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Estimation of Complete Period Life Tables for Singaporeans

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Abstract‡

Complete period life tables, with death rates for every year of age, are not available in Singapore. This study constructs such tables for Singaporeans from the limited mortality information contained in the abridged life tables provided by the Singapore Department of Statistics. We find that linear interpolation, Whittaker graduation, and the Coale-Kisker method together can generate complete life tables that are smooth and continuous. The validity of the complete life tables generated by our method is further confirmed by (1) comparing the life expectancies calculated from our estimated life tables with those provided by the Singapore Department of Statistics, and (2) comparing the shapes of the mortality functions derived from our life tables with those derived from the Commissioner's Valuation Tables for assured lives in Singapore.

Key words and phrases: *graduation, data-disaggregation, Whittaker-Henderson method, Coale-Kisker method*

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1 Introduction

Period life tables (also known as current life tables) represent the mortality profile of a community at a specific point of time. These tables show the conditional probabilities of death, denoted as q_x, for every single year of age from 0 to a maximum age such as 100 or 110 and can be varied by such factors as gender and race. In the United States, complete period life tables, by age, sex, and race are published from time to time by the National Center of Health Statistics (NCHS). In the United Kingdom, similar complete period life tables, known as English Life Tables, are prepared by the Government Actuary, based on the mortality experience of the general population in England and Wales. Complete period life tables, however, are not necessarily available in some developed countries, such as Singapore.

To glimpse the mortality pattern in Singapore, one can refer to two series of available tables. The first series is known as the Commissioner's Valuation Table (CVT), developed by the Monetary Authority of Singapore (Singapore's central bank), based on the mortality experience of the insured population. The CVT has been widely used in the insurance industry for the purpose of valuation. Regulation 26(5) of the Insurance Regulations 1992 in Singapore specifies that in respect to a policy other than an annuity, the minimum reserves for the valuation shall be made by using the 1992 Commissioner's Valuation Table for male lives and the 1992 Commissioner's Valuation Table with a three-year age setback for female lives. Similar to the English Life Tables and the U.S. life tables, the CVTs are complete in the sense that the conditional probabilities of death are shown for every single year of age. Despite their completeness, the use of CVTs is limited to the purpose of insurance valuation. At a specific point of time, the insured and general populations could share a similar shape of mortality profile, but the level of mortality for the insured is typically lower than that for the general population, mainly due to the underwriting procedures employed prior to issuing insurance policies. Using the CVT for mortality analyses of the general population is, therefore, inappropriate.

The second available series is the set of official life tables provided by the Singapore Department of Statistics. These official life tables are constructed based on the mortality experience of the entire Singaporean population and, therefore, applicable in mortality analyses of the general population. Unfortunately, for the purpose of easing workload and expenses, the mortality data in the official life tables are presented in an abridged form. Instead of every single year of age, values of the central death rates are shown at age 0, age group 1-4, quinquennial age groups 5–9, 10–14, and so on up to 65–69, and the open age group of 70 and over. Abridged life tables can allow the comparison of the mortality experiences of different territories and the longitudinal study of mortality levels of a population. In many actuarial and demographic applications, however, abridged life tables do not suffice.

In actuarial practice, complete life tables are always required for the computation of monetary functions involving life contingencies, and the absence of such tables often leads to problems. For example, Chen and Wong (1997) examined the adequacy of the benefits from the Central Provident Fund (CPF) savings in Singapore using actuarial simulations. They computed the monthly benefit to be received by a retiree by assuming that the accumulated value of the retiree's CPF savings was to be progressively liquidated by a life annuity. As no complete life table for the general Singaporean population existed at that time, they used an annuity certain with term equal to the expected future lifetime at the retirement age as a proxy for the value of the whole life annuity. It can be easily proved that the value of such an annuity certain is always less than that of its corresponding life annuity. In other words, all of the estimated monthly benefits in the study were overstated.

Another example occurs in the area of the assessment of future financial loss in personal injury litigation. In such cases courts often adopt the multiplicand and multiplier approach to determine the lump sum amount of compensation for the plaintiff for future loss of earnings and to cover consequential expenses. In the first stage of this approach, a multiplicand, which represents the future annual loss of income and consequential expenses, is decided. In the second stage, the multiplicand is multiplied by an appropriate multiplier, decided by the judge, to get the lump sum amount of compensation for the plaintiff. In the United Kingdom, tables of actuarially calculated multipliers, known as 'Ogden Tables', have been available since 1984. Sarony et al. (2003) also developed tables of multipliers for Hong Kong. The calculation of these actuarial multipliers, similar to the case for life annuities, requires the probabilities of death for all ages. Thus, tables of these multipliers are not available in Singapore at present.

The absence of appropriate complete period life tables for the general population inevitably handicaps demographers in Singapore and partly explains why mortality studies in Singapore have been scanty. In the implementation of the well-known Lee-Carter method of mortality forecasting, Lee and Carter (1992) pointed out that under the circumstances of population aging and demographic shifts, failing to consider the mortality distribution of the open age group would be uninformative and might even lead to serious distortion. This calls for life tables with more information in the open age group, preferably with death rates shown for every single year of age.

In this study, we develop a method to produce complete period life tables with q_xs for every single year of age from 0 to 99 from the limited mortality information shown in the abridged life tables published by the Singapore Department of Statistics using the ideas of interpolation, extrapolation, and graduation. To ensure that the estimated life tables are of high quality, the estimation method must consistently fulfill several criteria:

- First, the complete life expectancy at birth, denoted as $\mathring{e}_0$, calculated from our estimated complete period life table should be close to that officially publicized by the Singapore Department of Statistics. An absolute difference greater than one is regarded as unsatisfactory. In a statistical sense, this is to match the first moment derived from our estimated probabilities of death with the true first moment.

- Second, the mortality profiles described by our estimated life tables should share the properties of the typical laws of mortality—the probability of death should decrease from birth to a minimum value at around age 10 and then monotonically increase gradually.

- Third, the shapes of the plots of the mortality functions derived from our estimated life tables should be similar to those derived from the CVTs. In particular, the curves of the logarithms of the probabilities of death are compared.

2 The Estimation Procedure

The Singapore Department of Statistics' abridged life tables contain age-specific death rates for age 0, age group 1–4, quinquennial age groups 5–9, 10–14, etc., up to 65–69, and the open age group of 70 and over (Table 1). The so-called age-specific death rates here actually refer to the number of resident deaths per thousand residents of a specific age group during a given year.

Our estimation procedure is divided into three stages: (i) deriving the initial (raw) estimates using interpolative methods; (ii) graduation of these raw rates to produce a smooth and more reliable representation of the underlying pattern of mortality; and (iii) extrapolation of the rates to individual ages 70 and over.

Table 1
Abridged Life Table
For Singaporeans (2001)
Age-Specific Death Rates

	$1000 \times {}_nM_x$	
Age	Male	Female
0	2.4	2.1
1 – 4	0.3	0.3
5 – 9	0.1	0.1
10 – 14	0.1	0.1
15 – 19	0.4	0.3
20 – 24	0.7	0.2
25 – 29	0.7	0.2
30 – 34	0.7	0.5
35 – 39	1.0	0.6
40 – 44	1.6	0.9
45 – 49	2.5	1.5
50 – 54	4.6	2.6
55 – 59	8.1	4.6
60 – 64	13.2	7.2
65 – 69	23.2	12.8
70 +	58.3	47.5

2.1 Stage One: Initial Estimation

To obtain an initial estimate of the q_xs in the complete life tables, the central death rate for every single year of age (m_x) is used through an interpolation method. Let ${}_nM_x$ be the crude central death rate for persons in the integer age group x to $x + n - 1$ shown in the abridged life tables, and let m_x be the unknown single year central death rate for persons age x last birthday. The quantity m_x is to be obtained in this stage of the estimation.

Before performing the interpolation exercise, we treat ${}_nM_x$ as the single year central death rate at the mid-point of its corresponding age group. That is, we set m_7 as ${}_5M_5$, m_{12} as ${}_5M_{10}$, and so on. For the open age group, we assume that it closes at age 100 and, hence, m_{83} is taken as the central death rate for the open age group 70 and over shown on the abridged life tables. The infant mortality rate, m_0, is given exactly as ${}_1M_0$ in the abridged life tables; and m_1 is taken as ${}_4M_1$ without any interpolation (to avoid unwanted inflation due to the domination of m_0). The remaining values of the single year central death rates are determined using linear interpolation. Mathematically, we have

$$m_x = \begin{cases} \frac{1}{5}({}_5M_5 - {}_4M_1)(x-2) + {}_4M_1 & \text{for } x = 2,3,4,5,6; \\ \frac{1}{5}({}_5M_{k+3} - {}_5M_{k-2})(x-k) + {}_5M_{k-2} & \text{for } x = k,\ldots,k+4 \text{ and} \\ & k = 7,12,17,22,\ldots,62, \\ \frac{1}{16}({}_{30}M_{70} - {}_5M_{65})(x-67) + {}_5M_{65} & \text{for } x = 67,68,\ldots,82, \\ \frac{1}{17}(1 - {}_{30}M_{70})(x-83) + {}_{30}M_{70} & \text{for } x = 83,84,\ldots,100. \end{cases}$$

The initial estimates of m_x for males and females are given in Tables 2 and 3.

2.2 Stage Two: Graduation

The next step is to graduate the initial estimates to ensure they conform to the generally accepted beliefs about the underlying mortality rates: they are smooth, regular, and continuous.[1]

Graduation techniques have been extensively employed in dealing with the problem of raggedness in the initial estimates derived from mortality data. (See, for example, Renshaw et al., 1996; Renshaw, 1995; Renshaw and Hatzpoulos, 1995.) Though there are several methods of graduation, the two basic ones are:

1. Parametric gradation, which is performed by fitting a parametric formula (law of mortality) over the entire age range. An example is the Heligman and Pollard model (Heligman and Pollard, 1980); and

2. Non-parametric graduation—examples include moving weighted average graduation (Ramsay, 1991) and Whittaker graduation (Whittaker, 1923).

London (1985) provides a thorough discussion of the various types of graduation methods used by actuaries.

Among the various graduation techniques, Whittaker gradation (also known as Whittaker-Henderson graduation) performs well in terms of

[1]Miller (1946) stated that capricious irregularities in the tables from age to age, disturbing the orderly progression of premiums, etc., would be inconsistent with the common sense view that such figures should be reasonably regular and would tend to arouse an entirely justifiable skepticism.

our described estimation criteria. Whittaker graduation allows practitioners to customize the tradeoff between goodness of fit and smoothness and has been extensively employed. (See Guerrero et al., 2001 and Lowrie, 1993.) The objective of Whittaker graduation is to choose the $\{m_x^g\}$ sequence that minimizes the equation

$$J = F + hS \equiv \sum_{x=0}^{n} w_x (m_x^g - m_x)^2 + h \sum_{x=0}^{n-z} (\Delta^z m_x^g)^2 \qquad (1)$$

where n is the highest age used, m_x is the initial estimate of the central death rate at age x, m_x^g denotes the graduated value of the central death rate at age x, w_x is a positive weight imposed at age x, h is a nonnegative parameter, Δ is the forward difference operator such that $\Delta m_x^g = m_x^g - m_{x-1}^g$, and z is the number of times the difference operator is applied. The quantity $F \equiv \sum_{x=1}^{n} w_x (m_x^g - m_x)^2$ is a measure of fitness, $S \equiv \sum_{x=1}^{n-z} (\Delta^z m_x^g)^2$ is a measure of smoothness, and h reflects the relative importance of smoothness over fit. By minimizing F, $\{m_x^g\}$ will tend to be close to $\{m_x\}$, resulting in a better goodness of fit. By minimizing S, $\{m_x^g\}$ will tend to lie on an adaptive polynomial of degree z, resulting in a higher degree of smoothness. There is no optimal solution for z and h. They are determined subjectively by the practitioner.

In our estimation procedure, we use the special case of the Whittaker graduation, where $w_x = 1$ for all x. By trial and error, z is fixed to be 2, and the smoothing parameter, h, is chosen to be 0.5 for balancing goodness of fit and smoothness. Small changes in z or h have only a negligible effect on the overall result. The graduation is applied up to $x = 69$ (i.e., $n = 69$). For $x > 69$ (the open age group), central death rates are computed using an extrapolative procedure to be described in the next stage. Rewriting J in matrix form with the pre-determined values of w_x, z and h yields

$$J = (\mathbf{m}^g - \mathbf{m})^T (\mathbf{m}^g - \mathbf{m}) + \frac{1}{2} \mathbf{m}^{g\prime} \mathbf{k}_2{}' \mathbf{k}_2 \mathbf{m}^g \qquad (2)$$

where $\mathbf{m}^g = (m_1^g, \ldots, m_n^g)^T$, $\mathbf{m} = (m_1, \ldots, m_n)^T$ and $\mathbf{k}_2$ is an $(n-2) \times n$ matrix such that

$$\mathbf{k}_2(i,j) = (-1)^{2+i-j} \left[\frac{2!}{(j-i)!\,(2-j+i)!} \right]$$

for $i = 1, \ldots, n-2$, $j = 1, \ldots, n$ with $\mathbf{k}_2(i,j) = 0$ for $j < i$ or $j > 2+i$. Note that J is minimized when

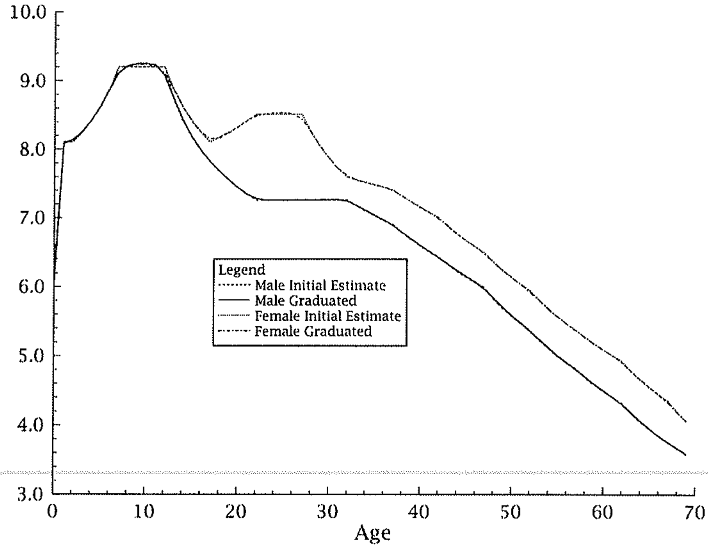

Figure 1: Initial Estimates and Graduated Values of $-\ln(m_x)$, Male 2001

$$\mathbf{m}^g = (\mathbf{I} + 0.5\mathbf{k}_2'\mathbf{k}_2)^{-1}\mathbf{m},$$

where $\mathbf{I}$ is an $n \times n$ identity matrix. A proof can be found in London (1985, page 57).

Tables 2 and 3 show the initial and graduated estimates of the central death rates obtained in this stage for males and females, respectively. Figure 1 displays the plot of $-\ln(m_x)$ for both the initial and graduated estimates. The notice that the graduated estimates show an improvement in smoothness without much deviation from the initial values. The shape of the plot agrees reasonably with our prior assumption of the underlying survival model.

2.3 Stage Three: Extrapolation

Table 4 shows the preliminary values of the single year central death rates obtained in the initial stage. Notice that at advanced ages, especially at ages over 90, the values are unexpectedly high. The disappointing performance of the interpolation at such advanced ages is in large part due to the lack of mortality information: only a single rate is provided for the open age group of 70 and over in the abridged life

Table 2

Initial Estimates and Graduated Values of m_x, Male 2001

Age	Initial Estimate	Graduated	Age	Initial Estimate	Graduated
0	0.002400	0.002400	35	0.000880	0.000875
1	0.000300	0.000307	36	0.000940	0.000938
2	0.000300	0.000290	37	0.001000	0.001014
3	0.000260	0.000260	38	0.001120	0.001118
4	0.000220	0.000221	39	0.001240	0.001235
5	0.000180	0.000179	40	0.001360	0.001355
6	0.000140	0.000139	41	0.001480	0.001478
7	0.000100	0.000109	42	0.001600	0.001614
8	0.000100	0.000098	43	0.001780	0.001777
9	0.000100	0.000096	44	0.001960	0.001950
10	0.000100	0.000095	45	0.002140	0.002125
11	0.000100	0.000098	46	0.002320	0.002313
12	0.000100	0.000114	47	0.002500	0.002557
13	0.000160	0.000158	48	0.002920	0.002911
14	0.000220	0.000217	49	0.003340	0.003318
15	0.000280	0.000278	50	0.003760	0.003737
16	0.000340	0.000339	51	0.004180	0.004171
17	0.000400	0.000400	52	0.004600	0.004666
18	0.000460	0.000461	53	0.005300	0.005290
19	0.000520	0.000522	54	0.006000	0.005975
20	0.000580	0.000583	55	0.006700	0.006675
21	0.000640	0.000642	56	0.007400	0.007391
22	0.000700	0.000686	57	0.008100	0.008176
23	0.000700	0.000702	58	0.009120	0.009105
24	0.000700	0.000703	59	0.010140	0.010094
25	0.000700	0.000702	60	0.011160	0.011095
26	0.000700	0.000701	61	0.012180	0.012150
27	0.000700	0.000700	62	0.013200	0.013431
28	0.000700	0.000699	63	0.015200	0.015171
29	0.000700	0.000698	64	0.017200	0.017139
30	0.000700	0.000697	65	0.019200	0.019161
31	0.000700	0.000698	66	0.021200	0.021188
32	0.000700	0.000714	67	0.023200	0.023248
33	0.000760	0.000758	68	0.025394	0.025390
34	0.000820	0.000815	69	0.027588	0.027569

Table 3
Initial Estimates and Graduated Values of m_x, Female 2001

Age	Initial Estimate	Graduated	Age	Initial Estimate	Graduated
0	0.002100	0.002100	35	0.000560	0.000559
1	0.000300	0.000307	36	0.000580	0.000579
2	0.000300	0.000290	37	0.000600	0.000609
3	0.000260	0.000260	38	0.000660	0.000658
4	0.000220	0.000221	39	0.000720	0.000716
5	0.000180	0.000179	40	0.000780	0.000775
6	0.000140	0.000139	41	0.000840	0.000838
7	0.000100	0.000109	42	0.000900	0.000914
8	0.000100	0.000099	43	0.001020	0.001018
9	0.000100	0.000097	44	0.001140	0.001134
10	0.000100	0.000097	45	0.001260	0.001253
11	0.000100	0.000098	46	0.001380	0.001377
12	0.000100	0.000109	47	0.001500	0.001524
13	0.000140	0.000139	48	0.001720	0.001716
14	0.000180	0.000179	49	0.001940	0.001929
15	0.000220	0.000222	50	0.002160	0.002147
16	0.000260	0.000261	51	0.002380	0.002374
17	0.000300	0.000286	52	0.002600	0.002643
18	0.000280	0.000282	53	0.003000	0.002994
19	0.000260	0.000263	54	0.003400	0.003386
20	0.000240	0.000241	55	0.003800	0.003789
21	0.000220	0.000220	56	0.004200	0.004196
22	0.000200	0.000205	57	0.004600	0.004629
23	0.000200	0.000199	58	0.005120	0.005113
24	0.000200	0.000197	59	0.005640	0.005616
25	0.000200	0.000196	60	0.006160	0.006123
26	0.000200	0.000198	61	0.006680	0.006664
27	0.000200	0.000214	62	0.007200	0.007342
28	0.000260	0.000259	63	0.008320	0.008296
29	0.000320	0.000318	64	0.009440	0.009376
30	0.000380	0.000381	65	0.010560	0.010484
31	0.000440	0.000441	66	0.011680	0.011650
32	0.000500	0.000491	67	0.012800	0.013055
33	0.000520	0.000521	68	0.014969	0.014939
34	0.000540	0.000541	69	0.017138	0.017033

Table 4
Initial Estimate of m_x at Advanced Ages, 2001

Age	Male	Female
70	0.01920	0.01056
75	0.02978	0.01931
80	0.04075	0.03015
85	0.05172	0.04099
90	0.16909	0.15956
95	0.44606	0.43971

tables. Thus, we must re-estimate the single year central death rate for ages over 69 through an extrapolative model that realistically reflects the underlying law of mortality at very advanced ages. The classical Gompertz curve is not an ideal choice, as empirical evidence indicates that mortality at older ages is not Gompertzian (Olshansky and Carnes, 1997). Coale and Kisker (1990) showed that, for a number of developed countries, mortality rates increase at a linearly decreasing rate, rather than at a constant rate as the Gompertz curve assumes. We apply the method suggested by Coale and Guo (1989), namely the Coale-Kisker method, which assumes a linearly decreasing rate, to extend the death rates up to age 99, based on the graduated values of m_x around age 70 obtained in the second stage of our estimation.

For $x \geq 70$, define

$$k(x) = k(x-1) - R, \tag{3}$$

where $k(x) = \ln(m_x / m_{x-1})$. Extending the formula up to $x = 110$ and summing, we get

$$k(70) + \cdots + k(110) = \ln\left(\frac{m_{110}}{m_{69}}\right) = 41k(69) - 861R.$$

Solving for R, we obtain

$$R = \frac{41k(69) + \ln(m_{69}) - \ln(m_{110})}{861}. \tag{4}$$

To minimize the effects of random fluctuations, $k(69)$ is replaced by $k^*(69)$, which is a moving average of $k(67)$ to $k(71)$, i.e.,

$$k^*(69) = \frac{k(67) + k(68) + k(69) + k(70) + k(71)}{5}$$
$$= \frac{k(67) + k(68) + 3k(69) - 3R}{5}.$$

Similarly, $\ln(m_{69})$ is replaced by $\ln(m^*_{69})$, which is defined as

$$\ln(m^*_{69}) = k^*(69) + \ln(m^*_{68}),$$

where

$$\ln(m^*_{68}) = \ln\left(\frac{m_{67} + m_{68} + m_{69}}{3}\right).$$

Substituting $k^*(69)$ and $\ln(m^*_{69})$ in R, we obtain

$$R = \frac{\frac{42}{5}\left[k(67) + k(68) + k(69)\right] + \ln\left(\frac{m_{67} + m_{68} + m_{69}}{3m_{110}}\right)}{861 + \frac{42 \times 3}{5}}. \quad (5)$$

According to the original Coale-Kisker method, m_{110} is assumed to be 1.0 for males. Such an assumption is based on the fact that there are almost no deaths occurring at ages greater than 110. For females, m_{110} is chosen as 0.8 to avoid imposing a crossover of male and female mortality at age 110. Coale and Kisker (1990) demonstrated that the choice of 1.0 or 0.8 for m_{110} has only a minor effect on the constructed life table for white males; we found that it is also true for the Singaporean life table. The values of m_{67}, m_{68}, and m_{69} used are the graduated values of the central death rates obtained in the second stage of our estimation.

3 Results

Finally we estimate the probability of death, q_x, using the assumption of uniform distribution of death, as (Bowers et al., 1997, page 90):

$$q_x = \frac{2m_x}{2 + m_x}. \quad (6)$$

Tables 5 and 6 show an example of a complete period life table estimated for males and females by our suggested procedures.[2] The mortality profiles that are described by our estimated life tables share the properties of the typical laws of mortality: mortality rates drop from a relatively large value at birth to a minimum value at around age 10 and then rises in a monotonic, continuous, and smooth manner. Table 7 presents a longitudinal comparison between the life expectancies at birth derived from our complete life table and those officially provided by the Singapore Department of Statistics. We could observe no systematic discrepancies for both sexes. The average difference between the two values is 0.26 and the maximum absolute difference is strictly less than one, showing that the criterion of first moment matching is satisfied.

Figures 2 compares the plots of $-\log(q_x)$ arising from our estimated complete life table with those from the Commissioner's Valuation Table (CVT) provided by the Monetary Authority of Singapore. Before age 50, their shapes are close to each other, but a parallel shift exists: the profile for the CVT is higher than that for our life table, reflecting the fact that the mortality level for assured lives is generally lower. After age 50, the curves coincide. The consistency further ensures that our complete life tables are representative of the mortality pattern of the general Singaporean population.

4 Concluding Remarks and Future Research

This paper proposes a simple method for estimating complete period life tables for Singaporeans from the abridged life tables that are provided by the Singapore Department of Statistics. We found that linear interpolation, supplemented with Whittaker graduation and Coale-Kisker extrapolation, could generate complete period life tables that are smooth and continuous and reasonably agree with the presumed underlying law of mortality. The reliability of our complete life tables is further substantiated by their consistency with the Commissioner's Valuation Tables and their agreement with the actual life expectancies.

[2]Complete period life tables from 1981 to 2001 for both sexes are available on request.

Table 5
Complete Period Life Table (Male, 2001)

Age	q_x	Age	q_x	Age	q_x	Age	q_x
0	0.0024						
1	0.0003	26	0.0007	51	0.0042	76	0.0480
2	0.0003	27	0.0007	52	0.0047	77	0.0521
3	0.0003	28	0.0007	53	0.0053	78	0.0564
4	0.0002	29	0.0007	54	0.0060	79	0.0612
5	0.0002	30	0.0007	55	0.0067	80	0.0663
6	0.0001	31	0.0007	56	0.0074	81	0.0719
7	0.0001	32	0.0007	57	0.0081	82	0.0779
8	0.0001	33	0.0008	58	0.0091	83	0.0844
9	0.0001	34	0.0008	59	0.0100	84	0.0914
10	0.0001	35	0.0009	60	0.0110	85	0.0990
11	0.0001	36	0.0009	61	0.0121	86	0.1072
12	0.0001	37	0.0010	62	0.0133	87	0.1161
13	0.0002	38	0.0011	63	0.0151	88	0.1256
14	0.0002	39	0.0012	64	0.0170	89	0.1359
15	0.0003	40	0.0014	65	0.0190	90	0.1469
16	0.0003	41	0.0015	66	0.0210	91	0.1588
17	0.0004	42	0.0016	67	0.0230	92	0.1716
18	0.0005	43	0.0018	68	0.0251	93	0.1853
19	0.0005	44	0.0019	69	0.0272	94	0.2001
20	0.0006	45	0.0021	70	0.0295	95	0.2158
21	0.0006	46	0.0023	71	0.0320	96	0.2327
22	0.0007	47	0.0026	72	0.0347	97	0.2507
23	0.0007	48	0.0029	73	0.0376	98	0.2699
24	0.0007	49	0.0033	74	0.0408	99	0.2904
25	0.0007	50	0.0037	75	0.0443		

Table 6
Complete Period Life Table (Female, 2001)

Age	q_x	Age	q_x	Age	q_x	Age	q_x
0	0.0021						
1	0.0003	26	0.0002	51	0.0024	76	0.0400
2	0.0003	27	0.0002	52	0.0026	77	0.0449
3	0.0003	28	0.0003	53	0.0030	78	0.0503
4	0.0002	29	0.0003	54	0.0034	79	0.0563
5	0.0002	30	0.0004	55	0.0038	80	0.0628
6	0.0001	31	0.0004	56	0.0042	81	0.0700
7	0.0001	32	0.0005	57	0.0046	82	0.0779
8	0.0001	33	0.0005	58	0.0051	83	0.0864
9	0.0001	34	0.0005	59	0.0056	84	0.0957
10	0.0001	35	0.0006	60	0.0061	85	0.1058
11	0.0001	36	0.0006	61	0.0066	86	0.1167
12	0.0001	37	0.0006	62	0.0073	87	0.1285
13	0.0001	38	0.0007	63	0.0083	88	0.1412
14	0.0002	39	0.0007	64	0.0093	89	0.1548
15	0.0002	40	0.0008	65	0.0104	90	0.1693
16	0.0003	41	0.0008	66	0.0116	91	0.1847
17	0.0003	42	0.0009	67	0.0130	92	0.2012
18	0.0003	43	0.0010	68	0.0148	93	0.2186
19	0.0003	44	0.0011	69	0.0169	94	0.2370
20	0.0002	45	0.0013	70	0.0192	95	0.2564
21	0.0002	46	0.0014	71	0.0218	96	0.2767
22	0.0002	47	0.0015	72	0.0247	97	0.2979
23	0.0002	48	0.0017	73	0.0279	98	0.3200
24	0.0002	49	0.0019	74	0.0315	99	0.3430
25	0.0002	50	0.0021	75	0.0355		

Table 7
Complete Life Expectancies at Birth for 1988–2001

	Males			Females		
Year	Est. $\mathring{e}_0$	$\mathring{e}_0$	DIFF	Est. $\mathring{e}_0$	$\mathring{e}_0$	DIFF
1988	73.35	72.60	0.75	77.14	76.90	0.24
1989	73.10	72.90	0.20	77.51	77.20	0.31
1990	73.83	73.10	0.73	77.90	77.60	0.30
1991	74.10	73.50	0.60	78.34	77.90	0.44
1992	74.06	73.80	0.26	78.41	78.20	0.21
1993	74.76	74.00	0.76	78.31	78.30	0.01
1994	74.55	74.10	0.45	78.46	78.40	0.06
1995	74.35	74.20	0.15	78.46	78.60	-0.14
1996	74.84	74.50	0.34	78.65	78.90	-0.25
1997	75.60	74.90	0.70	79.14	79.10	0.04
1998	75.72	75.30	0.42	79.10	79.40	-0.30
1999	76.24	75.60	0.64	79.49	79.70	-0.21
2000	76.49	76.10	0.39	79.85	80.10	-0.25
2001	77.10	76.40	0.70	80.22	80.40	-0.18

Our complete life tables can be used to compute values of life annuities, perform a more accurate analysis of retirement security, and derive tables of multipliers for the purpose of assessing the amount of pecuniary loss in personal injury litigations for the general Singaporean population. The proposed method can also be applied, with certain modifications, in countries other than Singapore where complete period life tables are not available.

Further work is needed to develop a better understanding of the demography of Singapore. As mortality has been improving continuously over the years in Singapore, it will be interesting to explore the way in which projected mortality improvements can be built into specialized complete life tables used for annuities.

References

Bowers, Jr., N.L., Gerber, H.U., Hickman, J.C., Jones, D.A., Nesbitt, C. J. *Actuarial Mathematics.* Society of Actuaries, Schaumburg, IL, 1997.

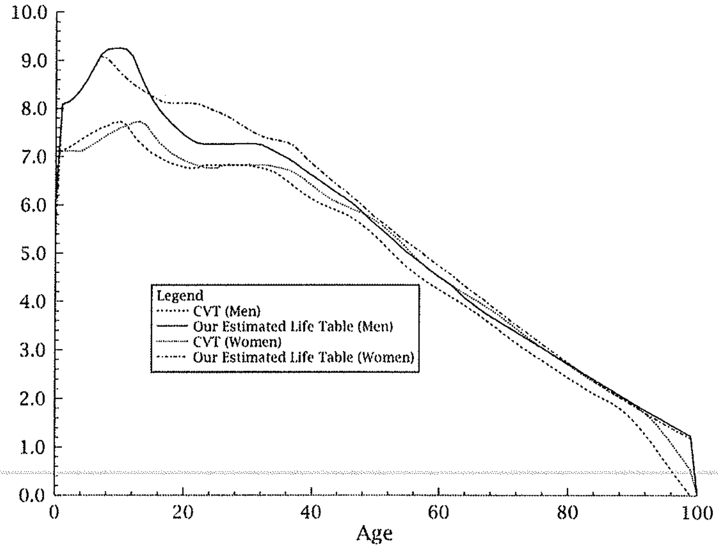

Figure 2: Male and Female $-\log(q_x)$ from Our Complete Life Table and CVT (1992)

Carter, L. and Lee, R. "Modeling and Forecasting U.S. Mortality." *Journal of the American Statistical Association* 87 (1992): 659-671.

Chen, R. and Wong, K.A. "The Adequacy of the CPF Account for Retirement Benefits in Singapore." *Singapore International Insurance and Actuarial Journal* 2 (1998): 121-138.

Coale, A. and Guo, G. "Revised Regional Model Life Tables at Very Low Levels of Mortality." *Population Index* 55 (1989): 613-643.

Coale, A. and Kisker, E. "Defects in Data on Old-Age Mortality in the United States: New Procedures for Calculating Mortality Schedules and Life Tables at the Highest Ages." *Asian and Pacific Population Forum* 4, no. 1 (1990): 1-31.

Guerrero, V.M., Juárez, R. and Poncela, P. "Data Graduation Based on Statistical Time Series Methods." *Statistics & Probability Letters* 52, no. 2 (2001): 169-175.

Heligman, L and Pollard, J.H. "The Age Pattern of Mortality." *Journal of the Institute of Actuaries* 107 (1980): 437-455.

London, D. *Graduation: The Revision of Estimates.* Winsted, CT: ACTEX Publications, 1985.

Lowrie, W.B. "Multidimensional Whittaker-Henderson Graduations with Constraints and Mixed Differences." *Transactions of the Society of Actuaries* (1993): 215-256.

Miller, M.D. *Elements of Graduation.* New York, NY: Actuarial Society of America and American Institute of Actuaries, 1946.

Olshansky, S.J. and Carnes, B.A. "Ever Since Gompertz." *Demography* 34, no. 1 (1997): 1-15.

Ramsay, C.M. "Minimum Variance Moving-Weighted-Average Graduation." *Transactions of the Society of Actuaries* (1991): 305-333.

Renshaw, A.E. "Graduation and Generalized Linear Models: An Overview." *City University, London, Actuarial Research Paper* 73 (1995).

Renshaw, A.E., Haberman, S., and Hatzopoulos, P. "The Modelling of Recent Mortality Trends in United Kingdom Male Assured Lives." *British Actuarial Journal* 2 (1996): 449-477.

Renshaw, A.E. and Hatzopoulos, P. "On the Graduation of 'Amounts'." *British Actuarial Journal* 2 (1995): 185-205.

Sarony, N., Chan, W.S. and Chan, F.W.H. *Personal Injury Tables Hong Kong: Tables for the Calculation of Damages.* Hong Kong: Sweet & Maxwell Asia, 2003.

Whittaker, E.T. "On a New Method of Graduation." *Proceedings of the Edinburgh Mathematical Society* 41 (1923): 63-75.