# Towards the Continuum Limit of the Overlap Quark Propagator in Landau Gauge * $\dagger$ 

J. B. Zhang ${ }^{\text {a }}$, F. D. R. Bonnet ${ }^{\text {a }}$, P. O. Bowman ${ }^{\text {b }}$, D. B. Leinweber ${ }^{\text {a }}$, A. G. Williams ${ }^{\text {a }}$<br>${ }^{a}$ CSSM Lattice Collaboration,<br>Special Research Center for the Subatomic Structure of Matter (CSSM) and Department of Physics and Mathematical Physics, University of Adelaide 5005, Australia<br>${ }^{\text {b }}$ Department of Physics and School for Computational Science and Information Technology, Florida State University, Tallahasse FL 32306, USA


#### Abstract

The properties of the momentum space quark propagator in Landau gauge are examined for the overlap quark action in quenched lattice QCD. Numerical calculations were done on two lattices with different lattice spacing $a$ and similar physical volumes to explore the quark propagator in the continuum limit. We have calculated the nonperturbative wavefunction renormalization function $Z(p)$ and the nonperturbative mass function $M(p)$ for a variety of bare quark masses and perform a simple linear extrapolation to the chiral limit. We find the behaviour of $Z(p)$ and $M(p)$ in the chiral limit are in good agreement between the two lattices.


## 1. Introduction

The quark propagator, is one of the fundamental quantities in QCD. By studying the momentum-dependent quark-mass function in the infrared region we can gain valuable insights into the mechanism of dynamical chiral symmetry breaking and the associated dynamical generation of mass. The ultraviolet behaviour of the propagator at large momentum can be used to extract the running quark mass .

There have been several studies of the momentum space quark propagator [1],2,3, [2, [5] using different gauge fixing and fermion actions. Here we focus on Landau gauge fixing and the overlapfermion action and extend previous work [5] to two lattices with different lattice spacings $a$ and very similar physical volumes. This allows us to probe the continuum limit of the quark propagator.

## 2. Quark Propagator on the Lattice

In a covariant gauge in the continuum the renormalized Euclidean-space quark propagator

[^0]must have the form
\[

$$
\begin{equation*}
S(\zeta ; p)=\frac{1}{i \not p A\left(\zeta ; p^{2}\right)+B\left(\zeta ; p^{2}\right)}=\frac{Z\left(\zeta ; p^{2}\right)}{i p p+M\left(p^{2}\right)} \tag{1}
\end{equation*}
$$

\]

where $\zeta$ is the renormalization point.
On the lattice the inverse lattice bare quark propagator takes the general form

$$
\begin{equation*}
\left(S^{\text {bare }}\right)^{-1}(p) \equiv i\left(\sum_{\mu} C_{\mu}(p) \gamma_{\mu}\right)+B(p) \tag{2}
\end{equation*}
$$

We use periodic boundary conditions in the spatial directions and anti-periodic in the time direction. The discrete momentum values for a lattice of size $N_{i}^{3} \times N_{t}$, with $n_{i}=1, . ., N_{i}$ and $n_{t}=1, . ., N_{t}$, are
$p_{i}=\frac{2 \pi}{N_{i} a}\left(n_{i}-\frac{N_{i}}{2}\right), \quad p_{t}=\frac{2 \pi}{N_{t} a}\left(N_{t}-\frac{1}{2}-\frac{N_{t}}{2}\right)$.
Defining the bare lattice quark propagator as
$S^{\text {bare }}(p) \equiv-i\left(\sum_{\mu} \mathcal{C}_{\mu}(p) \gamma_{\mu}\right)+\mathcal{B}(p)$,
we perform a spinor and color trace to identify
$\mathcal{C}_{\mu}(p)=\frac{i}{12} \operatorname{tr}\left[\gamma_{\mu} S^{\text {bare }}(p)\right], \quad \mathcal{B}(p)=\frac{1}{12} \operatorname{tr}\left[S^{\text {bare }}(p)\right]$.

The $C_{\mu}(p)$ and $B(p)$ in Eq.(2) can be written
$C_{\mu}(p)=\frac{\mathcal{C}_{\mu}(p)}{\mathcal{C}^{2}(p)+\mathcal{B}^{2}(p)}, \quad B(p)=\frac{\mathcal{B}(p)}{\mathcal{C}^{2}(p)+\mathcal{B}^{2}(p)},(6)$
where $\mathcal{C}^{2}(p)=\sum_{\mu}\left(\mathcal{C}_{\mu}(p)\right)^{2}$.
We can identify the appropriate kinematic lattice momentum $q$ directly from the definition of the tree-level quark propagator,
$q_{\mu} \equiv C_{\mu}^{(0)}(p)=\frac{\mathcal{C}_{\mu}^{(0)}(p)}{\left(\mathcal{C}^{(0)}(p)\right)^{2}+\left(\mathcal{B}^{(0)}(p)\right)^{2}}$.
Having identified the lattice momentum $q$, we can now define the bare lattice propagator as

$$
\begin{align*}
S^{\text {bare }}(p) & \equiv \frac{1}{i \not q A(p)+B(p)}=\frac{Z(p)}{i \not q+M(p)} \\
& =Z_{2}(\zeta ; a) S(\zeta ; p) \tag{8}
\end{align*}
$$

where $S(\zeta ; p)$ is the lattice version of the renormalized propagator in Eq. (1).

The overlap fermion formalism [6, 7] realizes an exact chiral symmetry on the lattice and is automatically $\mathcal{O}(a)$ improved. The massless coordinate-space overlap-Dirac operator can be written in dimensionless lattice units as
$D(0)=\frac{1}{2}\left[1+\gamma_{5} \epsilon\left(H_{w}\right)\right]$,
where $\epsilon\left(H_{w}\right)$ is the matrix sign function, where $H_{w}(x, y)=\gamma_{5} D_{w}(x, y)$ is the Hermitian WilsonDirac operator and where $D_{w}$ is the usual WilsonDirac operator on the lattice. However, in the overlap formalism the Wilson mass parameter $m_{w}$ enters with a negative sign.

The massless overlap quark propagator is given by
$S^{\text {bare }}(0)=\frac{1}{2 m_{w}}\left[D^{-1}(0)-1\right]=\frac{1}{2 m_{w}} \tilde{D}^{-1}(0)$.
This definition of the massless overlap quark propagator follows from the overlap formalism [8] and ensures that the massless quark propagator anticommutes with $\gamma_{5}$, i.e., $\left\{\gamma_{5}, S^{\text {bare }}(0)\right\}=0$ just as it does in the continuum [8]. The tree-level momentum-space massless quark propagator defines the kinematic lattice momentum $q$,
$S^{\text {bare }}(0, p) \equiv \tilde{D}_{c}^{-1}(0, p) \rightarrow S^{(0)}(0, p)=\frac{1}{i \not q q}$,

Table 1
Lattice parameters.

| Action | Volume | $\beta$ | $a(\mathrm{fm})$ | $u_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| Improved | $12^{3} \times 24$ | 4.60 | 0.125 | 0.88888 |
| Improved | $8^{3} \times 16$ | 4.286 | 0.194 | 0.87209 |

We can obtain $q$ numerically and analytically from the tree-level massless quark propagator [5].

Having identified the massless quark propagator in Eq. (10), we can construct the massive overlap quark propagator by simply adding a bare mass to its inverse, i.e.,
$\left(S^{\text {bare }}\right)^{-1}\left(m^{0}\right) \equiv\left(S^{\text {bare }}\right)^{-1}(0)+m^{0}$.

## 3. Numerical results

Here we work on two lattices with different lattice spacing $a$ and very similar physical volumes using a tadpole-improved plaquette plus rectangle gauge action. For each lattice size, we use 50 configurations . Lattice parameters are summarized in Table 1.
Our calculations use $\kappa=0.19$ for lattice 1 $\left(12^{3} \times 24\right)$ and $\kappa=0.1864$ for lattice $2\left(8^{3} \times 16\right)$ to make $m_{w} a=1.661$ on both lattices. We calculate at ten bare quark masses in physical units of $m^{0}=126,147,168,210,252,315,420,524$, 629 , and 734 MeV respectively.

The results of lattice 1 are presented in detail in Ref. [5]. It is satisfying that the results of lattice 2 are similar to those of lattice 1. Here we focus on the comparision of the results on these two lattices. All data has been cylinder cut (5] and extrapolated to the chiral limit using a simple linear chiral extrapolation . The mass function $M(p)$ for the two lattices is plotted in Fig. 1 and the renormalization function $Z(p)$ of the two lattices is plotted in Fig. 2. We can see that when the mass function $M(p)$ is plotted against the discrete lattice momentum $p$ the results of the two lattices are in good agreement, while for the renormalization function $Z(p)$, good agreement is reached on the two lattices if it is plotted against the kinematical lattice momentum $q$.


Figure 1. Comparison of the mass function $M(p)$ of two lattices in the chiral limit. The upper graph is plotted against the discrete lattice momentum $p$ and the lower graph is plotted against the kinematical lattice momentum $q$.

## 4. Summary

In this report, we use tadpole-improved quenched lattice configurations, and the overlap fermion operator with the Wilson overlap kernel. The momentum space quark propagator is calculated in Landau gauge on two lattices with different lattice spacing $a$ and very similar physical volumes to explore the continuum limit. We calculate the nonperturbative momentum-dependent wavefunction renormalization function $Z(p)$ and the nonperturbative mass function $M(p)$ for a variety of bare quark masses and perform a simple linear extrapolation to the chiral limit. We have seen that, the continuum limit for, $Z(p)$, is most rapidly approached when it is plotted against the kinematical lattice momentum $q$, whereas for the quark mass function, $M(p)$, we should plot against the discrete lattice momentum $p$. The agreement between the two lattices suggests that we are close to the continuum limit.


Figure 2. Comparison the renormalization function $Z^{(\mathrm{R})}(p)$ of two lattices in the chiral limit. The upper graph is plotted against the discrete lattice momentum $p$ and the lower graph is plotted against the kinematical lattice momentum $q$.

## REFERENCES

1. J. I. Skullerud and A. G. Williams, Phys. Rev. D 63, 054508 (2001) ; Nucl. Phys. Proc. Suppl. 83, 209 (2000) .
2. J. Skullerud, D. B. Leinweber and A. G. Williams, Phys. Rev. D 64, 074508 (2001) .
3. P. O. Bowman, U. M. Heller and A. G. Williams, Nucl. Phys. B (Proc. Suppl.) 106, 820 (2002) .
4. T. Blum et al., Phys. Rev. D66,014504(2002).
5. F. D. R. Bonnet, P. O. Bowman, D. B. Leinweber, A. G. Williams and J. B. Zhang, Phys. Rev. D 65, 114503 (2002).
6. R. Narayanan and H. Neuberger Nucl. Phys. B443, 305 (1995).
7. H. Neuberger, Phys. Lett. B427, 353(1998).
8. R. Narayanan and H. Neuberger, Nucl. Phys. B 443, 305 (1995) .
9. R.G. Edwards, U.M. Heller, and R. Narayanan, Phys. Rev. D 59, 094510(1999).

[^0]:    *Presented by J. B. Zhang at Lattice 2002
    ${ }^{\dagger}$ This work is supported by the Australia Research Council

