary construct reasonable development factors? (Addressed in Section 3.)
- How can the actuary determine development patterns that reflect the diversity of both deductible size and mix? (Addressed in Section 3.)
- How should the actuary determine consistent development factors between limited and excess values? (Addressed in Section 3.)
- What is a reasonable approach for indexing deductible limits over time? (Addressed in Section 3.)
- How can the actuary estimate the liability associated with aggregate loss limits, if any? (Addressed in Section 5.1.) and
- Is there a sound way to determine the proper asset value for associated service revenue?[1] (Addressed in Section 5.2.)

[1]Similar in usage to a loss conversion factor in retro rating, loss multipliers are applied to deductible losses to capture expenses that vary with loss.

2 Development Approaches

2.1 Overview

The approaches discussed in this paper rely on my company's full coverage workers compensation claim experience. In effect, I create deductible/excess development patterns as needed. Further, the approach benefits from credible histories of full coverage losses, although the techniques used do not necessarily require a large volume of claim experience.

Once I establish development factors that reflect deductible amounts, I apply them at the account level and determine the overall aggregate reserve by summarizing estimated ultimates for each account. This is a reasonable approach if you view each account as belonging to a cohort of policies with similar limit characteristics. Determining the overall reserve allows me to address the issue of deductible mix by reflecting each account's limits.

In Section 4 I describe the possible use of a loss distribution model to enforce consistent results between deductible/excess development factors. Once the parameters of the distribution are set, it is possible to determine development factors for any deductible size. Such a model may provide an alternative approach for determining tail factors through the projection of the distribution parameters.

2.2 Loss Ratio

In the absence of credible development histories, a common approach for determining liabilities is to apply loss ratios to premiums arising from the exposures. As this element historically was required to price the product, loss ratios for the various accounts written should be readily available. As an alternative, the reserve actuary could use published loss ratios of workers compensation excess writers or reinsurers.

For immature years where data are sparse, applying loss ratios is the most practical approach to take. Given the long-tailed nature of this business, experience over deductible limits emerges slowly over time. Expected experience is readily converted to an accident year basis based upon a pro rata earnings of the policy year exposures.

The loss ratio approach requires a database of individual accounts and pricing elements. The database should include an estimate of the full coverage loss ratio. From a pricing standpoint, that estimate can come from a variety of sources.[2] One approach is to use company experience by state, reflecting the individual account's premium distribution. This experience possibly can be blended with industry experience. As with other pricing efforts, experience ought to be developed to ultimate, brought on level, and trended to the appropriate exposure period.

In addition to an estimate of the full coverage loss ratio, the database should include estimates of excess losses for both occurrence and aggregate limits. For the occurrence limit, several approaches are possible, including estimating excess ratios based upon company experience. A potentially more credible approach uses excess loss pure premium ratios underlying industry-based excess loss factors used in retro rating. Beside their availability by multiple limits, excess loss factors can easily be adjusted to a loss basis and to reflect hazard groups with differing severity potential. Using account-based excess ratios reflecting unique state and hazard group characteristics leads to reasonable estimates of per occurrence excess losses:

$$\text{Per Occurrence Excess Losses} = P \times \text{ELR} \times \chi \qquad (1)$$

where

$$\begin{aligned} P &= \text{Premium;} \\ \text{ELR} &= \text{Expected loss ratio; and} \\ \chi &= \text{Per occurrence charge.} \end{aligned}$$

For the aggregate loss charge, I prefer a process similar to that used for determining insurance charges in a retro rating program. These charges rely on the National Council on Compensation Insurance's (NCCI) Table M.[3] The process reflects the size of the account, deductible, state severity relativities, prospective rating period, and appropriate rating plan parameters:

$$\text{Table M Aggregate Excess Losses} = P \times \text{ELR} \times (1 - \chi) \times \phi \qquad (2)$$

where ϕ is the per aggregate charge.

Applying equations (1) and (2) to each account and then aggregating leads to an estimate of ultimate accident year losses. Table 1 shows

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[2]Actuaries are generally familiar with techniques used to determine loss ratios for pricing purposes, and a detailed description is beyond the scope of this paper.

[3]I refer the interested reader to the Retrospective Rating Plan (1991) for further details.

a hypothetical case of how to apply both equations to determine the ultimate liabilities. Incurred but not reported (IBNR) amounts are determined by subtracting known losses from the ultimate estimate.

Again, the approach described in Table 1 is useful when no data are available or the data are too immature be useful. Loss ratio estimates can be consistently tied to pricing programs, at least at the outset. This approach also benefits from its reliance on a more credible pool of company and/or industry experience. On the negative side, the loss ratio approach has two shortcomings:

- It ignores emerging experience which may differ significantly from estimated ultimate losses. For this reason the loss ratio approach is not useful after several years of development; and
- It may not properly reflect account characteristics—development may emerge differently due to the exposures written.

2.3 Implied Development

There are many ways to incorporate emergence of losses in high deductible reserve estimates. Determining excess development implicitly is one possibility. The term *implied development* means an approach that works as follows:

- Develop full coverage losses to ultimate;
- Next develop deductible losses to ultimate by applying development factors reflecting various inflation indexed limits; and
- Determine ultimate excess losses by differencing the full coverage ultimate losses and the limited ultimate losses.

A variety of the usual development techniques can be applied to determine full coverage losses. These methods include paid and incurred techniques designed consistently with the company's reserving procedures for full coverage workers compensation. Care should be exercised in determining a full coverage tail factor consistent with the limited loss tail factors. The actuary should avoid developing limited losses beyond unlimited losses or even losses for lower limits beyond those of higher limits.

When calculating development factors for the various deductibles, it is appropriate to index the limits for inflationary effects. Adjusting the deductible by indexing keeps the proportion of deductible/excess losses fairly constant about the limit from year to year.

Table 1
Countrywide Insurance Enterprise
Account: Widget, Inc.
Expected Deductible/Aggregate Loss Charges

(1)	(2)	(3)	(4) $(2) \times (3)$	(5)	(6) $(4) \times (5)$	(7)	(8) $[(4) - (6)] \times (7)$
State	Premium	ELR	Expected Loss	Excess Ratio	Deductible Loss Charge	Aggregate Ratio	Aggregate Loss Charge
Arkansas	9,084	0.567	5,151	0.062	319	0.02	97
Illinois	573,066	0.532	304,871	0.105	32,011	0.02	5,457
Iowa	373,072	0.588	219,366	0.096	21,059	0.02	3,966
Kansas	70,549	0.644	45,434	0.071	3,226	0.02	844
Minnesota	1,012,622	0.457	462,768	0.143	66,176	0.02	7,932
South Carolina	22,980	0.522	11,996	0.048	576	0.02	228
South Dakota	94,401	0.697	65,797	0.211	13,883	0.02	1,038
Total	2,155,774	0.517	1,115,383	0.123	137,250	0.02	19,562

For example, if inflationary forces drive claim costs 10 percent higher each year, one would expect the percentage of losses over a $100,000 deductible for one year equate to those of a $110,000 deductible in the next. Indexing of deductible limits allows for the possibility of combining differing experience years in the determination of development factors.

There is no set method for determining the indexing value. One approach is to determine the index by fitting a line to average severities over a long-term history. Another simpler approach is to use an index that reflects the movement in annual severity changes. The actuary needs to be cognizant that a constant deductible over time usually implies increasing excess losses.

An advantage of the implied development approach is that it provides an estimate of excess losses at early maturities when excess losses have not emerged. The development factors for limited losses are more stable than those determined for losses above the deductible. This approach also provides an important byproduct in the estimation of assets under the high deductible program. Estimating deductible losses helps determine the asset represented by revenue collected from the application of a loss multiplier to future losses. Despite these advantages, the implied development approach appears to misplace its focus by *indirectly* calculating excess losses,[4] which can be problematic if one prefers to determine excess losses directly.

2.4 Direct Development

The direct development approach explicitly focuses on excess development, although it relies on development factors derived from the implied development approach. Given development factors for limited as well as full coverage losses, excess loss development factors can be calculated. Excess development is part of overall development, and the actuary should strive to determine excess factors in conjunction with limited development factors that balance to full coverage development. Reserve indications from implicit and explicit methods will not necessarily be the same, but the underlying loss development factors should be.

A variety of paid and incurred techniques are applicable here. I see several disadvantages to directly determining excess development factors and applying them to excess losses. These factors tend to be

[4]The excess losses are calculated indirectly by differencing ultimate unlimited losses with ultimate limited losses.

leveraged and extremely volatile, making them difficult to select. Additionally, if excess losses have not emerged at any particular stage of development, either the development factors do not exist or the indicated zero loss estimate is not particularly meaningful.

2.5 Credibility Weighting

There are significant drawbacks to the previous approaches (see Sections 2.2, 2.3, and 2.4) for determining excess liabilities. The credibility weighting approach offers more promise as it relies on credibility weighting indications based on experience with expected values, preferably based on pricing estimates. The actuary determines a suitable set of credibility weights then uses these weights to calculate the ultimate loss estimate (ULE_t), based on information at time t:

$$\text{ULE}_t = \text{OL}_t \times \text{LDF}_t \times Z_t + \text{EUL}_t \times (1 - Z_t) \tag{3}$$

where

$$\begin{aligned} \text{OL}_t &= \text{Observed loss at time } t; \\ \text{LDF}_t &= \text{Age to ultimate development factor at time } t; \\ Z_t &= \text{Credibility weight at time } t; \text{ and} \\ \text{EUL}_t &= \text{Expected ultimate loss at time } t. \end{aligned}$$

The Bornhuetter-Ferguson (1972) technique offers one approach for determining credibility weights that are specified as reciprocals of loss development factors. For the Bornhuetter-Ferguson approach, substituting $Z_t = 1/\text{LDF}_t$ into equation (3), yields:

$$\text{ULE}_t = \text{OL}_t + \text{ELR}_t \times \left(\frac{\text{LDF}_t - 1}{\text{LDF}_t}\right). \tag{4}$$

Using the Bornhuetter-Ferguson approach allows the actuary to determine liabilities either directly or indirectly and can tie into pricing estimates for recent years where excess losses have yet to emerge. Also, it provides more stable estimates over time, rather than the volatility arising from erratic emergence or leveraged development factors. A credibility weighting approach such as this provides better estimators of ultimate liabilities as well. A disadvantage of the Bornhuetter-Ferguson approach is that a portion of ULE_t, namely $\text{ULE}_t \times (1 - Z_t)$, ignores observed losses. That drawback suggests finding alternative credibility weights that are more responsive to the actual experience as desired.

3 An Overview of Development Models

I will now deal more specifically with a number of questions raised in the introduction: How can the actuary determine development factors in the absence of a long-term history under the deductible program? How can the actuary determine development patterns that reflect the diversity of both deductible size and mix? What is a reasonable approach for indexing deductible limits over time? How should the process relate development for various limits consistently? As determining development factors for a high deductible program often is an exercise in partitioning development about the deductible limit, one question often is: Is it possible to develop consistent tail factors among the limits to which the company is exposed?

In the absence of long-term experience under the deductible program, I suggest using a company's history of full coverage workers compensation claims. It is also appropriate to apply an indexed limit to the claims to determine a series of accident year loss development histories by limit. I examine limits ranging from $50,000 to $1,000,000. I focus, however, on the more common deductible sizes in the neighborhood of $250,000. I use case losses including indemnity, medical, and any subject allocated claim expense. The histories run for 25 years but are not separated by account, injury, or state. This suggests creating alternative development patterns that reflect these factors. Table 2 shows age to age development factors by indexed limit.

Table 2
Workers Compensation—High Deductibles
Limited Loss and ALAE Age to Age Development
Factors by Indexed Limit (Middle Six of Last Eight)

	Months:Months				
Limit	12:24	24:36	36:48	48:60	60:72
$50,000	1.5031	1.0418	1.0038	1.0025	1.0020
$100,000	1.6225	1.0727	1.0151	1.0063	1.0080
$250,000	1.6791	1.1300	1.0451	1.0207	1.0060
$500,000	1.6827	1.1393	1.0684	1.0322	1.0170
$750,000	1.6816	1.1408	1.0720	1.0359	1.0214
$1,000,000	1.6811	1.1411	1.0728	1.0371	1.0229
Unlimited	1.6876	1.1430	1.0749	1.0391	1.0196

In order to determine those development factors, I combine several years of experience based upon indexed limits. For example, for the most recent year limits are used as stated. But for the first prior year I adjust limits downward by an indexing factor of 1.095.[5] For the current year I assume a limit of $250,000 was the equivalent of a limit of $228,311 for the first prior year. Each limit is adjusted by the same index to generate the desired development factors. Figure 1 shows the exponential trend line fit through known data points determining the long-term indexing factor of 1.095. Also depicted is the indexed $250,000 loss limit.

I recommend separating claim count development from severity development when estimating high deductible liabilities. In this paper I focus on the counts for full coverage losses rather than the emergence of claims over a specific deductible limit. It is easier to recognize development in this fashion, as there is generally little true claim count IBNR after three years. This is true for larger claims, as they will be reported early (like other claims), but their severity will not be known for some time.

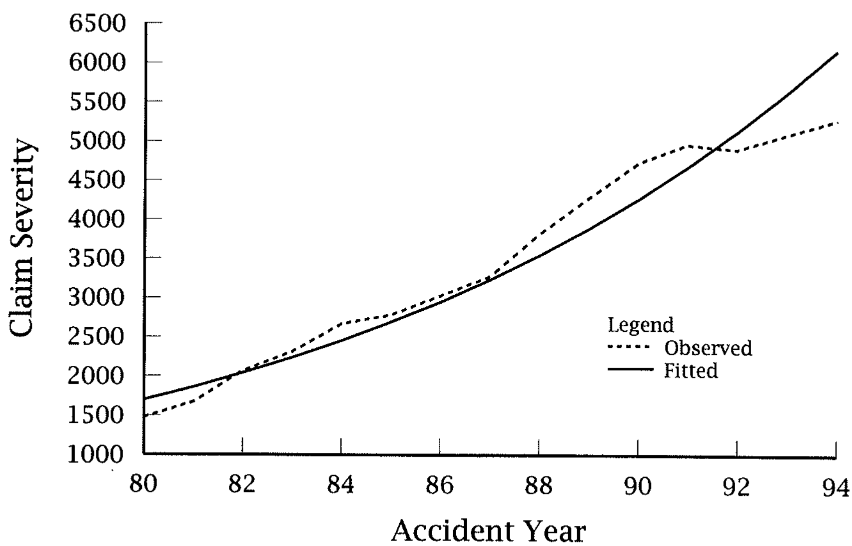
Table 3
Workers Compensation—Age to Age Development Factors
Full Coverage Claim Count

	Months:Months			
Accident Year	12:24	24:36	36:48	48:60
1988	-	-	-	0.9999
1989	-	-	0.9999	0.9994
1990	-	1.0026	0.9999	1.0001
1991	1.1111	1.0022	1.0002	-
1992	1.1305	1.0017	-	-
1993	1.1283	-	-	-
Last Three	1.1233	1.0022	1.0000	0.9998
Selected	1.1250	1.0025	1.0000	1.0000
Age to Ultimate	1.1278	1.0025	1.0000	1.0000

To develop limited losses to ultimate, I use a three parameter version of the inverse power curve recommended by Sherman (1984) to

[5]The selected indexing factor of 1.095 is based upon a long-term severity history. There may be, however, better approaches such as varying the indexing factor by year or adjusting for the distorting effects of larger claims.

Figure 1
Workers Compensation Loss and ALAE Severity Trend



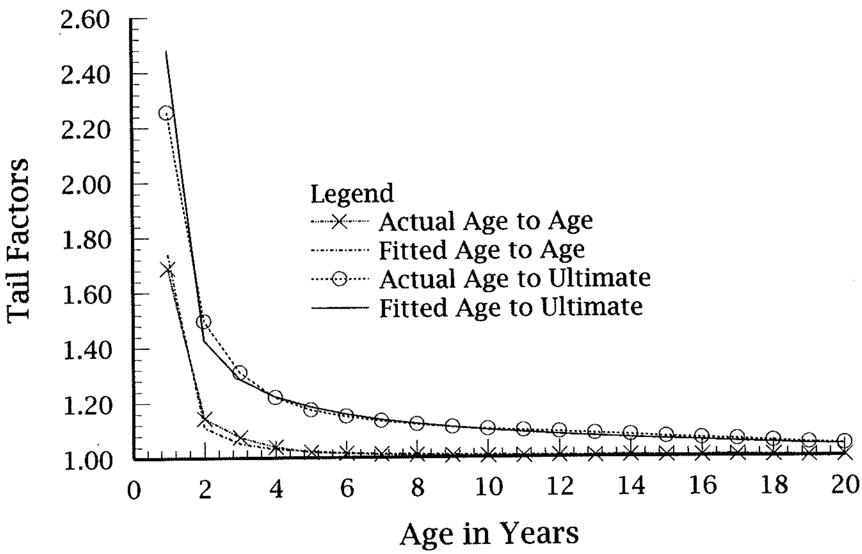
model the development arising in the tail. The curve can be written as a function of t as follows:

$$y(t) = 1 + \frac{a}{(t+c)^b} \quad t > 0, \tag{5}$$

where a, b, and c are constants. My concern is to determine consistent tail factors by limit. Starting with the unlimited loss development and fitting an inverse power curve to known age to age factors allows me to project ultimate unlimited losses. As the inverse power curve is defined for $t > 0$, a time to *end* the projection must be selected. I use a method that relies on extended development triangles. (The method is similar to the method used for determining our full coverage tail factor.) It turns out that the projected age to age development factors can be stopped at 40 years. Compounding the age to age factors from the fitted curve leads to the desired completion or tail factors.

Once the ultimate age is determined, I use a minimum chi-square (between observed and expected values) technique to fit inverse power curves to the age to age factors for the various deductible limits and extend to the common maturity. Although this approach generates

Figure 2
Workers Compensation Unlimited
Tail Factors: Actual vs. Fitted

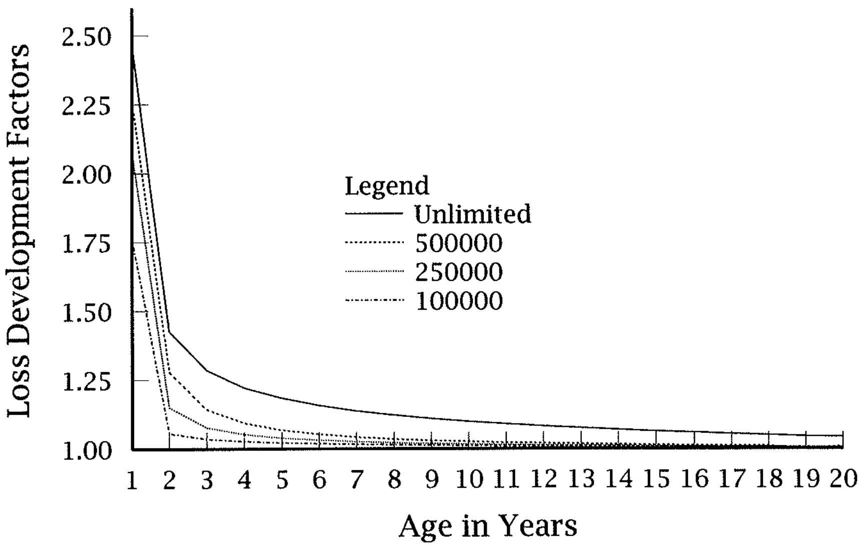


uniformly decreasing tail factors, it is not clear whether the bias in extending all curves to a common maturity is significant. (At lower limits, development likely ceases before 40 years.) Figure 2 depicts the age to age model determined for the unlimited loss development. Figure 3 shows the pattern of age to ultimate limited loss development factors resulting from the inverse power curve model.

Another issue is the relationship between loss development factors and limited severity relativities.[6] In some of my earlier efforts I attempted to develop losses by limit without regard to how they might relate to one another. This led to inconsistencies in development factors where completion factors for smaller deductibles, for example, sometimes exceeded factors for larger deductibles. I found that any attempts to determine deductible development factors need to address the relationship between the full coverage loss development and limited severity relativities.

[6]Limited severity relativities are defined simply as the ratio of the limited to unlimited severity.

Figure 3
Workers Compensation
High Deductibles Age to Ultimate Loss Development Factors



The following formulas show limited development factors (LDF_t^L) and excess development factors (XSLDF_t^L) at time t as functions of the unlimited loss development and limited severity relativities:

$$\mathrm{LDF}_t^L = \frac{C}{C_t} \times \frac{S}{S_t} \times \frac{R^L}{R_t^L} \quad (6)$$

$$\mathrm{XSLDF}_t^L = \frac{C}{C_t} \times \frac{S}{S_t} \times \frac{1 - R^L}{1 - R_t^L} \quad (7)$$

where t is the age and

L = Deductible limit;
C = Ultimate claim count;
C_t = Total claim count at time t;
S = Ultimate full coverage severity;
S_t = Full coverage severity at time t;

R = Ultimate limited severity relativity; and

R_t = Limited severity relativity at time t.

The motivation for equations (6) and (7) results from the desire to partition total loss development in a consistent fashion between limited and excess development. Note that

$$\begin{aligned} \text{LDF}_t^L &= R_t^L \times \text{LDF}_t^L + (1 - R_t^L) \times \text{XSLDF}_t^L \\ &= R_t^L \times \frac{C}{C_t} \times \frac{S}{S_t} \times \frac{R^L}{R_t^L} + (1 - R_t^L) \times \frac{C}{C_t} \times \frac{S}{S_t} \times \frac{1 - R^L}{1 - R_t^L} \\ &= \frac{C}{C_t} \times \frac{S}{S_t} \end{aligned} \quad (8)$$

as is expected.

Figure 4 shows how the historical limited severity relativities ought to relate to each other and how they change over time. The relativity curves cluster near unity initially and progressively decrease over time for smaller and smaller deductibles without crossing over one another.

Figure 4
Workers Compensation
High Deductibles Limited Severity Relativities

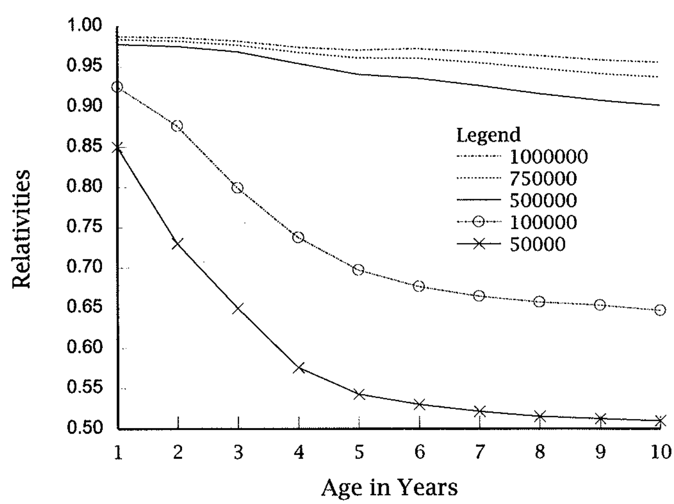


Table 4 shows age to age development about a \$250,000 deductible limit. Table 5 shows relativities and their changes for the selected deductible limit. Note how the change in limited loss development relates to the unlimited loss development. Actual case loss development does not always conform to expectations, as the limited loss development factor sometimes exceeds the unlimited, thus

$$\text{LDF}_t^L = \text{LDF}_t \times \Delta R_t^L \tag{9}$$

where $\Delta f_t^L = f_t^L / f_t$ for any function f. For example, for accident year 1993 moving from 12 to 24 months, a limited factor of 1.6229 is observed. This is equivalent to the unlimited loss development factor of 1.6044 compounded with the change in severity relativities for the same time period of 1.0116.

Note also the relationship of the excess development to the unlimited loss development for the same year:

$$\text{XSLDF}_t^L = \text{LDF}_t \times \Delta(1 - R_t^L). \tag{10}$$

The accident year 1993 excess development factor of 1.1684 is equivalent to the unlimited development factor times the ratio of the complements of the severity relativities moving from 12 to 24 months, i.e., $1.1684 = 1.6044 \times (1 - 0.9704)/(1 - 0.9593)$. The weighted average of the limited and excess development factors using the relativity as a weight gives the unlimited loss development factor in equation (7). Also $1.6044 = 0.9704 \times 1.6229 + (1 - 0.9704) \times 1.1684$ for accident year 1993.

4 Distributional Models

Statistical distributions are ideally suited for modeling loss development factors as they can tie the relativities to the severities and consequently provide consistent loss development factors. They model the development process by determining parameters of a distribution that vary over time.[7]

Once the distribution and its parameters are specified, it is possible to calculate the desired limited/excess severities. Comparing those severities over time leads to the needed development factors. Care has to be exercised to recognize claim count development at earlier maturities. Also, distributional models allow for interpolation among limits and years as needed.

[7] I use experience in the modeling process for known points in time. I rely upon techniques described previously to determine the projected ultimate losses for the final point in time.

Table 4
Workers Compensation—High Deductibles
Age to Age Loss and ALAE Development Factors

Accident	Month:Month				
Year	12:24	24:36	36:48	48:60	60:72
Panel A: Unlimited					
1989	1.7063	1.1756	1.0929	1.0359	1.0273
1990	1.8219	1.1574	1.0744	1.0387	-
1991	1.7724	1.1506	1.0737	-	-
1992	1.6912	1.1398	-	-	-
1993	1.6044	-	-	-	-
Average	1.7192	1.1559	1.0803	1.0373	1.0273
Panel B: $250,000 Deductible					
1989	1.7077	1.1598	1.0657	1.0221	1.0120
1990	1.7755	1.1509	1.0550	1.0247	-
1991	1.7734	1.1461	1.0643	-	-
1992	1.6750	1.1363	-	-	-
1993	1.6229	-	-	-	-
Average	1.7109	1.1483	1.0617	1.0234	1.0120
Panel C: Excess of $250,000 Deductible					
1989	1.6646	1.6582	1.6742	1.1927	1.2011
1990	4.4890	1.3049	1.3151	1.2411	-
1991	1.7373	1.3115	1.3675	-	-
1992	2.2474	1.2291	-	-	-
1993	1.1684	-	-	-	-
Average	2.2613	1.3759	1.4523	1.2169	1.2011

Table 5
Workers Compensation—High Deductibles
Limited Severity Relativities ($250,000 Deductible)

Accident	Month:Month					
Year	12	24	36	48	60	72
Panel A: Relativities						
1989	0.9675	0.9683	0.9553	0.9315	0.9191	0.9053
1990	0.9829	0.9578	0.9524	0.9353	0.9227	-
1991	0.9723	0.9728	0.9690	0.9605	-	-
1992	0.9717	0.9623	0.9594	-	-	-
1993	0.9593	0.9704	-	-	-	-
Average	0.9707	0.9663	0.9590	0.9424	0.9209	0.9053

Accident	Month:Month				
Year	12:24	24:36	36:48	48:60	60:72
Panel B: Changes in Relativities					
1989	1.0008	0.9866	0.9751	0.9867	0.9850
1990	0.9745	0.9944	0.9820	0.9865	-
1991	1.0005	0.9961	0.9912	-	-
1992	0.9903	0.9970	-	-	-
1993	1.0116	-	-	-	-
Average	0.9955	0.9935	0.9828	0.9866	0.9850

I use a Weibull distribution to specify the workers compensation claim loss distribution. The Weibull distribution is commonly used for workers compensation claims because it gives a reasonable depiction of the loss distributions. Some of the properties of the Weibull distribution are given in Hogg and Klugman (1984, Appendix, page 231). The cumulative distribution function (cdf), probability distribution function (pdf), moments and the truncated mean are shown below:

$$
\begin{aligned}
F(x) &= 1 - e^{-(x/\beta)^\alpha}, \quad x > 0 \quad \text{(the cdf)} \\
f(x) &= \frac{\alpha x^{\alpha-1}}{\beta^\alpha} e^{-(x/\beta)^\alpha}, \quad x > 0 \quad \text{(the pdf)} \\
E[X^n] &= \beta^n \Gamma(1 + \frac{n}{\alpha}) \quad \text{for } n = 1, 2, \ldots \\
\text{LEV}[X; d] &= \int_0^d x f(x)\, dx + d \times [1 - F(x)]
\end{aligned}
$$

$$= \beta\Gamma(1+\frac{1}{\alpha})P(1+\frac{1}{\alpha};\left(\frac{d}{\beta}\right)^{\alpha}) + de^{-(d/\beta)^{\alpha}}, \quad d > 0$$

where $\alpha > 0$ is the shape parameter, and $\beta > 0$ is the scale parameter. In addition, for $a > 0$, $\Gamma(a)$ is the gamma function, and $P(a,x)$ is the incomplete gamma function, i.e.,

$$\Gamma(a) = \int_0^{\infty} t^{a-1}e^{-t}\,dt = (a-1)\Gamma(a-1)$$
$$P(a,x) = \int_0^{x} t^{a-1}e^{-t}\,dt.$$

For accurate approximations to $\Gamma(a)$ and $P(a,x)$, see Abramowitz and Stegun (1965, Chapter 6).

The most difficult aspect of working with distributional models is estimating the unknown parameters involved. There are various statistical approaches that can be used, including the method of moments and the maximum likelihood estimation. I use an alternative approach called the minimum chi-square, which is based on the minimization of the sum of the squared deviations between actual and expected severity relativities around the $250,000 deductible size.

$$\chi^2 = \min_{\alpha,\beta}\left[\sum_i \frac{(\text{Actual}_i - \text{Expected}_i)^2}{\text{Expected}_i}\right].$$

The estimates of α and β are the parameter values that actually minimize chi-square (χ^2). I use a solver routine incorporated in Microsoft Excel's spreadsheet application, which allows me to constrain the optimization routine in such a fashion that the parameters generated produced the actual unlimited severity at the specified maturity.

Table 6 shows an example of results used to determine age to ultimate loss development factors by limit from 48 months to ultimate. In the table the limited and excess severities sum to the unlimited severity, as I base all severities upon total claim counts. I select 48 months to focus attention on changes in severity rather than changes in total claim counts assuming no IBNR count development after 36 months.

The following formulation shows how expected development at time t, ED_t can be partitioned about the deductible limit:

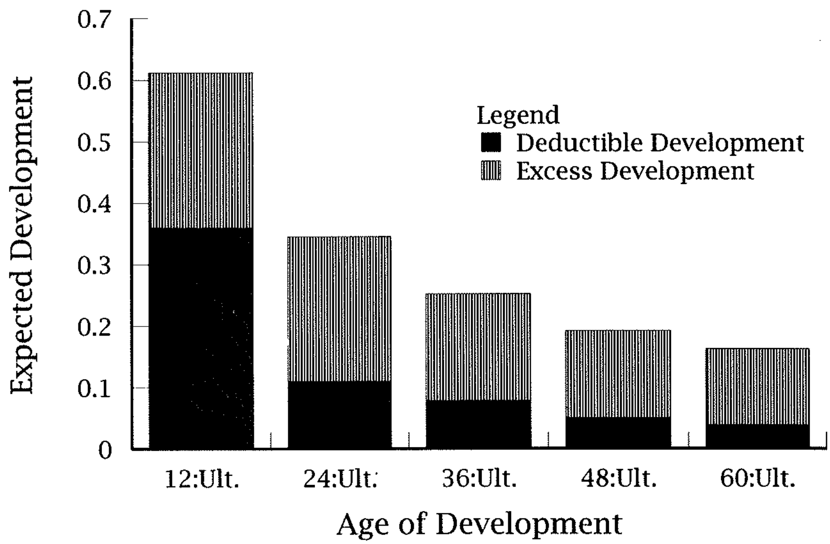
Table 6
Workers Compensation High Deductibles Actual Versus Fitted Limited/Excess Development Factors (at 48 Months) Using a Weibull Loss Distribution

Limit	Unlimited	$1,000,000	$750,000	$500,000	$250,000	$100,000	$50,000
Panel A: Ultimate							
Observed							
Limited Severity	6,846.4	6,159.2	5,980.4	5,714.4	5,094.8	3,939.6	3,036.5
Relativity	1.0000	0.8996	0.8735	0.8347	0.7442	0.5754	0.4435
Excess Severity	0.0	687.2	866.0	1,132.0	1,751.6	2,906.8	3,809.9
Fitted							
Limited Severity	6,846.4	6,295.2	6,106.5	5,778.7	5,064.4	3,926.7	3,043.8
Relativity	1.0000	0.9195	0.8919	0.8440	0.7397	0.5735	0.4446
Excess Severity	0.0	551.2	739.9	1,067.7	1,782.0	2,919.7	3,802.6
Weibull Parameters: Scale = 180.0, Shape = 0.2326, Mean = 6,846.4, Coefficient of Variation = 10.07							
Panel B: 48 Months							
Observed							
Limited Severity	5,530.2	5,346.6	5,288.5	5,182.3	4,824.0	3,807.5	2,937.1
Relativity	1.0000	0.9668	0.9563	0.9371	0.8723	0.6885	0.5311
Limited LDF	1.2380	1.1520	1.1308	1.1027	1.0561	1.0347	1.0338
Excess Severity	0.0	183.6	241.7	347.9	706.2	1,722.7	2,593.1
Excess LDF	-	3.7429	3.5830	3.2538	2.4803	1.6874	1.4692
Fitted							
Limited Severity	5,530.2	5,380.5	5,301.4	5,142.5	4,722.4	3,894.0	3,144.1
Relativity	1.0000	0.9729	0.9586	0.9299	0.8539	0.7041	0.5685
Limited LDF	1.2380	1.1700	1.1519	1.1237	1.0724	1.0084	0.9681
Excess Severity	0.0	149.7	228.8	387.7	807.8	1,636.2	2,386.1
Excess LDF	-	3.6820	3.2338	2.7539	2.2060	1.7844	1.5936
Weibull Parameters: Scale = 305.7, Shape = 0.2625, Mean = 5,530.2, Coefficient of Variation = 7.35							

$$
\begin{aligned}
\mathrm{ED}_t &= \frac{\mathrm{LDF}_t - 1}{\mathrm{LDF}_t} \\
&= \frac{R_t^L \times \mathrm{LDF}_t^L + (1 - R_t^L) \times \mathrm{XSLDF}_t^L - 1}{\mathrm{LDF}_t} \\
&= \frac{R_t^L \times (\mathrm{LDF}_t^L - 1)}{\mathrm{LDF}_t} + \frac{(1 - R_t^L) \times (\mathrm{XSLDF}_t^L - 1)}{\mathrm{LDF}_t}. \qquad (11)
\end{aligned}
$$

The first part of equation (11) represents the portion of expected development below the deductible limit (i.e., deductible development), while the second part of equation (11) represents the portion of expected development above the deductible limit (i.e., excess development). Figure 5 shows partitioned development above (excess) and below (deductible) a $250,000 deductible limit based upon the Weibull loss distribution model. Excess development represents the majority of development with increasing age.

Figure 5
Workers Compensation—High Deductibles
Partitioned Development Above & Below $250,000 Deductible Limit



5 Other Elements

Several other elements associated with high deductible plans call for further discussion: aggregate limits, service revenue, and allocated claim expense. Determining sound estimates for these items is complex. In the following discussion I recommend using the compound Poisson model of collective risk to estimate losses excess of aggregate limits. I also suggest an alternative procedure using the NCCI Table M, if collective risk modeling is impractical. The asset for service revenue, although not as difficult to determine, depends upon prior estimates of losses for deductible/aggregate limits. Treating allocated claim expense in a similar fashion to loss simplifies the estimation process for that liability, but separating the two pieces is problematic.

5.1 Aggregate Limits

Some risks, beside choosing to limit their per occurrence losses, desire to limit all losses under a high deductible program. Insurers satisfy that need by providing aggregate loss limits. These limits are conceptually similar to maximum premium limitations used in retro rating plans.

Determining loss development factors for losses excess of aggregate limits is more complicated than for per occurrence limitations. The obligations from these aggregate limits are generally less significant than for per occurrence limits. Beside the additional complexity, the data needed to determine development factors for these limits are generally sparse and not likely to be credible. Outside of attempting to gather data for development factors, I suggest using collective risk modeling techniques to determine the needed loss development factors. Specifically, I use the Heckman and Meyers (1983) collective risk model[8] with a Poisson claim frequency distribution and a Weibull claim severity distribution to determine development factors for losses excess of aggregate limits. Table 7 shows selected development factors using the Weibull loss distribution.

The sampling of development factors shows that development for losses in excess of aggregate limits decreases more rapidly over time with smaller deductibles than larger ones. This is not unexpected, as most of the later development occurs in the layers of loss above the

[8]Although I do not incorporate any parameter risk in determining the development factors, the model does allow for that possibility. Interested readers should see a discussion by Meyers and Schenker (1983) describing how to incorporate parameter risk into the collective risk model.

deductible limits which is not covered by the aggregate. Also, not unexpectedly, development is more leveraged for larger aggregate limits. There is one additional point the reader should note in reviewing Table 7. Although I show hypothetical results for risks of \$1 million and \$2.5 million in expected loss size, the limited expectations are considerably smaller.

Given the volatility of losses excess of aggregate limits, I recommend using a Bornhuetter-Ferguson method to smooth indications of ultimate liability. The example in Table 8 uses expected aggregate loss charges as well as expected development factors based on the collective risk modeling approach. The final indication adds known losses excess of aggregate limits and IBNR based on the modeled development patterns.

An alternative approach for determining IBNR estimates for aggregate excess of loss coverage merits consideration. The procedure uses the NCCI methodology (1991) for determining insurance charges in retrospective rating plans. It is a more practical approach than collective risk modeling, but its accuracy hinges on determining the proper insurance charge table.

The IBNR is determined by subtracting insurance charges at different maturities. The process used to determine the ultimate insurance charge is the same as that used for pricing purposes. The key to the NCCI procedure is the adjustment of expected losses reflecting loss limits. This adjustment increases expected losses used in determining the appropriate insurance charge table using the following formula:

$$\text{Adjustment Factor} = \frac{1 + 0.8\chi}{1 - \chi}.$$

The reason for increasing expected losses for the use of a per occurrence limit is to use a less dispersed loss ratio distribution and, consequently, a smaller insurance charge. Although this adjustment for a loss limit moves the selection of an insurance charge table in the right direction, we are not sure if the move has been made in an appropriate manner. Additionally, the procedure generates smaller insurance charges by using limited losses in the entry ratio calculation.

In order to calculate the insurance charge at earlier maturities I suggest determining the per occurrence charge used in the NCCI procedure by relating undeveloped, limited losses to ultimate, unlimited losses. For example, using the fitted results depicted in Table 6 for a \$250,000 deductible leads to a per occurrence charge of 31 percent (1 – 4722.4/6846.4) at 48 months.

Table 7
Workers Compensation High Deductibles
Development Factors for Losses Excess of Aggregate Limits
(Collective Risk Model Using Weibull Loss Distribution)

Aggregate Limit	Deductible	12 Months Excess Loss	12 Months LDF	48 Months Excess Loss	48 Months LDF	Ultimate Excess Loss
Panel A: Expected Unlimited Losses of $1,000,000						
$500,000	$100,000	9,253.6	13.024	114,646.0	1.051	120,523.3
	$250,000	22,882.5	12.007	228,070.7	1.205	274,761.6
	$500,000	28,653.6	13.255	289,389.2	1.312	379,794.3
$750,000	$100,000	155.1	136.451	18,005.9	1.175	21,163.6
	$250,000	1,844.9	63.845	84,475.1	1.394	117,788.5
	$500,000	4,257.2	49.763	138,526.3	1.529	211,851.8
$1,000,000	$100,000	0.8	2,242.750	1,274.7	1.408	1,794.2
	$250,000	94.5	418.531	23,343.1	1.694	39,551.2
	$500,000	494.5	213.275	57,471.2	1.835	105,464.6
Panel B: Expected Unlimited Losses of $2,500,000						
$1,000,000	$100,000	39,703.2	11.761	456,498.9	1.023	466,934.1
	$250,000	81,084.7	10.876	759,354.4	1.161	881,844.0
	$500,000	95,069.6	12.021	912,976.1	1.252	1,142,866.6
1,250,000	$100,000	3,829.0	64.779	236,271.2	1.050	248,037.5
	$250,000	17,740.7	36.191	522,364.3	1.229	642,046.5
	$500,000	26,520.1	33.986	674,759.3	1.336	901,315.4
1,500,000	$100,000	173.5	564.077	87,988.1	1.112	97,867.3
	$250,000	2,693.1	158.522	318,464.5	1.341	426,916.3
	$500,000	6,001.8	112.833	463,359.8	1.461	677,200.3

Table 8
Countrywide Insurance Enterprise
Workers Compensation—High Deductibles
Estimated Ultimate Aggregate Excess of Loss
(Utilizing Bornhuetter-Ferguson Methodology)

		Known Loss at 48 Mths		Aggregate Excess of Loss		
ACCT	DED	DED	EXAG	Expected	LDF	Indicated
Panel A: EUL = $1,000,000; AGL = $750,000						
A	100,000	581,252	-	21,164	1.175	3,152
B	250,000	703,027	-	117,789	1.394	33,292
C	500,000	764,493	14,493	211,852	1.529	87,789
Panel B: EUL = $2,500,000; AGL = $1,250,000						
X	100,000	1,453,169	203,169	248,038	1.050	214,980
Y	250,000	1,757,616	507,616	642,047	1.229	627,248
Z	500,000	1,911,285	661,285	901,315	1.336	887,963

EUL = Expected Unlimited Loss; AGL = Aggregate Limit; EXAG = Excess of Aggregate
ACCT = Account; DEDUCT = Deductible

In addition to reflecting the impact of the limit, this approach also captures the effects of development. Again, the issue is whether the adjustments for the limit and for development are appropriate.

Table 9 compares IBNR estimates determined using the NCCI Table M with estimates from the collective risk modeling approach depicted in Table 8. Table 10 contains further details for the IBNR estimates from the NCCI Table M procedure.

5.2 Service Revenue

A significant element that ought to be reflected on the asset side of the balance sheet is the revenue associated with servicing claims under a high deductible program. Service revenue is generated in an analogous fashion to the loss conversion factor in a retro rating plan. Generally, a factor is applied to deductible losses, limited by any applicable aggregate, to cover expenses that vary with these losses.

In practice, however, other elements are captured by the loss multiplier, reflecting the desire of the individual accounts to fund the cost of the program as losses emerge. The service revenue often is collected

Table 9
A Comparison of Aggregate Excess of Loss IBNR Estimates at 48 Months Collective Risk Model vs. NCCI Table M

Account	Deductible	Collective Risk Model	NCCI Table M
Panel A: EUL = $1,000,000; AGL = $750,000			
A	100,000	3,152	1,809
B	250,000	33,292	38,500
C	500,000	73,296	68,811
Panel B: EUL = $2,500,000; AGL = $1,250,000			
X	100,000	11,811	9,959
Y	250,000	119,633	103,000
Z	500,000	226,678	222,168

EUL = Expected Ultimate Loss; AGL = Aggregate Limit

as losses are paid, but it also may be gathered as a function of case-incurred losses.

I propose determining the asset in the following fashion:

- Determine ultimate deductible losses at the account level;
- Subtract ultimate losses excess of aggregate limits from ultimate deductible losses;
- Apply the selected loss multiplier to the difference determined in the second step to determine ultimate recoverables; and
- Determine the total asset by subtracting any known recoveries from the estimated ultimate recoverables and aggregate results for all accounts.

Table 11 shows an example of how in practice the asset for the service revenue may be determined.

Table 10
Determination of IBNR—Aggregate Excess of $1,250,000

		At 48 Months	Ultimate
(a)	Severity Deductible = L = $250,000	4722.4	5064.4
(b)	Frequency	365.2	365.2
(c)	Limited Loss: (a) × (b)	1,724,620.5	1,849,518.9
(d)	Entry Ratio: Aggregate	0.72	0.68
(e)	Loss Excess of Deductible: 1 - LEV(X;L) / E[X]	0.310	0.260
(f)	Adjustment for Limit: (1 + 0.8 × (e)) / (1 - (e))	1.810	1.633
(g)	Adjusted Limited Loss: EU × (f)	4,525,000	4,082,500
(h)	1994 Expected Loss Group	19	20
(i)	Insurance Charge Ratio	0.336	0.369
(j)	Insurance Charge Amount: (c) × (i)	579,472	682,472
(k)	IBNR - 682,472 - 579,472	103,000	

Risk Characteristics: Expected Unlimited Loss = $2,500,000; Severity = 6846.4; and Frequency = 365.2

Table 11
Countrywide Insurance Enterprise
Workers Compensation - High Deductibles
Estimated Ultimate Service Revenue

Account	Deductible	Excess of Aggregate	Net of Aggregate	Multiplier Revenue	Known Recoveries	Asset
Panel A: Expected Unlimited Loss - $2,500,000; Aggregate Limit - $1,250,000; Loss Multiplier - 10%						
X	1,465,376	214,980	1,250,396	125,040	96,960	28,080
Y	1,884,867	627,248	1,257,619	125,762	102,712	23,050
Z	2,147,711	887,963	1,259,748	125,975	106,912	19,063
Total	5,497,954	1,730,191	3,767,763	376,777	306,584	70,193

5.3 Allocated Claim Expense

There are two principal means of handling allocated claim expense under a high deductible program. Either the account manages this expense, or the expense is treated as loss and subjected to applicable limits. In the first instance development patterns reflecting loss only are appropriate for determining liabilities, while a combination of loss and expense is appropriate for the second case.

For this discussion I determine development factors by combining loss and expense components assuming expenses developed similar to losses. I apply the resulting development factors to experience arising from both types of plans, as most of the accounts we write choose to subject allocated claim expense to the deductible. Although different development patterns are likely for loss and expense versus loss only, the actuary needs to decide based upon the mix of plans whether using both development patterns is worth the effort.

A remaining issue is how best to split loss and allocated claim expense for financial reporting purposes. Although splitting them proportionately based upon their full coverage counterparts is expeditious, other more actuarially sound approaches, even if available, may not be cost justifiable.

6 Conclusion

This discussion suggests some possible approaches for estimating liabilities under a high deductible program. As with many actuarial procedures, much work and improvement still are needed. I hope my suggestions provoke further discussion about how to better estimate these liabilities.

Although the reader may have his or her own ideas on how to improve upon my suggestions, I think several of the following suggestions warrant further consideration:

- Obtain longer histories of experience under the program better reflecting risk characteristics;
- Derive (select) parameters (distributions) that provide better fits to the actual data;
- Determine better tail factors and/or parameters of the utilized loss distribution; and
- Develop more advanced approaches to index loss limits.

These are known issues for actuaries, who long have confronted either limited or excess loss development. More comprehensive data in a workers compensation program allows the application of more sophisticated loss distributional approaches which affords greater consistency to all of the pieces involved. To that end I hope this paper provides a few steps toward developing sounder actuarial techniques for analyzing workers compensation high deductible loss development.

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