PUBLISHED VERSION

Maltman, Kim
The mixed isospin vector current correlator reexamined Physical Review D, 1996; 53(5):2563-2572

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http://link.aps.org/doi/10.1103/PhysRevD.53.2563

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QCD sum rule analysis of the mixed-isospin vector current correlator reexamined

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(Received 5 April 1995)

The mixed-isospin vector current correlator $\langle 0|T(V^{\rho}_{\mu}V^{\omega}_{\nu})|0\rangle$ is evaluated using QCD sum rules. The sum rule treatment is a modification of previous analyses necessitated by the observation that those analyses produce forms of the correlator that fail to be dominated, near $q^2 = 0$, by the most nearby singularities. The inclusion of contributions associated with the ϕ meson rectifies this problem. The resulting sum rule fit provides evidence for a significant direct $\omega \to \pi\pi$ coupling contribution in $e^+e^- \to \pi^+\pi^-$. It is also pointed out that results for the q^2 dependence of the correlator cannot be used to provide information about the (off-shell) q^2 dependence of the off-diagonal element of the vector meson propagator unless a very specific choice of interpolating fields for the vector mesons is made.

PACS number(s): 11.55.Hx, 12.39.Fe, 14.40.Cs, 24.85.+p

I. INTRODUCTION

Nonelectromagnetic isosopin breaking is well established in many strongly interacting systems (e.g., splittings in the hadron spectrum, binding energy differences in mirror nuclei, asymmetries in polarized np scattering, binding energies and level splittings of light Λ hypernuclei [1]). In few-body systems, an important source of this breaking has been thought to be the mixing of isoscalar and isovector mesons appearing in meson exchange diagrams. In particular, the bulk of the non-Coulombic contributions to the charge symmetry-breaking nn-pp scattering length difference and to the $A = 3$ binding energy difference, and of the np asymmetry at 183 MeV, can be explained [2,3] using the value of ρ-ω mixing extracted from an analysis of $e^+e^- \to \pi^+\pi^-$ in the ρ-ω interference region [4,5]. The plausibility of this explanation (which employs the observed mixing, measured at $q^2 = m^2_{\omega}$, unchanged in the spacelike region $q^2 < 0$) has, however, recently been called into question by Goldman, Henderson, and Thomas [6] who pointed out that, in the context of a particular model, the relevant ρ-ω mixing matrix element has significant q^2 dependence. Subsequently, various authors, employing various computational and/or model frameworks, have showed that the presence of such q^2 dependence appears to be a common feature of isospin breaking in both meson-propagator and current-correlator matrix elements [7–16].

In the present paper we will concentrate on the isospin-breaking vector current correlator

$$\Pi_{\mu\nu}(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|T[V^{\rho}_{\mu}(x)\, V^{\omega}_{\nu}(0)]|0\rangle\ , \quad (1.1)$$

where

$$V^{\rho}_{\mu} = (\bar{u}\gamma_{\mu}u - \bar{d}\gamma_{\mu}d)/2\ , \quad V^{\omega}_{\mu} = (\bar{u}\gamma_{\mu}u + \bar{d}\gamma_{\mu}d)/6\ . \quad (1.2)$$

This correlator was first analyzed using QCD sum rules in Ref. [17], and the analysis updated by the authors of Ref. [12] who, in particular, stressed the q^2 dependence of the correlator implicit in the results of this analysis. As will be shown below, a worrisome feature of the resulting fit is that the phenomenological representation of the correlator near $q^2 = 0$ is not dominated by the most nearby singularities, suggesting that some ingredient may be missing from the form chosen for this representation. This missing ingredient is identified below and it is shown that a reanalysis of the correlator, which includes it, rectifies the problem. The resulting correlator still displays a very strong q^2 dependence, and, in addition, provides evidence for the presence of significant "direct" $\omega \to \pi\pi$ coupling in $e^+e^- \to \pi^+\pi^-$.

The paper is organized as follows. In Sec. II, those features of the behavior of quantum field theories under field redefinitions relevant to attempts in the literature to relate meson propagators and current correlators are discussed, and it is explained why the freedom of field redefinition implies that (1) one cannot obtain off-shell information about the off-diagonal element of the vector meson propagator from the off-diagonal element of the vector current correlator without making specific choices for the vector meson interpolating fields, and (2) if one writes the off-diagonal element of the vector meson propagator as

$$\Delta^{\rho\omega}_{\mu\nu}(q^2) \equiv -(g_{\mu\nu} - q_{\mu}q_{\nu}/q^2)\frac{\theta^{\rho\omega}(q^2)}{(q^2 - m^2_{\rho})(q^2 - m^2_{\omega})} \quad (1.3)$$

$\theta^{\rho\omega}(q^2)$ cannot, in general, be q^2 independent. In Sec. III we return to the QCD sum rule analysis of the vector current correlator, first explaining why certain features of the existing analyses suggest the need for a modified analysis, and then performing this analysis. The results both correct the apparently unphysical features of the previous analyses and provide evidence for non-negligible direct $\omega \to \pi^+\pi^-$ contributions to $e^+e^- \to \pi^+\pi^-$ in the ρ-ω interference region. In Sec. IV, we point out why, unlike the case of the analogous axial vector correlator (see Ref. [16]), chiral perturbation theory (ChPT) to one

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loop cannot be used to constrain the sum rule analysis. Finally, in Sec. V, a brief summary of the main results of the paper is given.

II. CONSEQUENCES OF THE FREEDOM OF FIELD REDEFINITION

Let us begin by clarifying the relation (or lack thereof) between the vector-meson-propagator and vector-current-correlator matrices. The former is an, in general, off-shell Green function, which we may think of as being associated with some low-energy effective Lagrangian $\mathcal{L}_{\rm eff}$ in which the vector meson degrees of freedom have been made explicit. As is well known [18–20], the form of such a Lagrangian is not unique: if ϕ and χ are two possible field choices describing a given particle, related by $\phi = \chi F(\chi)$, with $F(0) = 1$, then $\mathcal{L}_{\rm eff}[\phi]$ and $\mathcal{L}'_{\rm eff}[\chi] \equiv \mathcal{L}_{\rm eff}[\chi F(\chi)]$ produce exactly the same experimental observables [18]. However, while the S-matrix elements of the two theories are identical, this is not true of the general off-shell Green functions. One is free to make field redefinitions of the form above (as is done, e.g., in order to obtain the canonical form of the effective Lagrangian for ChPT [19–21]) without changing the physical consequences of the theory; the Green functions, however, are not in general invariant under such field redefinitions. Useful pedagogical illustrations of this general principle, for pion Compton scattering and the linear σ model, are given in Ref. [22] and Chap. IV of Ref. [23], respectively. In the case of interest to us, what this means is that the off-shell behavior of the vector meson propagator is dependent on the particular choice of fields used to represent the vector mesons (the choice of "interpolating field"). It is not a physical observable. In contrast, the vector current correlators $\Pi^{ab}_{\mu\nu}(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|T[V^a_\mu(x)\, V^b_\nu(0)]|0\rangle$ are physical objects, independent of the interpolating field choice. The spectral functions for $\Pi^{33}_{\mu\nu}$ and $\Pi^{88}_{\mu\nu}$ are, for example, accessible from a combination of $\tau^- \to \nu_\tau\pi^-\pi^0$ and $e^+e^- \to \pi\pi\,,\, \pi\pi\pi\pi$ data, and that for $\Pi^{38}_{\mu\nu}$ could in principle be obtained from a careful analysis of the deviation of the ratio of the differential decay rates for $\tau^- \to \nu_\tau\pi^-\pi^0$ and $e^+e^- \to \pi^+\pi^-$ from that predicted by isospin symmetry. As such there can be no general (i.e., valid for all choices of interpolating field) relation between the correlator and propagator matrices. This point is the source of some confusion in Ref. [12] where an attempt is made to obtain the off-shell propagator based on an analysis of the correlator.

Before proceeding to the reanalysis of the correlator, let us be more precise about the problems with the interpretation of the results of Ref. [12], in the light of the above comments. The authors begin by writing a general form for the spectral function of the correlator:

$$\operatorname{Im}\Pi_{\mu\nu}(q^2) = A_0\operatorname{Im}\Pi^{\rho\omega}_{\mu\nu}(q^2) + A_1\operatorname{Im}\Pi^{\rho'\omega'}_{\mu\nu} + \cdots\ , \tag{2.1}$$

where the superscripts on the right-hand side (RHS) should, for the moment, be taken only as labeling the region of spectral strength, and where the ellipsis refers to all other contributions (we return to this below). Equation (2.1) is, of course, completely general. The authors of Ref. [12], however, then identify A_0 with $m^2_\rho m^2_\omega/g_\rho g_\omega$, where $g_{\rho,\omega}$ are the vector meson decay constants, defined by

$$\langle 0|V^{\rho,\omega}_\mu|\rho,\omega\rangle \equiv \frac{m^2_{\rho,\omega}}{g_{\rho,\omega}}\epsilon_\mu\ , \tag{2.2}$$

and $\Pi^{\rho\omega}_{\mu\nu}$ with the off-diagonal element of the vector meson propagator. This amounts to assuming that the isospin-unmixed $I = 1$ ρ state, $\rho^{(0)}$, couples only to V^ρ_μ, and the isospin-unmixed $I = 0$ ω state, $\omega^{(0)}$, only to V^ω_μ, the isospin-breaking contribution to $\Pi_{\mu\nu}$ of Eq. (2.2) from the ρ,ω region then resulting solely from the $\rho^{(0)}$-$\omega^{(0)}$ mixing in the meson propagator. In this interpretation, fixing the imaginary part of the correlator in the ρ,ω region (via the sum rule analysis) allows one to obtain the isospin-breaking parameters of the imaginary part of the vector meson propagator, and, via a dispersion relation, the behavior of the off-diagonal element of the propagator off shell. As explained above, however, such a possibility is excluded on general grounds. The problem with the interpretation of Ref. [12] is that not one, but three sources of isospin breaking exist in the contributions to $\Pi_{\mu\nu}$ from the ρ,ω region: that due to $\rho^{(0)}$-$\omega^{(0)}$ mixing (discussed above), that due to the direct coupling of V^ρ_μ to $\omega^{(0)}$, and that due to the direct coupling of V^ω_μ to $\rho^{(0)}$. The physical matrix elements between V^ω_μ and the physical ρ or V^ρ_μ and the physical ω would be described by new isospin-breaking parameters $\phi^{(\rho)\omega}$ and $\phi^{(\omega)\rho}$,

$$\begin{aligned}\langle 0|V^\omega_\mu|\rho\rangle &\equiv \frac{m^2_\omega}{g_\omega}\phi^{(\omega)\rho}\epsilon_\mu\ ,\\ \langle 0|V^\rho_\mu|\omega\rangle &\equiv \frac{m^2_\rho}{g_\rho}\phi^{(\rho)\omega}\epsilon_\mu\ ,\end{aligned} \tag{2.3}$$

where $\phi^{(\omega)\rho}$ and $\phi^{(\rho)\omega}$ receive contributions both from mixing and from the direct couplings, and are, in general, q^2 dependent, and also interpolating-field-dependent off shell. Thus, off shell, the ρ-ω region contribution to $\Pi_{\mu\nu}$ depends not only on the (interpolating-field-choice-dependent) isospin-breaking parameters of the off-diagonal element of the vector meson propagator, but also on the (interpolating-field-choice-dependent) isospin-breaking parameters $\phi^{(\omega)\rho}$ and $\phi^{(\rho)\omega}$ (which contain contributions from the direct couplings). The total correlator is independent of the interpolating field choice, but the individual contributions are not. One is, of course, free to choose a convenient set of ρ,ω interpolating fields and work with these, provided one calculates contributions to S-matrix elements. Since, to $O(m_d - m_u)$, Eq. (2.2) remains valid when we replace ρ and ω with $\rho^{(0)}$ and $\omega^{(0)}$, the fields

$$\begin{aligned}\rho^{(0)c}_\mu &\equiv \frac{g_\rho}{m^2_\rho}V^\rho_\mu\ ,\\ \omega^{(0)c}_\mu &\equiv \frac{g_\omega}{m^2_\omega}V^\omega_\mu\ ,\end{aligned} \tag{2.4}$$

satisfy $\langle 0|\rho_\mu^{(0)c}|\rho^{(0)}\rangle = \epsilon_\mu$ and $\langle 0|\omega_\mu^{(0)c}|\omega^{(0)}\rangle = \epsilon_\mu$, and hence serve as possible choices of interpolating fields for $\rho^{(0)}$ and $\omega^{(0)}$. With this choice of interpolating fields (and not with others) one obtains

$$\Pi_{\mu\nu}(q^2) = \frac{m_\rho^2}{g_\rho}\frac{m_\omega^2}{g_\omega}\Delta_{\mu\nu}^{(c)\rho\omega}(q^2) \qquad (2.5)$$

where $\Delta_{\mu\nu}^{(c)\rho\omega}(q^2)$ is the off-diagonal element of the vector meson propagator for the interpolating field choice above. For a general choice of interpolating field, however, neither $\Pi_{\mu\nu}$ nor $\Pi_{\mu\nu}^{\rho\omega}$ is proportional to $\Delta_{\mu\nu}^{\rho\omega}$.

Note that the above discussion also clarifies one ongoing point of debate in the literature, namely, that concerning the q^2 dependence of the quantity $\theta^{\rho\omega}(q^2)$ appearing in Eq. (1.3). Defining $\hat\Pi(q^2)$ by

$$\Pi_{\mu\nu}(q^2) \equiv (g_{\mu\nu} - q_\mu q_\nu/q^2)\hat\Pi(q^2) \ , \qquad (2.6)$$

the absence of massless singularities implies that $\hat\Pi(0) = 0$ [14]. This in turn implies, with

$$\Delta_{\mu\nu}^{(c)\rho\omega}(q^2) \equiv -(g_{\mu\nu} - q_\mu q_\nu/q^2)\Delta^{(c)\rho\omega}(q^2) \ , \qquad (2.7)$$

$\Delta^{(c)\rho\omega}(q^2) = 0$, and hence $\theta^{\rho\omega}(0) = 0$. Since this is true for one choice of the vector meson interpolating fields, it is incumbent upon those advocating

$$\theta^{\rho\omega}(q^2) = \theta^{\rho\omega}(m_\omega^2) \qquad (2.8)$$

to explicitly demonstrate the existence of an interpolating field choice for the vector mesons for which Eq. (2.8) is valid; the relation cannot be true in general.

III. THE QCD SUM RULE ANALYSIS OF $\Pi_{\mu\nu}(q^2)$ REEXAMINED

With the above discussion in mind, let us turn to the sum rule analysis of the vector correlator, first briefly reviewing the treatment and results of Refs. [12,17]. The sum rule approach consists of writing an operator product expansion (OPE) representation for the correlator, valid in the region of validity of perturbative QCD, and a second, phenomenological, representation in terms of hadronic parameters, and then Borel transforming both. The Borel transform serves to extend the ranges of validity of both representations and, in addition, to (1) emphasize the operators of lowest dimension in the OPE representation and (2) give higher weight to the parameters of the lowest-lying resonances in the phenomenological representation. One then matches the transformed representations in order to make predictions for the relevant hadronic parameters.

The OPE for the correlator of interest was performed long ago [17]. Truncating the expansion at operators of dimension 6, one finds that, defining $\Pi(q^2)$ by

$$\Pi_{\mu\nu} \equiv (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2) \ , \qquad (3.1)$$

one has

$$\Pi^{\rm OPE}(Q^2) = \frac{1}{12}\left[-c_0\ln(Q^2) + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \frac{2c_3}{Q^6}\right] \qquad (3.2)$$

where $Q^2 = -q^2$ and

$$\begin{aligned} c_0 &= \frac{\alpha_{\rm EM}}{16\pi^3} \ , \\ c_1 &= \frac{3}{2\pi^2}(m_d^2 - m_u^2) \ , \\ c_2 &= \left(\frac{m_d - m_u}{m_d + m_u}\right) 2f_\pi^2 m_\pi^2\left[1 + \left(\frac{\gamma}{2+\gamma}\right)\left(\frac{m_d + m_u}{m_d - m_u}\right)\right] \ , \\ c_3 &= -\frac{224}{81}\pi\left[\alpha_s\langle\bar q q\rangle_0\right]^2\left[\frac{\alpha_{\rm EM}}{8\alpha_s(\mu^2)} - \gamma\right] \ , \end{aligned} \qquad (3.3)$$

with $\gamma \equiv \langle\bar d d\rangle_0/\langle\bar u u\rangle_0 - 1$. Taking for the phenomenological representation (in the narrow resonance approximation)

$$\begin{aligned} {\rm Im}\,\Pi^{\rm phen}(q^2) = \frac{\pi}{12}[&f_\rho\delta(q^2 - m_\rho^2) - f_\omega\delta(q^2 - m_\omega^2) \\ &+ f_{\rho'}\delta(q^2 - m_{\rho'}^2) - f_{\omega'}\delta(q^2 - m_{\omega'}^2)] \\ &+ \frac{\alpha_{\rm EM}}{192\pi^2}\theta(q^2 - s_0) \end{aligned} \qquad (3.4)$$

(where f_ρ, f_ω, $f_{\rho'}$, and $f_{\omega'}$ may be thought of as the parameters to be determined from the sum rule analysis) one finds, upon Borel transformation and matching,

$$\begin{aligned} \frac{1}{M^2}[f_\rho\exp(-m_\rho^2/M^2) - f_\omega\exp(-m_\omega^2/M^2) + f_{\rho'}\exp(-m_{\rho'}^2/M^2) - f_{\omega'}\exp(-m_{\omega'}^2/M^2)] + \frac{\alpha_{\rm EM}}{16\pi^3}\exp(-s_0/M^2) \\ = c_0 + \frac{c_1}{M^2} + \frac{c_2}{M^4} + \frac{c_3}{M^6} \ , \end{aligned} \qquad (3.5)$$

where M is the Borel mass. As pointed out in Ref. [12], to $O(\delta m^2,\ \delta m'^2)$, where $\delta m^2 = m_\omega^2 - m_\rho^2$ and $\delta m'^2 = m_{\omega'}^2 - m_{\rho'}^2$, Eq. (3.5) can be rewritten in terms of the parameters ξ, β, ξ', and β', where

$$\begin{aligned} \xi &= \frac{\delta m^2}{m^4}\left(\frac{f_\rho + f_\omega}{2}\right) \ , \\ \beta &= \frac{(f_\omega - f_\rho)}{m^2\,\xi} \ , \\ \xi' &= \frac{\delta m'^2}{m'^4}\left(\frac{f_{\rho'} + f_{\omega'}}{2}\right) \ , \\ \beta' &= \frac{(f_{\omega'} - f_{\rho'})}{m'^2\,\xi'} \ , \end{aligned} \qquad (3.6)$$

with $m^2 \equiv (m_\rho^2 + m_\omega^2)/2$ and $m'^2 \equiv (m_{\rho'}^2 + m_{\omega'}^2)/2$, as

$$\xi \frac{m^2}{M^2}\left(\frac{m^2}{M^2} - \beta\right)\exp(-m^2/M^2) + \xi' \frac{m'^2}{M^2}\left(m'^2 - \beta'\right)\exp(-m'^2/M^2) + \frac{\alpha_{EM}}{16\pi^3}\exp(-s_0/M^2) = c_0 + \frac{c_1}{M^2} + \frac{c_2}{M^4} + \frac{c_3}{M^6} \,. \tag{3.7}$$

If c_{0-3} were precisely known, Eq. (3.5) or Eq. (3.7) could, in principle, be used to determine the parameters ξ, β, ξ', and β'. There are, however, some uncertainties in the values of the c_i, associated with the imprecision in our knowledge of the values of the four-quark condensates and of the isospin-breaking ratio of the $\langle \bar{u}u\rangle_0$ and $\langle \bar{d}d\rangle_0$ condensates. The authors of Ref. [12] (which updates Ref. [17]) consider a range of possibilities for these quantities, and also take for $r \equiv (m_d - m_u)/(m_d + m_u)$ the value $r = 0.28$, obtained from an analysis of pseudoscalar isomultiplet splittings [24] employing Dashen's theorem [25] for the electromagnetic contributions to these splittings. The last ingredient of the analysis of Ref. [12] is the imposition of an external constraint on the hadronic parameter ξ, based on the observed interference in the ρ-ω interference region in $e^+e^- \to \pi^+\pi^-$. This constrained value, $\xi = 1.13 \times 10^{-3}$, is based on (1) the assumed connection between the correlator and the propagator (presumably valid for the essentially on-shell value of the mixing, though not elsewhere) and (2) the assumption that direct $\omega^{(0)} \to \pi\pi$ contributions to $e^+e^- \to \pi^+\pi^-$ can be neglected (see Ref. [26] for a discussion of these issues). There appears to be no particularly good reason for the latter assumption, and, indeed, it would seem appropriate to allow ξ to be fitted by the sum rule analysis as a test of this assumption (as will be done below), but let us follow the analysis of Ref. [12] for the moment. Using the sum rule, Eq. (3.7), and imposing the constraint $\xi = 1.13 \times 10^{-3}$, as discussed above, the authors of Ref. [12] solve for β, ξ', and β' for four different input sets $\{c_i\}$. Using the expression (3.4) for $\mathrm{Im}\,\Pi^{\rm phen}(q^2)$ and the fact that $\Pi(q^2)$ satisfies an unsubtracted dispersion relation, one may show that, to first order in δm^2 and $\delta m'^2$,

$$\mathrm{Re}\,\Pi(0) = \frac{1}{12}\left[\xi(1-\beta) + \xi'(1-\beta')\right] \,. \tag{3.8}$$

Using the values of the parameters obtained in Ref. [12], one finds that the ratios of the contributions to $\mathrm{Re}\,\Pi(0)$ from the ρ'-ω' region to those from the ρ-ω region are 1.8, 0.8, 0.3, and 0.8 for input sets I, II, III, and IV, respectively. The failure of the results to be dominated by the nearby (ρ,ω) singularities suggests that the phenomenological form employed for the spectral function may well be incomplete, either in missing low-lying contributions or in failing to include the effect of even more distant singularities. If we consider Eqs. (3.4) and (3.8) for a moment an interesting possibility becomes evident. If one had all isospin-breaking effects generated solely by $\rho^{(0)}$-$\omega^{(0)}$ mixing, and if the physical vector mesons were a simple rotation of the isospin-pure basis (not in general true when the wave-function renormalization matrix of the system is nondiagonal), we would have $f_\rho = f_\omega$ for f_ρ and f_ω as written in Eq. (3.4). While the assumptions required to arrive at this conclusion are certainly not satisfied in general, this nonetheless indicates that there should be significant cancellation between the ρ and ω contributions to the correlator. Thus a single isolated resonance, even with a coupling much smaller than that of the ρ or ω, could in fact contribute significantly to $\Pi_{\mu\nu}$. This suggests that the ϕ contribution to $\mathrm{Im}\,\Pi_{\mu\nu}$, neglected in Ref. [12], may well be non-negligible. In fact we can make a rough estimate of the expected size of f_ϕ [where f_ϕ is defined by adding a contribution $\frac{\pi}{12} f_\phi \delta(q^2 - m_\phi^2)$ to $\mathrm{Im}\,\Pi^{\rm phen}(q^2)$ in Eq. (3.4)] as follows. ϕ is known to be not quite pure $\bar{s}s$. If, e.g., we take the Particle Data Group (PDG) [27] value for the octet-singlet mixing angle, $\theta = 39°$ (quadratic fit), $\phi \simeq \phi^{(0)} - \delta\omega^{(0)}$, where $\phi^{(0)}$ is the pure $\bar{s}s$ state and $\delta = 0.065$ rad is the deviation of θ from ideal mixing. The contribution of the ϕ pole term to $\Pi_{\mu\nu}$ due to mixing in the propagator should then be of order $-\delta$ times that associated with the ω pole, i.e., $\simeq 0.065\, f_\omega \simeq 0.065\, f_\rho$. There will, of course, also, in general, be isospin-breaking contributions from direct couplings to the current vertices, not just from mixing in the propagator, but the above discussion shows that $f_\phi \simeq (0.05$–$0.10)\, f_{\rho,\omega}$ should be a reasonable expectation. As we will see below, this (rather crude) estimate is indeed borne out by the sum rule analysis.

Let us, therefore, add a term $\frac{\pi}{12} f_\phi \delta(q^2 - m_\phi^2)$ to $\mathrm{Im}\,\Pi^{\rm phen}(q^2)$ on the RHS of Eq. (3.4), and perform a reanalysis of that equation. We will follow Ref. [12] in choosing the range of input values for the $\{c_i\}$, with, however, the following modifications. First, the small c_1 term dropped in Ref. [12] will be retained, though, as pointed out there, it in fact has little effect on the final results. The numerical value is obtained by using $(m_d + m_u)(1\ \mathrm{GeV}) = 12.5 \pm 2.5$ MeV from Ref. [28] and the updated value of r discussed below. The main modification to the input is in the parameter r. There is now considerable evidence that Dashen's theorem is significantly violated [29–31], Refs. [30,31], in particular, suggesting that

$$(m_{K^+}^2 - m_{K^0}^2)_{\rm EM} \simeq 1.9\,(m_{\pi^+}^2 - m_{\pi^0}^2)_{\rm expt} \tag{3.9}$$

[where the factor 1.9 on the RHS of Eq. (3.9) is absent in Dashen's theorem]. Using Eq. (3.9) in place of Dashen's theorem for the electromagnetic contribution to the kaon mass splitting produces a rescaling of r by 1.22. The resulting change in the c_i is essentially to rescale the values of c_2 in Ref. [12] by this same factor. In assessing the effect of the uncertainties in the values of the $\{c_i\}$ for a given input set, the input errors on c_2 have also been rescaled by this factor of 1.22. Finally, since the masses of all the resonances appearing above, including the ρ' and ω', are known, we may take these as input

TABLE I. Sum rule fit for the parameters ξ, β, ξ', β', and f_ϕ.

Input	$\xi \times 10^3$	β	$\xi' \times 10^5$	β'	$f_\phi \times 10^3$
Set I	2.18±0.39	1.49±0.06	-2.63±0.79	-5.84±0.12	2.30±0.52
Set III	3.10±0.39	1.62±0.02	-4.57±0.69	-5.72±0.01	3.57±0.52
Set IV	2.59±0.39	1.55±0.04	-3.47±0.61	-5.78±0.04	2.86±0.45

and use the sum rule to extract the isospin-breaking parameters $\{f_k\}$, where $i = 1, \dots, 5$ correspond to ρ, ω, ρ', ω', and ϕ, respectively. Note that, in taking this approach, we are abandoning the constraint on ξ employed in Ref. [12]. If the direct $\omega^{(0)} \to \pi^+\pi^-$ coupling is, indeed, negligible in $e^+e^- \to \pi^+\pi^-$, this will manifest itself by the value of ξ resulting from the sum rule analysis being near 1.13×10^{-3}.

The analysis of the modified version of the sum rule, (3.5), proceeds as follows. First, from the terms of $O(M^0)$, $c_0 = \alpha_{\rm EM}/16\pi^3$. One may check that, as in Ref. [12], the analysis is very insensitive to the value of the EM threshold parameter s_0. We will, therefore, quote all results below for the value, $s_0 = 1.8$ GeV, employed in a number of the results quoted in Ref. [12]. Second, again as in Ref. [12], we impose the local duality relation

$$\int_0^\infty ds\, \mathrm{Im}\,\Pi^{\rm phen}(s) = O(\alpha_{\rm EM}, m_q^2) \tag{3.10}$$

[which is equivalent to matching the coefficients of the $O(1/M^2)$ terms in Eq. (3.5)]. With the index $k = 1, \dots, 5$ labeling ρ, ω, ρ', ω', and ϕ, respectively, as above, this relation is

$$\sum_k (-1)^{k+1} f_k = c_0 s_0 + c_1 \ . \tag{3.11}$$

[Note that the c_i tabulated in Ref. [12] have had the appropriate factors of m^2 required to leave the remaining coefficient dimensionless factored out of them. Thus, e.g., c_1 in Eq. (3.11) is m^2 times that tabulated in Ref. [12].] The remaining four relations required to obtain a solution for the five unknowns $\{f_k\}$ are obtained by acting on Eq. (3.5) with $(-1)^n \frac{\partial^n}{\partial(1/M^2)^n}$ for $n = 1, \dots, 4$. One may check that the results are not sensitive to using precisely the PDG values for the ρ' and ω' masses. Indeed, shifting either mass by 50 MeV induces changes of $< 4\%$ in ξ, $< 2.5\%$ in β, $< 5\%$ in β', and $< 20\%$ in ξ'. The resulting changes in the correlator itself are even smaller: e.g., $\Pi(0)$ and $\frac{d\Pi}{dq^2}(0)$ are changed by $< 2\%$ by the above mass shifts.

In Table I, the results of the modified sum rule analysis are displayed for the input sets I, III, and IV of Ref. [12], modified as described above. The errors shown in the table correspond to the uncertainties in the input parameters c_2 and c_3 (those quoted in Ref. [12] in the case of c_3 and the rescaled version thereof in the case of c_2). The stability of the analysis is illustrated, for input set IV, in Figs. 1–5, which display the parameters ξ, β, ξ', β', and f_ϕ as a function of the Borel mass M in the range 1–10 GeV (the choice of the first four parameters, rather than corresponding f_k values, is made in order to facilitate comparison with Ref. [12]). Set I generates results of comparable stability, while the results of set III are even more stable than those of set IV. In all three cases a wide stability window exists in the Borel mass for all five output parameters. This stability window, moreover, occurs without the necessity of using unphysical values for the average of the ρ' and ω' masses. As noted previously in Ref. [12], results for input set II are considerably less stable than for the other sets: in fact, no stability window exists anywhere in the range $M = 1$ and $M = 10$ GeV,

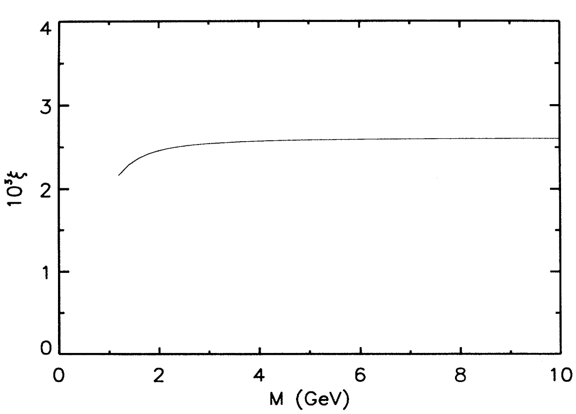

FIG. 1. Dependence of ξ on the Borel mass M for modified input set IV.

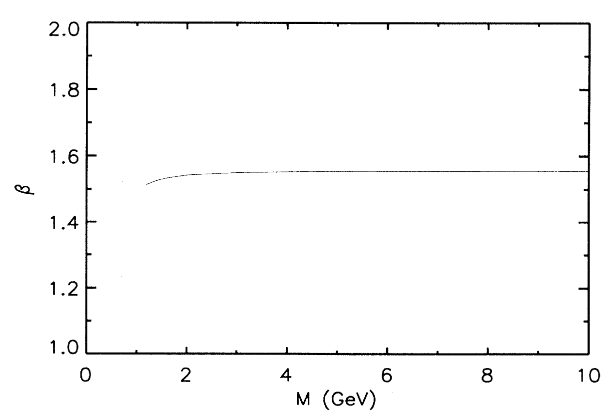

FIG. 2. Dependence of β on the Borel mass M for modified input set IV.

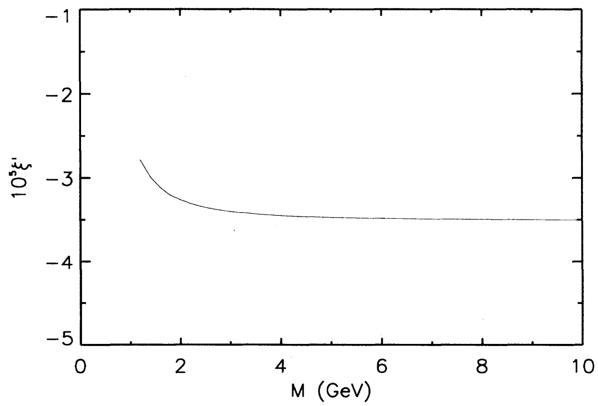

FIG. 3. Dependence of ξ' on the Borel mass M for modified input set IV.

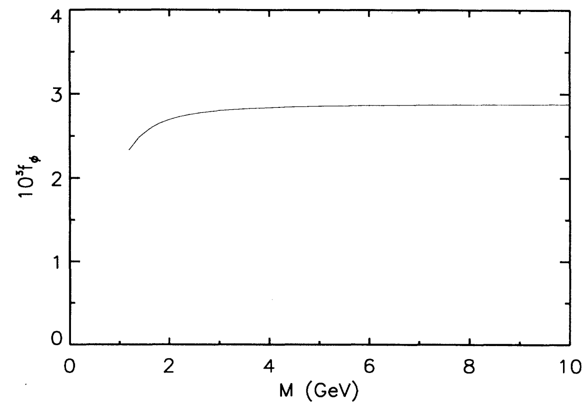

FIG. 5. Dependence of f_ϕ on the Borel mass M for modified input set IV.

apart from for the very lower edge of the error band for the magnitude of c_3, for which values input set II is very close to the upper end of the corresponding error band for input set I. The instability of the analysis for input set II is illustrated (for the central values of c_2 and c_3) in Fig. 6, where the parameter f_ϕ is plotted as a function of the Borel mass M. As a result of this instability, results corresponding to input set II are not quoted in the table; for most of the input range (i.e., for larger values of the magnitude of c_3) the input set appears, from the sum rule analysis, to be unphysical.

A number of features are evident from the results of the above analysis. First, from Table I, we see that the magnitude of ξ differs significantly from that which would be expected from the analysis of $e^+e^- \to \pi^+\pi^-$, neglecting $\omega^{(0)} \to \pi^+\pi^-$ contributions, suggesting that the latter are, indeed, not negligible. It should be stressed that the errors quoted in the table correspond to varying c_2 and c_3 separately within the range of quoted errors, and taking the maximum variation of the resulting output. One can obtain even lower values of ξ, i.e., closer to that expected, if one can indeed neglect $\omega^{(0)} \to \pi^+\pi^-$ contributions to $e^+e^- \to \pi^+\pi^-$, by letting c_2 lie at the bottom of its error band and, simultaneously, the magnitude of c_3 lie at the top of its error band in set I. However, such a combination (which produces $\xi = 1.43 \times 10^{-3}$) is quite unstable, the values of ξ', e.g., varying by more than 20% between $M = 3$ and 5 GeV. A similar result, $\xi = 1.48 \times 10^{-3}$, can be obtained from set II for the central value of c_2 and the lower edge of the error band for the magnitude of c_3, with comparable ($\simeq$ 20% over the range $M = 3$ to 5 GeV) instability. All other portions of the set II error band are even more unstable. Thus it appears very clear that the value $\xi = 1.13 \times 10^{-3}$ is excluded by the present sum rule analysis. The second observation is that the inclusion of the ϕ pole term in the phenomenological representation of the correlator rectifies the problem of the strength of the distant singularities. This can be seen from the relative size of ξ and ξ' in Table I, but is more evident in Table II, where the output values for the

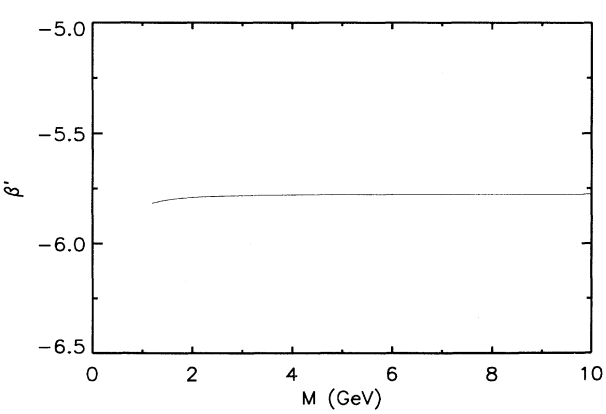

FIG. 4. Dependence of β' on the Borel mass M for modified input set IV.

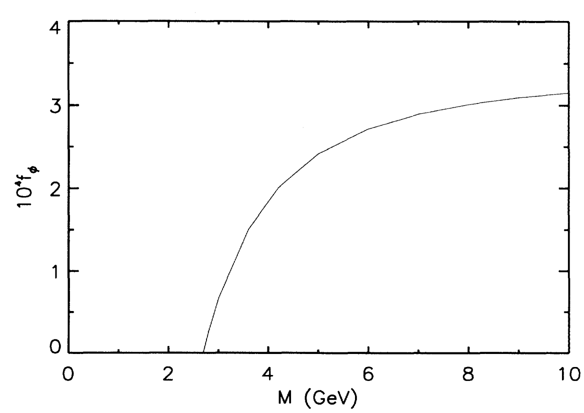

FIG. 6. Dependence of f_ϕ on the Borel mass M for modified input set II.

TABLE II. Sum rule fit for the isospin-breaking parameters $\{f_k\}$. Values are quoted for the central values of the input parameters $\{c_i\}$. The units are GeV^2.

Input	$f_\rho \times 10^2$	$f_\omega \times 10^2$	$f_\phi \times 10^3$	$f_{\rho'} \times 10^4$	$f_{\omega'} \times 10^4$
Set I	3.53	3.73	2.30	5.34	8.45
Set III	5.00	5.30	3.57	9.32	14.6
Set IV	4.18	4.42	2.86	7.06	11.1

parameters $\{f_k\}$ are tabulated, for the central values of the input parameters $\{c_i\}$, for input sets I, III, and IV. The ratios of f_ϕ to f_ω are 0.062, 0.068, and 0.066 for sets I, III, and IV, respectively. This is in (better than should be expected) agreement with the rough estimate given above, confirming the physical plausibility of the solutions obtained. Moreover, $f_{\rho'}$ and $f_{\omega'}$ are now a factor of 40–60 smaller than f_ρ and f_ω. The structure of the resulting contributions to the correlator near $q^2 = 0$ is shown in Table III, where the ρ, ω and also the ρ', ω' contributions have been combined. Note that the individual ρ and ω contributions are a factor of $\simeq 13$ larger than the ϕ contribution, but the cancellation between them is such that the ϕ contribution is approximately twice as large as their sum. The ρ'-ω' region contribution is then less than 10% of the ϕ contribution. The more distant singularities, thus, have only a small effect, justifying, *a posteriori*, the neglect of yet more distant singularities in the phenomenological side of the sum rule analysis. The fact that, after including the ϕ contributions, the ρ', ω' contributions are now so small, the high degree of stability of the analysis, and the smallness of the shifts in β, ξ which resulted from shifting the input ρ', ω' masses as described above suggest, moreover, that the simplified treatment of the higher, continuum contributions in terms of just the ρ', ω' pole terms is a safe one. Given that the results satisfy all the above tests for being physically sensible and stable, it appears that the resulting values for the correlator and its slope with respect to q^2 at $q^2 = 0$ should be taken as good estimates, within the uncertainties resulting from those in the input parameters. The fact that, due to cancellation between the otherwise dominant ρ and ω contributions, the ϕ contribution is actually dominant no doubt accounts for the unphysical behavior of the spectral distribution of the correlator obtained in the absence of the ϕ term. Note that, despite the significant changes in the fit, as compared to Ref. [12], the slope of the correlator remains large in the present results. We would also like to stress that the possibility of negligible direct $\omega^{(0)} \to \pi\pi$ contribution to $e^+e^- \to \pi^+\pi^-$ is incompatible with the present sum rule analysis. A similar conclusion results from a recent treatment of the direct coupling in a field-theoretic model employing confining quark propagators motivated by quark and gluon Schwinger-Dyson equation studies, together with the chiral limit Bethe-Salpeter amplitude for the pion [32].

TABLE III. Behavior of the correlator near $q^2 = 0$. Contributions to $12\,\Pi(0)$ from the ρ-ω, ϕ, and ρ'-ω' regions are quoted for central values of the input parameters $\{c_i\}$ for each input set, while the effect of the uncertainties in these values is displayed explicitly for $12\,\Pi(0)$ and $12\frac{d\Pi}{dq^2}(0)$. All entries are in units of 10^{-3}, except for $12\frac{d\Pi}{dq^2}(0)$, which is in units of $10^{-3}\ \text{GeV}^{-2}$.

Input	ρ-ω	ϕ	ρ'-ω'	$12\Pi(0)$	$12\frac{d\Pi}{dq^2}(0)$
Set I	-1.06	2.21	-0.18	0.96±0.14	3.88±0.61
Set III	-1.92	3.44	-0.31	1.22±0.14	5.10±0.60
Set IV	-1.43	2.75	-0.24	1.08±0.14	4.43±0.61

IV. THE CORRELATOR TO ONE-LOOP ORDER IN ChPT

In Ref. [16], the one-loop ChPT analysis of the mixed-isospin axial vector current correlator analogous to $\Pi_{\mu\nu}(q^2)$ above, i.e., $\langle 0|T(A^3_\mu A^8_\nu)|0\rangle$, was shown to place important constraints on the sum rule treatment of the correlator. One might, therefore, expect to obtain similarly useful constraints on the vector current correlator. We show, in this section, that the situation for the two correlators is actually rather different and that, although one can easily work out $\Pi_{\mu\nu}(q^2)$ to one loop in ChPT, the form of the result obtained clearly indicates that yet higher-order corrections must be expected to be large. The one-loop result, therefore, in this case, provides no useful constraints for the sum rule treatment.

The techniques for computing the vector correlator of interest are detailed in Ref. [21] and straightforward to apply. One finds that, to one loop and $O(m_d - m_u)$, the correlator is given by

$$\Pi(q^2) = \frac{1}{12}\left[\frac{\ln(m^2_{K^0}/m^2_{K^+})}{48\pi^2} + \left(\frac{4m^2_{K^0}}{3q^2} - \frac{1}{3}\right)\bar{J}_{K^0}(q^2) - \left(\frac{4m^2_{K^+}}{3q^2} - \frac{1}{3}\right)\bar{J}_{K^+}(q^2)\right] \quad (4.1)$$

where

$$\bar{J}_P(q^2) = -\frac{1}{16\pi^2}\int_0^1 dx\ \ln\left[1 - x(1-x)q^2/m_P^2\right] \quad (4.2)$$

and $m^2_{K^0,K^+}$ are the leading-order expressions for the kaon squared masses, $m^2_{K^0} = B_0(m_s + m_d)$ and $m^2_{K^+} = B_0(m_s + m_u)$, in the notation of Ref. [21]. For our purposes we will not need the general expression for $\bar{J}$ (which is quoted in Appendix A of Ref. [21]), but only the behavior near $q^2 = 0$, which is given by

$$\bar{J}_P(q^2) = \frac{1}{96\pi^2}\frac{q^2}{m_P^2} + \frac{1}{960\pi^2}\frac{q^4}{m_P^4} + \cdots\ . \quad (4.3)$$

Expanding (4.1) and (4.2) in powers of q^2 and using (4.3), we obtain, for the behavior of the correlator in the vicinity of $q^2 = 0$,

$$\Pi(q^2) = \frac{1}{12}\left(\frac{(m_{K^0}^2 - m_{K^+}^2)}{48\pi^2 \bar{m}_K^2}\right)\left(1 + \frac{q^2}{10\bar{m}_K^2} + \cdots\right), \tag{4.4}$$

where $\bar{m}_K^2$ is the average of the K^+ and K^0 squared masses. Thus, $12\Pi(0) = (m_{K^0}^2 - m_{K^+}^2)/48\pi^2\bar{m}_K^2$, where the kaon mass difference is that due to the strong isospin breaking, i.e., with the electromagnetic contribution removed. Using Eq. (3.9) for the electromagnetic contribution, we find that the RHS of this expression is 5.5×10^{-5}, to be compared with the results of the sum rule analysis, $\simeq 1\times 10^{-3}$. The one-loop ChPT result is a factor of $\simeq 20$ smaller than the sum rule result.

The discrepancy between the one-loop ChPT and sum rule analyses for the correlator near $q^2 = 0$ is, in fact, already to be anticipated from the absence of the low-energy constants (LEC's) L_i^r of Ref. [21] in Eq. (4.1) and the fact that the one-loop result, (4.1), contains only contributions from one-loop graphs having internal kaon loops. The absence of the L_i^r, as is well known, signals the absence of resonance (in particular, vector meson) contributions to the correlator in question, heavy resonance exchange being known to saturate the values of the $O(p^4)$ LEC's (i.e., after one first couples the heavy resonances to the pseudoscalars in the standard manner and then integrates out the heavy fields [20,33,34]). Moreover, the noncontact kaon loop graphs are known to be suppressed in size [the coefficient of q^2 in the leading term of $\bar{J}_K$ in Eq. (4.5), e.g., is a factor of m_π^2/m_K^2 smaller than for the corresponding π loop integral $\bar{J}_\pi$] and, moreover, in the case at hand, i.e., the correlator $\Pi(q^2)$, those terms in which this suppression would be lifted by the presence of the m_K^2/q^2 factor in the coefficient multiplying $\bar{J}_K(q^2)$ cancel, since the expression for the correlator involves the difference of the K^+ and K^0 loop contributions.

The slow convergence of the chiral series when the leading contribution vanishes and the next-to-leading-order contribution results purely from loop graphs (i.e., is independent of the fourth-order LEC's L_i^r) has already been seen in other processes. For example, for $\gamma\gamma \to \pi^0\pi^0$, which, to one-loop order, receives contributions only from loop graphs (though in this case, loop graphs with internal π lines), the one-loop expression [35,36] deviates from the experimental amplitude [37] even very close to threshold, and two-loop corrections (sixth order in the chiral expansion) are required to bring the amplitude into agreement with experiment [38]. Similarly, for $\eta \to \pi^0\gamma\gamma$, one finds a one-loop amplitude with no leading term, no contributions from the fourth-order LEC's, and π loop contributions suppressed by a factor $(m_d - m_u)$. Together with the natural suppression of the K loop contributions noted above, the result is that the one-loop prediction for the partial rate [39] is a factor of $\simeq 170$ smaller than observed experimentally [27], as expected from the independent information that the dominant contribution to the amplitude is due to vector meson exchange [40,41]. The form of the one-loop expression for the correlator, (4.1), is especially analogous to the $\eta \to \pi^0\gamma\gamma$ case.

It should also be noted that, in addition to similarities to processes known to involve significant $O(p^6)$ (and higher) contributions, there is concrete evidence for the unreliability of (4.1) based on a recent study of the related correlator $\Pi_{\mu\nu}^{38}(q^2)$ to two-loop order [42]. This correlator, which is identical to $\Pi_{\mu\nu}(q^2)$ to one-loop order, involves only a single combination of the $O(p^6)$ LEC's, $Q^0(\mu) - 3L_9^{(-1)}(\mu) - 3L_{10}^{(-1)}(\mu)$ in the notation of Ref. [43], (where μ is the renormalization scale), this combination being, in principle, obtainable from experimental data using the chiral sum rules of Ref. [43]. [For the full $O(p^6)$ Lagrangian see Ref. [44].] One finds that, independent of a knowledge of this new LEC, the two-loop corrections are necessarily large on the scale of the one-loop result (4.1), as expected from the more general arguments above (see Ref. [42] for further details).

Thus we see that, unlike the case of the mixed-isospin axial vector current correlator, where one-loop ChPT results allowed one to uncover an error in the chiral behavior of the sum rule result [13], we cannot use (4.1) to obtain any useful constraints on the behavior of the correlator $\Pi_{\mu\nu}$ extracted from the sum rule analysis. Moreover, since, unlike the sum rule result for the slope of the axial correlator with respect to q^2, which did not display any stability plateau with respect to the Borel mass [13,16], the present results show excellent stability windows and, moreover, reproduce the physically expected scale for the ϕ contributions to the correlator given by the estimate discussed above, it appears likely that the sum rule analysis is reliable in the present case. One thus would seem justified in, as suggested above, turning the tables and using the sum rule result as a means of constraining the $O(q^6)$ LEC's that would occur in a two-loop calculation of the correlator. This approach has, in fact, been employed in the case of the analogous vector correlator $\Pi_{\mu\nu}^{38}$ to provide a first estimate of the $O(q^6)$ LEC $Q^0(\mu) - 3L_9^{(-1)}(\mu) - 3L_{10}^{(-1)}(\mu)$, mentioned above [42].

V. SUMMARY AND DISCUSSION OF RESULTS

The basic results of the paper are as follows. We have demonstrated that (1) in making a sum rule analysis of the mixed-isospin vector current correlator, it is necessary to include the ϕ pole term in the phenomenological form of the representation of the correlator, and that, when one does so, the spectral structure of the correlator becomes physically sensible; (2) the expression for the correlator away from $q^2 = m_\omega^2$ has no general interpretation as the off-diagonal element of a vector meson propagator except for a particular vector meson interpolating field choice; (3) the freedom of field redefinition shows that the isospin-breaking factor $\theta^{\rho\omega}(q^2)$, which occurs in the numerator of the expression (1.3) for the off-diagonal element of the vector meson propagator, cannot, in general, be taken to be independent of q^2; (4) the behavior of the correlator near $q^2 = m_\omega^2$ suggests that the direct $\omega^{(0)} \to \pi^+\pi^-$ contribution to $e^+e^- \to \pi^+\pi^-$ is not negligible in the ρ-ω interference region; (5) the possibility exists of using the sum rule result for the correlator near $q^2 = 0$ to obtain information on the $O(p^6)$ LEC's of ChPT.

A few words are, perhaps, in order concerning the fourth point above. It is usually thought that, although a direct $\omega \to \pi^+\pi^-$ coupling would induce an imaginary part in the off-diagonal element of the inverse vector-meson-propagator matrix, the extraction of the magnitude of the real part of this matrix element from $e^+e^- \to \pi^+\pi^-$ data should be safe. The reason for this belief is that, in the limit that $m_\rho^2 - m_\omega^2$ is taken to be purely imaginary (where m_ρ and m_ω are the complex ρ, ω pole positions), the direct ω coupling contribution to the $e^+e^- \to \pi^+\pi^-$ amplitude precisely cancels that from the imaginary part of the off-diagonal element of the inverse propagator matrix associated with the $\pi\pi$ intermediate state (induced by the presence of the direct ω coupling) [45]. However, if one takes the values for the ρ, ω pole positions from analyses using an S-matrix type parametrization, one finds that the real part of $m_\rho^2 - m_\omega^2$ is not negligible. This in turn produces a contribution proportional to the direct $\omega \to \pi\pi$ coupling constant $g_{\omega\pi\pi}$, which, if $g_{\omega\pi\pi}$ and the corresponding $\rho\pi\pi$ coupling constant $g_{\rho\pi\pi}$ are relatively real and of the same sign, interferes destructively with the contribution from the real part of the inverse propagator mixing matrix element. The value extracted from the sum rule analysis would, in this case, then be expected to be larger than that obtained from experiment. As an example, if $g_{\omega\pi\pi} = 0.05 g_{\rho\pi\pi}$, the true value of the real part of the off-diagonal element of the inverse vector meson propagator at the ω pole can be as much as 60% higher than that usually quoted (see Ref. [46] for further discussion).

ACKNOWLEDGMENTS

The hospitality of the Department of Physics and Mathematical Physics of the University of Adelaide and the continuing financial support of the Natural Sciences and Research Engineering Council of Canada are gratefully acknowledged. The author also wishes to thank Tony Williams, Tony Thomas, Terry Goldman, and Jerry Stephenson for numerous useful discussions on the issues of charge symmetry breaking in few-body systems, and Peter Tandy for making available the results of Ref. [32] before publication.

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