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Abstract†

Though actuaries have developed several types of stochastic investment models for inflation, stock market returns, and interest rates, there are two commonly used in practice: autoregressive time series models with normally distributed errors, and autoregressive conditional heteroscedasticity (ARCH) models. ARCH models are particularly suited when there is heteroscedasticity in inflation and interest rate series. In such cases nonnormal residuals are found in the empirical data. This paper examines whether Australian univariate inflation and interest rate data are consistent with autoregressive time series and ARCH model assumptions.

Key words and phrases: *stochastic investment models, heteroscedasticity, unit roots, ARCH, inflation, interest rates*

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1 Introduction to ARCH Models

In recent years actuaries have developed and applied time series models of inflation, interest rates, and stock market returns to assist with pension and insurance financial management. Some of the earliest work in developing models for actuarial applications was performed by Wilkie (1986, refined in 1995). Carter (1991) develops an Australian version of the Wilkie model using traditional time series analysis of Australian time series data for inflation, equity markets, and interest rates. See Geoghegan *et al.*, (1992), Daykin and Hey (1989, 1990), and Boyle *et al.*, (1998, Chapter 9) for a discussion of these and other models and their actuarial applications.

The standard assumption in actuarial models is that the model errors are independent and identically distributed (i.i.d.) normal random variables. Inflation rates and interest rates are then modeled using *autoregressive time series.* A discrete time stochastic process $\{Y_t, t = 0, 1, \ldots, n, \ldots\}$, where Y_t is a real valued random variable at time t, is called an *autoregressive process* of order p, AR(p), if it can be represented as

$$Y_t = \mu + \sum_{k=1}^{p} \phi_1 (Y_{t-k} - \mu) + \epsilon_t \tag{1}$$

where $\mu = E[Y_t]$, p is a positive integer, and $\phi_1, \ldots, \phi_p$ are constants with $\phi_p \neq 0$. In addition, the ϵ_ts form a sequence of uncorrelated normal random variables with mean 0 and variance σ^2. The time series in equation (1) is stationary in the sense that it has a constant unconditional mean and variance. In practice the series used in actuarial applications, such as the inflation or interest rate, are assumed to be autoregressive and have constant unconditional means.

If the level of a series in equation (1) is not stationary, but the difference of the series (i.e., ΔY_t) is stationary, then the series is said to contain a unit root (or said to be integrated or order 1, or to be difference stationary). The existence of unit roots determines the nature of the trends in the series. If a series contains a unit root, then the trend in the series is stochastic and shocks to the series will be permanent. If the series does not contain a unit root, then the series is trend stationary. The trend in the series will be deterministic, and shocks to the series will be transitory.

When the i.i.d. error assumption is not practical, other models must be considered. One such model is the autoregressive conditional heteroscedasticity (ARCH) model. The ARCH model, introduced by Engle

(1982), allows for time-varying conditional variance by modeling the variance of the errors of a series, ν_t, as a function of past model errors, ϵ_t, using the equation:

$$\nu_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 \tag{2}$$

where q is the order of the ARCH process, or simply an ARCH(q) process. The errors of the series are obtained after fitting a mean equation to allow for mean reversion.

The GARCH model, introduced by Bollerslev (1986), allows the variance of the errors to depend on previous values of the variance as well as past errors using the equation:

$$\nu_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{q} \phi_j \nu_{t-j}$$

which is referred to as a GARCH(p, q) process. Many other volatility models have been proposed: the exponential GARCH model (Nelson, 1991) and the nonlinear asymmetric GARCH model (Engle and Ng, 1993).

The models used for scenario generation as described in the actuarial literature typically use ARCH models. For example, Mulvey (1996) describes the Towers Perrin model where inflation is modeled as an autoregressive process with ARCH errors. Sherris, Tedesco, and Zehnwirth (1996), Harris (1994, 1995), and others support the need to model heteroscedasticity in Australian inflation and interest rates.

This paper will consider using ARCH models for Australian time series data. Specifically, the models assume ARCH and normal distribution of errors using Australian inflation, stock market, and interest rate time series data. The paper does not examine assumptions of independence of errors or model selection, and models will need to satisfy wider criteria than are examined in this paper. Carter (1991) and Harris (1994, 1995) have considered some of these issues for Australian data.

2 Australian Time Series Data

The data used for the empirical analysis in this paper are taken from the *Reserve Bank of Australia Bulletin* database. The study uses quarterly data. This is the highest frequency for which the inflation series is available in Australia. The Australian Consumer Price Index is determined quarterly—a frequency suitable for many actuarial applications.

Different series are available over different time periods. The longest time period for which data are available on a quarterly basis for all of the financial and economic series is from September 1969.[1] The series considered are:

- The Consumer Price Index-All Groups (CPI);
- The All Ordinaries Share Price Index (SPI);
- Share dividend yields;
- The 90 day bank bill yields;
- The two year Treasury bond yields;
- The five year Treasury bond yields; and
- The ten year Treasury bond yields.

An index of dividends is constructed from the dividend yield and the Share Price Index series. Logarithms and differences of the logarithms are used in the analysis of the CPI, SPI, and dividends. The difference in the logarithms of the level of a series is the continuously compounded equivalent growth rate of the series.

Figures 1 through 8 provide time series plots of the series. An examination of the plots for the CPI, SPI and the Dividend Index series shows exponential growth. The plot of the logarithms of these series suggests that the series could be fluctuations around a linear trend in the logarithms. Such a series is referred to as *trend stationary.* The plot of the differences of the logarithms of these series appears to indicate a nonconstant variance or heterogeneity. Table 1 provides summary statistics for all of the series.

The interest rate series all show a changing level as interest rates rose during the 1970s and 1980s. Models of interest rates that incorporate mean-reversion, i.e., models that assume that the level of interest rates has constant unconditional mean and variance, are often used. This is not intuitive from our examination of the time series plots of the interest rates. The differences in the levels of the interest rates seem to fluctuate around a constant value, but the series appear to be heteroscedastic.

[1] Individual series are available for differing time periods. For example, Phillips (1994) fits Bayes models to Australian macroeconomic time series. The data used are similar to those used here but cover different time periods.

Figure 1
Consumer Price Index

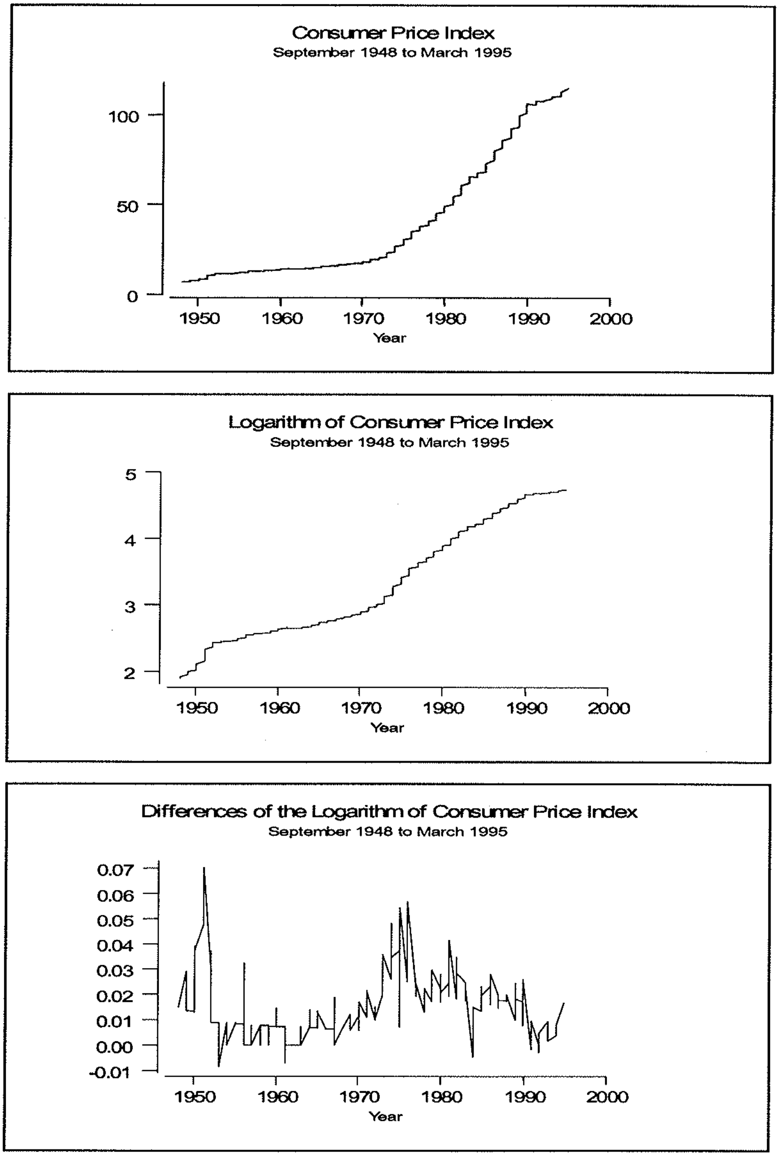


Figure 2
All Ordinaries Share Price Index

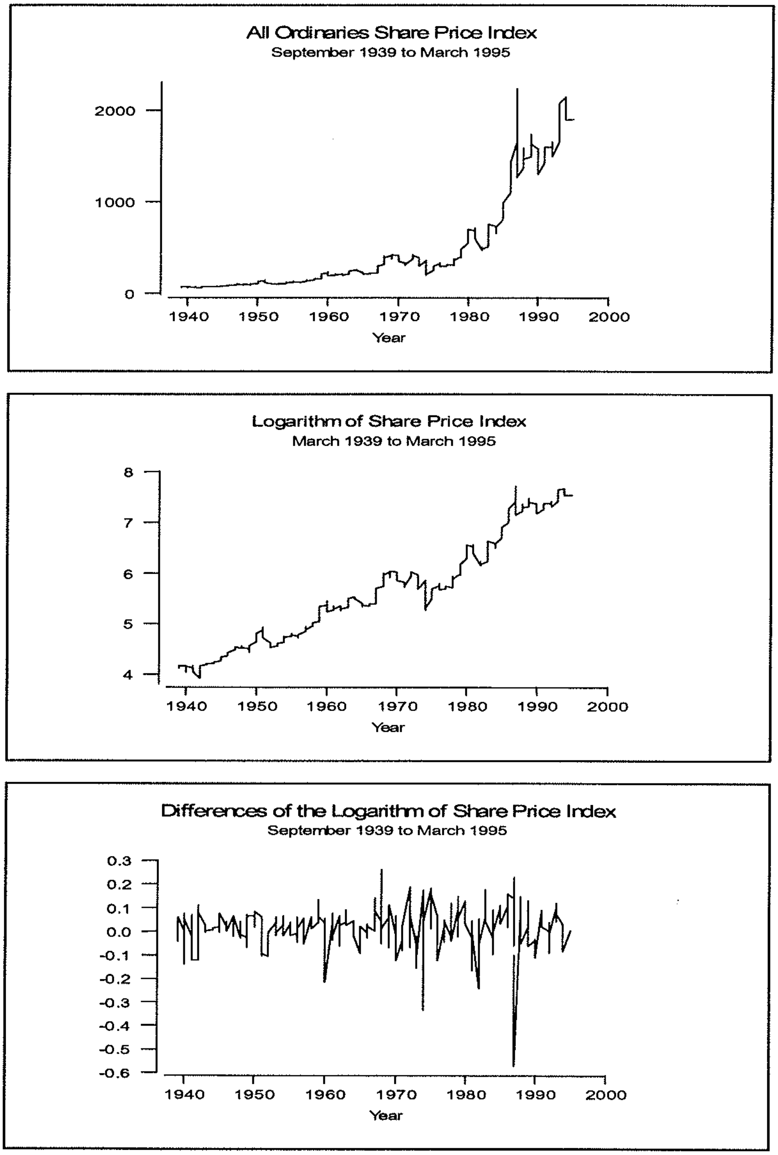


Figure 3
Share Price Dividend Index

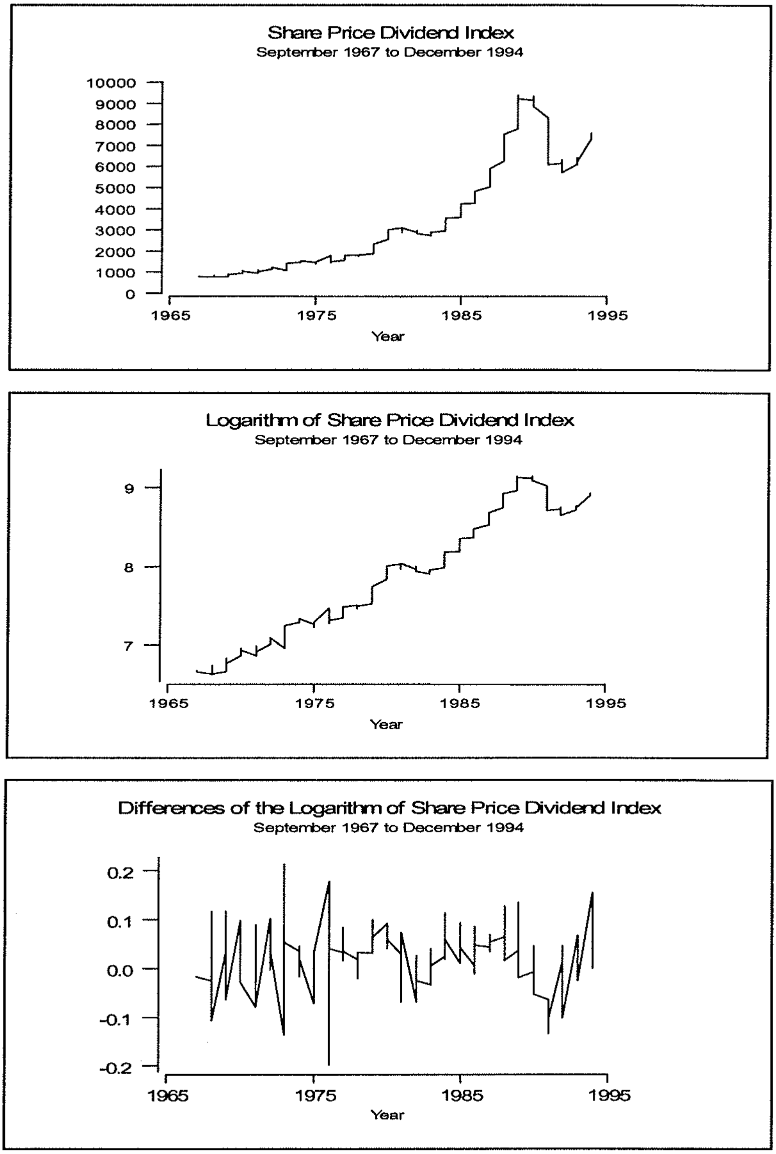
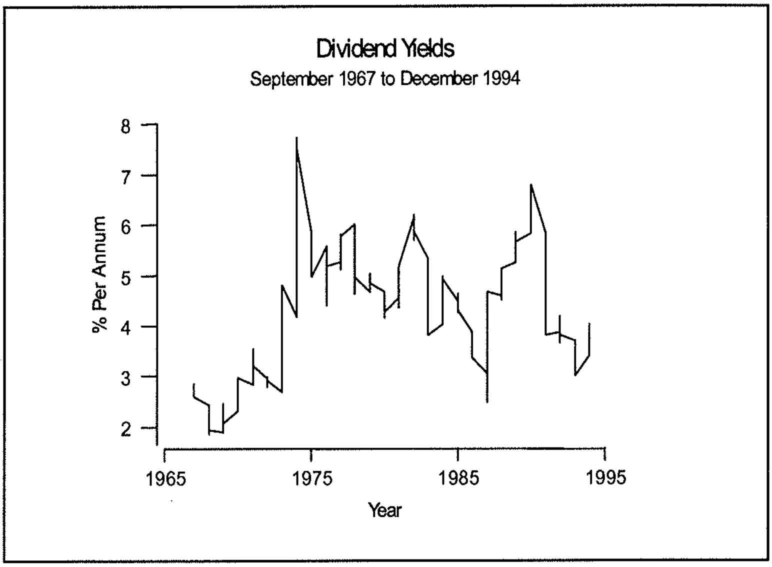


Figure 4
Dividend Yields



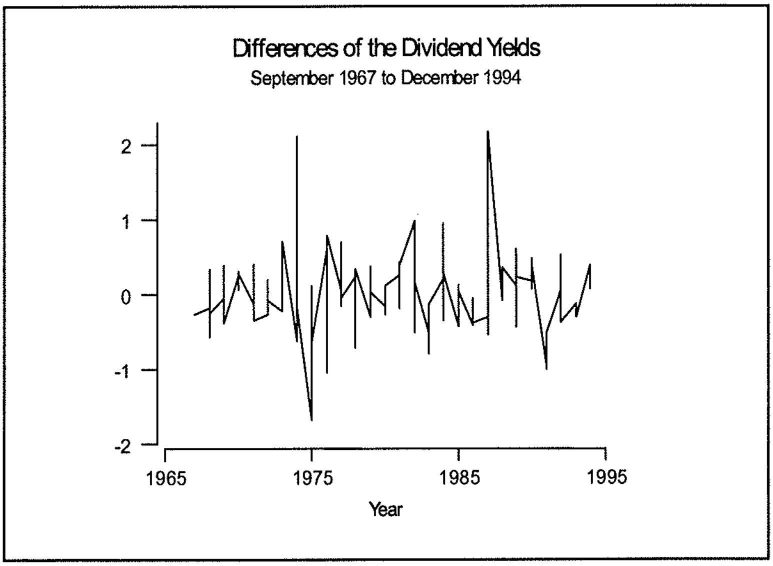
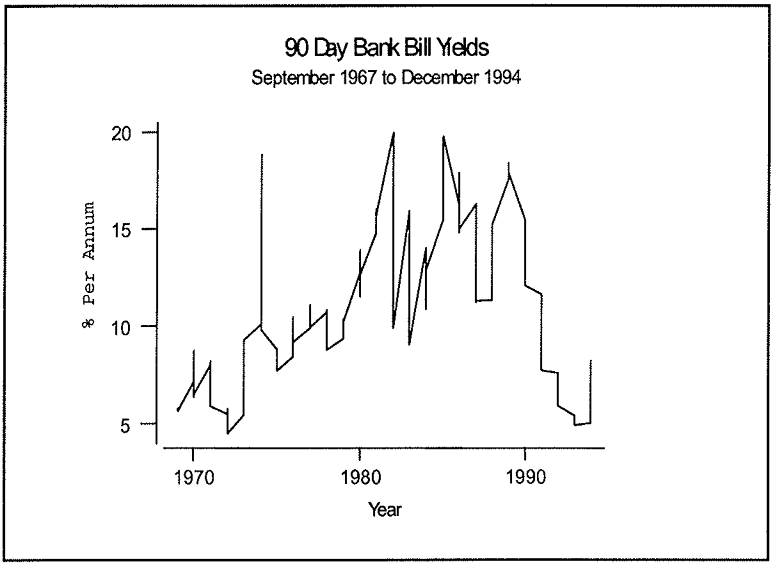


Figure 5
90 Day Bank Bill Yields



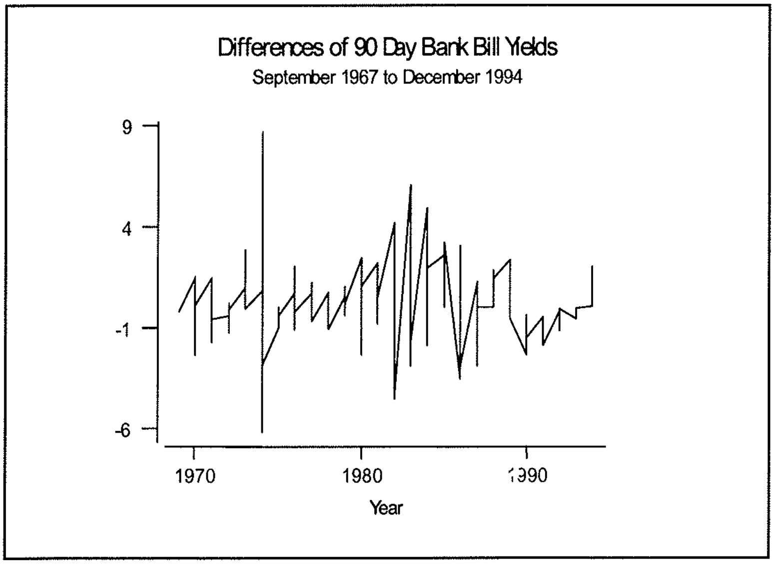
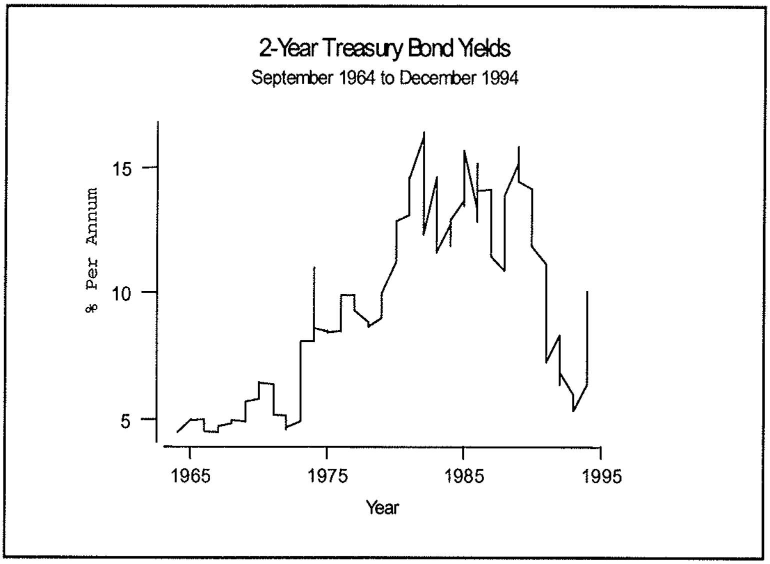


Figure 6
Two Year Treasury Bond Yields



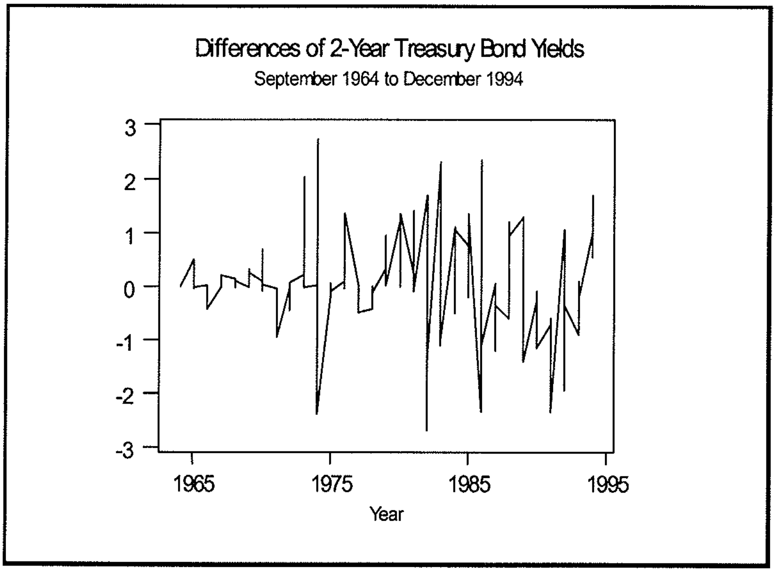
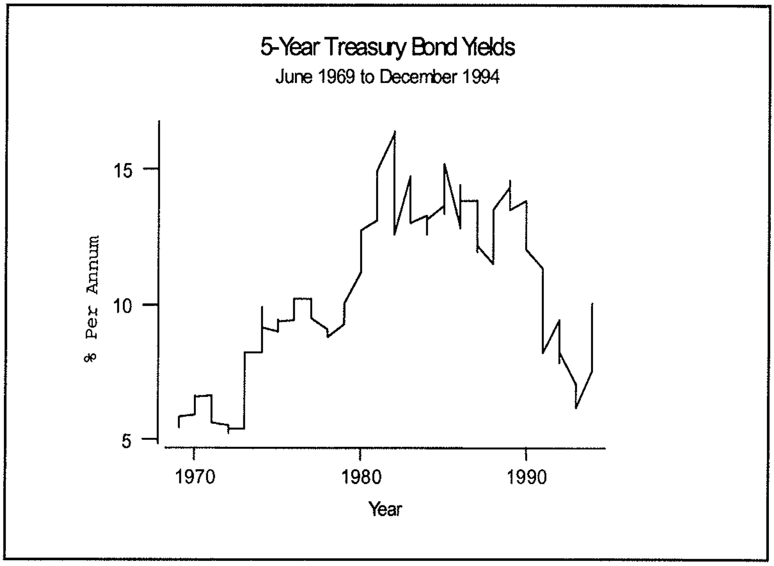


Figure 7
Five Year Treasury Bond Yields



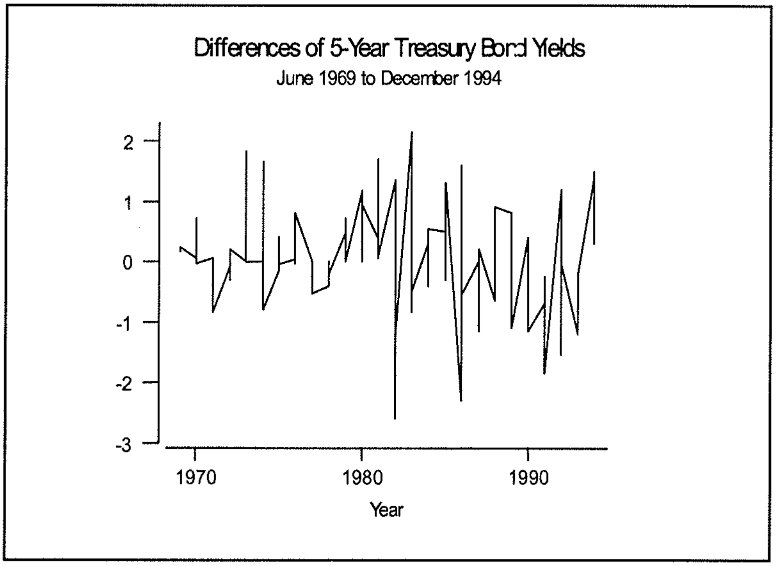
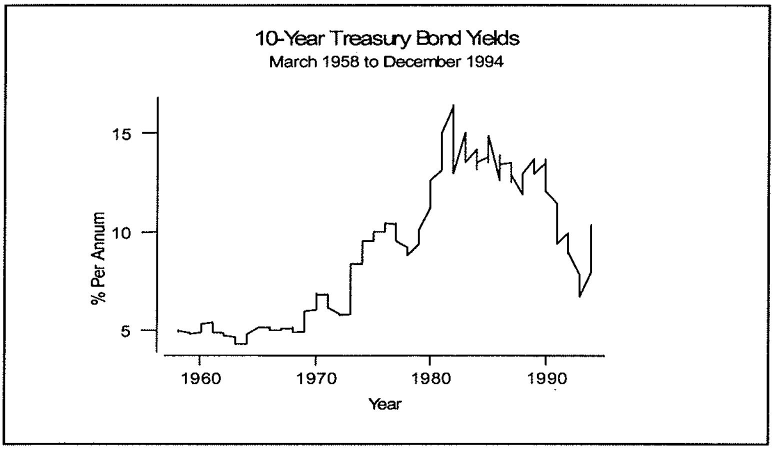


Figure 8
Ten Year Treasury Bond Yields



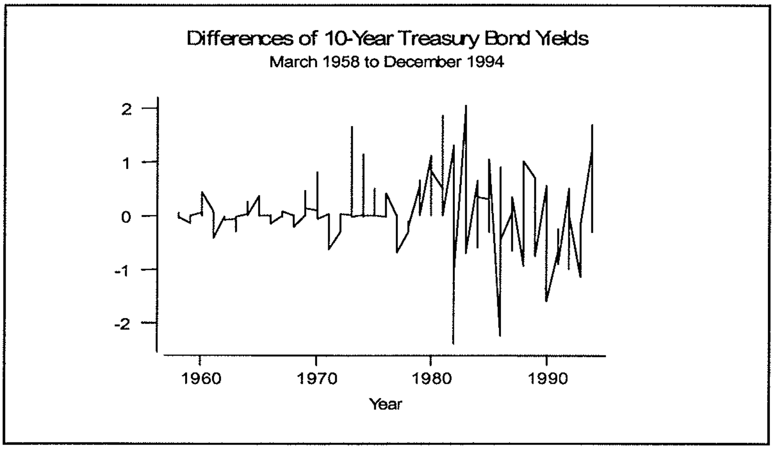


Table 1
Summary Statistics of All Series
Quarterly Data from September 1969 to December 1994

	Mean	STDEV	Max	Min	Median	Mode	SKEW	KURT
CPI	60.074	32.462	112.80	17.000	55.300	107.60	0.2375	-1.3631
C	3.9220	0.62386	4.7256	2.8332	4.0128	4.6784	-0.3408	-1.2288
SPI	865.01	595.05	2238.7	194.30	603.40	2238.7	0.6797	-1.0008
S	6.5177	0.71137	7.7137	5.2694	6.4026	7.7137	0.1667	-1.4523
DVY	4.4506	1.1496	7.7300	2.0700	4.5000	5.8500	0.2237	-0.1128
DVS	3741.5	2584.0	9398.3	861.74	2877.4	9398.3	0.7365	-0.7603
BB90	10.909	4.1029	19.950	4.4500	10.350	15.450	0.3310	-0.8313
TB2	10.185	3.2623	16.400	4.6000	9.9400	15.150	0.0137	-1.1443
TB5	10.465	2.9845	16.400	5.2000	10.030	13.850	-0.0775	-1.0843
TB10	10.648	2.8299	16.400	5.7500	10.180	9.5000	-0.0997	-1.0091

Notes: Quarterly data for all series were available from September 1969 to December 1994. The data are *CPI* = Consumer Price Index; *C* = ln(*CPI*); *SPI* = Share Price Index; *S* = ln(*SPI*); *DVY* = Share dividend yields; *DVS* = Share dividends series; *BB90* = 90 day bank bills yields; *TB2* = Two year treasury bond yields; *TB5* = Five year treasury bond yields; *TB10* = Ten year treasury bond yields. In addition, STDEV = Standard Deviation; SKEW = Coefficient of skewness; and KURT = Coefficient of excess kurtosis.

The following notation is used throughout the rest of the paper:

$$
\begin{aligned}
t &= \text{Number of quarters since January 1, 1969, } t = 1, 2, \ldots; \\
\epsilon_t &= \text{The error term at } t \text{, for } t = 1, 2, \ldots; \\
CPI_t &= \text{Consumer Price Index for quarter } t; \\
C_t &= \ln(CPI_t); \\
\Delta f_t &= f_t - f_{t-1} \quad \text{for any function } f; \\
SPI_t &= \text{Share Price Index for quarter } t; \\
S_t &= \ln(SPI_t); \\
DVY_t &= \text{Dividend yield for quarter } t; \\
Y_t &= \ln(DVY_t); \\
DVI_t &= \text{Dividend index for the Australian data for quarter } t; \\
I_t &= \ln(DVI_t); \\
F_t &= \text{Force of interest for quarter } t.
\end{aligned}
$$

3 Analysis of the Australian Data

3.1 Inflation

Sherris, Tedesco, and Zehnwirth (1996) provide empirical evidence that the C_t series contains a unit root for Australian data. Although unit root tests can erroneously reject the hypothesis of a unit root in the presence of structural breaks[2] (Silvapulle, 1996) and are affected by additive outliers[3] (Shin, Sarkar, and Lee, 1996), this is not taken into account. Structural changes can lead to erroneous rejection of the hypothesis of a unit root.

An AR(1) model is fitted[4] (with a log-likelihood value of 331.778) to the CPI series to give

$$\Delta C_t = 0.0187 + 0.802(\Delta C_{t-1} - 0.0187) + 0.0090\epsilon_t. \tag{3}$$

This AR(1) model is examined first because it is used in actuarial applications with the assumption that the errors are normally distributed and with constant variance. Diagnostics for these model assumptions

[2] A structural break occurs in the series where there is a discontinuity in the mean or the trend.

[3] An additive outlier is a single observation which is not consistent with the other observations in the series usually indicated by a highly significant t-ratio.

[4] All equations were fitted with the SHAZAM (1993) econometrics package.

are given in Table 2. The ARCH test of Engle (1982), is based on a regression of ϵ_t^2 on ϵ_{t-1}^2 and is a test for nonlinear dependence in the residuals.

The ARCH test regresses the squared residuals from the AR(1) model on a constant and the lagged squared residuals. The number of observations times the R^2 of this regression ($N \times R^2$) has an asymptotic χ^2 distribution with 1 degree of freedom.

The Jarque-Bera test is based on the statistic

$$N \times [\frac{\gamma_1^2}{6} + \frac{\gamma_2^2}{24}]$$

where γ_1 is defined as the skewness and γ_2 is defined as the excess kurtosis. This statistic has a χ^2 distribution with 2 degrees of freedom for large N. Skewness and excess kurtosis are defined as:

$$\gamma_1 = \frac{m_3}{m_2^{3/2}} \quad \text{and} \quad \gamma_2 = \frac{m_4}{m_2^2} - 3$$

where m_k is the k-th sample central moment, i.e.,

$$m_k = \frac{1}{N}\sum_{t=1}^{N}(\epsilon_t - \bar{\epsilon}).$$

Table 2
Quarterly Inflation Rate Autoregressive Model
AR(1) Model for C_t

Log-Likelihood Function Value		331.778
ARCH Test	2.535	(χ^2, 1 df, - 5% critical value 3.841)
Skewness	0.7781	(std. dev. is 0.240)
Excess Kurtosis	3.1785	(std. dev. is 0.476)
Jarque-Bera Test	46.8732	(χ^2, 2 df, 5% critical value 5.991)

The residuals for equation (3) are leptokurtic.[5] The statistical evidence for ARCH in this data series over this time period is not strong, although Sherris, Tedesco, and Zehnwirth (1996) find that a GARCH(1, 1) model fits ΔC_t well for the period September 1948 to March 1995.

[5]A leptokurtic distribution is more peaked than the normal distribution and thus has fatter tails.

The inflation model described in Mulvey (1996) uses an ARCH model for volatility. An ARCH(1) model is fitted to the Australian quarterly CPI data to obtain

$$\Delta C_t = 0.0187 + 0.675(\Delta C_{t-1} - 0.0187) + \sigma_t \epsilon_t \qquad (4)$$
$$\sigma_t^2 = 0.00007 + 0.31\epsilon_{t-1}^2 \qquad (5)$$

with a log-likelihood value of 329.792. Diagnostics for ARCH and normal distribution of errors for this model are reported in Table 3.

Table 3
Quarterly Inflation Rate Autoregressive Model
AR(1) Model–ARCH(1) Model for C_t

Log-Likelihood Function Value		329.792
ARCH Test	0.294	(χ^2, 1 df)
Skewness	0.6550	(std. dev. is 0.240)
Excess Kurtosis	3.7087	(std. dev. is 0.476)
Jarque-Bera Test	57.6464	(χ^2, 2 df)

Although the model appears to capture ARCH in the volatility of the rate of inflation, the errors are still significantly nonnormal. The log-likelihood decreases. These results suggest that if an autoregressive model for the rate of inflation is used, the normality assumption for the errors will not be appropriate. An ARCH model with the assumption that errors are normally distributed is also not supported as an appropriate model for Australian inflation data. Because such an ARCH model is often used by actuaries in practice for inflation, some caution about the results from such a model is warranted.

3.2 Stock Market Series

The Wilkie (1986) approach to modeling stock returns uses a dividend yield and a dividend index. The model described in Mulvey (1996) divides stock returns into dividends and price appreciation. We consider models for price appreciation, dividend yields, and a dividend index for the Australian data. Sherris, Tedesco, and Zehnwirth (1996) present the results from unit root tests for the data considered here which indicate that the logarithm of the Australian Share Price Index, the logarithm of the dividends series, and dividend yields are difference stationary. An important issue in equity market data is the allowance for share market crashes. In this paper we consider them as additive outliers.

Growth in an equity index and dividends are the two components of the return from equities that require modeling for actuarial applications. In this section models for the Australian equity market index and for dividends on the index are considered.

3.3 Share Price Index

Because we are interested in using volatility models for stock market returns we consider the following model for the Share Price Index:

$$\Delta S_t = \mu_S + \epsilon_t \sqrt{\nu_t} \quad (6)$$

$$\nu_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 \quad (7)$$

where $\mu_S = E[\Delta S_t]$.

Table 4 reports the results from fitting this model with ARCH(1) volatility. Note the α_1 parameter for ARCH volatility is significant at the 5 percent significance level. Based on the tests on the residuals given in Table 4, however, the residuals do not appear to be from a normal distribution. We have not tested these residuals for independence. Thus, although scenarios generated from a model using ARCH errors appear to be supported by the historical data, we should not use such a model in practice with the normal distribution of errors.

Because the quarter December 1987 appears in the residuals as an outlier corresponding to a stock market crash, it is of interest to determine the impact that this observation has on the results. This particular quarter is modeled as an additive outlier using a dummy variable denoted by $D(4,87)$, i.e.,

$$D_t(4,87) = \begin{cases} 1 & t \text{ denotes the quarter is December 1987;} \\ 0 & \text{otherwise.} \end{cases}$$

The AR(1) model is modified as:

$$\Delta S_t = \mu_S + \beta D_t(4,87) + \epsilon_t. \quad (8)$$

Table 5 reports the results of fitting equation (8) assuming constant variance.

The ARCH test indicates that an ARCH model should be considered for the volatility even after adjusting for the market crash outlier. The model used is

$$\Delta S_t = \mu_S + \beta D_t(4,87) + \epsilon_t\sqrt{\nu_t} \tag{9}$$

with equation (7) representing the ARCH(1) component. Table 6 reports the results of fitting equation (9). The ARCH parameter is not significant, and the results do not support ARCH errors in SPI returns after adjusting for the market crash using an additive outlier.

Table 4
ΔS_t with ARCH Errors

Log-Likelihood Function Value	83.992	
Mean Equation	Constant	
Coefficient	0.01634	
t-ratio	1.720	
Variance Equation ARCH	α_0	α_1
Coefficient	0.00795	0.40661
t-ratio	4.921	1.985
Diagnostics of Errors		
ARCH Test	0.105	(χ^2, 1 df)
Skewness	-0.6818	(std. dev. is 0.240)
Excess Kurtosis	1.2789	(std. dev. is 0.476)
Jarque-Bera Test	13.2316	(χ^2, 2 df)

3.4 Dividend Yields

Preliminary analysis using the unit root tests indicate that the logarithms of the dividend yields are difference stationary, so we consider the model:

$$\Delta Y_t = \mu_Y + \beta D_t(4,87) + \epsilon_t\sqrt{\nu_t} \tag{10}$$

with the ARCH(1) component as in equation (7). This model is fitted, and the ARCH test gives a significant result. An ARCH model is fitted for ν_t, and the results for the variance equation are reported in Table 7. The model appears satisfactory from the point of view of ARCH errors.

Autoregressive models for dividend yields are used in scenario generation for actuarial modeling. With this in mind, the following AR(1) model is used:

$$\Delta Y_t = \mu_Y + \psi\Delta Y_{t-1} + \beta D_t(4,87) + \epsilon_t\sqrt{\nu_t} \tag{11}$$

Table 5
ΔS_t with Constant Mean and Variance and December 1987 Dummy Variable for Market Crash

Log-Likelihood Function Value	92.238	
Mean Equation	μ_S	β
Coefficient	0.02195	-0.59475
t-ratio	2.231	-6.035
Diagnostics of Errors		
ARCH Test	4.164	(χ^2, 1 df)
Skewness	-0.7679	(std. dev. is 0.240)
Excess Kurtosis	1.4587	(std. dev. is 0.476)
Jarque-Bera Test	17.0620	(χ^2, 2 df)

with the ARCH(1) component as in equation (7). Note that ψ is a constant.

We fit an AR(1) model to the dividend yield and check for outliers and ARCH. As would be expected given the share market index results, an outlier in the December 1987 quarter is detected corresponding to the share market crash. A dummy intervention variable is included for this observation and the residuals are tested for ARCH. The test is significant, so we fit an autoregressive model with ARCH errors as in equation (11). The residuals from this model do not reject the normal distribution assumption.

As noted earlier, in the actuarial literature models for scenario generation are based on autoregressive models for dividend yields and a normal distribution of errors. Such a model would have been considered satisfactory if no test for unit roots had been performed. Unit root tests, however suggest that the series is difference stationary and the difference stationary model would be preferred in this case.

3.5 Share Dividends

Sherris, Tedesco, and Zehnwirth (1996) construct a dividend index (DVI_t) for the Australian data. This index is defined as:

$$DVI_t = SPI_t \times DVY_t. \tag{12}$$

Modeling the rate of growth of dividends, $I_t = \ln(DVI_t)$, is difficult because dividends contain seasonal patterns. The difference series, ΔI_t,

Table 6
ΔS_t with Constant Mean, ARCH Errors, and December 1987 Dummy Variable for Market Crash

Log-Likelihood Function Value	93.7266	
Mean Equation	μ_S	β
Coefficient	0.02195	-0.59475
t-ratio	2.395	-4.641
Variance Equation ARCH	α_0	α_1
Coefficient	0.00752	0.20405
t-ratio	5.262	1.346
Diagnostics of Errors		
ARCH Test	0.456	(χ^2, 1 df)
Skewness	-0.4209	(std. dev. is 0.240)
Excess Kurtosis	0.2998	(std. dev. is 0.476)
Jarque-Bera Test	3.10087	(χ^2, 2 df)

is first modeled as an AR(1) time series. The residuals from this model indicate ARCH and an outlier in the series in the June quarter of 1976. The cause of this outlier is not known. A dummy variable, $D_t(2,76)$, is defined as:

$$D_t(2,76) = \begin{cases} 1 & t \text{ denotes the quarter is June 1976;} \\ 0 & \text{otherwise.} \end{cases}$$

After including a dummy variable for the outlier, the model becomes:

$$\Delta I_t = \mu_I + \psi \Delta I_{t-1} + \beta D_t(2,76) + \epsilon_t \sqrt{\nu_t} \tag{13}$$

with the ARCH(1) component as in equation (7). In this model of equation (13) the ARCH effect diminishes in significance. These results for the equity series are displayed in Table 8 support the point made in Chan and Wang (1996) that ARCH effects in share investment returns series are magnified by observations such as the crash that may be outliers.

3.6 Interest Rates

The interest rate series is transformed into a force of interest, F_t, using the transformations:

$$F_t = \begin{cases} \ln(1 + 90i_t/36500) & \text{for 90 day bank bill yields;} \\ \ln(1 + i_t/200) & \text{for 2, 5, and 10 year bond yields} \end{cases} \tag{14}$$

where i_t is the per annum percentage yield to maturity for the 90 day bank bill, two, five, and ten year bond for quarter t.

Sherris, Tedesco, and Zehnwirth (1996) present statistical support for these Australian bond yields containing a unit root and hence being difference stationary. In contrast, the assumption often used for scenario generation of future bill and bond yields in actuarial investment models is an autoregressive model. The standard unit root tests do not provide support for an autoregressive model for the Australian data series examined in this paper. These tests may have low power against close-to-stationary models.

For the interest rate series we consider models for the transformed interest rate series of the form

$$\Delta F_t = \mu_F + \epsilon_t \sqrt{\nu_t} \tag{15}$$

As before, models with constant volatility are considered initially.

For 90 day bank bills there is an outlier for the June 1994 quarter. This corresponds to a quarter when there was a significant tightening of monetary policy with the government raising short-term official interest rates dramatically. The series is adjusted for the effect of this outlier as follows:

$$F_t = \mu_I + \psi \Delta F_{t-1} + \beta D_t(2,94) + \epsilon_t \sqrt{\nu_t} \tag{16}$$

where

$$D_t(2,94) = \begin{cases} 1 & t \text{ denotes the quarter is June 1994;} \\ 0 & \text{otherwise.} \end{cases}$$

The adjusted series shows evidence of ARCH, so an ARCH model is fitted. Although this captures the ARCH effect, the normal distribution assumption for the residuals still is rejected.

Table 9 reports the fitted model and diagnostics for ARCH and normality for all of the bond series. For the two year bond yields there are no outliers and no evidence of ARCH, and the residuals appear to satisfy the normal distribution assumption. For the five year bond yields there are no outliers and no significant evidence of ARCH, but the residuals are negatively skewed and fat-tailed and reject the normal distribution assumption. In the case of the ten year bond yields there are no outliers

Table 7
ΔD_t (After Adjustment for Crash Dummy Variable)

Log-Likelihood Function Value	85.0154	
Variance Equation ARCH	α_0	α_1
Coefficient	0.00865	0.22236
t-ratio	5.006	1.448
Diagnostics of Errors		
ARCH Test	0.000	(χ^2, 1 df)
Skewness	0.170	(std. dev. is 0.240)
Excess Kurtosis	-0.0194	(std. dev. is 0.476)
Jarque-Bera Test	0.4984	(χ^2, 2 df)

and no evidence of ARCH. The residuals reject the normal distribution even more strongly than for the five year bond yields.

Autoregressive models are commonly used for interest rates in actuarial modeling. An AR(1) model of the form:

$$F_t = a_0 + a_1(F_{t-1} - a_0) + \epsilon_t \tag{17}$$

is fitted to the transformed yields for the Australian series. For the two year bond yields the parameter estimates (standard errors in parentheses) are $a_0 = 0.0534$ (0.0084) and $a_1 = 0.943$ (0.0301) with log-likelihood 399.3. This autoregressive model is used as the null hypothesis in a likelihood ratio test against the alternative of $a_1 = 1.0$ (a unit root), but the standard critical values reject the null hypothesis.

The AR(1) residuals reject the normal distribution assumption but show no significant statistical evidence of ARCH. This result holds for all of the autoregressive models fitted to the bond yield series. If an autoregressive model is used, then these results indicate that these interest rate models are not adequate and that adding ARCH volatility does not produce a better model.

4 Conclusions

The main aim of this paper has been to examine standard assumptions used in actuarial models for economic scenario generation. Quarterly Australian data for inflation, stock market, and interest rate series are examined to see if simple autoregressive models and ARCH models

Table 8

ΔI_t is AR(1) with ARCH errors and June, 1976 Dummy Variable

Log-Likelihood Function Value		142.19	
Mean Equation	μ_I	ψ	β
Coefficient	0.02419	-0.01256	-0.2206
t-ratio	4.175	-1.416	-2.584
Variance Equation ARCH		α_0	α_1
Coefficient		0.00299	0.17822
t-ratio		5.548	1.315
Diagnostics of Errors			
ARCH Test		0.001	(χ^2, 1 df)
Skewness		-0.1549	(std. dev. is 0.240)
Excess Kurtosis		0.7948	(std. dev. is 0.476)
Jarque-Bera Test		2.4374	(χ^2, 2 df)

of volatility with the assumption of a normal distribution of errors are reasonable. All of the analysis has been based on univariate series.

The results do not suggest that volatility in the series can be successfully modeled using an ARCH process. After allowing for additive outliers, some series do not show evidence of ARCH (for example, the rate of change of (transformed) bond yields). Equity returns show evidence of ARCH, even after adjusting for the effect of outliers such as the market crash. Outliers also increase the ARCH effect in the equity series.

The distribution assumed for errors in models used in practice must be considered carefully because the normal distribution assumption is not appropriate for errors based on the time series data for most of the models considered here. Alternative models and error distributions for economic scenario generation for actuarial applications require further investigation. It is not necessarily sufficient to use simple autoregressive models and a normal distribution for the errors. Even adding ARCH volatility in the hope that the normal distribution for errors will be adequate for modeling is not satisfactory.

This paper further demonstrates the need to model volatility in these series but indicates that the ARCH and normal distribution assumptions often used in practice and the actuarial literature are not supported by Australian historical data.