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Abstract§

A measure of level of post-retirement standard of living is the replacement ratio, i.e., percentage of final salary received as annual retirement income derived from savings. The replacement ratio depends on many factors including salary, salary increases, investment returns, and post-retirement mortality. Elementary life contingencies techniques are used to develop a replacement ratio formula and analyze its sensitivity to these factors.

Key words and phrases: *retirement planning, savings, interest rates, life annuity, social security*

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1 Introduction

Planning for retirement is increasingly important because individuals are living longer, demanding a better quality of life, and are facing a higher cost of living. Individuals must continually assess how much to save during their working years and how much retirement income will be sufficient. These types of decisions are generally influenced by several factors including the education, level of income, age at entry into the work force, expected retirement age, investment returns, survivorship trends, inflation, wage increases, and expense pattern.

In retirement planning, the individual first must consider the various sources of income he or she will have at retirement. In several countries, the primary sources of income are generally derived from social security benefits, employer-provided retirement benefits, and personal savings. As illustrated in Rejda (1988, Chapter 4, pages 64–65), retirement benefits from social security or other public pension schemes and personal savings and investments generally form a large proportion of post-retirement income in the U.S.

Once the various sources of retirement income have been identified, the individual must determine whether he or she will have sufficient income for retirement. As Cordell (1999) observes, post-retirement financial adequacy is often measured by the level of standard of living enjoyed by the individual during the period just prior to retirement. This level of standard of living is best quantified in terms of the annual salary of the individual just prior to retirement.

In this paper, we will use the replacement ratio, RR_x, to express the amount of annual retirement income as a fraction of the individual's final annual salary just prior to retirement. The amount of annual retirement income can be determined by converting savings into a life annuity and aggregating all annuity payments. In effect, we have

$$\mathrm{RR}_x = \frac{\text{Annual Retirement Income}}{\text{Final Annual Salary}}. \tag{1}$$

So, for example, consider a retiree age 65 with a final salary of \$100,000. The retiree has savings of \$50,000 in a bank account, \$575,000 in a defined contribution plan,[1] receives \$7,200 per year from a social insurance scheme, and receives \$12,000 per year from a defined benefit

[1] In a defined contribution plan, the employer and employee make specific contributions into a fund for the employee. The accumulated contributions at retirement are used to purchase an annuity pension benefits.

plan.[2] If it costs \$11.541 to purchase an annuity due paying \$1 per year for life, then the annual retirement income is:

$$\begin{aligned}\text{Annual Retirement Income} &= \$7,200 + \$12,000 + \frac{\$50,000 + \$575,000}{11.541} \\ &= \$73,354.75,\end{aligned}$$

yielding a replacement ratio of 73.355%.

There are several reasons to purchase a fixed life annuity at retirement. Alternatives to purchasing a life annuity, however, do exist. For example, the retiree can purchase an annuity certain for a period that is equal to the retiree's life expectancy at retirement. An obvious problem with this choice is that the retiree may outlive the annuity and be left with no source of income beyond that age. On the other hand, it can be argued that a larger amount of income can be derived with an annuity-certain if the retired individual does not live beyond the life expectancy at retirement. With a life annuity, the risk resulting from survivorship beyond the life expectancy at age of retirement is transferred to the company issuing the life annuity. Furthermore, any investment risk also is borne by the issuing company. With a fixed life annuity, the interest rate generally is guaranteed from issue by the company. From the perspective of the retired individual, there is no uncertainty that may arise from interest rate changes that may affect his or her annuity income. See Borch (1984) for more on these issues.

Inflation is another consideration. In the annuity market, it is possible to purchase an annuity contract that provides some floor of protection against inflation. In some countries, the law may require that income arising from disbursements of retirement benefits come in the form of a life annuity (McGill 1984, Chapter 6, pages 125–129).

We will now develop replacement ratio formulas and explore the various factors that can affect the replacement ratio at retirement. The results are general and intuitively appealing. They are not country-specific as we take into account that the various sources of income that are common to most countries (although the proportions derived from the various sources may differ significantly). We do not recommend the level of replacement ratio that is adequate for an individual's retirement. Our hope is to provide a tool that can be used to assist in retirement planning.

[2] In a defined benefit plan the employer promises to pay an annual pension benefit to the employee for life starting at retirement. The size of the benefit is determined by a specified mathematical formula.

The paper is organized as follows. In Section 2 we provide a simple model for calculating replacement ratios where the sole source of retirement income is derived from personal savings. Section 3 contains an analysis of the various partial derivatives of the replacement ratio. Section 4 gives several modifications of the simple model. In section 5, the case of Singapore is considered. The paper concludes in Section 6.

The following standard actuarial notation are used; see, for example, Kellison (1991). For $j \geq 0$ and $n = 1, 2, \ldots,$

$$\ddot{s}_{\overline{n}|j} = \sum_{t=0}^{n-1} (1+j)^{n-t};$$

$$s_{\overline{n}|j} = \sum_{t=1}^{n} (1+j)^{n-t};$$

$$\ddot{a}_{\overline{n}|j} = \sum_{t=0}^{n-1} (1+j)^{-t};$$

$$a_{\overline{n}|j} = \sum_{t=1}^{n} (1+j)^{-t};$$

$$(Ia)_{\overline{n}|j} = \sum_{t=1}^{n} t(1+j)^{-t};$$

$$(D\ddot{a})_{\overline{n}|j} = \sum_{t=0}^{n-1} (n-t)(1+j)^{-t};$$

$$^{k}\ddot{a}_r = \sum_{t=0}^{\infty} (1+k)^{-t}\,{}_tp_r;$$

where $^{k}\ddot{a}_r$ is the present value of a life annuity due paying 1 annually beginning age r and discounted at the effective rate of interest k, and ${}_tp_r$ is the probability that a person age r survives to age $r+t$, and

$$^{k}(Ia)_r = \sum_{t=1}^{\infty} t(1+k)^{-t}\,{}_tp_r.$$

2 Calculating Replacement Ratios: A Simple Model

Consider an individual currently age x whose sole source of retirement income is derived from personal savings and saves a percentage s of annual salary for retirement. We assume that the person will retire at the normal retirement age r. Let $\mathrm{AS}(x)$ be this individual's actual annual salary at age x and assume that salaries increase at the annual rate of w. Salaries are assumed to be paid at the start of the year. So the expected annual salary from age $x+t$ to $x+t+1$ for a person currently age x is paid at age $x+t$ and is given by

$$\mathrm{ES}_x\,(x+t) = \mathrm{AS}\,(x)\,(1+w)^t\,. \tag{2}$$

The individual will save the amount of $s\,\mathrm{ES}(x+t)$ at age $x+t$. If savings accumulate at the annual effective interest rate of i, then the portion of the retirement income derived from savings at age $x+t$ will be

$$Z\,(x+t,r) = s\,\mathrm{AS}\,(x)\,(1+w)^t\,(1+i)^{r-x-t}\,. \tag{3}$$

Let FS_x denote the accumulated future savings resulting from the annual savings made from age x to retirement, then

$$\begin{aligned} \mathrm{FS}_x &= \sum_{t=0}^{r-x-1} Z\,(x+t,r) \\ &= s\,\mathrm{AS}\,(x)\,G\,(i,w)\,, \end{aligned} \tag{4}$$

where

$$G\,(i,w) = (1+i)^{r-x} \sum_{t=0}^{r-x-1} \left(\frac{1+w}{1+i}\right)^t.$$

It is straightforward to show that:

$$G\,(i,w) = \begin{cases} (1+w)^{r-x}\,\ddot{s}_{\overline{r-x}|\,j_1} & \text{if } i > w \\ (1+i)^{r-x}\,(r-x) & \text{if } i = w \\ \frac{(1+i)^{r-x}}{(1+j_2)}\,\ddot{s}_{\overline{r-x}|\,j_2} & \text{if } i < w \end{cases} \tag{5}$$

where

$$\begin{aligned} j_1 &= \left(\frac{1+i}{1+w}\right) - 1, \\ j_2 &= \left(\frac{1+w}{1+i}\right) - 1. \end{aligned}$$

Suppose that at age x, the individual has saved accumulated past of PS_x. At retirement, the individual can expect to have saved a total of

$$\mathrm{TS}_x(r) = \mathrm{PS}_x(1+i)^{r-x} + \mathrm{FS}_x. \tag{6}$$

The amount TS_x will then be used at age r to purchase a retirement life annuity with level annual payments of $B_x(r)$ given by

$$B_x(r) = \frac{\mathrm{TS}_x(r)}{{}^k\ddot{a}_r} \tag{7}$$

where ${}^k\ddot{a}_r$ is the actuarial present value of a life annuity due.

The replacement ratio at normal retirement age r for a person currently age x, $\mathrm{RR}_x(r)$, is given by

$$\mathrm{RR}_x(r) = \frac{B_x(r)}{\mathrm{ES}_x(r)}. \tag{8}$$

Note that

$$\begin{aligned}
\mathrm{RR}_x(r) &= \frac{\mathrm{PS}_x(1+i)^{r-x} + \mathrm{FS}_x}{\mathrm{AS}(x)(1+w)^{r-x}\,{}^k\ddot{a}_r} \\
&= \frac{\mathrm{PS}_x[(1+i)/(1+w)]^{r-x}}{\mathrm{AS}(x)\,{}^k\ddot{a}_r} + \frac{sG(i,w)(1+w)^{-(r-x)}}{{}^k\ddot{a}_r} \\
&= \mathrm{ps}_{x,r} + \mathrm{fs}_{x,r}
\end{aligned} \tag{9}$$

where

$$\mathrm{ps}_{x,r} = \left[\frac{1+i}{1+w}\right]^{r-x} \frac{c_x}{{}^k\ddot{a}_r} \tag{10}$$

and

$$\mathrm{fs}_{x,r} = \frac{s\,G(i,w)}{(1+w)^{r-x}\,{}^k\ddot{a}_r}. \tag{11}$$

The term $\mathrm{ps}_{x,r}$ can be intuitively interpreted as the portion of replacement ratio contributed by savings already made at age x accumulated to retirement, and the term $\mathrm{fs}_{x,r}$ can be intuitively interpreted as that portion contributed by savings from salary beginning from age x until retirement. Note that

$$c_x = \frac{\mathrm{PS}_x}{\mathrm{AS}(x)}$$

from equation (10) represents the proportion of past savings expressed as a percentage of salary at age x.

Some interesting results can be derived from $y(r)$

$$y(r) = \frac{\mathrm{FS}_x}{\mathrm{PS}_x (1+i)^{r-x}},$$

which is the ratio of future expected savings to past savings. If $i > w$, then

$$y(r) = \frac{s\,\mathrm{AS}(x)}{\mathrm{PS}_x}\left(\frac{1+w}{1+i}\right)^{r-x} \ddot{s}_{\overline{r-x}|\,j_1} = \frac{s}{c_x}\ddot{a}_{\overline{r-x}|\,j_1}$$

and therefore

$$\frac{\partial y(r)}{\partial r} = \left(\frac{s}{c_x}\right)\left(\frac{\delta_1}{d_1}\right)\left(\frac{1}{1+j_1}\right)^{r-x} \le \frac{s}{c_x},$$

which gives an upper bound on the change y for increases in the normal retirement age. Here, $d_1 = j_1/(1+j_1)$ and $\delta_1 = \ln(1+j_1)$ are the usual discount and force of interest corresponding to interest rate j_1. If $i = w$, it is straightforward to show that

$$y(r) = \frac{s}{c_x}(r-x)$$

so that we have

$$\frac{\partial y(r)}{\partial r} = \frac{s}{c_x}.$$

If $i < w$, then

$$y(r) = \frac{s\,\mathrm{AS}(x)}{\mathrm{PS}_x}\left(\frac{\ddot{s}_{\overline{r-x}|\,j_2}}{1+j_2}\right) = \frac{s}{c_x} s_{\overline{r-x}|\,j_2}$$

and therefore

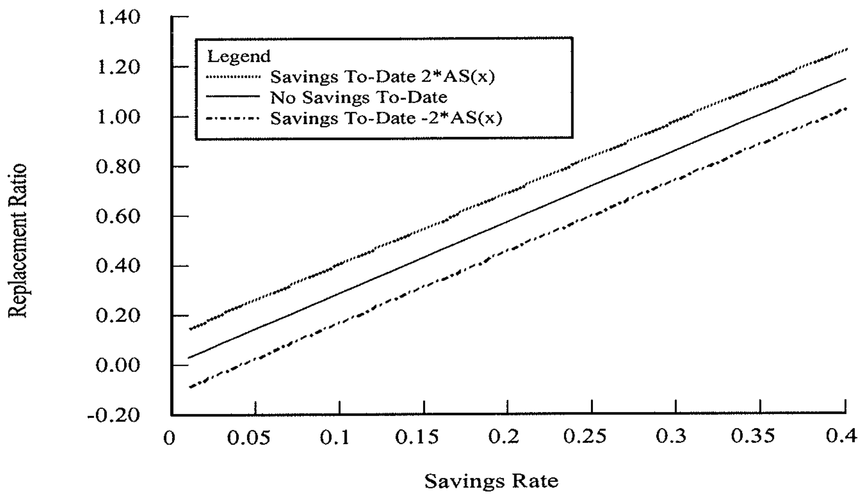
$$\frac{\partial y(r)}{\partial r} = \frac{s}{c_x}\frac{\delta_2}{j_2}(1+j_2)^{r-x} \geq \frac{s}{c_x},$$

which gives a lower bound on the change of the ratio for increases in the normal retirement age. To interpret these results, when the increase in the wages is greater than the interest earned on savings, then salary increases will raise FS_x relative to PS_x. For the other cases, a similar interpretation will hold.

For purposes of illustration, Figure 1 displays the relationship between the savings rate and the replacement ratio. In Figure 1, we assume the following: $x = 25$, $r = 65$, $i = 3$ percent, $w = 4$ percent, and a life annuity factor of $\ddot{a}_{65} = 11.541$ based on the United Kingdom's A1967-70 Mortality Table (ultimate). Savings to-date vary between -200 percent (a negative savings represents a net debt) and +200 percent of current annual salary. Notice the relationship between savings rate and replacement ratio is linear, with a greater intercept for larger savings-to-date. The linear relationship is positively sloping, which demonstrates the fact that increasing savings rate tends to increase the income replacement ratio at retirement. If the assumptions hold, a 25 year old individual with zero savings to-date must save about 35 percent of his or her annual salary to achieve a 100 percent replacement ratio at retirement at age 65.

Figure 2 displays the relationship of the age at retirement and the replacement ratio assuming $x = 25$, $i = 3$ percent, $w = 4$ percent, $s = 20$ percent, and life annuity factors based on the A1967-70 Mortality Table (ultimate). Savings to-date vary between -200 percent and +200 percent of current annual salary. Unlike Figure 1, Figure 2 displays a nonlinear relationship between retirement age and replacement ratio. It demonstrates that deferring the retirement, a choice that many can make today because of increasing life expectancy, will help increase the replacement ratio at retirement. According to the figure, a 25 year old individual with zero savings who intends to save 20 percent of salary annually can achieve a 100 percent replacement ratio when retirement is made at a little below age 75. Furthermore, for the same individual, if retirement were chosen at age 65, the replacement ratio will be below 60 percent.

Figure 1
Replacement Ratio $\mathrm{RR}_x(65)$ and Savings Rate s



3 Sensitivity of the Replacement Ratio

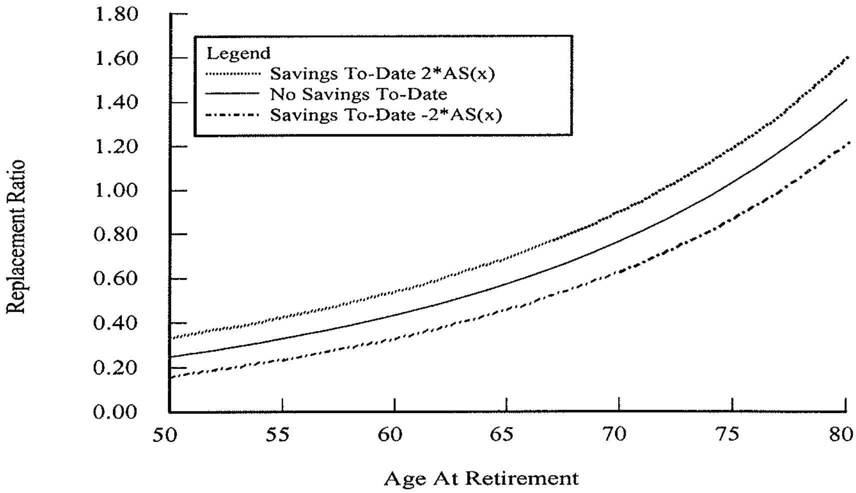
We now investigate the sensitivity of the RR_x to r, i, w, and x, respectively. To illustrate, the replacement ratio depends on the age at retirement r. Intuitively, for an individual who decides to retire at a later age, his or her replacement ratio is expected to increase because he or she will have a longer employment period which allows him or her to save more wealth for retirement. Furthermore, a later retirement age allows for a smaller life annuity factor ${}^{k}\ddot{a}_r$ (which decreases as r increases) and, as a result, a larger replacement ratio because the annuity is spread over a shorter expected retirement period.

3.1 The Effect of r on RR_x

We now show that, under certain conditions $\partial\,\mathrm{RR}_x(r)/\partial r \geq 0$. Three different cases are considered.

Case 1: $i > w$. If the interest rate is larger than the rate of wage increase, the replacement ratio in equation (9) becomes

Figure 2
Replacement Ratio $\mathrm{RR}_x(r)$ and Age at Retirement r



$$\mathrm{RR}_x(r) = \frac{c_x (1+j_1)^{r-x} + s\,\ddot{s}_{\overline{r-x}|\,j_1}}{{}^k\ddot{a}_r} = \frac{f}{g} \tag{12}$$

where

$$f = c_x (1+j_1)^{r-x} + s\,\ddot{s}_{\overline{r-x}|\,j_1}$$

and

$$g = {}^k\ddot{a}_r.$$

Note that

$$\frac{\partial g}{\partial r} = g' = \sum_{t=0}^{\infty} v^t\,{}_tp_r\,(\mu_r - \mu_{r+t}) \tag{13}$$

where μ_y is the force of mortality at age $y \geq r$; see Jordan (1967). Now, consider the first derivative of equation (12):

$$\frac{\partial}{\partial r}\mathrm{RR}_x(r) = \frac{\partial}{\partial r}\left(\frac{f}{g}\right) = \left(\frac{f'g - fg'}{g^2}\right), \tag{14}$$

where

$$f' = \frac{\partial f}{\partial r} = (1+j_1)^{r-x}\,\delta_1\left(c_x + \frac{s}{d_1}\right) = \delta_1\left(f + \frac{s}{d_1}\right).$$

It is apparent that $f > 0$ and $g > 0$; similarly, $f' > 0$. If the post-retirement force of mortality is a nondecreasing function of age, i.e., $\mu_r \leq \mu_{r+t}$ for any $t > 0$, then $g' \leq 0$. Thus, from equation (14), $\partial\,\mathrm{RR}_x(r)/\partial r > 0$.

Case 2: $i = w$. If the interest rate and the rate of wage increase are the same, the replacement ratio in equation (9) becomes

$$\mathrm{RR}_x(r) = \frac{c_x + s(r-x)}{{}^k\ddot{a}_r} = \frac{f}{g}, \tag{15}$$

where

$$f = c_x + (r-x)s \quad \text{and } g = {}^k\ddot{a}_r.$$

Note that $f' = s > 0$. Under a nondecreasing force of mortality requirement, it is straightforward to see that $\partial\,\mathrm{RR}_x(r)/\partial r > 0$.

Case 3: $i < w$. If the interest rate is smaller than the rate of wage increase, the replacement ratio in equation (9) becomes

$$\mathrm{RR}_x(r) = \frac{c_x(1+j_1)^{r-x} + s(1+j_1)^{r-x}\,s_{\overline{r-x}|\,j_2}}{{}^k\ddot{a}_r} = \frac{f}{g} \tag{16}$$

where

$$f = c_x(1+j_1)^{r-x} + s(1+j_1)^{r-x}\,s_{\overline{r-x}|\,j_2} \quad \text{and } g = {}^k\ddot{a}_r.$$

It follows that

$$f' = \frac{\delta_2}{(1+j_2)^{r-x}} \left(\frac{s - c_x j_2}{j_2} \right). \tag{17}$$

Under the condition $s > c_x j_2$, we have $f' > 0$. The reverse is true when $s \leq c_x j_2$. Consider the first derivative of equation (16). Both f and g are positive terms. Under the condition $s > c_x j_2$ we also have $f' > 0$. Furthermore, if the force of mortality is increasing with age, we have $g' \leq 0$. Thus, $\partial \mathrm{RR}_x(r)/\partial r > 0$.

We are now ready to state our first result.

Proposition 1. *If the post-retirement force of mortality μ_y is a nondecreasing function of $y \geq r$, then $\partial RR_x(r)/\partial r > 0$ if (i) $i \geq w$, or (ii) $i < w$ and $s > c_x j_2$.*

According to Proposition 1, there will be possible situations where $\mathrm{RR}_x(r)$ is a decreasing function of r. This does not mean, however, that the annuity payment is also a decreasing function of r. In other words, the retiree is not necessarily getting a smaller annuity payment by delaying retirement.

3.2 The Effect of i on $\mathrm{RR}_x(r)$

From equation (9), we have

$$\frac{\partial \mathrm{RR}_x(r)}{\partial i} = \frac{\partial \mathrm{ps}_{x,r}}{\partial i} + \frac{\partial \mathrm{fs}_{x,r}}{\partial i} \tag{18}$$

where

$$\begin{aligned} \frac{\partial \mathrm{ps}_{x,r}}{\partial i} &= \frac{c_x(r-x)}{{}^k\ddot{a}_r} \frac{1}{(1+w)} \left(\frac{1+i}{1+w} \right)^{r-x-1} \\ &= \frac{c_x(r-x)}{{}^k\ddot{a}_r} \frac{(1+j_1)^{r-x-1}}{(1+w)} \end{aligned} \tag{19}$$

and

$$\frac{\partial \mathrm{fs}_{x,r}}{\partial i} = \frac{s}{(1+w)^{r-x}\, {}^k\ddot{a}_r} \frac{\partial G(i,w)}{\partial i}. \tag{20}$$

We can easily see that

$$\frac{\partial \ddot{s}_{\overline{n}|j}}{\partial j} = \sum_{t=1}^{n-1} (n-t)\,(1+j)^{n-t-1} = (1+j)^{n-1}\left[n\ddot{a}_{\overline{n}|j} - (Ia)_{\overline{n}|j}\right]. \quad (21)$$

Equation (21) is obviously nonnegative because $n\ddot{a}_{\overline{n}|j} > (Ia)_{\overline{n}|j}$. We now consider the three cases: $i > w$, $i = w$, and $i < w$.

Case 1: $i > w$. If the interest rate is larger than the rate of wage increase, from equations (5) and (21), we have

$$\begin{aligned}\frac{\partial G\,(i,w)}{\partial i} &= (1+w)^{r-x}\,\frac{\partial \ddot{s}_{\overline{r-x}|j_1}}{\partial j_1}\,\frac{\partial j_1}{\partial i}\\ &= (1+w)^{r-x-1}\,\frac{\partial \ddot{s}_{\overline{r-x}|j_1}}{\partial j_1}\\ &= (1+i)^{r-x-1}\left[(r-x)\ddot{a}_{\overline{r-x}|j_1} - (Ia)_{\overline{r-x}|j_1}\right]\end{aligned} \quad (22)$$

which is obviously nonnegative.

Case 2: $i = w$. If the interest rate and the rate of wage increase are the same, from equation (5), we have

$$\frac{\partial G\,(i,w)}{\partial i} = (r-x)^2\,(1+i)^{r-x-1} \quad (23)$$

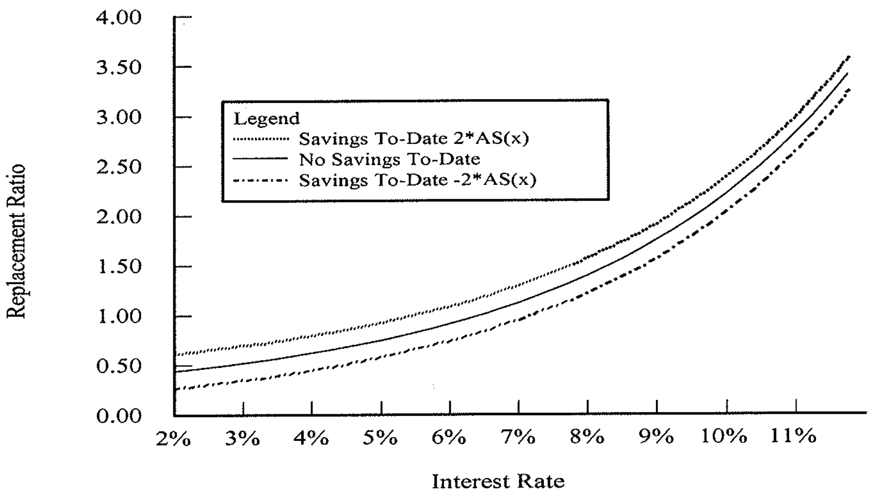
which is also obviously nonnegative.

Case 3: $i < w$. If the interest rate is smaller than the rate of wage increase from equations (5) and (21), we have

$$\begin{aligned}\frac{\partial G\,(i,w)}{\partial i} &= (1+w)^{r-x}\,\frac{\partial a_{\overline{r-x}|j_2}}{\partial j_2}\,\frac{\partial j_2}{\partial i}\\ &= (1+w)^{r-x}\,\frac{-1}{(1+j_2)}\,(Ia)_{\overline{r-x}|j_2}\left(\frac{-(1+w)}{(1+i)^2}\right)\\ &= (1+w)^{r-x}\,\frac{1}{1+i}\,(Ia)_{\overline{r-x}|j_2}\end{aligned} \quad (24)$$

which is also nonnegative.

Figure 3
Replacement Ratio $\mathrm{RR}_x(65)$ and Interest Rate i



In summary, combining equations (22) through (24) and the result from the three cases examined above, we have both

$$\frac{\partial\, \mathrm{ps}_{x,r}}{\partial i} \geq 0 \quad \text{and} \quad \frac{\partial\, \mathrm{fs}_{x,r}}{\partial i} \geq 0$$

which implies that

$$\frac{\partial \mathrm{RR}_x(r)}{\partial i} \geq 0$$

under all circumstances.

Figure 3 shows the relationship of the interest rate on savings and the replacement ratio. Figure 3 is based on the following information $x = 25, r = 65, w = 5$ percent, AS(25) = 20,000, $s = 20$ percent, and a life annuity factor of $\ddot{a}_x = 11.541$ based on the A1967-70 Mortality Table (ultimate). Savings to-date vary between −200 percent and +200 percent of current annual salary. Notice that there is a sharper increase for larger interest rates. This means that one way to increase income replacement ratio is to increase the interest earnings, assuming all other factors are fixed.

Due to the scaling of the graph, there appears to be little difference in the replacement ratio for differing amounts of savings, but such is not actually the case. For example at $i = 4$ percent, with savings of twice the current salary, the replacement ratio will be about 76 percent. With savings of minus twice the current salary (the individual is in debt), the replacement ratio is 42 percent and is almost halved.

3.3 The Effect of w on $\mathrm{RR}_x(r)$

From equation (9), we have

$$\frac{\partial \mathrm{RR}_x(r)}{\partial w} = \frac{\partial \mathrm{ps}_{x,r}}{\partial w} + \frac{\partial \mathrm{fs}_{x,r}}{\partial w}. \tag{25}$$

It is clear that

$$\frac{\partial \mathrm{ps}_{x,r}}{\partial w} < 0.$$

Turning to the second term, we have

$$\frac{\partial \mathrm{fs}_{x,r}}{\partial w} = \left(\frac{s}{k\ddot{a}_r}\right)\frac{\partial}{\partial w}\left[\frac{G(i,w)}{(1+w)^{r-x}}\right].$$

Consider the three cases: $i > w$, $i = w$, and $i < w$. It turns out that in all three cases,

$$\frac{\partial \mathrm{fs}_{x,r}}{\partial w} \le 0 \quad \text{which implies that} \quad \frac{\partial \mathrm{RR}_x(r)}{\partial w} < 0.$$

Case 1: $i > w$. If the interest rate is larger than the rate of wage increase, from equations (5) and (21), we have

$$\frac{\partial \mathrm{fs}_{x,r}}{\partial w} = -\frac{s(1+j_1)^{r-x}}{(1+w)k\ddot{a}_r}\left((r-x)\ddot{a}_{\overline{r-x}|j_1} - (Ia)_{\overline{r-x}|j_1}\right). \tag{26}$$

Equation (26) is always negative.

Case 2: $i = w$. If the interest rate and the rate of wage increase are the same, from equation (5), we have

$$\frac{\partial\, \mathrm{fs}_{x,r}}{\partial w} = 0. \tag{27}$$

Case 3: $i < w$. If the interest rate is smaller than the rate of wage increase from equations (5) and (21), we have

$$\frac{\partial\, \mathrm{fs}_{x,r}}{\partial w} = -\frac{s\,(Ia)_{\overline{r-x}|\,j_2}}{(1+w)^k \ddot{a}_r}, \tag{28}$$

which is also always negative. In this case, when the investment earnings rate is lower than the wage increase rate, then a further increase in wages will decrease the replacement ratio. This is true because the replacement ratio is expressed as a percentage of the retiree's final salary just prior to retirement. In this case, accumulation from savings is not increasing fast enough to keep pace with increases in wages, which may be a reflection of inflation.

To summarize, we have $\partial\, \mathrm{RR}_x(r)/\partial w \leq 0$.

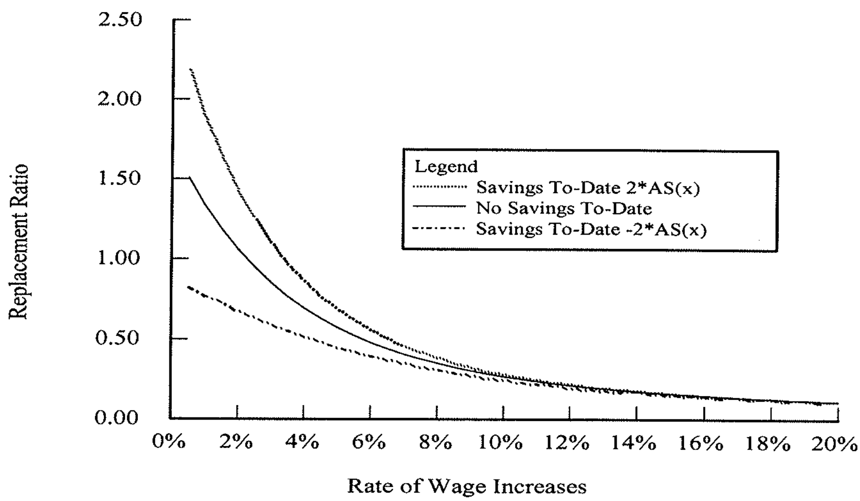
Figure 4 displays the relationship of the replacement ratio and the rate of increase in wages. We observe a declining replacement ratio under all circumstances. The decline in the replacement ratio caused by increasing rates of wages, however, should not be viewed negatively. When wages are particularly high, there is usually a tendency for individuals to increase their savings rate. In this analysis, we have assumed that the savings rate is constant regardless of the level of income. In reality, this may not be the case. We therefore caution the reader to carefully interpret these results.

The opposite effect that i and w have on the replacement ratio can be expected because of their impact on the numerator and denominator in the replacement ratio formula. The net impact of i and w on the replacement ratio depends on the change in the value of the composite rate of j_1 when $i > w$ or j_2 when $i < w$.

3.4 The Effect of x on $\mathrm{RR}_x(r)$

The current age can be viewed as the starting age for which the individual consciously saves for retirement. It is expected that delaying savings for retirement will lower the replacement ratio. We consider the following three cases.

Figure 4
Replacement Ratio $\mathrm{RR}_x(65)$ and Wage Increase w



Case 1: $i > w$. If interest rate is larger than the rate of wage increase, the replacement ratio in equation (9) becomes that in equation (12). We express it as f/g with

$$f = c_x (1 + j_1)^{r-x} + s\ddot{s}_{\overline{r-x}|j_1} \quad \text{and} \quad g = {}^k\ddot{a}_r.$$

Therefore, we have

$$\frac{\partial}{\partial x} \mathrm{RR}_x(r) = \frac{1}{g}\frac{\partial f}{\partial x} = -\frac{\delta_1}{g}(1 + j_1)^{r-x}\left(c_x + \frac{s}{d_1}\right). \qquad (29)$$

Using this result, we have $\partial \mathrm{RR}_x(r)/\partial x < 0$. When investments are earning at a larger rate than the rate of increase of wages, a delay in savings for retirement will lower the replacement ratio. This is intuitively true—a delay in savings means forfeiting the opportunity to earn more through investing. A delay also means a lower amount of savings because of the shorter period to save.

Case 2: $i = w$. If the interest rate and the rate of wage increase are the same, the replacement ratio in equation (9) becomes equation (15) which can be expressed as f/g with

$$f = c_x + (r - x)s \quad \text{and} \quad g = {}^k\ddot{a}_r.$$

Consider the first derivative of equation (15):

$$\frac{\partial}{\partial x}\mathrm{RR}_x(r) = -\frac{s}{g} < 0. \tag{30}$$

While in both cases 1 and 2, the replacement ratio decreases with increasing age x, the rate of decrease is greater when $i > w$ than when $i = w$. This follows directly from

$$(1 + j_1)^{r-x}\,\delta_1\left(c_x + \frac{s}{d_1}\right) > s.$$

Case 3: $i < w$. If interest rate is smaller than the rate of wage increase, the replacement ratio in equation (9) becomes (16) which can be expressed as f/g with

$$f = c_x\,(1 + j_1)^{r-x} + s\,(1 + j_1)^{r-x}\,s_{\overline{r-x}|\,j_2} \quad \text{and} \quad g = {}^k\ddot{a}_r.$$

Thus, we have

$$\frac{\partial}{\partial x}\mathrm{RR}_x(r) = \delta_2\,(1 + j_2)^{x-r}\left(c_x - \frac{s}{j_2}\right). \tag{31}$$

It is also obvious that $\partial\,\mathrm{RR}_x(r)/\partial x < 0$ under the condition that $s > c_x j_2$.

We now state this as our next proposition.

Proposition 2. *$\partial\,RR_x(r)/\partial x < 0$ if (i) $i \geq w$; or (ii) $i < w$ and $s > c_x j_2$.*

The conditions in Propositions 1 and 2 are identical, but the signs of their respective partial derivatives are opposite. This makes sense because x and r would have the opposite effect on the savings accumulation period prior to retirement. Nevertheless, it should be noted that the impact of r on RR_x is much greater than the impact x has on RR_x. In addition to the impact on the savings accumulation period prior to retirement, r also has an effect on the cost in providing the retirement benefit.

4 Extensions to the Replacement Ratio

We now offer some suggestions on ways of extending the concept of replacement ratios introduced in Section 2. These extensions include considering (i) the impact of inflation during retirement, (ii) payments other than annual payments, and (iii) the incorporation of other possible sources of retirement income.

4.1 The Inflation-Adjusted Replacement Ratio, $\mathrm{IARR}_x(r+z)$

Recall the central idea behind the replacement ratio: replace a fraction of the salary at retirement age r with a level annuity income. Suppose, however, after age r, the retiree could have continued working and could have expected annual salary increases of $100w'\%$. It seems reasonable to calculate the replacement ratio at age $r+z$, that is $z \geq 0$ years after retirement age r, based on the projected retirement benefit at age $r+z$ and the hypothetical post-retirement salary at age $r+z$.

Another factor to consider is the impact of inflation on both benefits and salaries. To ensure that there is no deterioration in the standard of living after the retirement age, it is important to have a benefit that increases annually to compensate for the corrosive effects of inflation and hence provide a measure of financial stability throughout the retirement period.

In defining the inflation-adjusted replacement ratio at age $r+z$ there are two factors to consider: (i) the hypothetical projected wage increases (that would have occurred had the retiree kept on working), and (ii) the expected inflation during the post retirement years. Specifically, we use the ratio of projected benefits to projected salary where both quantities are expressed in constant dollars to define the new replacement ratio.

Let $B_x(r+z)$ denote the annual retirement annuity income to be received at age $r+z$. Given a projected total savings of $\mathrm{TS}_x(r)$, the benefits satisfy

$$\mathrm{TS}_x(r) = \sum_{t=0}^{\infty} B_x(r+t)\, v^t\, {}_tp_r \tag{32}$$

where the right side of the equation gives the expected present value of the annuity benefits to be received during retirement.

Assuming a constant inflation rate of ζ per annum throughout the retirement period, the benefit at age $r+z$ expressed in inflation-adjusted (i.e., constant) dollars is $B_x(r+z)(1+\zeta)^{-z}$, while the projected hypothetical salary expressed in inflation-adjusted dollars is $\text{ES}_x(r)(1+w')^z(1+\zeta)^{-z}$.

The inflation-adjusted replacement ratio at age $r+z$, $\text{IARR}_x(r+z)$, is thus defined as:

$$\begin{aligned}\text{IARR}_x(r+z) &= \frac{B_x(r+z)(1+\zeta)^{-z}}{\text{ES}_x(r)(1+w')^z(1+\zeta)^{-z}}\\ &= \frac{B_x(r+z)}{\text{ES}_x(r)(1+w')^z},\end{aligned} \tag{33}$$

which appears to be independent of inflation, ζ, even though it implicitly does depend on inflation.

To reduce the effects of inflation on the retiree's standard of living during retirement, the retiree may purchase an annuity with a benefit that increases annually at a constant rate b'.[3] Thus

$$B_x(r+z) = B_x(r)\left(1+b'\right)^z, \tag{34}$$

where

$$\begin{aligned}\text{TS}_x(r) &= \sum_{t=0}^{\infty} B_x(r)(1+b')^t v^t {}_tp_r\\ &= B_x(r)\sum_{t=0}^{\infty}\left(\frac{1+b'}{1+k}\right)^t {}_tp_r = B_x(r)\,{}^{k'}\ddot{a}_r\end{aligned} \tag{35}$$

and

$$1+k' = \frac{1+k}{1+b'}.$$

Thus, we can solve for

$$B_x(r) = \frac{\text{TS}_x(r)}{{}^{k'}\ddot{a}_r}.$$

It follows that

[3]Another option is to purchase a variable annuity that is linked to inflation. See, for example Black and Skipper (1994, pp. 159–161).

$$\begin{aligned}\mathrm{IARR}_x(r+z) &= \frac{B_x(r)}{\mathrm{ES}_x(r)}\left(\frac{1+b'}{1+w'}\right)^z \\ &= \mathrm{RR}_x(r)\left(\frac{1+b'}{1+w'}\right)^z &(36)\\ &= \frac{\mathrm{TS}_x(r)}{\mathrm{ES}_x(r)\,{}^{k'}\ddot{a}_r}\left(\frac{1+b'}{1+w'}\right)^z. &(37)\end{aligned}$$

To consider the effects of inflation on IARR_x, we split interest into two components: inflation and real interest. In addition, we split wage increases and benefit increases into two components: increases due to inflation and real increases. Thus,

$$k' = \zeta + k^* \tag{38}$$

$$b' = \zeta + b^* \tag{39}$$

and

$$w' = \zeta + w^*. \tag{40}$$

where k^* is the real rate of interest, b^* the real rate of benefit increases and w^* is the real rate of wage increases. (Note that $-\infty < k^*, b^*, w^* < \infty$.) The IARR_x is now given by

$$\mathrm{IARR}_x(r+z) = \frac{\mathrm{TS}_x(r)}{\mathrm{ES}_x(r)\,{}^{k'}\ddot{a}_r}\left(\frac{1+\zeta+b^*}{1+\zeta+w^*}\right)^z \tag{41}$$

with

$$1+k' = \frac{1+\zeta+k^*}{1+\zeta+b^*}.$$

Consider the rate of change of the replacement ratio with respect to ζ, and applying the chain rule of differentiation, we get

$$\begin{aligned}\frac{\partial}{\partial\zeta}\mathrm{IARR}_x(r+z) = -\frac{\mathrm{IARR}_x(r+z)}{(1+\zeta+b^*)}\Bigg[&z\frac{(b^*-w^*)}{(1+\zeta+w^*)} \\ &+\frac{(k^*-b^*)\,{}^{k'}(Ia)_r}{(1+\zeta+k^*)\,{}^{k'}\ddot{a}_r}\Bigg] \qquad (42)\end{aligned}$$

where ${}^{k'}(Ia)_r$ is the expected present value of an increasing life annuity immediate issued to a person age r. We have used the result

$$\frac{\partial k'}{\partial \zeta} = \frac{-(k^* - b^*)}{(1 + \zeta + b^*)^2}$$

$$\frac{\partial}{\partial \zeta}\left(\frac{1 + \zeta + b^*}{1 + \zeta + w^*}\right)^z = -z\frac{(b^* - w^*)}{(1 + \zeta + w^*)^2}\left(\frac{1 + \zeta + b^*}{1 + \zeta + w^*}\right)^{z-1}$$

and from Bowers, et al., (1997, Chapter 5), we have

$$\frac{\partial}{\partial k'}\left({}^{k'}\ddot{a}_r\right) = -\frac{1}{(1 + k')}{}^{k'}(Ia)_r .$$

The sign of the partial derivative is not clear as it depends on the sign of $(b^* - w^*)$ and $(k^* - b^*)$. For example, for U.S. hourly paid workers, the average annual percentage change in hourly earnings in constant dollars (w^*) ranged from -4.5 percent in 1980 to 1.5 percent in 1997.[4] On the other hand, as k^* is set by the insurer, it would reflect the insurer's conservative estimate of its expected real rate of return on its investments. In 1990, U.S. insurers earned on average 8.89 percent on its assets, which decreased to 7.17 percent in 1996. During the same period the annual rate of inflation dropped from 5.4 percent to 2.3 percent, so their real rate of return ranged between 3 and 6 percent.[5] The retiree sets b^*.

4.2 Adjustments for Monthly Factors

The definition of the replacement ratio in equation (8) assumes that

(1) The savings arising from salary are made at once at the beginning of each year, and

(2) The life annuity payable at retirement commences immediately and is paid also once a year.

In reality, employees are paid more frequently than once a year and the retired individuals want to receive income more often than once a

[4]Source: *Statistical Abstract of the Unites States 1998.* U.S. Department of Commerce, page 434, Table No. 693.

[5]Source: *Statistical Abstract of the Unites States 1998.* U.S. Department of Commerce, page 489, Table No. 772 (for inflation) and page 536, Table No. 854 (for insurers' returns).

year. It is easy to extend the replacement ratio definition to reflect such frequent payments. For the numerator, we adjust FS_x by multiplying it by $a_{\overline{1}|}^{(m)}$ assuming that salaries are paid m times during the year and the first such salary in the year commences at the end of the first $1/m$th of the year. Similarly, for the denominator, the life annuity factor ${}^k\ddot{a}_r$ will be replaced by ${}^k\ddot{a}_r^{(m)}$, where the annuity income is payable m times during the year and the first such income commences at the start of the first $1/m$th of the year.

Thus, from equations (8) and (9), the replacement ratio definition becomes

$$\mathrm{RR}_x^{(m)}(r) = \frac{\mathrm{PS}_x (1+i)^{r-x} + \mathrm{FS}_x a_{\overline{1}|}^{(m)}}{\mathrm{AS}(x)(1+w)^{r-x}\, {}^k\ddot{a}_r^{(m)}}. \tag{43}$$

In the case of monthly contributions and benefits, $m = 12$, and the m during the saving stage (numerator of equation (43)) is not necessarily the same as the m during the payout stage (denominator of equation (43)). The sensitivity analysis in Section 3 can therefore be repeated using equation (43) for the replacement ratio.

4.3 Accounting for Other Sources of Retirement Income

In the previous development of replacement ratios, it has been assumed that the sole source of retirement income is personal savings. This is unrealistic. In several countries, primarily in well-developed ones, other sources of retirement income include social security benefits and benefits from employer-sponsored pension plans. For either the public or private pension plan, employers generally make a contribution and "a significant part of labor's compensation consists of pension benefits" (Hsiao 1984).

Social security programs of several nations share common characteristics. Coverage is usually compulsory, i.e., every working individual is required to make a contribution, together with a portion of the employer's share, toward providing pension benefits. Although benefits are generally related to earnings, some programs pay benefits so that there is a certain level of standard of living attained by participants. For our purposes, we will assume that pension benefits derived from social security are fixed and pre-determined at retirement. We shall denote the annual social security benefit payable at age $r + z$ by SS_{r+z}.

With respect to employer-sponsored pension plans, there are two main types of plans: defined benefit plans and defined contribution plans. To simplify introducing these other sources of income into the replacement ratio formula, we shall denote the defined benefit payable at age $r + z$ as BB_{r+z}.

For the defined contribution pension plan, the contribution of the total employer and employee contribution at age $x + t$ is BC_{x+t} where

$$\mathrm{BC}_{x+t} = c\,\mathrm{AS}\,(x+t)$$

and c is the rate of total contributions as a percentage of salary. This amount will be assumed to earn interest at the annual rate of i'. Thus, the amount of benefit derived from this defined contribution plan is expected to accumulate to

$$\begin{aligned}\sum_{t=0}^{r-x-1} BC_{x+t}\left(1+i'\right)^{r-(x+t)} &= \sum_{t=0}^{r-x-1} c\,\mathrm{AS}\,(x+t)\left(1+i'\right)^{r-x-t} \\ &= c\,\mathrm{AS}\,(x)\,G\left(i', w\right) \qquad (44)\end{aligned}$$

at retirement age r.

In summary, the equivalent amount of annual retirement benefit at r can then be expressed as:

$$B_x\,(r) = \frac{\mathrm{TS}_x\,(r)}{{}^k\ddot{a}_r} + (SS_r + \mathrm{BB}_r) + \frac{c\,\mathrm{AS}\,(x)\,G\left(i', w\right)}{{}^k\ddot{a}_r}.$$

5 The Case of Singapore

In Singapore, there are two unique elements that must be incorporated in the calculation of replacement ratios. First, employees and employers are required to make periodic contributes to the Singapore's Central Provident Fund (CPF), which is Singapore's major social security program to provide income at retirement. These contributions then accumulate with interest until the employee's retirement. The accumulated contributions are then used to purchase a retirement annuity. It is therefore similar to a defined-contribution plan. The CPF provides a major source of income during retirement.

The contributions are directly tied to earnings and the contribution rates are reviewed periodically and are generally linked to the country's economic performance. In good economic times, the contribution rates are higher and vice versa. For our purposes, CPF contribution rates can be viewed as forced savings, thus helping the individual boost his or her replacement ratio. For an in-depth discussion of the CPF program in Singapore, see Chen and Wong (1998).

The second unique element that must be considered when planning for retirement in Singapore is home ownership. Singapore is believed to have one of the highest percentage of home ownership in the world. According to the *1998 Singapore Yearbook of Statistics*, more than 90 percent of Singaporeans own a home in Singapore. Most homes in Singapore are generally referred to as HDB flats; the term HDB refers to the Housing Development Board, a government agency that develops and manages housing for Singaporeans. Because housing is generally considered an essential commodity in Singapore, the premium paid for owning one will decrease the amount that can be saved from personal income for retirement purposes. This is of major concern because the price of a house in Singapore is expensive relative to income. For example, a four-room HDB flat approximately costs $140,000, a five-room HDB flat can be double that amount, and landed properties can range in prices exceeding $1 million.[6]

Suppose that at age x, an individual purchases a house and borrows an amount of HL_x for a period of $r - x$ years at the housing loan rate of h. Assuming level annual amortization repayments of P_x, then

$$P_x = \frac{\mathrm{HL}_x}{\ddot{a}_{\overline{r-x}|h}}. \tag{45}$$

For simplicity, equation (45) assumes the first loan repayment is made at the time of the origination of the loan. Each year's salary therefore reduces the disposable income by an amount of P_x. Furthermore, assume that the rate of contribution to the CPF will be constant each year at c and that investments on CPF will earn an effective rate of i'. In Singapore, CPF members are allowed to repay their housing loans out of their accounts. Thus, accumulated future savings, FS'_x, become

$$\begin{aligned} \mathrm{FS}'_x = & \sum_{t=0}^{r-x-1} s\,\mathrm{ES}\,(x+t)\,(1+i)^{r-x-t} \\ & + \sum_{t=0}^{r-x-1} [c\,\mathrm{ES}\,(x+t) - P_x]\left(1+i'\right)^{r-x-t}. \end{aligned} \tag{46}$$

[6]As of October 2000, the average annual income in Singapore was approximately $36,000 (Singapore dollars), with US$1 = $1.75 (Singapore dollars).

Equation (46) can be simplified to:

$$\begin{aligned}
\mathrm{FS}_x' &= s\,\mathrm{AS}(x) \sum_{t=0}^{r-x-1} (1+i)^{r-x} \left(\frac{1+w}{1+i}\right)^t \\
&\quad + c\,\mathrm{AS}(x) \sum_{t=0}^{r-x-1} \left(1+i'\right)^{r-x} \left(\frac{1+w}{1+i'}\right)^t \\
&\quad - P_x \sum_{t=0}^{r-x-1} \left(1+i'\right)^{r-x-t} \\
&= s\,\mathrm{AS}(x)\, G(i,w) + c\,\mathrm{AS}(x)\, G\left(i',w\right) - P_x \ddot{s}_{\overline{r-x}|\,i'} \\
&= \mathrm{AS}(x)\left[sG(i,w) + cG\left(i',w\right)\right] \\
&\quad - \mathrm{HL}_x \left(1+i'\right)^{r-x} \frac{\ddot{a}_{\overline{r-x}|\,i'}}{\ddot{a}_{\overline{r-x}|\,h}}.
\end{aligned} \tag{47}$$

Again, similar to Section 2, we suppose that at age x the individual has saved a total of PS_x. At retirement, the individual can expect to have saved a total of

$$\mathrm{TS}_x(r) = \mathrm{PS}_x (1+i)^{r-x} + \mathrm{FS}_x'. \tag{48}$$

This amount will then be used to purchase a life annuity at retirement for which the level annual payments will be as given in equation (7). Therefore, the replacement ratio becomes

$$\begin{aligned}
\mathrm{RR}_x(r) = \frac{\mathrm{PS}_x (1+i)^{r-x}}{\mathrm{AS}(x)\,(1+w)^{r-x}\, k\ddot{a}_r} &+ \frac{\mathrm{AS}(x)\left[sG(i,w) + c\,G\left(i',w\right)\right]}{\mathrm{AS}(x)\,(1+w)^{r-x}\, k\ddot{a}_r} \\
&- \frac{\mathrm{HL}_x \left(1+i'\right)^{r-x} \ddot{a}_{\overline{r-x}|\,i'}}{\mathrm{AS}(x)\,(1+w)^{r-x}\, k\ddot{a}_r \ddot{a}_{\overline{r-x}|\,h}}.
\end{aligned} \tag{49}$$

Using symbols earlier defined, we can express equation (49) as

Table 1
Effect of c on Replacement Ratios
In Singapore, with $h_x = 10$

	Retirement Age					
c	50	55	60	65	70	75
10%	-35.1%	-34.9%	-34.0%	-31.8%	-27.0%	- 17.5%
20%	-19.2%	-12.8%	-3.4%	10.7%	32.5%	67.0%
30%	-3.3%	9.4%	27.2%	53.2%	92.1%	151.4%
40%	12.6%	31.5%	57.9%	95.7%	151.7%	235.8%
50%	28.4%	53.6%	88.5%	138.3%	211.2%	320.3%

Table 2
Effect of h_x on Replacement Ratios
In Singapore, with $c = 30\%$

	Retirement Age					
h_x	50	55	60	65	70	75
0	74.8%	103.5%	143.1%	198.9%	279.6%	398.4%
5	35.7%	56.5%	84.2%	126.1%	185.9%	274.9%
10	-3.3%	9.4%	27.2%	53.2%	92.1%	151.4%
15	-42.4%	-37.7%	-30.7%	-19.6%	-1.7%	27.9%
20	-81.4%	-84.8%	-88.6%	-92.5%	-95.5%	-95.6%

$$\mathrm{RR}_x(r) = \frac{(1+j_1)^{r-x}}{{}^k\ddot{a}_r}\left[c_x + \frac{sG(i,w) + cG\left(i',w\right)}{(1+j_1)^{r-x}(1+w)^{r-x}} - \frac{\left(1+j_1'\right)^{r-x}\ddot{a}_{\overline{r-x}|i'}h_x}{(1+j_1)^{r-x}\ddot{a}_{\overline{r-x}|h}}\right] \tag{50}$$

where

$$h_x = \frac{HL_x}{\mathrm{AS}(x)}$$

is the housing loan expressed as a multiple of salary at age x.

For purposes of illustration, consider the case where we have a Singapore individual who is currently age 25 and is receiving an annual salary of \$20,000. Assume he or she has saved \$20,000 and that his or her salary is expected to increase at the rate of 3 percent per annum. Apart from contributions to the CPF, he or she expects to save annually 10 percent for retirement. Interest rates will be assumed at $i = 5$ percent, $i' = 4$ percent, and $h = 5$ percent.

Tables 1 and 2 display the result of varying the CPF contribution rates and the ratio of housing loans to salary, respectively, on the value of the replacement ratio at different retirement ages. Table 1 shows the effect of varying the contribution rates to the CPF, and Table 2 shows the effect of varying the housing loan ratio. To further illustrate the results, at a CPF contribution rate of say 30 percent, this same individual can retire at age 65 with a replacement ratio of 53.2 percent. Similarly, if he or she borrows money for housing at the amount five times current salary and CPF contribution rate stays at 30 percent, he or she can retire at age 65 with replacement ratio of a whopping 126.1 percent. The results in Tables 1 and 2 generally demonstrate that to improve replacement ratios, one can either:

- Decide to delay retirement;
- Increase contribution rates to the CPF scheme; or
- Control the budget for housing.

6 Concluding Remarks and Future Research

This paper proposes a financial measure, the replacement ratio, that can be used for individuals in planning for retirement. The replacement ratio, which is expressed as the proportion of the retirement income to that of the final salary at retirement, is an intuitively appealing financial construct. The paper does not recommend a suitable level of the replacement ratio: it is left to each person to determine the level that is appropriate.

The size of a person's retirement income depends on several factors such as wage increases, savings rate, interest rates, and inflation both prior to and during retirement. The replacement ratio developed in this paper readily allows us to examine the effect of changes of these factors to the replacement ratio. In Section 3, we examined the sensitivity of the replacement ratio with respect to changes in the retirement age r, the investment earnings rate i, the rate of wage increases w during employment periods, and the current age x (the age at which the individual consciously begins to set aside funds specifically for retirement purposes).

We found that under certain conditions, delaying the retirement age and increasing the return on investments have the effect of increasing the replacement ratio. On the other hand, delaying the age at which savings for retirement commences has the effect of reducing the replacement ratio. For wage increases, the replacement ratio will either increase or decrease depending upon the relationship of the rate of i and w. We caution the reader when interpreting the results when wage increases are concerned. Large increases in wages can cause the replacement ratio to be smaller—this should not be interpreted to mean that a lower replacement ratio leads to a deterioration of retirement income. When the base salary is large and yet savings have not been accumulating, the replacement ratio may be relatively low but the amount of retirement income can still be large.

Our work here is only a beginning to better understanding the various factors that can affect income at retirement. The discussion has been simplified to facilitate understanding. For example, we have assumed constant interest rates and constant rate of wage increases. Furthermore, we have assumed that interest rates, savings rates, and the rates of wage increases are all independent. In reality, this is not true. One possible future research is to examine empirical data that may support evidence of dependence and that may account for the time series nature of these variables. Some recent papers by Knox (1993) and Booth and Yakoubov (2000) that suggest the use of a stochastic approach may be helpful in these instances. It will be interesting to explore the link of these stochastic approaches in further developing the replacement ratio model recommended in this paper. Another possible future research area is the impact of mortality improvement, a phenomenon that is observed worldwide.

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