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FLIC-Overlap Fermions and Topology

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APE smearing the links in the irrelevant operators of clover fermions (Fat-Link Irrelevant Clover (FLIC) fermions) provides significant improvement in the condition number of the Hermitian-Dirac operator and gives rise to a factor of two savings in computing the overlap operator. This report investigates the effects of using a highly-improved definition of the lattice field-strength tensor $F_{\mu\nu}$ in the fermion action, made possible through the use of APE-smeared fat links in the construction of the irrelevant operators. Spurious double-zero crossings in the spectral flow of the Hermitian-Wilson Dirac operator associated with lattice artifacts at the scale of the lattice spacing are removed with FLIC fermions composed with an $\mathcal{O}(a^4)$-improved lattice field strength tensor. Hence, FLIC-Overlap fermions provide an additional benefit to the overlap formalism: a correct realization of topology in the fermion sector on the lattice.

1. SPECTRAL FLOW

Overlap fermions [1] are a realisation of chiral symmetry on the lattice. Given some reasonable Hermitian-Dirac operator H, we can deform H into a chiral action through the overlap formalism,

$$D_o = \frac{1}{2}\big(1 + \gamma_5\,\epsilon(H)\big), \quad \epsilon(H) = \frac{H}{\sqrt{H^2}}\,, \tag{1}$$

and H will be referred to as the overlap kernel.

Examinations of the spectral flow of the overlap kernel can shed considerable light on the nature of the associated overlap operator. Consider for example the spectral flow of Figure 1 for overlap fermions based on the standard Wilson kernel, reproduced from Ref. [2]. As the regulator-mass parameter of the Wilson kernel is varied over the doubler-free region ($0 < m < 2$ at tree level) zero crossings occur, signaling the point at which the associated overlap operator becomes sensitive to the topology giving rise to the zero mode. The sign of the slope of the kernel flow at the zero crossing indicates the sign of the topological charge giving rise to the zero mode. As such the flow should only cross zero once.

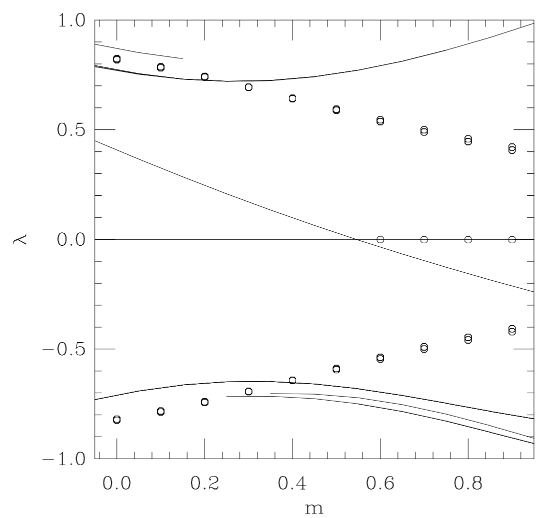

Figure 1. Low-lying eigenmode flow of the Hermitian-Wilson kernel (curves) and the flow of the associated overlap operator (circles) from Ref. [2].

The position of the zero crossing reflects the size of the underlying topological object [3]. Smaller objects give rise to zero crossings at larger regulator mass m.

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2. SPURIOUS DOUBLE CROSSINGS

In our examinations of the spectral flow of the standard Wilson kernel [3] and the correlation of the eigenmodes with their underlying topological gauge field structures, we encountered a few configurations in which the spectral flow of a single mode would cross zero twice in the doubler free region. Figure 2 illustrates the low-lying eigenmode spectrum for two configurations which display a double crossing in one of the modes. The relation between the hopping parameter κ and the regulator mass of Fig. 1 is $\kappa = 1/(-2\,m + 8\,r)$ such that the tree-level doubler-free region is $1/8 < \kappa < 1/4$ for $r = 1$. The $SU(3)$ gauge-field configurations are generated on a $16^3 \times 32$ lattice with a mean-field improved plaquette plus rectangle action at $\beta = 4.60$ providing a lattice spacing of 0.122(2) fm. Modes are tracked in the spectral flow by examining their position in space-time. Modes belonging to the same underlying topological structure are plotted with a common symbol in the spectral flow plots. Our experience is that about one in five configurations displays such a double crossing.

Figure 3 displays the spectral flow for the same configurations but this time evaluated with the Fat-Link Irrelevant Clover (FLIC)-fermion kernel [4,5] constructed with four-sweep APE-smeared fat links with a smearing fraction [6] of $\alpha = 0.7$. FLIC fermions provide an effective alternative to the Wilson kernel as they have an improved condition number which gives rise to a factor of two reduction in compute time. Here the double crossing is removed for one configuration but survives for the other.

The plaquette-based clover estimate of the lattice field-strength tensor $F_{\mu\nu}$ is known to be rather poor. The associated topological charge typically differs from integer values by 10%, even on very smooth configurations [6]. Since the irrelevant operators of FLIC fermions are constructed from APE-smeared links, one may use highly-improved $\mathcal{O}(a^4)$-improved definitions of the lattice field strength tensor [7]; $\mathcal{O}(g^2\,a^2)$ errors are removed in the APE-smearing process. Figure 4 illustrates the spectral flow obtained from FLIC fermions incorporating a three-loop $\mathcal{O}(a^4)$-improved definition of the lattice field strength tensor [7]. The spurious double crossings are eliminated. It is perhaps interesting to note that the double-crossings can be eliminated with the Wilson kernel provided the irrelevant Wilson term is constructed with fat links. However, a factor of two speedup is not realized for the fat-Wilson kernel.

3. CONCLUSION

We have been successful in creating spectral flows with a spurious double crossing [8] by cooling t'Hooft-ansatz instantons on the lattice until their topological charge is reduced to 0.4. This suggests to us that the observed double crossings seen with the standard Wilson kernel are simply lattice artifacts associated with dislocations. The fact that the zero crossings always occur at very large values of the regulator parameter further confirms that these double crossings are spurious lattice artifacts due to physics at the scale of the lattice spacing and should be removed. The three-loop improved FLIC fermion kernel achieves this goal.

REFERENCES

1. R. Narayanan and H. Neuberger, Nucl. Phys. B443 (1995) 305, hep-th/9411108.
2. R.G. Edwards, U.M. Heller and R. Narayanan, Nucl. Phys. B540 (1999) 457, hep-lat/9807017.
3. D. J. Kusterer, J. Hedditch, W. Kamleh, D. B. Leinweber and A. G. Williams, Nucl. Phys. B **628** (2002) 253 [arXiv:hep-lat/0111029].
4. CSSM Lattice, J.M. Zanotti et al., Phys. Rev. D65 (2002) 074507, hep-lat/0110216.
5. W. Kamleh et al., Phys. Rev. D66 (2002) 014501, hep-lat/0112041.
6. F. D. Bonnet, P. Fitzhenry, D. B. Leinweber, M. R. Stanford and A. G. Williams, Phys. Rev. D **62** (2000) 094509 [arXiv:hep-lat/0001018].
7. S. O. Bilson-Thompson, D. B. Leinweber and A. G. Williams, arXiv:hep-lat/0203008.
8. D. Kusterer, *et al.*, in preparation.

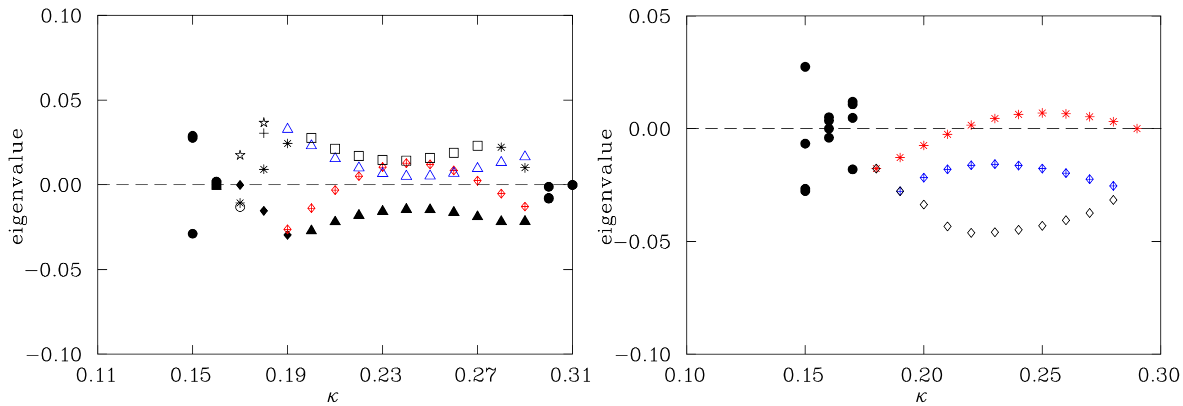

Figure 2. Low-lying eigenmode flow of the standard Hermitian-Wilson kernel for two selected configurations displaying double crossings of a single mode.

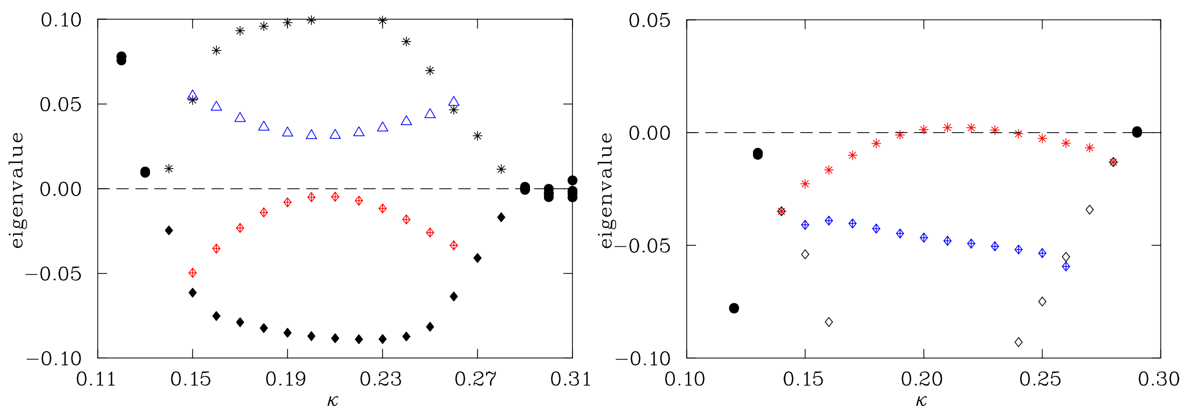

Figure 3. Low-lying eigenmode flow of the Hermitian-FLIC kernel with $F_{\mu\nu}$ estimated via the standard clover-plaquette links paths. Selected configurations are as in Fig. 2.

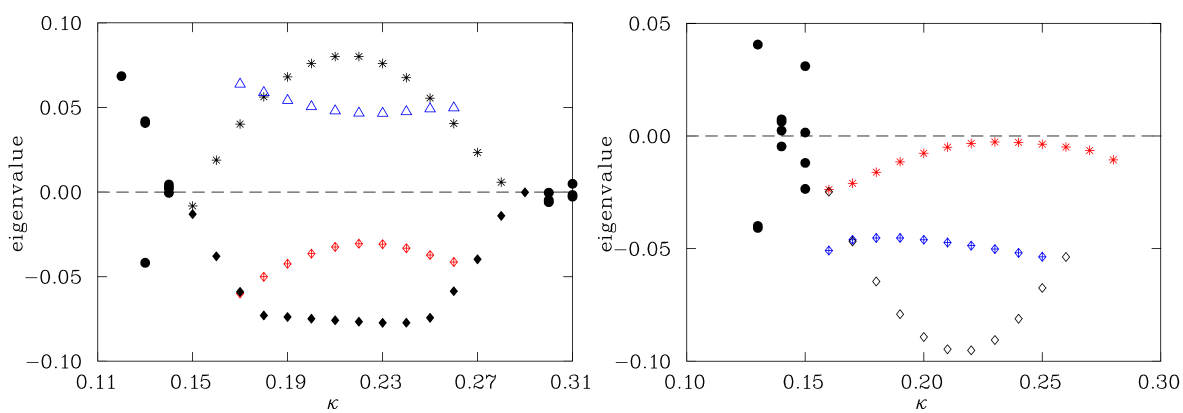

Figure 4. Low-lying eigenmode flow of the Hermitian-FLIC kernel with $F_{\mu\nu}$ determined via the $\mathcal{O}(a^4)$-improved three-loop definition [7]. Selected configurations are as in Figs. 2 and 3.