ADP-02-89/T528

FLIC Overlap Fermions

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The action of the overlap-Dirac operator on a vector is typically implemented indirectly through a multi-shift conjugate gradient solver. The compute-time required depends upon the condition number, κ, of the matrix that is used as the overlap kernel. While the Wilson action is typically used as the overlap kernel, the FLIC (Fat Link Irrelevant Clover) action has an improved condition number and provides up to a factor of two speedup in evaluating the overlap action. We summarize recent progress on the use of FLIC overlap fermions.

1. Introduction

Overlap fermions [1] are a realisation of chiral symmetry on the lattice. Given some reasonable Hermitian-Dirac operator H, we can deform H into a chiral action through the overlap formalism,

$$D_o = \frac{1}{2}\big(1 + \gamma_5\,\epsilon(H)\big), \quad \epsilon(H) = \frac{H}{\sqrt{H^2}}. \tag{1}$$

Unfortunately, the matrix sign function $\epsilon(H)$ is difficult to evaluate and is typically approximated by a sum over poles [2] which can be evaluated using a multi-shift conjugate gradient (CG) solver [3]. This is an iterative approximation where the number of iterations for a given accuracy increases with the condition number of the kernel, $\kappa(H) = |\lambda_{\max}/\lambda_{\min}|$.

Usually the Hermitian Wilson-Dirac operator is used as the overlap kernel. Its low-lying spectrum is characterised by a handful of isolated eigenmodes which can be very small, increasing the condition number, κ, unacceptably. These eigenmodes can be projected out of the basic operator, reducing its condition number to a numerically acceptable level, and then dealt with explicitly [4]. Unfortunately, as the spectrum rapidly becomes dense, projecting out low-lying modes can only help one so far.

An alternative is to use a kernel with an improved spectrum, that is, where the region of dense modes is shifted away from zero. The FLIC (Fat Link Irrelevant Clover) [5,6] action possesses this property.

2. Spectral Flow and Condition Number

Spectral flow diagrams give us a good comparison of the two different kernels, FLIC and Wilson, as they allow us to see directly the difference in the qualitative structure of the low-lying spectra of the two actions [6]. Figure 1 displays the flow of the lowest 15 eigenvalues as a function of m for an ensemble of 10 mean-field improved Symanzik configurations at $\beta = 4.60$ and size $12^3 \times 24$, with $a = 0.122(2)$. As we are interested in the magnitude of the low-lying values rather than their sign, we illustrate $|\lambda|$ vs m.

We see the Wilson spectrum is very poor, with a high density of very small eigenmodes and no gap away from zero. By contrast, the FLIC action (which has a clover term, and irrelevant operators constructed from four-sweep APE-smeared links) possesses a spectrum which is clearly superior to that of the Wilson. Not only has the density of very small modes been significantly reduced, the region where the spectrum becomes dense has been shifted much further away from zero.

This improvement in the spectrum results in the FLIC action being much better conditioned than the Wilson action. In Figure 2 we examine the condition number, κ, of the two actions as a function of m. The condition number is calculated after having projected out the lowest 15 eigenmodes on the 2 lattices that are shown. The

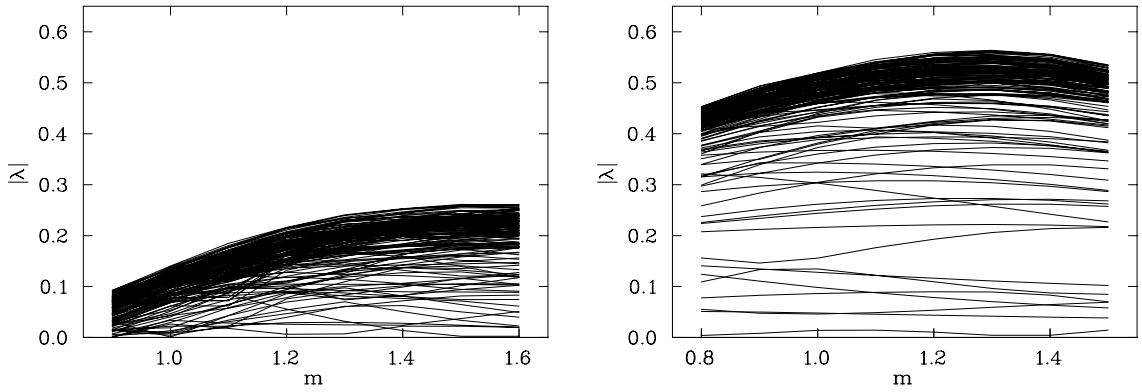

Figure 1. Spectral flow of the Wilson action (left) and the FLIC action (right) at $\beta = 4.60$.

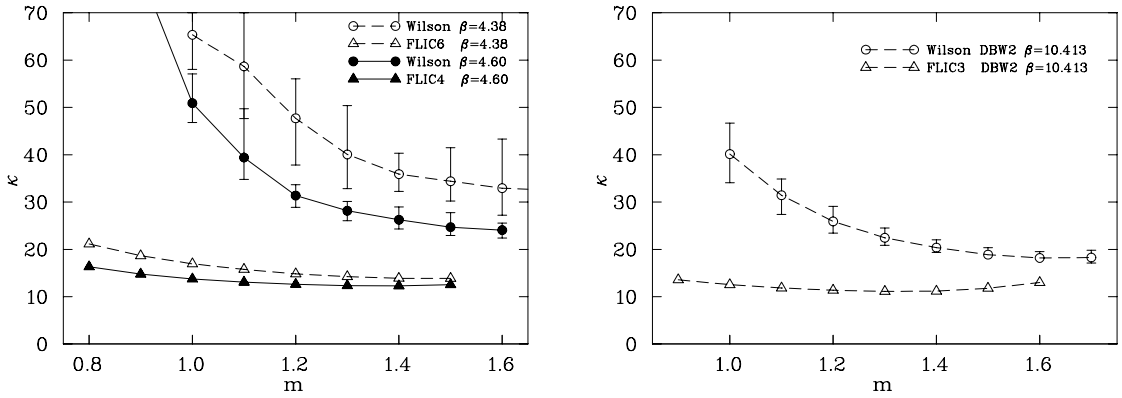

Figure 2. Comparison of the condition number of the FLIC and Wilson fermion actions. Symanzik improved glue (left plot) at both $a = 0.165$ and $a = 0.122$ with 15 modes projected out, and DBW2 glue (right plot) at $a = 0.165$ with 10 modes projected out are illustrated.

finer lattice is the same as the one used in the spectral flow plots, and the coarser lattice uses the same gluonic action, but is an $8^3 \times 16$ lattice at $\beta = 4.38$, corresponding to a lattice spacing of a=0.165(2). The points are the mean condition numbers across the ensembles, and the error bars indicate the minimum and maximum condition numbers, giving an idea of the variation in κ.

3. Gluonic Action

There have been suggestions to accelerate the computation of the sign function by using non-pertubatively improved gauge actions [7]. Our results are based on Symanzik improved gauge configurations and further improvements arising from the use of the FLIC action are in addition to that of using improved gluon-field configurations. This is verified by performing a similar analysis on gluonic configurations using a Monte-Carlo Renormalisation Group improved action. Some preliminary results of this investigation using DBW2 glue [8] are displayed in Figure 2, with a full report given elsewhere [9]. We note that at a fixed lattice spacing, DBW2 glue improves the condition number for both actions, although the effect is much more pronounced for the Wilson action.

4. Compute Time

Saving iterations (by reducing the condition number) does not necessarily reduce the most important quantity, compute time. Shifting from a standard Wilson action to a partially smeared action means that we now have two sets of gauge fields, the standard and smeared links. Additionally, the standard spin-projection trick is no longer applicable, possibly providing an additional factor of two in compute time needed. However, it can be shown that the spin projection trick can be generalised to include partially smeared actions as well [10]. This results in paying at most a single factor of two compute-time for a FLIC-fermion matrix-vector multiplication. As there is significant additional expense in the evaluation of the overlap sign function, we get to keep the majority of the speedup gained by reducing the number of iterations [6]. This results in FLIC-Overlap fermions being approximately twice as fast as the standard Wilson-kernel formulation.

5. Physical Results

Regardless of the kernel used, all overlap fermions are free from $O(a)$ errors. However, different kernels may in general produce actions which differ at $O(a^2)$. As a first investigation into this matter, we have calculated the quark propagator in momentum space using FLIC-Overlap fermions, essentially performing the same calculation that has been done earlier with the standard overlap action [11]. The chirally extrapolated mass function is shown in Figure 3, with a full report given elsewhere [12].

6. Conclusion

In the overlap formalism one is free to choose the argument of the sign function, the overlap kernel, so long as one uses a reasonable Dirac operator. The standard kernel choice is the Wilson action. By choosing the FLIC action, one can obtain a significant reduction in the compute time needed for overlap fermions. We have reviewed the work that has been done to date using FLIC Overlap fermions, including some preliminary results into the physical structure of the quark propagator.

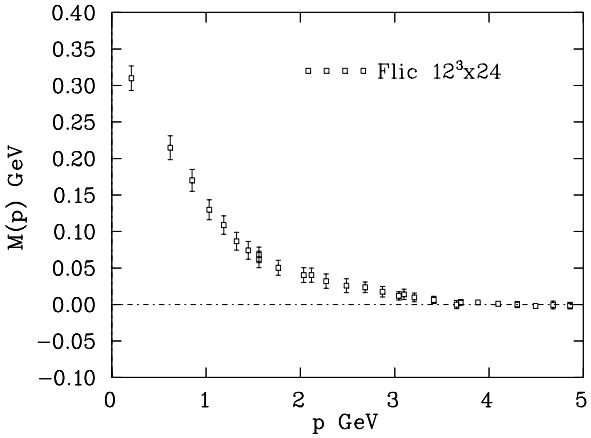

Figure 3. The chirally extrapolated mass function as a function of p, at $\beta = 4.60, L = 12^3 \times 24$.

REFERENCES

1. R. Narayanan and H. Neuberger, Nucl. Phys. B443 (1995) 305, hep-th/9411108.
2. H. Neuberger, Phys. Rev. Lett. 81 (1998) 4060, hep-lat/9806025.
3. R.G. Edwards, U.M. Heller and R. Narayanan, Nucl. Phys. B540 (1999) 457, hep-lat/9807017.
4. R.G. Edwards, U.M. Heller and R. Narayanan, (1999), hep-lat/9905028.
5. CSSM Lattice, J.M. Zanotti et al., Phys. Rev. D65 (2002) 074507, hep-lat/0110216.
6. W. Kamleh et al., Phys. Rev. D66 (2002) 014501, hep-lat/0112041.
7. R.G. Edwards and U.M. Heller, Phys. Rev. D65 (2002) 014505, hep-lat/0105004.
8. A. Borici and R. Rosenfelder, Nucl. Phys. Proc. Suppl. 63 (1998) 925, hep-lat/9711035.
9. W. Kamleh, D.B. Leinweber and A.G. Williams, In preparation.
10. W. Kamleh, hep-lat/0209154.
11. CSSM Lattice, F.D.R. Bonnet et al., Phys. Rev. D65 (2002) 114503, hep-lat/0202003.
12. W. Kamleh et al., In preparation.