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Near-thermal radiation in detectors, mirrors, and black holes: A stochastic approach

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In analyzing the nature of thermal radiance experienced by an accelerated observer (Unruh effect), an eternal black hole (Hawking effect), and in certain types of cosmological expansion, one of us proposed a unifying viewpoint that these can be understood as arising from the vacuum fluctuations of the quantum field being subjected to an exponential scale transformation in these systems. This viewpoint, together with our recently developed stochastic theory of particle-field interaction understood as quantum open systems described by the influence functional formalism, can be used effectively to address situations where the spacetime possesses an event horizon only asymptotically, or none at all. Examples studied here include detectors moving at uniform acceleration only asymptotically or for a finite time, a moving mirror, and a two-dimensional collapsing mass. We show that in such systems radiance indeed is observed, albeit not in a precise Planckian spectrum. The deviation therefrom is determined by a parameter which measures the departure from uniform acceleration or from exact exponential expansion. The methodology illustrated here is expected to be useful for the investigation of nonequilibrium black hole thermodynamics and the linear response regime of back reaction problems in semiclassical gravity. [S0556-2821(97)07308-6]

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I. INTRODUCTION

Particle production [1] with a thermal spectrum from black holes [2–4], moving mirrors [5], accelerated detectors [6], observers in a de Sitter universe [7], and certain cosmological spacetimes [8] has been a subject of continual discussion since the mid 1970's because of its extraordinary nature and its basic theoretical value. The mainstream approaches to these problems rely on thermodyamic arguments [9,10], finite-temperature field theory techniques [11–13], or geometric constructions (event horizon as a global property of spacetime) [14] or pairwise combinations thereof. (The status of work on quantum field theory in curved spacetimes up to 1980 can be found in [15].) The 1980's saw attempts and preparations for the back reaction problem [16] (for cosmological back reaction problems, see [17]), i.e., the calculation of the energy-momentum tensor (see [18] and earlier references), the effect of particle creation on a black hole (in a box, to ensure quasiequilibrium with its radiation) [14], and the dynamical origin of black hole entropy [19]. These inquiries are mainly confined to equilibrium thermodynamics or finite-temperature field theory conditions.[1] To treat problems of a dynamical nature such as the backreaction of Hawking radiation on black hole collapse, one needs a new conceptual framework and a more powerful formalism for tackling nonequilibrium conditions and high-energy (trans-Planckian) processes. A new viewpoint which stresses the local, kinematic nature of these processes rather than the traditional global geometric properties has been proposed [31–34] which regards the Hawking-Unruh thermal radiance observed in one vacuum as resulting from exponential redshifting of quantum noise of another. This view puts the nature of thermal radiance in the two classes of spacetimes on the same footing [35] and empowers one to tackle situations which do not possess an event horizon at all, as the examples in this and a companion paper will show.

Such a formalism of statistical field theory has been developed by one of us and co-workers in recent years [36–40]. This approach aims to provide the statistical mechanical underpinnings of quantum field theory in curved spacetime, and strives at a microscopic and elemental description of the structure and dynamics of matter and spacetime. The starting point is the quantum and thermal fluctuations for fields, and the focus is on the evolution of the reduced density matrix of an open system (or the equivalent Wigner distribution functions); the quantities of interest are the noise and dissipation kernels contained in the influence functional [42], and the equation of motion takes the form of a master, Langevin, Fokker-Planck, or stochastic Schrödinger equation describing the evolution of the quantum statistical state of the system, including, in addition to the quantum field effects like

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[1]Among other notable alternatives, we would like to mention Sciama's dissipative system approach [20], Unruh's [21] and Jacobson's work on sonic black holes [22], Zurek and Thorne's degree of freedom counts [23], Sorkin's geometric or ''entanglement'' entropy [24,25] (see also [26]), the Bekenstein-Page information theory approach [27,28], the views expressed by Stephens, 't Hooft, and Whiting [29], and most significantly, the string-theoretical origin of black hole entropy [30].

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radiative corrections and renormalization, also statistical dynamical effects like decoherence, correlation, and dissipation. Since it contains the causal (Schwinger-Keldysh) effective action [43], it is a generalization of the traditional scheme of thermal field theory [13] and the ''in-out'' (Schwinger-DeWitt) effective action [44], and is particularly suited for treating fluctuations and dissipation in back reaction problems from semiclassical gravity [40] to mesoscopic physics [41].

The foundation of this approach has been constructed recently based on open system concepts and the quantum Brownian model [45,46]. The method has since been applied to particle creation and back reaction processes in cosmological spacetimes [47–49]. For particle creation in spacetimes with event horizons, such as for an accelerated observer and black holes, this method derives the Hawking-Unruh effect [50,46] from the viewpoint of exponential amplification of quantum noise [34]. It can also describe the linear response regime of back reaction viewed in the context of a fluctuation-dissipation relation [49,52].

This paper is a continuation of our earlier work [46,50,52] to present *an approximation scheme* to show that near-thermal radiation is emitted from systems undergoing near-uniform acceleration or in slightly perturbed spacetimes. We wish to demonstrate the relative ease to treat such problems using quantum field theory methods aided by statistical-mechanical considerations. This approach also highlights *a unified viewpoint* towards thermal particle creation from spacetimes with and without event horizons based on the interpretation that the thermal radiance can be viewed as resulting from quantum noise of the field being amplified by an exponential scale transformation in these systems (in specific vacuum states) [34]. In contradistinction to viewing these as global, geometric effects, this viewpoint emphasizes the kinematic effect of scaling on the vacuum in altering the relative weight of quantum versus thermal fluctuations [35].

It may appear that this approximation scheme can be equally implemented by taking the conventional viewpoints (notably the geometric viewpoint), and the perturbative calculation can be performed by other existing methods (notably the thermal field theory method). But as we will show here, it is not as easy as it appears. *Conceptually*, the geometric viewpoint assumes that a sufficient condition for the appearance of Hawking radiation is the existence of an event horizon, which is considered as a global property of the spacetime or the system.[2] When the spacetime deviates from the eternal black hole or that the trajectory deviates from the uniformly accelerated one, physical reasoning tells us that the Hawking or Unruh radiation should still exist, albeit with a nonthermal spectrum. But even if an event horizon exists for the perturbed spacetime, it may not be so easily describable in geometric terms. And for time-dependent perturbations of lesser symmetry or for situations where uniform acceleration occurs only for a finite interval of time, it is not easy to deduce the form of Hawking radiation in terms of purely global geometric quantities (see, however, Wald [53] and Bonados *et al.* [54]). The concept of an approximate event horizon, which exists for a finite period of time or only asymptotically, is difficult to define and, even if it is possible (by apparent horizons, e.g., [55]), rather unwieldy to implement in the calculation of particle creation and back reaction effects.

Technically one may think calculations via the thermal field theory are equally possible. Indeed this has been tried before by one of us and others. One way is to assume a quasiperiodic condition on the Green function, making it near thermal [13]. But this is not a good solution, as the deviation from eternal black hole or uniform acceleration disables the Euclidean section in the spacetimes (Kruskal or Rindler), and the imaginary-time finite-temperature theory is not well defined any more. Besides, to deal with the statistical dynamics of the system, one should use an in-in boundary condition and work with causal Green functions. The lesson we learned from treating the back reaction problems of particle creation in cosmological spacetimes [36,47] is that one can no longer rely on methods which are restricted to equilibrium conditions (like the imaginary-time or thermofield dynamics methods), but should use nonequilibrium methods such as the Schwinger-Keldysh (closed time path) effective action [43] for the treatment of dynamical back reactions. Its close equivalent, the influence functional method [42], is most appropriate for investigating the quantum-statistical dynamics of matter and geometry, like the entropy of quantum fields and spacetimes, information flow, coherence loss, etc. [56].

In this paper we shall use these methods to analyze particle creation in perturbed situations whose background spacetime possesses an event horizon, such as an asymptotically uniformly accelerated detector (Sec. II), or one accelerated in a finite time interval (Sec. III), the moving mirror, and the asymptotically Schwarzschild spacetime (Sec. IV). In a follow-up paper [57] we shall study near-thermal particle creation in an exponentially expanding universe, a slow-roll inflationary universe, and a universe in asymptotically exponential expansion. What ties the problem of thermal radiation in cosmological as well as black hole spacetimes together is the exponential scale-transformation viewpoint expressed earlier [31–35]. The stochastic field theory approach is capable of implementing this view. One can describe all these systems with a single parameter measuring the deviation from uniformity or stationarity, and show that the same parameter also appears in the near-thermal behavior of particle creation in all these systems. This result is relevant to our exploration of the linear-response regime of the back reaction problem in semiclassical gravity.

A. Stochastic approach

Consider a particle detector linearly coupled to a quantum field. The dynamics of the internal coordinate Q of the detector can be described by Langevin equations of the form [46]

[2]Note that for an extreme Reissner-Nordström spacetime this is not the case, as there exists an event horizon but no radiation. This example arose in the discussion between Hu and Unruh (private communication) who shared the somewhat unconventional view that the exponential redshifting is a more basic mechanism than event horizons responsible for thermal radiance.

$$\frac{\partial L}{\partial Q}-\frac{d}{dt}\frac{\partial L}{\partial \dot{Q}}-2\int_{t_i}^{t}\mu(t,s)Q(s)ds=\xi(t), \qquad (1.1)$$

where $\xi(t)$ is a stochastic force with correlator $\langle\xi(t)\xi(t')\rangle=\hbar\nu(t,t')$. The overdot denotes a derivative with respect to t. The trajectory $x^\mu(s)$ of the detector, parametrized by a suitable parameter s, will be denoted simply by $x(s)$ for convenience.

For the special case of linear coupling between a field ϕ and the detector of the form $L_{\rm int}=eQ\phi(x(s))$, the kernels μ and ν, called the *dissipation* and *noise* kernels, respectively, are given by

$$\mu(s,s')=\frac{e^2}{2}G(x(s),x(s'))\equiv-i\frac{e^2}{2}\langle[\hat{\phi}(x(s)),\hat{\phi}(x(s'))]\rangle, \qquad (1.2)$$

$$\nu(s,s')=\frac{e^2}{2}G^{(1)}(x(s),x(s'))\equiv\frac{e^2}{2}\langle\{\hat{\phi}(x(s)),\hat{\phi}(x(s'))\}\rangle, \qquad (1.3)$$

where G and $G^{(1)}$ are the Schwinger and the Hadamard functions of the free field operator $\hat{\phi}$ evaluated for two points on the detector trajectory, angular brackets denote the expectation value with respect to a vacuum state at some arbitrarily chosen initial time t_i, and square and curly brackets denote the commutator and anticommutator, respectively. This result may be obtained either by integrating out the field degrees of freedom as in the Feynman-Vernon influence functional approach [40] or via manipulations of the coupled detector-field Heisenberg equations of motion in the canonical operator approach.

It will often be convenient to express the kernels μ and ν as the real and imaginary parts of a complex kernel $\zeta\equiv\nu+i\mu$, called the *influence kernel*. For linear couplings, it follows from the above expressions that ζ is given by the Wightman function G^+:

$$\zeta(s,s')=e^2G^+(x(s),x(s'))\equiv e^2\langle\hat{\phi}(x(s))\hat{\phi}(x(s'))\rangle. \qquad (1.4)$$

The influence kernel thus admits the mode function representation

$$\zeta(s,s')=e^2\sum_k u_k(x(s))u_k^*(x(s')), \qquad (1.5)$$

the u_k's being the mode functions satisfying the field equations and defining the particular Fock space whose vacuum state is the one chosen above. Note that this method of evaluating the kernels μ and ν is only applicable for linear coupling cases.

An alternative approach [46] consists of decomposing the field Lagrangian into parametric oscillator Lagrangians at the very outset, thus converting a quantum-field-theoretic problem to a quantum-mechanical one. Denoting the kth parametric oscillator degrees of freedom by q_k and their masses and frequencies by m_k and ω_k, respectively, the detector-field interaction mentioned earlier is generally given by $L_{\rm int}=\Sigma_k c_k(s)Qq_k$, where the coupling ''constants'' c_k now become time dependent and contain information about the detector trajectory. In this approach, the influence kernel is given in terms of the oscillator mode functions X_k as

$$\zeta(s,s')=\int_0^\infty dkI(k,s,s')X_k(s)X_k^*(s'), \qquad (1.6)$$

where the X_k's satisfy the parametric oscillator equations

$$\ddot{X}_k+\omega_k^2(t)X_k=0, \qquad (1.7)$$

with initial conditions $X_k(t_i)=1$ and $\dot{X}_k(t_i)=-i\omega_k(t_i)$. The *spectral density* function $I(k,s,s')$ is defined as

$$I(w,s,s')=\sum_k\delta(\omega-\omega_k)\frac{c_k(s)c_k(s')}{2m_k(t_i)\omega_k(t_i)}. \qquad (1.8)$$

One may decompose the influence kernel into its real and imaginary parts, thus obtaining the dissipation and noise kernels:

$$\mu(s,s')=\frac{i}{2}\int_0^\infty dkI(k,s,s')[X_k^*(s)X_k(s')-X_k^*(s')X_k(s)], \qquad (1.9)$$

$$\nu(s,s')=\frac{1}{2}\int_0^\infty dkI(k,s,s')[X_k^*(s)X_k(s')+X_k^*(s')X_k(s)]. \qquad (1.10)$$

By expressing the field as a collection of parametric oscillators, it can be explicitly verified that the two approaches mentioned above lead to the same result for the influence kernel ζ. For the purpose of calculating it in a specific case, we will find it more convenient to use the second approach.

To study the thermal properties of the radiation measured by a detector, the influence kernel is compared to that of a thermal bath of static oscillators each in a coherent state [46]:

$$\zeta=\int_0^\infty dkI_{\rm eff}(k,\Sigma)[C_k(\Sigma)\cos k\Delta-i\sin k\Delta], \qquad (1.11)$$

where

$$\Sigma=(t+t')/2,\quad \Delta=t-t',$$

and for a thermal bath at temperature $T=\beta^{-1}$, the function $C_k=\coth(\hbar k/2k_BT)$. We will show in the specific cases discussed below that the unknown function C_k indeed has a coth form in the leading order, and can then deduce the temperature of the radiation seen by the detector. Here $I_{\rm eff}(k,\Sigma)$ is the effective spectral density, also to be determined by formal manipulations of Eq. (1.6). We can always write ζ in this way since ν is even in Δ while μ is odd. By equating the real and imaginary parts of the two forms of ζ and Fourier inverting, we obtain

$$I_{\rm eff}C_k=\frac{1}{\pi}\int_{-\infty}^{\infty}d\Delta\cos k\Delta\,\nu(\Sigma,\Delta), \qquad (1.12)$$

$$I_{\rm eff}=-\frac{1}{\pi}\int_{-\infty}^{\infty}d\Delta\sin k\Delta\,\mu(\Sigma,\Delta). \qquad (1.13)$$

We will now consider various examples where ζ is evaluated and shown to have, to zeroth order, a thermal form. Higher-order corrections to ζ give a *near-thermal* spectrum. In principle, the real and imaginary parts of the influence kernel may be substituted in the Langevin equation (1.1) to yield stochastic near-thermal fluctuations of the detector coordinate Q. This procedure will be demonstrated in the example of a finite-time uniformly accelerated detector (Sec. III below).

B. Relation to perturbative approach

The methodology presented above describes a *stochastic field theory approach* to the problem of detector response, as opposed to the usual perturbation theory approach (where the perturbation parameter is e^2) in the calculation of detector transition probabilities. It should be emphasized that Eq. (1.1) is exact for linear coupling and does not involve a perturbation expansion in e^2 (for linear systems, such an expansion is, in fact, unnecessary because they are exactly solvable).

However, the relationship (1.4) between the influence kernel ζ and the Wightman function G^+ allows us to connect the stochastic approach to usual perturbation theory. In this case, the quantity of interest is the *detector response function* $\mathcal{F}(E)$ [6], given, to lowest order in perturbation theory, by the Fourier transform of the Wightman function, and hence the function ζ, as

$$\mathcal{F}(E-E_0)=\frac{1}{e^2}\int_{-\infty}^{\infty}ds\int_{-\infty}^{\infty}ds'\,e^{-i(E-E_0)(s-s')}\zeta(s,s'), \tag{1.14}$$

where s,s' are proper time parameters along the detector trajectory. The limits of integration in the above equation should be modified if the detector is switched on for a finite time. This function is proportional to the transition probability of the detector to excited states of energy E. However, it has the disadvantage of being a perturbative result, and furthermore, involves an integration over the entire history of the detector. The stochastic approach, on the other hand, leads to the evaluation of detector observables *as a function of proper time*, and can be employed to ultimately obtain the time-dependent density matrix of the detector [42].

II. ASYMPTOTICALLY UNIFORMLY ACCELERATED OBSERVER

As a starter, we first consider the case of a nonuniformly accelerated monopole detector in $1+1$ dimensions. The special case of trajectories which are asymptotically inertial in the far past and asymptotically uniformly accelerated in the far future has been analyzed using methods of field quantization in curvilinear coordinates by Costa [58] and by Percoco and Villalba [59].

For a general detector trajectory $(x(\tau),t(\tau))$ parametrized by the proper time τ, it has been shown [52] that the function $\zeta(\tau,\tau')$ is

$$\zeta(\tau,\tau')\equiv\nu+i\mu$$
$$=\frac{e^2}{2\pi}\int_0^{\infty}\frac{dk}{k}e^{-ik(t(\tau)-t(\tau'))}\cos k[x(\tau)-x(\tau')]. \tag{2.1}$$

Here e is the coupling constant of the detector to a massless scalar field (initially in its ground state). The initial state of the detector is unspecified at the moment and would appear as a boundary condition on the equation of motion of the detector. Here, however, we are primarily interested in the noise and dissipation kernels themselves, as properties of the field, and not in the state of the detector.

First, we note that the function ζ can be separated into advanced and retarded parts, in terms of the advanced and retarded null coordinates $v(\tau)=t(\tau)+x(\tau)$ and $u(\tau)=t(\tau)-x(\tau)$, respectively:

$$\zeta^a(\tau,\tau')=\frac{e^2}{4\pi}\int_0^{\infty}\frac{dk}{k}e^{-ik[v(\tau)-v(\tau')]},$$

$$\zeta^r(\tau,\tau')=\frac{e^2}{4\pi}\int_0^{\infty}\frac{dk}{k}e^{-ik[u(\tau)-u(\tau')]},$$

$$\zeta(\tau,\tau')=\zeta^a(\tau,\tau')+\zeta^r(\tau,\tau'). \tag{2.2}$$

In the case when the detector is uniformly accelerated with acceleration a, its trajectory is given by

$$v(\tau)=\frac{1}{a}e^{a\tau},\quad u(\tau)=-\frac{1}{a}e^{-a\tau}. \tag{2.3}$$

Substitution of the above trajectory into Eqs. (2.2) yields a thermal, isotropic detector response at the Unruh temperature $a/(2\pi)$ [46,52].

A. Perturbation increasing with time

The above analysis is now applied to the case of near-uniform acceleration by introducing a dimensionless h parameter which measures the departure from exact uniform acceleration:

$$h=\frac{\dot a}{a^2}, \tag{2.4}$$

where the overdot indicates a derivative with respect to the proper time. The trajectory of the detector is now chosen to be

$$v(\tau)=\frac{1}{a(\tau)}\exp\left(\int a(\tau)d\tau\right),$$

$$u(\tau)=\frac{1}{a(\tau)}\exp\left(-\int a(\tau)d\tau\right). \tag{2.5}$$

One can expand $a(\tau)$ in a Taylor series about the acceleration at $\tau=0$:

$$a(\tau)=a_0+\sum_{n=1}^{\infty}\frac{\tau^n}{n!}a_0^{(n)}, \tag{2.6}$$

where $a_0^{(n)}$ denotes the nth derivative of a at $\tau=0$. We make the assumption of ignoring second and higher derivatives of a. This implies

$$a(\tau)=a_0+h_0\tau a_0^2, \tag{2.7}$$

where $h_0=\dot{a}_0/a_0^2$.

Hereafter, we shall also make the further assumption of evaluating quantities to first order in h_0. In this approximation, $h=h_0$ to first order in h_0. Then there is no distinction between h and h_0 (h is essentially constant), and we can safely drop the subscript and work with h alone. It should be noted that expanding quantities to first order in h actually involves expansion of quantities to first order in $h\tau a_0$, and hence, for arbitrary trajectories, the final results are to be considered valid over time scales τ such that $\tau \ll (ha_0)^{-1}$. Alternatively, Eq. (2.7) can be taken to define a family of trajectories for which this analysis applies.

Using the linearized form of $a(\tau)$, one can now obtain the trajectory explicitly, to first order in h. The result is

$$v(\tau)=a_0^{-1}e^{a_0\tau}\left[1+h\tau a_0\left(\frac{a_0\tau}{2}-1\right)\right],$$

$$u(\tau)=-a_0^{-1}e^{-a_0\tau}\left[1-h\tau a_0\left(\frac{a_0\tau}{2}+1\right)\right]. \tag{2.8}$$

One also finds, to first order in h,

$$e^{-ik[v(\tau)-v(\tau')]}=\exp\left[-\frac{2ik}{a_0}e^{a_0\Sigma}\sinh\left(\frac{a_0\Delta}{2}\right)\right]\left\{1-ikhe^{a_0\Sigma}\left[\left(\frac{a_0\Delta^2}{4}+a_0\Sigma^2-2\Sigma\right)\sinh\left(\frac{a_0\Delta}{2}\right)+\Delta(a_0\Sigma-1)\cosh\left(\frac{a_0\Delta}{2}\right)\right]\right\}, \tag{2.9}$$

$$e^{-ik[u(\tau)-u(\tau')]}=\exp\left[-\frac{2ik}{a_0}e^{-a_0\Sigma}\sinh\left(\frac{a_0\Delta}{2}\right)\right]\left\{1+ikhe^{-a_0\Sigma}\left[\left(\frac{a_0\Delta^2}{4}+a_0\Sigma^2+2\Sigma\right)\sinh\left(\frac{a_0\Delta}{2}\right)-\Delta(a_0\Sigma+1)\cosh\left(\frac{a_0\Delta}{2}\right)\right]\right\}, \tag{2.10}$$

where $\Delta=\tau-\tau'$, $\Sigma=\frac{1}{2}(\tau+\tau')$.

Using the identities [50,51]

$$\exp\left[-\frac{2ik}{a_0}e^{-a_0\Sigma}\sinh\left(\frac{a_0\Delta}{2}\right)\right]=\frac{4}{\pi}\int_0^\infty d\nu K_{2i\nu}\left(\frac{2k}{a_0}e^{-a_0\Sigma}\right)[\cosh(\pi\nu)\cos(\nu a_0\Delta)-i\sinh(\pi\nu)\sin(\nu a_0\Delta)], \tag{2.11}$$

$$\int_0^\infty dx x^\mu K_{i\nu}(bx)=2^{\mu-1}b^{-\mu-1}\Gamma\left(\frac{1+\mu+i\nu}{2}\right)\Gamma\left(\frac{1+\mu-i\nu}{2}\right), \tag{2.12}$$

and

$$|\Gamma(i\nu)|^2=\frac{\pi}{\nu\sinh\pi\nu},\quad |\Gamma(\tfrac{1}{2}+i\nu)|^2=\frac{\pi}{\cosh\pi\nu}, \tag{2.13}$$

one finally obtains, after some simplification,

$$\zeta^a(\tau,\tau')=\frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}\left[\coth\left(\frac{\pi k}{a_0}\right)\cos(k\Delta)(1+h\Gamma_1)-i\sin(k\Delta)\right], \tag{2.14}$$

$$\zeta^r(\tau,\tau')=\frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}\left[\coth\left(\frac{\pi k}{a_0}\right)\cos(k\Delta)(1+h\Gamma_2)-i\sin(k\Delta)\right], \tag{2.15}$$

with

$$\Gamma_1=-k\tan(k\Delta)\tanh^2\left(\frac{\pi k}{a_0}\right)\left[\left(\frac{a_0\Delta^2}{4}+a_0\Sigma^2-2\Sigma\right)\sinh\left(\frac{a_0\Delta}{2}\right)+\Delta(a_0\Sigma-1)\cosh\left(\frac{a_0\Delta}{2}\right)\right],$$

$$\Gamma_2=k\tan(k\Delta)\tanh^2\left(\frac{\pi k}{a_0}\right)\left[\left(\frac{a_0\Delta^2}{4}+a_0\Sigma^2+2\Sigma\right)\sinh\left(\frac{a_0\Delta}{2}\right)-\Delta(a_0\Sigma+1)\cosh\left(\frac{a_0\Delta}{2}\right)\right]. \tag{2.16}$$

The advanced and retarded parts of Re(ζ) being unequal, the noise is anisotropic. Adding expressions (2.12) and (2.13), we have

$$\zeta(\tau,\tau')=\frac{e^2}{2\pi}\int_0^\infty\frac{dk}{k}\times\left[\coth\left(\frac{\pi k}{a_0}\right)\cos(k\Delta)(1+h\Gamma)-i\sin(k\Delta)\right], \tag{2.17}$$

where

$$\Gamma=\frac{\Gamma_1+\Gamma_2}{2}=k\Sigma\tan(k\Delta)\tanh^2\left(\frac{\pi k}{a_0}\right)\left(2\sinh\frac{a_0\Delta}{2}-a_0\Delta\cosh\frac{a_0\Delta}{2}\right). \tag{2.18}$$

The noise experienced by the detector is thus identical to the noise experienced in a heat bath, with a small correction Γ. The accelerated detector, therefore, has a near-thermal response at the Unruh temperature $a_0/(2\pi)$ with an order-h correction which increases with time.

B. Perturbation exponentially decreasing with time

We will now consider a trajectory for the accelerated detector which exponentially approaches the uniformly accelerated trajectory at late times. This trajectory, in null coordinates, has the form

$$v(\tau)=a_0^{-1}e^{a_0\tau}(1+\alpha e^{-\gamma\tau}),$$

$$u(\tau)=-a_0^{-1}e^{-a_0\tau}(1+\alpha e^{-\gamma\tau}). \tag{2.19}$$

In this case, the magnitude of the proper acceleration is, to first order in α,

$$a(\tau)=a_0\left\{1+\alpha e^{-\gamma\tau}\left(1+\frac{\gamma^2}{a_0^2}\right)\right\}+O(\alpha^2). \tag{2.20}$$

The influence kernel is obtained in a manner similar to the treatment of the previous subsection. Here, we get

$$\zeta^a(\tau,\tau')=\frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}\times\left[\coth\left(\frac{\pi k}{a_0}\right)\cos(k\Delta)(1+\alpha\Gamma_1)-i\sin(k\Delta)\right], \tag{2.21}$$

$$\zeta^r(\tau,\tau')=\frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}\times\left[\coth\left(\frac{\pi k}{a_0}\right)\cos(k\Delta)(1+\alpha\Gamma_2)-i\sin(k\Delta)\right], \tag{2.22}$$

with

$$\Gamma_1=-2ka_0^{-1}e^{-\gamma\Sigma}\sinh\frac{(a_0-\gamma)\Delta}{2}\tan(k\Delta)\tanh^2\left(\frac{\pi k}{a_0}\right),$$

$$\Gamma_2=-2ka_0^{-1}e^{-\gamma\Sigma}\sinh\frac{(a_0+\gamma)\Delta}{2}\tan(k\Delta)\tanh^2\left(\frac{\pi k}{a_0}\right). \tag{2.23}$$

The noise is again seen to be anisotropic. Adding ζ^a and ζ^r, we have

$$\zeta(\tau,\tau')=\frac{e^2}{2\pi}\int_0^\infty\frac{dk}{k}\times\left[\coth\left(\frac{\pi k}{a_0}\right)\cos(k\Delta)(1+\alpha\Gamma)-i\sin(k\Delta)\right], \tag{2.24}$$

where

$$\Gamma=\frac{\Gamma_1+\Gamma_2}{2}=-2ka_0^{-1}e^{-\gamma\Sigma}\sinh\frac{a_0\Delta}{2}\cosh\frac{\gamma\Delta}{2}\tan(k\Delta)\tanh^2\left(\frac{\pi k}{a_0}\right). \tag{2.25}$$

In this case, the correction to the thermal spectrum is exponentially suppressed at late times. This feature will distinguish the behavior of quantum fields in the vicinity of a moving mirror and a collapsing mass, as shown in later sections.

III. FINITE-TIME UNIFORMLY ACCELERATED DETECTOR

In this section, we consider a detector trajectory which is a uniformly accelerated one for a finite interval of time $(-t_0,t_0)$. Before and after this interval, the trajectory is taken to be inertial, at uniform velocity. To ensure continuity of the proper time along this trajectory, the velocity of the detector is assumed to vary continuously at the junctions $\pm t_0$.

With these constraints, the trajectory is chosen to be

$$\begin{aligned} x(t) &= x_0^{-1}(a^{-2}-t_0 t) \quad (t<-t_0) \\ &= (a^{-2}+t^2)^{1/2} \quad (-t_0<t<t_0) \\ &= x_0^{-1}(a^{-2}+t_0 t) \quad (t>t_0). \end{aligned} \tag{3.1}$$

The trajectory is symmetric under the interchange $t\to -t$. a is the magnitude of the proper acceleration during the uniformly accelerated interval $(-t_0,t_0)$ of Minkowski time and x_0 is the position of the detector at time t_0. x_0 and t_0 are related by $x_0^2-t_0^2=a^{-2}$. Before the uniformly accelerated interval, the detector has a uniform velocity $-t_0/x_0$ (we have chosen units such that $c=1$; if one keeps factors of c, the velocity is $-c^2t_0/x_0$), and after this interval, its velocity is t_0/x_0. This trajectory thus describes an observer traveling at constant velocity, then turning around and traveling with the same speed in the opposite direction. The ''turnaround'' interval corresponds to the interval of uniform acceleration. This example could thus be viewed as a quantum description of the classical twin paradox scenario in special relativity, where two twins, one on an inertial trajectory and one on a trajectory which is accelerated for a finite amount of time, compare their experiences at a future spacetime point where they meet.

We may also define null coordinates $u=t-x$ and $v=t+x$. In terms of these, the time at which the trajectory crosses the future horizon $u=0$ of the uniformly accelerated interval is $t_H=-(a^2u_0)^{-1}$.

If we choose to parametrize the trajectory by the proper time τ, it can be expressed as (with the zero of proper time chosen at $t=0$)

$$\begin{aligned} u(\tau) &= \theta(-\tau_0-\tau)v_0\{a(\tau+\tau_0)-1\}-a^{-1}\theta(\tau_0+\tau) \\ &\quad \times\theta(\tau_0-\tau)e^{-a\tau}+\theta(\tau-\tau_0)u_0\{1+a(\tau_0-\tau)\}, \end{aligned} \tag{3.2}$$

$$\begin{aligned} v(\tau) &= -\theta(-\tau_0-\tau)u_0\{a(\tau+\tau_0)+1\}+a^{-1}\theta(\tau_0+\tau) \\ &\quad \times\theta(\tau_0-\tau)e^{a\tau}+\theta(\tau-\tau_0)v_0\{1-a(\tau_0-\tau)\}, \end{aligned} \tag{3.3}$$

where $\pm\tau_0$ is the proper time of the trajectory when it exits (enters) the uniformly accelerated phase. It satisfies the relations

$$v_0\equiv t_0+x_0=a^{-1}e^{a\tau_0},$$

$$u_0\equiv t_0-x_0=-a^{-1}e^{-a\tau_0}. \tag{3.4}$$

Another convenient definition is the horizon-crossing proper time $\pm\tau_H=\pm(a^{-1}+\tau_0)$.

The function $\zeta(\tau,\tau')$ can be found in a standard way. If both points lie on the inertial sector of the trajectory, it has the usual zero-temperature form in two-dimensional Minkowski space. If both points lie on the uniformly accelerated sector, it has a finite-temperature form exhibiting the Unruh temperature. It is therefore straightforward to obtain the following. If $\tau,\tau'>\tau_0$ or $\tau,\tau'<-\tau_0$,

$$\zeta(\tau,\tau')=\frac{e^2}{2\pi}\int_0^\infty\frac{dk}{k}e^{ik(\tau'-\tau)}. \tag{3.5}$$

If $-\tau_0<\tau,\tau'<\tau_0$,

$$\begin{aligned} \zeta(\tau,\tau') &= \frac{e^2}{2\pi}\int_0^\infty\frac{dk}{k}\left\{\coth\left(\frac{\pi k}{a}\right)\cos k(\tau'-\tau)\right. \\ &\quad \left.+i\sin k(\tau'-\tau)\right\}. \end{aligned} \tag{3.6}$$

Also, if $\tau<-\tau_0,\quad \tau'>\tau_0$,

$$\begin{aligned} \zeta(\tau,\tau') &= \frac{e^2}{2\pi}\int_0^\infty\frac{dk}{k}\cos k[(\tau'+\tau)\tanh(a\tau_0)] \\ &\quad \times e^{ik\{\tau'-\tau+2[a^{-1}\tanh(a\tau_0)-\tau_0]\}}. \end{aligned} \tag{3.7}$$

Of interest is this function evaluated for one point on the inertial sector and the other on the uniformly accelerated sector. We will show that this function has a thermal form if the point on the inertial sector is sufficiently close to (t_0,x_0) and departs smoothly from the thermal form away from it. It is also found that the horizons of the uniformly accelerated sector (which are *not* horizons for the entire trajectory) are the points where the near-thermal expansion breaks down.

Consider, for example, the case when $-\tau_0<\tau'<\tau_0$ and $\tau<-\tau_0$. Then the function ζ is expressed as

$$\begin{aligned} \zeta(\tau,\tau') &= \frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}\{e^{-ik\{a^{-1}e^{-a\tau'}+v_0[a(\tau+\tau_0)-1]\}} \\ &\quad +e^{ik\{a^{-1}e^{a\tau'}+u_0[a(\tau+\tau_0)+1]\}}\}. \end{aligned} \tag{3.8}$$

Introducing the Fourier transforms

$$e^{(ik/a)e^{a\tau}}=\frac{1}{2\pi a}\int_{-\infty}^{\infty}d\omega e^{i\omega\tau}\Gamma\left(-\frac{i\omega}{a}\right)\left(\frac{k}{a}\right)^{i\omega/a}e^{\pi\omega/2a},$$

$$k>0,$$

$$e^{(-ik/a)e^{-a\tau}}=\frac{1}{2\pi a}\int_{-\infty}^{\infty}d\omega e^{i\omega\tau}\Gamma\left(\frac{i\omega}{a}\right)\left(\frac{k}{a}\right)^{-i\omega/a}e^{\pi\omega/2a},$$

$$k>0, \tag{3.9}$$

we obtain, after some simplification,

$$\zeta(\tau,\tau')=\frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}\left\{\cos k\left(\tau'+\tau_0+\frac{1}{a}\ln[1-a(\tau+\tau_0)]\right)\coth\left(\frac{\pi k}{a}\right)+i\sin k\left(\tau'+\tau_0+\frac{1}{a}\ln[1-a(\tau+\tau_0)]\right)\right.$$
$$+\cos k\left(\tau'+\tau_0-\frac{1}{a}\ln|a(\tau+\tau_H)|\right)\left[\coth\left(\frac{\pi k}{a}\right)\theta(\tau_H+\tau)+\theta(-\tau_H-\tau)\right]$$
$$\left.+i\sin k\left(\tau'+\tau_0-\frac{1}{a}\ln|a(\tau+\tau_H)|\right)\theta(\tau_H+\tau)\right\}. \tag{3.10}$$

If we further restrict our attention to the case $\tau>-\tau_H$ i.e., both points lie inside the Rindler wedge—the above expression simplifies to

$$\zeta(\tau,\tau')=\frac{e^2}{2\pi}\int_0^\infty\frac{dk}{k}\cos\left(\frac{k}{2a}\ln[1-a^2(\tau+\tau_0)^2]\right)\left\{\coth\left(\frac{\pi k}{a}\right)\cos k\left[\tau'+\tau_0+\frac{1}{2a}\ln\left(1-2\frac{\tau+\tau_0}{\tau+\tau_H}\right)\right]\right.$$
$$\left.+i\sin k\left[\tau'+\tau_0+\frac{1}{2a}\ln\left(1-2\frac{\tau+\tau_0}{\tau+\tau_H}\right)\right]\right\}. \tag{3.11}$$

It is clear from this expression that an exact thermal spectrum is recovered in the limit of $\tau\to-\tau_0$, as expected. Suppose we now define $\tau+\tau_0\equiv\epsilon$ as the time difference between the proper time τ and the proper time of entry into the accelerated phase, $-\tau_0$. Then $a\epsilon$ will be the appropriate dimensionless parameter characterizing a near-thermal expansion. Note that $\epsilon<0$.

From the above expression for ζ, we find that there is no correction to the thermal form of $\zeta(\tau,\tau')$ to first order in ϵ. This can be understood from the fact that the coordinate difference between the point $\tau=-\tau_0-\epsilon$ and a corresponding point on a globally uniformly accelerated trajectory with the *same* proper time is of order ϵ^2. Indeed, we may define Rindler coordinates (ξ,η) on the right Rindler wedge by $v=\xi^{-1}e^{\xi\eta}$ and $u=-\xi^{-1}e^{-\xi\eta}$. Then the Rindler coordinates for the point $\tau=-\tau_0-\epsilon$ on the trajectory we consider are found to be $\xi=a+O(\epsilon^2)$ and $\eta=-\tau_0-\epsilon+O(\epsilon^2)$, which are exactly the coordinates, to order ϵ, of a corresponding point with the same proper time on a globally uniformly accelerated trajectory with acceleration a. It is thus no surprise that the spectrum is exactly thermal up to order ϵ.

Furthermore, it can be shown in a straightforward way from the above expression that the spectrum is also thermal up to $O(\epsilon^2)$, although the above-mentioned coordinate difference does have terms of order ϵ^2. Then the first correction to the thermal spectrum is of order ϵ^3 and has the form

$$\zeta(\tau,\tau')=\frac{e^2}{2\pi}\int_0^\infty\frac{dk}{k}\left\{\coth\left(\frac{\pi k}{a}\right)\cos k\left(\tau'-\tau+\frac{a^2\epsilon^3}{3}\right)\right.$$
$$\left.-i\sin k\left(\tau'-\tau+\frac{a^2\epsilon^3}{3}\right)\right\}+O(\epsilon^4). \tag{3.12}$$

The validity of such a near-thermal expansion is characterized by the requirement that $|a\epsilon|$ is small. This translates to $-1<a(\tau+\tau_0)$ or equivalently, $\tau>-\tau_H$. The expansion thus breaks down for $\tau<-\tau_H$, for which case the two-point function may be called strictly nonthermal. This is the case when one of the points lies outside the right Rindler wedge while the other point is still inside it. The two-point function in such a situation will contain nontrivial correlations across the Rindler horizon, as was pointed out before [52].

The response of the detector is governed by the Langevin equation (1.1). This equation may be formally integrated to yield

$$\langle Q(\tau)Q(\tau')\rangle=\frac{\hbar}{\Omega^2}\int_{-\infty}^{\tau}ds\int_{-\infty}^{\tau'}ds'\,\nu(s,s')e^{-\gamma(\tau-s)}$$
$$\times e^{-\gamma(\tau'-s')}\sin\Omega(\tau-s)\sin\Omega(\tau'-s'), \tag{3.13}$$

where $\Omega=(\Omega_0^2-\gamma^2)^{1/2}$, Ω_0 is the natural frequency of the internal detector coordinate, and $\gamma=e^2/4$ is the dissipation constant arising out of the detector's coupling to the field. The double integral in the above equation may be computed by splitting each integral into a part which lies completely in the uniformly accelerated sector and parts which lie in the inertial sectors. For example, suppose we wish to compute the above correlation function for the case $-\tau_0<\tau,\tau'<\tau_0$; i.e., both points lie in the uniformly accelerated sector. Then each integral can be split into two parts ($\int_{-\infty}^{\tau}=\int_{-\infty}^{-\tau_0}+\int_{-\tau_0}^{\tau}$) and the resulting double integral therefore has four terms:

$$\langle Q(\tau)Q(\tau')\rangle=F_1+F_2+F_3+F_4. \tag{3.14}$$

Writing $\nu\equiv\mathrm{Re}(\zeta)$, we obtain, after straightforward manipulations,

$$F_1 \equiv \frac{2\hbar\gamma}{\pi\Omega^2}\mathrm{Re}\int_{-\infty}^{-\tau_0} ds \int_{-\infty}^{-\tau_0} ds' \int_0^\infty \frac{dk}{k} e^{ik(s'-s)} e^{-\gamma(\tau-s)} e^{-\gamma(\tau'-s')} \sin\Omega(\tau-s)\sin\Omega(\tau'-s')$$

$$= \frac{\hbar\gamma}{\pi\Omega^2} e^{-\gamma(\tau+\tau'+2\tau_0)} \int_0^\infty \frac{dk}{k} [(\gamma^2-k^2+\Omega^2)^2+4\gamma^2k^2]^{-1}\{(\Omega^2+\gamma^2+k^2)\cos\Omega(\tau-\tau')+(\Omega^2-\gamma^2-k^2)$$

$$\times\cos\Omega(\tau+\tau'+2\tau_0)+2\gamma\Omega\sin\Omega(\tau+\tau'+2\tau_0)\} \tag{3.15}$$

and

$$F_4 \equiv \frac{2\hbar\gamma}{\pi\Omega^2}\mathrm{Re}\int_{-\tau_0}^{\tau} ds \int_{-\tau_0}^{\tau'} ds' \int_0^\infty \frac{dk}{k} e^{ik(s'-s)} \coth\left(\frac{\pi k}{a}\right) e^{-\gamma(\tau-s)} e^{-\gamma(\tau'-s')} \sin\Omega(\tau-s)\sin\Omega(\tau'-s')$$

$$= \frac{\hbar\gamma}{\pi\Omega^2} e^{-\gamma(\tau+\tau'+2\tau_0)} \int_0^\infty \frac{dk}{k}\coth\left(\frac{\pi k}{a}\right) [(\Omega^2+\gamma^2-k^2)^2+4\gamma^2k^2]^{-1}\{(\gamma^2+k^2+\Omega^2)\cos\Omega(\tau-\tau')+(\Omega^2-\gamma^2-k^2)$$

$$\times\cos\Omega(\tau+\tau'+2\tau_0)+2\gamma\Omega[\sin\Omega(\tau+\tau'+2\tau_0)-\sin\Omega(\tau+\tau_0)-\sin\Omega(\tau'+\tau_0)]$$

$$-\Omega^2[\cos\Omega(\tau+\tau_0)+\cos\Omega(\tau'+\tau_0)-1]\}, \tag{3.16}$$

where ''Re'' stands for the real part.

The functions F_2 and F_3, in which one of the integration variables runs over the inertial sector and the other over the uniformly accelerated sector, are difficult to evaluate. We shall simply express them here in the form

$$F_2 = \frac{\hbar\gamma}{\pi\Omega^2} e^{-\gamma(\tau+\tau')} \mathrm{Re}\int_{-\infty}^{-\tau_0} ds\, e^{\gamma s}\sin\Omega(\tau-s) \int_0^\infty \frac{dk}{k} [e^{ik[au_0 s+u_0(1+a\tau_0)]} A_1(k;\tau') + e^{-ik[av_0 s - v_0(1-a\tau_0)]} A_2(k;\tau')], \tag{3.17}$$

$$F_3 = \frac{\hbar\gamma}{\pi\Omega^2} e^{-\gamma(\tau+\tau')} \mathrm{Re}\int_{-\infty}^{-\tau_0} ds\, e^{\gamma s}\sin\Omega(\tau'-s) \int_0^\infty \frac{dk}{k} [e^{-ik[au_0 s+u_0(1+a\tau_0)]} A_1(k;\tau) + e^{ik[av_0 s - v_0(1-a\tau_0)]} A_2(k;\tau)], \tag{3.18}$$

where the functions A_1 and A_2 are

$$A_1(k;s) = \int_{-\tau_0}^{s} ds'\, e^{ika^{-1}e^{as'}} e^{\gamma s'} \sin\Omega(s-s'),$$

$$A_2(k;s) = \int_{-\tau_0}^{s} ds'\, e^{-ika^{-1}e^{-as'}} e^{\gamma s'} \sin\Omega(s-s'). \tag{3.19}$$

Similarly, if one wishes to compute the detector correlation function for two points in the late inertial sector ($\tau,\tau' > \tau_0$), then one has nine terms similar in form to the ones displayed above.

A. Response function for the finite-time accelerated detector

The calculation of the response function for the finite-time accelerated detector is very similar to the previous calculation of detector correlation functions. We will assume that the detector is switched on *before* the uniformly accelerated phase, at a proper time $\tau = -T < -\tau_0$, and switched off after the end of this phase, at $\tau = T$. Then the response function separates into the following nine terms:

$$\mathcal{F}(E) = \sum_{i=1}^{9} T_i, \tag{3.20}$$

where

$$T_1 = \frac{1}{e^2}\int_{-T}^{-\tau_0} d\tau \int_{-T}^{-\tau_0} d\tau'\, e^{-iE(\tau-\tau')}\zeta(\tau,\tau'),$$

$$T_2 = \frac{1}{e^2}\int_{-T}^{-\tau_0} d\tau \int_{-\tau_0}^{\tau_0} d\tau'\, e^{-iE(\tau-\tau')}\zeta(\tau,\tau'),$$

$$T_3 = \frac{1}{e^2}\int_{-T}^{-\tau_0} d\tau \int_{\tau_0}^{T} d\tau'\, e^{-iE(\tau-\tau')}\zeta(\tau,\tau'),$$

$$T_4 = \frac{1}{e^2}\int_{-\tau_0}^{\tau_0} d\tau \int_{-T}^{-\tau_0} d\tau'\, e^{-iE(\tau-\tau')}\zeta(\tau,\tau'),$$

$$T_5 = \frac{1}{e^2}\int_{-\tau_0}^{\tau_0} d\tau \int_{-\tau_0}^{\tau_0} d\tau'\, e^{-iE(\tau-\tau')}\zeta(\tau,\tau'),$$

$$T_6 = \frac{1}{e^2}\int_{-\tau_0}^{\tau_0} d\tau \int_{\tau_0}^{T} d\tau'\, e^{-iE(\tau-\tau')}\zeta(\tau,\tau'),$$

$$T_7 = \frac{1}{e^2}\int_{\tau_0}^{T} d\tau \int_{-T}^{-\tau_0} d\tau'\, e^{-iE(\tau-\tau')}\zeta(\tau,\tau'),$$

$$T_8=\frac{1}{e^2}\int_{\tau_0}^{T}d\tau\int_{-\tau_0}^{\tau_0}d\tau' e^{-iE(\tau-\tau')}\zeta(\tau,\tau'),$$

$$T_9=\frac{1}{e^2}\int_{\tau_0}^{T}d\tau\int_{\tau_0}^{T}d\tau' e^{-iE(\tau-\tau')}\zeta(\tau,\tau'). \qquad (3.21)$$

Of all the terms displayed above, only T_5 is an exact thermal response, since it is evaluated on the uniformly accelerated sector. The remaining terms constitute the near-thermal corrections.

We now turn to the simplification of these terms. The symmetry of the trajectory under reflection about the x axis is expressed in null coordinates as the relation $u(-\tau)=-v(\tau)$, for all τ. This relation leads to the identity $\zeta(-\tau,-\tau')=\zeta^*(\tau,\tau')$. Furthermore, ζ always obeys the identity $\zeta(\tau',\tau)=\zeta^*(\tau,\tau')$. It can be shown in a straightforward manner that these two identities combine to yield $T_4=T_8=T_2^*=T_6^*$, $T_7=T_3^*$, and $T_9=T_1^*$. Thus we may rewrite the response function as the sum of four independent terms:

$$\mathcal{F}(E)=2\mathrm{Re}T_1+2\mathrm{Re}T_3+4\mathrm{Re}T_2+T_5, \qquad (3.22)$$

where ''Re'' stands for the real part. We have thus manifestly shown that the response function is real, as expected (straightforward changes of the integration variables shows that T_5 is real). Now we evaluate each of these four terms.

First, T_5 is easy to evaluate because it involves integrating the two-point function along the uniformly accelerated sector. We get the result

$$T_5=4\tau_0\int_0^\infty dk\frac{1}{k(k+E)}\frac{1}{e^{2\pi k/a}-1}\frac{\sin 2\tau_0(k-E)}{\pi(k-E)}. \qquad (3.23)$$

As $\tau_0\to\infty$, this contribution reduces to the usual thermal result:

$$T_5\to\frac{2\tau_0}{E^2}\frac{1}{e^{2\pi E/a}-1}. \qquad (3.24)$$

The terms involving T_1 and T_3 involve integration of the two-point function over purely inertial sectors of the trajectory. Therefore, they can also be easily evaluated to yield

$$2\mathrm{Re}T_1=\frac{2}{\pi}(T-\tau_0)\int_0^\infty dk\frac{1}{k(E+k)}\sin[(T-\tau_0)(E+k)] \qquad (3.25)$$

and

$$2\mathrm{Re}T_3=\frac{4}{\pi}\int_0^\infty\frac{dk}{k}\frac{1}{(E+k)^2-k^2\tanh^2(a\tau_0)}\cos[E(T+\tau_0)+k(T-\tau_0)+2ka^{-1}\tanh(a\tau_0)]$$
$$\times\sin\frac{1}{2}((T-\tau_0)\{E+k[1+\tanh(a\tau_0)]\})\sin\frac{1}{2}((T-\tau_0)\{E+k[1-\tanh(a\tau_0)]\}). \qquad (3.26)$$

The $2\mathrm{Re}T_1$ term above represents correlations on each of the inertial sectors. As expected, it vanishes in the limiting cases $T\to\tau_0$ and $T\to\infty$ [in the latter case, we use the identity $\sin(T\alpha)/(\pi\alpha)\to\delta(\alpha)$ as $T\to\infty$, and the fact $E>0$]. The second term above, $2\mathrm{Re}T_3$, represents correlations *between* the two asymptotically inertial sectors. As expected, it also vanishes in the limits $T\to\tau_0$ and $T\to\infty$ (using the same δ function representation in the latter case, and $E>0$, $\tau_0>0$). Thus, if the detector is switched on for a sufficiently long time, there is no contribution to the response function from the purely inertial sectors of the trajectory. It should also be noted that the dependence of the two terms above on the proper time difference $T-\tau_0$ may be exploited to develop a near-thermal expansion of the response function, with $T-\tau_0$ being a small parameter. This would correspond to the case of the detector being switched on (off) just before (after) the uniformly accelerated phase of the trajectory.

The calculation of the remaining term $4\mathrm{Re}T_2$ is the crux of this analysis. This term represents correlations between the inertial and accelerated sectors. From the definition of T_2, we can write it as

$$T_2=\int_{-\tau_0}^{\tau_0}d\tau' e^{iE\tau'}\int_{-T}^{-\tau_0}d\tau\int_0^\infty\frac{dk}{4\pi k}e^{-iE\tau}(\exp\{-ika^{-1}e^{-a\tau'}+ikv_0[1-a(\tau+\tau_0)]\}+\exp\{ika^{-1}e^{a\tau'}+iku_0[1+a(\tau+\tau_0)]\}). \qquad (3.27)$$

In the above expression, we may explicitly perform the integration over τ and rescale the variable τ' in order to extract the dependence on T and τ_0. Then we get

$$T_2=i\tau_0\int_{-1}^{1}dy e^{iE\tau_0 y}\int_0^\infty\frac{dk}{4\pi k}\Bigg[\frac{\exp[-ika^{-1}e^{-a\tau_0 y}-ikv_0(a\tau_0-1)]}{(kv_0a+E)}\{e^{i(kv_0a+E)\tau_0}-e^{i(kv_0a+E)T}\}$$
$$-\frac{\exp[ika^{-1}e^{a\tau_0 y}+iku_0(a\tau_0+1)]}{(ku_0a-E)}\{e^{-i(ku_0a-E)\tau_0}-e^{-i(ku_0a-E)T}\}\Bigg]. \qquad (3.28)$$

We see from the above expression that T_2 vanishes, as expected, in the limits $\tau_0 \to 0$ (because the slope of the trajectory is stipulated to vary continuously in the setup of the problem, this limit actually corresponds to the everywhere-inertial trajectory $x = a^{-1}$) and $T \to \tau_0$.

In order to examine the limit $T \to \infty$, we first take the real part of the above expression. Then the required limit gives rise to δ functions in the T-dependent terms, which do not contribute to the integral. The remaining terms give, as $T \to \infty$, the nonzero result

$$4\mathrm{Re}T_2 \to \tau_0 \int_{-1}^{1} dy \int_0^\infty \frac{dk}{k} \left\{ \frac{\sin[E\tau_0(1+y) + ka^{-1}e^{a\tau_0 y} + ku_0]}{\pi(ku_0 a - E)} - \frac{\sin[E\tau_0(1+y) - ka^{-1}e^{-a\tau_0 y} + kv_0]}{\pi(kv_0 a + E)} \right\}. \tag{3.29}$$

When T is finite, a near-thermal expansion of the term $4\mathrm{Re}T_2$ will be obtained by expressing it as a sum over Rindler modes, rather than the Minkowski mode sum in Eq. (3.28). We will therefore take Eq. (3.28) and reexpress it in Rindler modes using the Fourier transform relations (3.9), and then try to perform the integral over y and k. This procedure, after carrying out the y integration, yields

$$\begin{aligned} T_2 = \frac{i\tau_0}{4\pi^2 a} \int_{-\infty}^{\infty} d\omega \int_0^\infty \frac{dk}{k} e^{\pi\omega/2a} \frac{\sin[(E+\omega)\tau_0]}{(E+\omega)} \Bigg[& \frac{\Gamma(i\omega a^{-1})(ka^{-1})^{-i\omega a^{-1}}}{(kv_0 a + E)} \{ e^{i(E\tau_0 + kv_0)} - e^{i\{ET + kv_0[1 + a(T-\tau_0)]\}} \} \\ & - \frac{\Gamma(-i\omega a^{-1})(ka^{-1})^{i\omega a^{-1}}}{(ku_0 a - E)} \{ e^{i(E\tau_0 + ku_0)} - e^{i\{ET + ku_0[1 - a(T-\tau_0)]\}} \} \Bigg]. \end{aligned} \tag{3.30}$$

In order to accomplish the integral over Minkowski modes k in the above expression, we will now use the following integral formulas:

$$\int_0^\infty \frac{dk}{k} e^{ik\alpha} \frac{(ka^{-1})^{-i\omega a^{-1}}}{(kv_0 a + E)} = (a^2 v_0)^{i\omega a^{-1}} E^{-i\omega a^{-1} - 1} e^{-i(E/v_0 a)\alpha} \Gamma(-i\omega a^{-1}) \Gamma(1 - i\omega a^{-1}, -iE(v_0 a)^{-1}(\alpha + i\epsilon)), \tag{3.31}$$

and, similarly,

$$\int_0^\infty \frac{dk}{k} e^{ik\alpha} \frac{(ka^{-1})^{i\omega a^{-1}}}{(-ku_0 a + E)} = (-a^2 u_0)^{-i\omega a^{-1}} E^{i\omega a^{-1} - 1} e^{i(E/u_0 a)\alpha} \Gamma(i\omega a^{-1}) \Gamma(1 + i\omega a^{-1}, iE(u_0 a)^{-1}(\alpha + i\epsilon)), \tag{3.32}$$

where $\Gamma(,)$ is the incomplete gamma function. This function is multivalued in its second argument with a branch cut along the imaginary axis, which is why it is necessary to introduce a small positive quantity ϵ to make the function well defined. The equalities in the above expressions therefore hold in the limit $\epsilon \to 0^+$.

These formulas may be used to carry out the k integration in T_2. One then obtains

$$\begin{aligned} T_2 = {} & \frac{i\tau_0}{4\pi E} \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \frac{e^{\pi\omega/2a}}{\sinh(\pi\omega a^{-1})} \frac{\sin[(E+\omega)\tau_0]}{(E+\omega)} \\ & \times \Bigg[\left(\frac{E}{a^2 v_0} \right)^{-i\omega a^{-1}} \{ e^{iE(\tau_0 - a^{-1})} \Gamma(1 - i\omega a^{-1}, -iEa^{-1}(1 + i\epsilon)) - e^{iE(2T - \tau_H)} \Gamma(1 - i\omega a^{-1}, -iEa^{-1}[1 + i\epsilon + a(T - \tau_0)]) \} \\ & + \left(\frac{E}{-a^2 u_0} \right)^{i\omega a^{-1}} e^{iE\tau_H} \{ \Gamma(1 + i\omega a^{-1}, iEa^{-1}(1 - i\epsilon)) - \Gamma(1 + i\omega a^{-1}, iEa^{-1}[1 - i\epsilon - a(T - \tau_0)]) \} \Bigg]. \end{aligned} \tag{3.33}$$

This accomplishes the task of expressing T_2 as a sum over Rindler modes. We readily see from the above expression that the quantities in both curly brackets vanish in the limit $T \to \tau_0$, and hence the entire expression vanishes in that limit. This will facilitate an expansion of the above quantity in $(T - \tau_0)$. To do so, however, it will be convenient to consider the limiting cases of the above exact expression in the high- and low-energy regimes.

Firstly, at high energies Ea^{-1}, $ET \gg 1$, we may use the asymptotic result $\Gamma(x,y) \sim y^{x-1} e^{-y}$ for large values of $|y|$. Then we may simplify the above expression to yield

$$\begin{aligned} 4\,\mathrm{Re}T_2 \sim {} & \frac{\tau_0}{E} \int_{-\infty}^{\infty} \frac{d\omega}{\pi\omega} \frac{1}{\sinh(\pi\omega a^{-1})} \frac{\sin[(E+\omega)\tau_0]}{(E+\omega)} [-2\cos[2\omega a^{-1}\ln(Ea^{-1})] \sin[(E+\omega)\tau_0] + \sin(2ET + (\omega - E)\tau_0 \\ & - \omega a^{-1} \ln\{(Ea^{-1})^2 [1 + a(T - \tau_0)]\}) + \sin(ET + \omega\tau_0 + \omega a^{-1} \ln\{E^2 a^{-1} |\tau_H - T|\}) [\theta(\tau_H - T) + e^{\pi\omega/a} \theta(T - \tau_H)]]. \end{aligned} \tag{3.34}$$

The last term in the above equation points to a qualitatively different behavior of the response function according to whether T is less than or greater than τ_H, i.e., whether the detector is switched on after or before crossing the horizon. This is also seen at the more primitive level of the Wightman function, Eq. (3.10), where the θ function dependence on $\tau_H+\tau$ is displayed.

The above expression may be simplified even further if one assumes that the detector is switched on after crossing the horizon, i.e., $\tau_H>T$. This is consistent with the limiting near-thermal behavior as $T\to\tau_0$, which we wish to finally obtain. It is then convenient to define the dimensionless parameter $\alpha=a(T-\tau_0)$, which is chosen to be small. We thus get the near-thermal result, to first order in α, and at high energies, as

$$4\,\mathrm{Re}T_2\sim\alpha\frac{\tau_0}{a}\int_{-\infty}^{\infty}\frac{d\omega}{\pi\omega}\frac{1}{\sinh(\pi\omega a^{-1})}\frac{\sin[(E+\omega)\tau_0]}{(E+\omega)}$$
$$\times\left\{\cos\left[(E+\omega)\tau_0-2\frac{\omega}{a}\ln\left(\frac{E}{a}\right)\right]+2\left(1-\frac{\omega}{E}\right)\cos[(E+\omega)\tau_0]\cos\left[2\frac{\omega}{a}\ln\left(\frac{E}{a}\right)\right]\right\}+O(\alpha^2). \tag{3.35}$$

To obtain the low-energy behavior of T_2, we consider the following series representation of the incomplete gamma function

$$\Gamma(x,y)=\Gamma(x)-\sum_{n=0}^{\infty}\frac{(-1)^n y^{x+n}}{n!(x+n)}, \tag{3.36}$$

where $\Gamma(x)$ is the ordinary gamma function. This may be used to express the incomplete Gamma functions in T_2 as follows:

$$\Gamma(1\pm i\omega a^{-1},\mp iEa^{-1}(1\pm i\epsilon))=\Gamma(1\pm i\omega a^{-1})-\sum_{n=0}^{\infty}\frac{(-1)^n(Ea^{-1})^{1+n\pm i\omega a^{-1}}e^{\mp i(\pi/2)(1+n\pm i\omega a^{-1})}}{n!(1+n\pm i\omega a^{-1})} \tag{3.37}$$

and

$$\Gamma(1+i\omega a^{-1},-iEa^{-1}[1+i\epsilon+a(T-\tau_0)])$$
$$=\Gamma(1+i\omega a^{-1})-\sum_{n=0}^{\infty}\frac{(-1)^n\{Ea^{-1}[1+a(T-\tau_0)]\}^{1+n+i\omega a^{-1}}e^{-i(\pi/2)(1+n+i\omega a^{-1})}}{n!(1+n+i\omega a^{-1})}, \tag{3.38}$$

$$\Gamma(1-i\omega a^{-1},iEa^{-1}[1-i\epsilon-a(T-\tau_0)])$$
$$=\Gamma(1-i\omega a^{-1})-\sum_{n=0}^{\infty}\frac{(-1)^n(E|T-\tau_H|)^{1+n-i\omega a^{-1}}}{n!(1+n-i\omega a^{-1})}[e^{i(\pi/2)(1+n-i\omega a^{-1})}\theta(\tau_H-T)+e^{-i(\pi/2)(1+n-i\omega a^{-1})}\theta(T-\tau_H)]. \tag{3.39}$$

All of the above expressions are exact, and again show a qualitatively different behavior of the response function according to whether the detector is switched on before or after the horizon crossing time $-\tau_H$, at all energies.

To extract the low-energy behavior we will keep the leading term ($n=0$) in the above expansions, and substitute back into the expression for T_2, Eq. (3.33). This will yield the low-energy result. However, since this expression is rather lengthy and not very illuminating, we will further restrict ourselves to the near-thermal approximation. That is, we switch off the detector before horizon crossing ($T<\tau_H$), and expand to first order in α. This procedure yields, after much simplification, the following result, valid at low energies:

$$4\,\mathrm{Re}T_2\sim\alpha\frac{2\tau_0}{a}\int_{-\infty}^{\infty}\frac{d\omega}{\pi\omega}\frac{1}{\sinh(\pi\omega a^{-1})}\frac{\sin[(E+\omega)\tau_0]}{(E+\omega)}\left[\frac{1}{2}e^{\pi\omega/2a}\{\Gamma(1-i\omega a^{-1})\exp i[(E+\omega)\tau_0-Ea^{-1}-\omega a^{-1}\ln(Ea^{-1})]\right.$$
$$+\Gamma(1+i\omega a^{-1})\exp-i[(E+\omega)\tau_0-Ea^{-1}-\omega a^{-1}\ln(Ea^{-1})]\}+\cos[(E+\omega)\tau_0]\cos[Ea^{-1}+2\omega a^{-1}\ln(Ea^{-1})]$$
$$\left.-\frac{Ea^{-1}}{1+\omega^2a^{-2}}\{\sin[(E+\omega)\tau_0-Ea^{-1}-2\omega a^{-1}\ln(Ea^{-1})]+\omega a^{-1}\cos[(E+\omega)\tau_0-Ea^{-1}-2\omega a^{-1}\ln(Ea^{-1})]\}\right]$$
$$+O(\alpha^2). \tag{3.40}$$

To summarize, in this subsection we have simplified each term in the response function of a detector accelerated for a finite time, and expressed the individual terms as appropriate mode sums. The most complicated of these terms, $4\,\mathrm{Re}T_2$, involving correlations between the inertial and accelerated sectors of the detector's trajectory, can be expressed analytically as a sum over Rindler modes [Eq. (3.33)], and facilitates a near-thermal expansion at high and low excitation energies, in terms of the dimensionless parameter $a(T-\tau_0)$, which measures how long the detector remains switched on beyond the uniformly accelerated regime. We have displayed results to leading order in this parameter. These results could be used as a starting point towards further numerical and analytical studies of this system, including a more detailed investigation of various limiting cases offered by the three independent time scales in this problem, namely, a^{-1}, τ_0, and T.

IV. MOVING MIRROR AND COLLAPSING MASS

A. Moving mirror in Minkowski space

The relation between radiance from a moving mirror and a black hole is well known. As a warm-up preparation, let us first study the motion of a mirror following a trajectory $z(t)$ in Minkowski space. A massless scalar field ϕ is coupled to the mirror via a reflection boundary condition. It obeys the Klein-Gordon equation

$$\frac{\partial^2\phi}{\partial t^2}-\frac{\partial^2\phi}{\partial x^2}=0, \tag{4.1}$$

subject to the boundary condition

$$\phi(t,z(t))=0. \tag{4.2}$$

For a general mirror path this equation is difficult to solve; however, we can exploit the invariance of the wave equation under a conformal transformation to change to simpler coordinates. We follow the treatment of [5]. To this end, we introduce a transformation between the null coordinates u,v and $\overline{u,v}$ defined as

$$u=t-x,\quad v=t+x,$$
$$u=f(\overline{u}),\quad v=\overline{v}. \tag{4.3}$$

The function f is chosen such that the mirror trajectory is mapped to $\overline{z}=0$. To do this, we relate the two sets of coordinates as follows:

$$t=\tfrac{1}{2}[\overline{v}+f(\overline{u})],$$
$$x=\tfrac{1}{2}[\overline{v}-f(\overline{u})]. \tag{4.4}$$

On the mirror path, setting $\overline{z}=0$ means that the trajectory can be expressed as

$$\tfrac{1}{2}[\overline{t}-f(\overline{t})]=z(\tfrac{1}{2}[\overline{t}+f(\overline{t})]), \tag{4.5}$$

which allows f to be implicitly determined. In the new coordinates the wave equation is unchanged; however, it now has a time-independent boundary condition, meaning the mirror is static, while the detector moves along some more complicated path. Thus the wave equation with boundary condition can easily be solved to give the mode solutions

$$u_k^{in}(\overline{t,x})\simeq\sin\omega\overline{x}e^{-i\omega\overline{t}}, \tag{4.6}$$

where the mode functions are orthonormal in the Klein-Gordon inner product. In these barred coordinates, ζ is proportional to the two-point function in the presence of a *static* reflecting boundary at $\overline{x}=0$.

Also, in these coordinates, the time-dependent modes of the field are just exponentials. That is, the field can be described by simple harmonic oscillators with unit mass. This can be obtained most simply by expanding the field as

$$\phi(\overline{t,x})=\sqrt{\frac{2}{L}}\sum_k{}'\; q_k(\overline{t})\sin k\overline{x}, \tag{4.7}$$

where Σ_k' indicates that the summation is restricted to modes $k>0$, and identifying q_k as the oscillator degrees of freedom of the field.

We then find that $X_k(\overline{t})$ is a solution to the oscillator equation (1.4), and by satisfying the initial conditions $X_k(0)=1$, $X_k'(0)=-ik$ we obtain

$$X_k(\overline{t})=e^{-ik\overline{t}}. \tag{4.8}$$

We now consider a detector placed in the vicinity of the mirror. The spectral density function I is determined by the path of the detector and its coupling to the field. Denoting the detector position by $r(t)$ and the field modes by $q_k(t)$ and assuming the monopole interaction

$$\begin{aligned}L_{\text{int}}&=-\int eQ\phi(\overline{t,x})\delta(\overline{r}-\overline{x})d\overline{x}\\&=-eQ\phi(\overline{t,r})\\&=-\sqrt{\frac{2}{\pi}}\int eQq_k(\overline{t})\sin k\overline{r}dk,\end{aligned} \tag{4.9}$$

we have

$$\begin{aligned}I(k,\overline{t,t'})&=\int\frac{dk_n}{2k_n}\delta(k-k_n)e^2\sin k_n\overline{r(t)}\sin k_n\overline{r(t')}\\&=\frac{e^2}{\pi k}\sin k\overline{r(t)}\sin k\overline{r(t')}.\end{aligned} \tag{4.10}$$

Defining $\overline{u}=\overline{t}-\overline{r}(t)$ and $\overline{v}=\overline{t}+\overline{r}$, we can now express the function ζ as

$$\begin{aligned}\zeta=-\frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}[&e^{ik(\overline{u}'-\overline{u})}-e^{ik(\overline{u}'-\overline{v})}-e^{ik(\overline{v}'-\overline{u})}\\&+e^{ik(\overline{v}'-\overline{v})}].\end{aligned} \tag{4.11}$$

Since only the outgoing modes have reflected off the mirror, only the outgoing part of the correlations ζ will give appropriate thermal behavior. Thus, from now on, we focus on the correlation

$$\zeta_{uu} = -\frac{e^2}{4\pi}\int_0^\infty \frac{dk}{k} e^{ik(\overline{u'}-\overline{u})}. \tag{4.12}$$

It remains to evaluate the above function. To do this, we specify the function f by considering a specific mirror trajectory. A convenient choice of the mirror path is

$$z(t) = -t - Ae^{-2\kappa t} + B \tag{4.13}$$

for A, B, κ positive. This path possesses a future horizon in the sense that there is a last ingoing ray which the mirror will reflect; all later rays never catch up with the mirror and so are not reflected. It is this aspect which enables the moving mirror to emulate a black hole. Equation (4.5) can now be solved to give

$$f(\overline{t}) = -\overline{t} - \frac{1}{\kappa}\ln\frac{B-\overline{t}}{A}. \tag{4.14}$$

In the late time limit $(\overline{t} \simeq B)$, f^{-1} has the behavior

$$f^{-1}(x) \simeq B - Ae^{-\kappa(B+x)} + \alpha, \tag{4.15}$$

where α is taken to be small in the sense that terms of order α^2 are ignored. In this approximation, one finds

$$\alpha = -\kappa A^2 e^{-2\kappa(B+x)} \tag{4.16}$$

and the transformation from barred to unbarred coordinates becomes

$$\overline{u} = B - Ae^{-\kappa(B+u)} - \kappa A^2 e^{-2\kappa(B+u)}, \tag{4.17}$$

plus terms of higher powers in $e^{-\kappa(B+u)}$.

We now need an explicit form for the detector trajectory $u(t)$ since this is what appears in the function ζ. Choosing it to be inertial, we have $r(t) = r_* + wt$, which gives $u(t) = t(1-w) - r_*$. In terms of the proper time of the detector, this becomes $u(\tau) = \tau\sqrt{[(1-w)/(1+w)]} - r_*$.

Defining the sum and difference $\Sigma = \frac{1}{2}(\tau+\tau')$ and $\Delta = \tau - \tau'$, and $z = \sqrt{[(1-w)/(1+w)]}$, we obtain

$$\begin{aligned}\overline{u'} - \overline{u} = &-2Ae^{-\kappa(B+r_*+\Sigma z)}\sinh\left(\frac{\kappa z \Delta}{2}\right)\\ &-2\kappa A^2 e^{-2\kappa(B+r_*+\Sigma z)}\sinh(\kappa z\Delta).\end{aligned} \tag{4.18}$$

This is substituted in ζ_{uu}, and, after some simplification we obtain the near-thermal form

$$\begin{aligned}\zeta_{uu}(\tau,\tau') = &-\frac{e^2}{4\pi}\int_0^\infty \frac{dk}{k}\\ &\times\left[\coth\left(\frac{\pi k}{\kappa z}\right)\cos(k\Delta)(1+\Gamma) - i\sin(k\Delta)\right],\end{aligned} \tag{4.19}$$

with

$$\Gamma = -2kz^{-1}Ae^{-\kappa(B+r_*+\Sigma z)}\tanh^2\left(\frac{\pi k}{\kappa z}\right)\tan k\Delta \sinh(\kappa z\Delta). \tag{4.20}$$

Thus a thermal detector response, at the temperature $\kappa/2\pi$, Doppler shifted by a factor z depending on the speed of the detector, is observed, with a correction that exponentially decays to zero at late times.

B. Collapsing mass in two dimensions

We now study radiance from a collapsing mass, using the analogy of the moving mirror model. We essentially follow the method of [15], but using stochastic analysis, and generalizing it to include higher-order terms in the various Taylor expansions involved, thus exhibiting the near-thermal properties of detector response.

We will exploit the conformal flatness of two-dimensional spacetime in the subsequent analysis. Outside the body the metric is expressed as

$$ds_o^2 = C(r)\,du\,dv, \tag{4.21}$$

where u, v are the null coordinates,

$$\begin{aligned}u &= t - r^* + R_0^*,\\ v &= t + r^* + R_0^*,\end{aligned} \tag{4.22}$$

and r^* is the Regge-Wheeler coordinate:

$$r^* = \int^r \frac{dr'}{C(r')}, \tag{4.23}$$

with R_0^* being a constant. The metric outside the body is thus assumed to be static in order to mimic the four-dimensional spherically symmetric case (for which Birkhoff's theorem holds). The point at which the conformal factor $C=0$ represents the horizon, and the asymptotic flatness condition is imposed by $C \to 1$ as $r \to \infty$.

On the other hand, the metric inside the ball is for now assumed to be a completely general conformally flat metric:

$$ds_i^2 = A(U,V)\,dU\,dV, \tag{4.24}$$

with

$$\begin{aligned}U &= \tau - r + R_0,\\ V &= \tau - r - R_0,\end{aligned} \tag{4.25}$$

and R_0 and R_0^* are related in the same way as r and r^*. The surface of the collapsing ball will be taken to follow the world line $r = R(\tau)$, such that, for $\tau < 0$, $R(\tau) = R_0$. Thus, at the onset of collapse, $\tau = t = 0$, $U = V = u = v = 0$ on the surface of the ball.

We will let the two sets of coordinates be related by the transformation equations

$$\begin{aligned}U &= \alpha(u),\\ v &= \beta(V).\end{aligned} \tag{4.26}$$

The functions α and β are not independent of each other because one coordinate transformation has already been specified by the definition of r^* in Eq. (4.23).

Without as yet determining the precise form of α and β, we will consider a massless scalar field ϕ propagating in this spacetime subject to a reflection condition $\phi(r=0,\tau)=0$. Such a field propagates in a similar fashion to the field in the vicinity of a moving mirror. To make this explicit, we introduce a new set of barred coordinates

$$\overline{u}=\beta[\alpha(u)-2R_0],$$

$$\overline{v}=v. \tag{4.27}$$

In terms of these, we also define the coordinates $\overline{r}=\frac{1}{2}(\overline{v}-\overline{u})$, and $\overline{t}=\frac{1}{2}(\overline{v}+\overline{u})$.

These new coordinates have the properties (a) $r=0\Rightarrow\overline{r}=0$ and (b) the field equations have incoming mode solutions of the form e^{ikv}. Thus the left-moving parts of the correlation functions of the ''in'' vacuum defined in terms of barred coordinates are identical to those of the vacuum defined with respect to unbarred coordinates.

Keeping these properties in mind, we may expand the field in terms of standard modes obeying the reflection boundary condition (by conformal invariance of the massless scalar field equation) as

$$\phi(\overline{r},\overline{t})=\sqrt{\frac{2}{L}}\sum_{k>0}\overline{q_k}(\overline{t})\sin k\overline{r}, \tag{4.28}$$

just as in the moving mirror case.

We now consider a detector placed outside the collapsing ball at fixed r (or r^*), namely, $r=r_0$ (or $r^*=r_0^*$). The interaction between detector and field is described by the interaction Lagrangian

$$L_{\rm int}=-\epsilon Q\phi(\overline{s,r}), \tag{4.29}$$

where

$$\begin{aligned}\overline{r}&=\tfrac{1}{2}\{v-\beta[\alpha(u)-2R_0]\}\\&=\tfrac{1}{2}\{t+r_0^*-R_0^*-\beta[\alpha(t-r_0^*+R_0^*)-2R_0]\},\\ \overline{s}&=\tfrac{1}{2}\{t+r_0^*-R_0^*+\beta[\alpha(t-r_0^*+R_0^*)-2R_0]\},\end{aligned} \tag{4.30}$$

and Q is the internal detector coordinate.

The influence kernel ζ, due to a reflection condition at $\overline{r}=0$, has the same form as the moving mirror case, in barred coordinates. Its outgoing part is therefore given by

$$\zeta_{uu}=\frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}e^{ik(\overline{u'}-\overline{u})}, \tag{4.31}$$

where

$$\overline{u}=\overline{s}-\overline{r}=\beta[\alpha(t-r_0^*-R_0^*)-2R_0] \tag{4.32}$$

and $\overline{u'}$ is the same function of t'.

We will now determine the functions α and β and show that, to zeroth order in an appropriate parameter, $\overline{u}$ is an exponential function of t, and thus ζ_{uu} has a thermal form. The correction to the exponential form, obtained by including higher-order terms, will lead to a near-thermal spectrum.

To determine α and β we match the interior and exterior metrics at the collapsing surface $r=R(\tau)$. Then we have[3]

$$\alpha'(u)\equiv\frac{dU}{du}=-C\frac{(1-\dot R)}{\dot R}\left[1+\left(1+\frac{AC}{\dot R^2}[1-\dot R^2]\right)^{1/2}\right]^{-1}, \tag{4.33}$$

$$\beta'(V)\equiv\frac{dv}{dV}=C^{-1}\frac{\dot R}{1+\dot R}\left[1-\left(1+\frac{AC}{\dot R^2}[1-\dot R^2]\right)^{1/2}\right], \tag{4.34}$$

where $\dot R=dR/d\tau$.

Now we expand these quantities about the horizon. We recall the definition of the horizon radius R_h as $C(R_h)=0$. We may further define τ_h as $R(\tau_h)=R_h$. Then we obtain the Taylor expansions

$$R(\tau)=R_h+\nu(\tau_h-\tau)+\beta(\tau_h-\tau)^2+\cdots, \tag{4.35}$$

where $\nu=-\dot R(\tau_h)$, $\beta=\frac{1}{2}\ddot R(\tau_h)$, and

$$\begin{aligned}C&=\left.\frac{\partial C}{\partial r}\right|_{R_h}(R-R_h)+\left.\frac{1}{2}\frac{\partial^2 C}{\partial r^2}\right|_{R_h}(R-R_h)^2+\cdots\\&=2\kappa\nu(\tau_h-\tau)+(2\kappa\beta+\gamma\nu^2)(\tau_h-\tau)^2+\cdots,\end{aligned} \tag{4.36}$$

where $\kappa=\frac{1}{2}\partial C/\partial r|_{R_h}$, the surface gravity, and $\gamma=\frac{1}{2}\partial^2C/\partial r^2|_{R_h}$. Since the ball is collapsing, $\nu>0$.

Substituting the above expansions in the expression for $\alpha'(u)$, we obtain, to order $(\tau_h-\tau)^2$,

$$\frac{dU}{du}=a(R_0-R_h+\tau_h-U)+b(R_0-R_h+\tau_h-U)^2, \tag{4.37}$$

where

$$a=(\nu+1)\kappa, \tag{4.38}$$

$$b=\frac{\kappa}{\nu}\left\{(3+\nu)\beta+(1+\nu)\frac{\gamma\nu^2}{2\kappa}-\frac{1}{2}A\kappa(1-\nu^2)(1+\nu)\right\}. \tag{4.39}$$

Note that, for a slowly collapsing ball, $\nu\ll1$, and hence a reduces to the surface gravity κ.

Also, to order $(\tau_h-\tau)$,

$$\frac{dv}{dV}=c+d(\tau_h+R_h-R_0-V), \tag{4.40}$$

where

[3]Note that the formulas for α' and β' do not agree with the corresponding formulas in Ref. [15] [Eqs. (8.17) and (8.18)]. The formulas in [15] have sign errors for the quantities within square brackets.

$$c=\frac{A(1+\nu)}{2\nu}, \tag{4.41}$$

$$d=\frac{A}{\nu^2}\left(\beta-\frac{A\kappa}{4}(1-\nu^2)(1+\nu)\right). \tag{4.42}$$

We consider a regime in which $(\tau_h-\tau)d \ll c$ so that we may ignore the second term in Eq. (4.40). Then we can integrate this equation to give

$$v(V)\equiv\beta(V)=c_1+cV, \tag{4.43}$$

where c_1 is an integration constant.

Similarly to lowest order in b/a^2 (which turns out to be the appropriate dimensionless parameter describing deviations from exact exponential scaling or exact thermal behavior), we integrate Eq. (4.37) to give

$$U(u)\equiv\alpha(u)$$
$$=R_0-R_h+\tau_h+a^{-1}e^{-a(u-c_2)}\left(1+\frac{b}{a^2}e^{-a(u-c_2)}\right), \tag{4.44}$$

c_2 being another integration constant.

We are now in a position to obtain explicitly the transformation between barred and unbarred coordinates, to lowest order in b/a^2. Thus we have

$$\overline{u}=\beta[\alpha(u)-2R_0]$$
$$=M_1+M_2e^{-a(u-c_2)}\left(1+\frac{b}{a^2}e^{-a(u-c_2)}\right), \tag{4.45}$$

where

$$M_1=c_1-c(R_0+R_h-\tau_h), \tag{4.46}$$

$$M_2=\frac{c}{a}. \tag{4.47}$$

At the position r_0^* of the detector, $u=t-r_0^*$. Therefore, defining $\Delta=u'-u$ and $\Sigma=\frac{1}{2}(u'+u)+r_0^*$, we may perform the above transformation to obtain

$$\overline{u'}-\overline{u}=-2M_2e^{ac_2}\left\{e^{-a(\Sigma-r_0^*)}\sinh\frac{a\Delta}{2}\right.$$
$$\left.+\frac{b}{a^2}e^{-2a(\Sigma-r_0^*-c_2/2)}\sinh a\Delta\right\}. \tag{4.48}$$

Invoking the identities (2.11) and (2.13), the function ζ_{uu} can now be simplified to yield the near-thermal form

$$\zeta_{uu}=\frac{e^2}{4\pi}\int_0^\infty\frac{dk}{k}\left\{\coth\left(\frac{\pi k}{a}\right)\cos k\Delta(1+\Gamma)-i\sin k\Delta\right\}, \tag{4.49}$$

where

$$\Gamma=-\frac{2bk}{a^3}e^{a(c_2-\Sigma-r_0^*)}\tanh^2\left(\frac{\pi k}{a}\right)\tan k\Delta\sinh a\Delta. \tag{4.50}$$

The function Γ vanishes at late times ($\Sigma\to\infty$). Thus the exact thermal spectrum is recovered at the Hawking temperature redshifted by the velocity of the surface of the ball, on a time scale defined by the surface gravity a.

V. DISCUSSION

We now summarize our findings and discuss their implications. There are four main points made or illustrated here

(1) This paper gives *a stochastic field theoretical derivation of particle creation* in the class of spacetimes which possess an event horizon in some limit. This approach generalizes the established methods of quantum field theory and thermal field theory (in curved spacetimes) to statistical and stochastic field theory. The exact thermal radiance cases arising from an exact exponential scale transformation such as is found in a uniformly accelerated detector, the Schwarzschild black hole and the de Sitter universe, have been treated in the stochastic theoretical method before [46,50]. Here we give the treatment of the moving mirror and the collapsing mass as further examples. (Thermal radiation in certain classes of cosmological spacetimes including the inflationary universe will be studied in a following paper [57].)

(2) We have shown that in all the examples considered in this class of spacetimes, i.e., accelerated observers, moving mirrors, and collapsing masses (black holes), those which yield *a thermal spectrum of created particles all involve an exponential scale transformation*. Thermal radiance observed in one vacuum arises from the exponential scaling of the quantum fluctuations (noise) in another vacuum. This view espoused by one of us [31–35] is illustrated in the examples treated here.

(3) A practical aim of this paper is to show how one can use quantum field theory techniques aided by statistical-mechanical concepts to calculate particle creation in the near-exponential cases, yielding near-thermal spectra. These cases are not so easy to formulate conceptually using the traditional methods: The geometric picture in terms of the properties of the event horizons as global geometric entities works well for equilibrium thermodynamics (actually thermostatics) conditions, so does thermal field theory which assumes *a priori* a finite-temperature condition (e.g., periodic boundary condition on the imaginary time). However, they cannot be easily generalized to nonequilibrium dynamical conditions. In the stochastic theory approach we used, the starting point is the vacuum fluctuations of quantum fields subjected to kinematical or dynamical excitations. There is no explicit use of the global geometric properties of spacetimes: The event horizons arise from exponential scaling. (Thus, for example, this method can describe the situations where a detector is accelerated only for a short duration, whereas one cannot easily describe in geometric terms the scenario of an event horizon appearing and disappearing.) There is also no *a priori* assumption of equilibrium conditions: The concept of temperature is neither viable nor necessary, as is expected in all nonequilibrium conditions. Thermal or near-thermal radiance is a result of some specific

conditions acting on the vacuum fluctuations in the system.

(4) We restrict our attention in this paper to near-thermal conditions because of technical rather than conceptual limitations. In the near-thermal cases treated here, we want to add that the stochastic theoretical method is not the only way to derive these results. One can alternatively approach with the global geometric or thermal field methods, say, by working with generalized definitions of event horizons or quasiperiodic Green functions. However, we find it logically more convincing and technically more rigorous to use the stochastic field theory method to define and analyze field theoretical and statistical concepts like fluctuations and dissipation, correlation, and coherence. We believe that in the fully dynamical and nonequilibrium cases, such as will be encountered in the full back reaction problem (not just confined to the linear response regime), this method is more advantageous than the existing ones. Even though the technical problems will likely be grave (just the built-in balance between dissipation and fluctuations alone requires a self-consistent treatment *ab initio*), there are no conceptual pitfalls or intrinsic shortcomings.

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