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Charged dilaton black holes with a cosmological constant

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The properties of static spherically symmetric black holes, which are either electrically or magnetically charged, and which are coupled to the dilaton in the presence of a cosmological constant Λ , are considered. It is shown that such solutions do not exist if $\Lambda > 0$ (in arbitrary spacetime dimensions ≥ 4). However, asymptotically anti-de Sitter black hole solutions with a single horizon do exist if $\Lambda < 0$. These solutions are studied numerically in four dimensions and the thermodynamic properties of the solutions are derived. The extreme solutions are found to have zero entropy and infinite temperature for all nonzero values of the dilaton coupling constant.

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I. INTRODUCTION

Over the past few years much interest has been focused on the properties of charged black holes coupled to the dilaton field, generally in a manner dictated by the low-energy limit of string theory, with a massless dilaton. A number of black hole and related solutions have been derived. These include the static spherically symmetric solutions [1,2], dilatonic versions of the C -metric solutions [3] which represent oppositely charged black holes undergoing uniform acceleration, and the generalization of the Majumdar-Papapetrou metric which represents a collection of maximally charged black holes in an asymptotically flat background [4,5]. Time-dependent Kastor-Traschen-type cosmological multi-black-hole solutions have been discussed by Horne and Horowitz [5] and by Maki and Shiraiishi [6]. However, exact solutions have been constructed only for certain special values of the dilaton coupling and for special powers of a Liouville-type dilaton potential [6], which excluded the case that the potential is simply a cosmological constant.

It is therefore natural to ask whether static spherically symmetric solutions representing charged black holes coupled to the dilaton also exist in the presence of a cosmological constant. To date this question has not been answered although some attempts have been made to understand the problem. Okai has examined the problem using series solutions [7], and has placed limits on the number of possible horizons. Furthermore, in a recent paper [8] two of us have derived the global properties of solutions in the related model described by the action

$$\begin{aligned}
 S = \int d^D x \sqrt{-g} & \left\{ \frac{\mathcal{R}}{4} - \frac{1}{D-2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \mathcal{V}(\phi) \right. \\
 & - \frac{1}{4} \exp\left(\frac{-4g_0\phi}{D-2}\right) F_{\mu\nu} F^{\mu\nu} \\
 & \left. - \frac{1}{2(D-2)!} \exp\left(\frac{-4g_0\phi}{D-2}\right) F_{\mu_1\mu_2\cdots\mu_{D-2}} \right. \\
 & \left. \times F^{\mu_1\mu_2\cdots\mu_{D-2}} \right\}, \tag{1}
 \end{aligned}$$

where the dilaton potential $\mathcal{V}(\phi)$ was chosen to be of Liouville form, $\mathcal{V} = (\Lambda/2) \exp[-4g_1\phi/(D-2)]$. Here $F_{\mu\nu}$ is the field strength of the electromagnetic field, and $F_{\mu_1\mu_2\cdots\mu_{D-2}}$ is the $(D-2)$ -form field strength of an Abelian gauge field. In [8] it was shown that charged black hole solutions with a realistic asymptotic behavior do not exist for the Liouville-type potential.¹ The one exception to this result was the case $g_1 = 0$, where it was found that both asymptotically de Sitter and asymptotically anti-de Sitter solutions do exist, and that the corresponding critical point is an attractor in the phase space. To show that black hole solutions exist one must further demonstrate that integral curves connect these critical points to regular horizons. That is the object of the present paper. We will demonstrate that black hole spacetimes do exist in the case of a negative cosmological

¹Similar results apply to the uncharged case [9].

constant Λ , but do not exist if $\Lambda > 0$. This result stands in sharp contrast to the standard Reissner–Nordström–de Sitter solutions.

II. NONEXISTENCE OF BLACK HOLES WITH A POSITIVE COSMOLOGICAL CONSTANT

In order to demonstrate that black holes with a positive cosmological constant do not exist in dilaton gravity, it is convenient to adopt the coordinates used by Garfinkle, Horowitz, and Strominger [2] in their discussion of the black hole solutions with a massless dilaton [1]: namely,

$$ds^2 = -f dt^2 + f^{-1} dr^2 + R^2 d\Omega_{D-2}^2, \quad (2)$$

where $f = f(r)$ and $R = R(r)$, and $d\Omega_{D-2}^2$ is the standard round metric on a $(D-2)$ sphere, with angular coordinates θ_i , $i = 1, \dots, D-2$.

In the present paper, as in [8], we will consider cases in which only $F_{\mu\nu}$ is present with

$$\mathbf{F} = \exp\left(\frac{4g_0\phi}{D-2}\right) \frac{Q_e}{R^{D-2}} dt \wedge dr, \quad (3)$$

corresponding to an isolated electric charge, or else only $F_{\mu_1\mu_2\dots\mu_{D-2}}$ is present with components

$$F_{\hat{\theta}_1\hat{\theta}_2\dots\hat{\theta}_{D-2}} = \frac{Q_m}{R^{D-2}} \epsilon_{\hat{\theta}_1\hat{\theta}_2\dots\hat{\theta}_{D-2}} \quad (4)$$

in an orthonormal frame, which is a magnetic monopole ansatz if $D = 4$. Since the field equations are invariant under the duality transformation [1] $Q_e \rightarrow Q_m$, $\phi \rightarrow -\phi$, it is convenient to define a constant

$$a \equiv \begin{cases} +2g_0/(D-2) & \text{electric ansatz (3),} \\ -2g_0/(D-2) & \text{magnetic ansatz (4).} \end{cases} \quad (5)$$

The field equations derived from (1) may then be written

$$\frac{2}{(D-2)R^{D-2}} [R^{D-2} f \phi']' = \frac{d\mathcal{V}}{d\phi} + \frac{aQ^2 e^{2a\phi}}{R^{2(D-2)}}, \quad (6)$$

$$\frac{R''}{R} = -\frac{4\phi'^2}{(D-2)^2}, \quad (7)$$

$$\frac{1}{R^{D-2}} [f (R^{D-2})']' = (D-2)(D-3) \frac{1}{R^2} - 4\mathcal{V} - \frac{2Q^2 e^{2a\phi}}{R^{2(D-2)}}, \quad (8)$$

which applies both to the electric and magnetic cases, with $Q = Q_e$ or $Q = Q_m$ as appropriate. Here $' \equiv d/dr$. One further field equation follows from (6)–(8) by virtue of the Bianchi identity.

The asymptotic properties of the solutions of these field equations were discussed in [8] for potentials $\mathcal{V}(\phi)$ of Liouville type. In the case of a simple cosmological constant $\mathcal{V} \equiv \Lambda/2$, it was demonstrated that the only possible “realistic” asymptotics are de Sitter or anti–de Sitter type, depending on the sign of Λ . Furthermore, de Sitter asymptotics are obtained only in the region in which

the Killing vector $\partial/\partial t$ is spacelike, and consequently any black hole solutions in such a model must possess at least two horizons. It is quite straightforward to show that in fact there are no such solutions. We prove the result by contradiction.

Suppose that asymptotically de Sitter solutions exist with at least two horizons, and let the two outermost horizons be labeled r_{\pm} , with $r_- < r_+$. The requirement of regularity at the horizon means that near $r = r_+$, $f \propto (r-r_+)$, and $\phi(r_+)$ and $R(r_+)$ are bounded with $R(r_+) \neq 0$, and similarly for r_- . In the case of a cosmological constant $\mathcal{V} \equiv \Lambda/2$, Eq. (6) then implies that, at both horizons,

$$\phi' f' \Big|_{r_{\pm}} = \frac{a(D-2)Q^2 e^{2a\phi}}{2R^{2(D-2)}} \Big|_{r_{\pm}}. \quad (9)$$

Since $\partial/\partial t$ is spacelike in the asymptotic region, asymptotically de Sitter solutions must have $f'(r_-) > 0$ and $f'(r_+) < 0$. For the moment let us assume that $a > 0$. Then (9) implies that $\phi'(r_-) > 0$ and $\phi'(r_+) < 0$. These two values of ϕ' must be smoothly connected and thus $\phi'(r)$ must go through zero at least once in the interval (r_-, r_+) at a point r_0 such that $\phi''(r_0) < 0$. However, since $f(r) > 0$ on the interval (r_-, r_+) , it follows from (6) that, if $\phi'(r_0) = 0$, then $\text{sgn}\phi''(r_0) = \text{sgn}a > 0$. We thus obtain a contradiction. If $a < 0$, then each of the signs $\phi'(r_-)$, $\phi'(r_+)$, and $\phi''(r_0)$ is reversed in the argument above and we once again obtain a contradiction. Finally, in the case of anti–de Sitter asymptotics two horizons are also ruled out, as one must then simultaneously change the signs of $f'(r_-)$, $f'(r_+)$, and $f(r)$ on the interval (r_-, r_+) .

We note in passing that our argument is readily extended to rule out static spherically symmetric solutions with two horizons in the case that $\mathcal{V}(\phi)$ is a monotonic function with $\text{sgn}\frac{d\mathcal{V}}{d\phi} = \text{sgn}a$. For example, in the case of a Liouville-type potential $\mathcal{V} = (\Lambda/2) \exp[-4g_1\phi/(D-2)]$, such solutions do not exist if $a/(g_1\Lambda) < 0$. This accords with the results of [8], since it was found there that Robinson–Bertotti-type solutions can only exist if $a/(g_1\Lambda) > 0$, and it is well known that these latter solutions only exist in the same circumstances as solutions with two degenerate horizons. Of course, as was observed in [8], general models with a Liouville potential do not possess realistic asymptotics.

III. ASYMPTOTICALLY ANTI–de SITTER BLACK HOLES

Let us now turn to the case of a negative cosmological constant $\mathcal{V} \equiv \Lambda/2$, with $\Lambda < 0$. Since $\partial/\partial t$ is timelike in the asymptotic region, black hole solutions with a single horizon can exist in this case, and as was shown in the preceding section this is in fact the maximum number of horizons possible. As a starting point, it is straightforward to determine the large r behavior of the asymptotically anti–de Sitter solutions. If we make the expansions $\phi = \sum_{i=0}^{\infty} \phi_i r^{-i}$, $f = \sum_{i=-2}^{\infty} f_i r^{-i}$, $R = r + \sum_{i=0}^{\infty} R_i r^{-i}$, and furthermore use the freedom of translating the origin

in the radial direction to set $R_0 = 0$, we find

$$\begin{aligned} \phi &= \phi_0 + \frac{\phi_{D-1}}{r^{D-1}} + \dots, \\ f &= \frac{-2\Lambda r^2}{(D-1)(D-2)} + 1 - \frac{2M}{r^{D-3}} \\ &\quad + \frac{2Q^2 e^{2a\phi_0}}{(D-2)(D-3)r^{2D-6}} \\ &\quad + \frac{8\Lambda\phi_{D-1}^2}{(2D-3)(D-2)^3 r^{2D-4}} + \dots, \\ R &= r - \frac{2(D-1)\phi_{D-1}^2}{(2D-3)(D-2)^2 r^{2D-3}} + \dots. \end{aligned} \quad (10)$$

The constants M , ϕ_0 , and ϕ_{D-1} are free, M being proportional to the Arnowitt-Deser-Misner (ADM) mass.

One should compare these results to those of Gregory and Harvey [10] and Horne and Horowitz [11], who investigated black holes coupled to a massive dilaton with a quadratic potential. As in the models of [10,11] the force associated with the dilaton here is short range, but the strength of its contribution is (for $D = 4$) one power of r stronger here than for the massive dilaton. Furthermore, at this stage ϕ_{D-1} is a free parameter, whereas it is fixed in terms of the other charges in the massive dilaton models simply on the basis of solving the asymptotic field equations. However, although ϕ_{D-1} is a free parameter as far as the asymptotic series is concerned, if we further demand that a particular solution with an asymptotic expansion (10) correspond to a black hole, then we can integrate Eq. (6) between the horizon r_μ and infinity to obtain an integral relation

$$\phi_{D-1} = \frac{a(D-2)^2 Q^2}{4\Lambda} \int_{r_\mu}^{\infty} dr \frac{e^{2a\phi}}{R^{D-2}}. \quad (11)$$

Consequently, for black hole solutions ϕ_{D-1} is constrained to depend on the other charges of the theory, and cannot be regarded as an independent ‘‘hair.’’

Unfortunately there is no transparent means for obtaining the general solution to the field equations in closed form. We therefore turn to numerical integration. Following [11] we will change coordinates and use R as the radial variable, so that the metric becomes

$$ds^2 = -f dt^2 + h^{-1} dR^2 + R^2 d\Omega_{D-2}, \quad (12)$$

where $h(R) \equiv f \left(\frac{dR}{dr}\right)^2$, and now $f = f(R)$. The advantage of working with these coordinates is that by suitably combining the appropriate differential equations one can solve for f in terms of h and ϕ . One finds

$$f = h \exp \left[\frac{8}{(D-2)^2} \int^R d\bar{R} \dot{\phi}^2 \bar{R} \right], \quad (13)$$

where an overdot denotes d/dR . There are then just two independent field equations remaining, which (with $\mathcal{V} \equiv \Lambda/2$ and \mathbf{F} corresponding to an electric field) are

$$\begin{aligned} -R\dot{h} + (D-3)(1-h) \\ = \frac{4R^2 h \dot{\phi}^2}{(D-2)^2} + \frac{2\Lambda R^2}{D-2} + \frac{2Q^2 e^{2a\phi}}{(D-2)R^{2D-6}}, \end{aligned} \quad (14)$$

$$\begin{aligned} Rh\ddot{\phi} + \dot{\phi} \left[D-3+h - \frac{2\Lambda R^2}{D-2} - \frac{2Q^2 e^{2a\phi}}{(D-2)R^{2D-6}} \right] \\ = \frac{1}{2}(D-2)aQ^2 \frac{e^{2a\phi}}{R^{2D-5}}. \end{aligned} \quad (15)$$

The asymptotic series (10) become

$$\begin{aligned} \phi &= \phi_0 + \frac{\phi_{D-1}}{R^{D-1}} + \dots, \\ h &= \frac{-2\Lambda R^2}{(D-1)(D-2)} + 1 - \frac{2M}{R^{D-3}} \\ &\quad + \frac{2Q^2 e^{2a\phi_0}}{(D-2)(D-3)R^{2D-6}} - \frac{8\Lambda\phi_{D-1}^2}{(D-2)^3 R^{2D-4}} + \dots, \end{aligned} \quad (16)$$

in terms of the new variables.

Since the equations are invariant under the rescaling $R \rightarrow cR$, $\Lambda \rightarrow \Lambda/c^2$, $Q \rightarrow c^{D-3}Q$, it is possible to eliminate Λ . We will therefore set $\Lambda = -1$. Furthermore, the equations are invariant under $\phi \rightarrow \phi - \phi_0$, $Q \rightarrow Qe^{a\phi_0}$, and so one can also set $\phi_0 = 0$ with no loss of generality. For numerical integration it is of course necessary to choose a particular spacetime dimension, and so we will henceforth take $D = 4$.

Equations (14), (15) are equivalent to three first-order ordinary differential equations and thus generally have a three-parameter set of solutions. However, many of these will correspond to naked singularities. The requirement that solutions have a regular horizon reduces the three parameters to two, which may be taken to be the radial position of the horizon R_μ and $\phi_\mu \equiv \phi(R_\mu)$. Since the equations are singular on the horizon, we start the integration a small distance from R_μ , the initial values of h , ϕ , and $\dot{\phi}$ being determined in terms of R_μ and ϕ_μ by solving for the coefficients \tilde{h}_i and $\tilde{\phi}_i$ in the power series expansions, $h = \sum_{i=1}^{\infty} \tilde{h}_i (R - R_\mu)^i$, $\phi = \phi_\mu + \sum_{i=1}^{\infty} \tilde{\phi}_i (R - R_\mu)^i$. Since the solutions are rather cumbersome, we will not list them here. As shown above, black hole solutions can only have one horizon, and so we may restrict the initial conditions to those with $\dot{h}(R_\mu) > 0$.

As was shown in [8], the critical point of the field equations corresponding to asymptotically anti-de Sitter solutions is a strong attractor, and thus integrating out from a regular horizon it is possible to find solutions for which $h(R)$ and $a\phi(R)$ increase until $h(R)$ and $\phi(R)$ eventually agree with the asymptotic expansions (16) to arbitrary accuracy. The system is thus far more amenable to numerical analysis than the corresponding system with a quadratic dilaton potential [11]. Not all initial conditions with $\dot{h}(R_\mu) > 0$ lead to asymptotically anti-de Sitter solutions, however. Typically, we find that in some instances $h(R)$ increases to a maximum, decreases to a small positive minimum, and then finally both $h(R)$ and $|\phi(R)|$ diverge to $+\infty$ at a finite value of R . Essentially, $h(R)$ comes close to displaying a second horizon — how-

ever, as was shown above, no double horizon solutions exist, and thus, from (15), $\dot{\phi}$ must diverge as h comes close to zero the second time. The final behavior of such solutions, when examined in the coordinates (2), is found to correspond to a central singularity.

In order to study the properties of the solutions we performed a large number ($\gtrsim 10^4$) of integrations. Rather than varying the charge Q as an initial condition, we adopted the approach of initially fixing $Q = 1$ and integrating out until the solutions matched the expansions (16) for some arbitrary $\phi_0 = k$. If we then subtract the constant k from the solution for ϕ thus obtained, by the invariance of the equations mentioned above we have equivalent solutions with $\phi_0 = 0$ and $Q = e^{a\phi_0}$. Figure 1 is a contour plot² showing the integration constant M as a function of $R_{\mathcal{H}}$ and $\phi_{\mathcal{H}}$ (unrescaled). An interesting feature of the diagram is the critical line which separates the parameter space into a region which admits black hole solutions (on the right) from the region with no black hole solutions. As the critical line is approached both M and the effective charge at infinity, $e^{a\phi_0}$, become infinite. It is clear from the plot that if $R_{\mathcal{H}}$ and $-a\phi_{\mathcal{H}}$ are both large, then M is almost independent of $\phi_{\mathcal{H}}$. However, for small $R_{\mathcal{H}}$ the relation between M , $\phi_{\mathcal{H}}$, and $R_{\mathcal{H}}$ is quite complicated. If the solutions are rescaled so that $\phi_0 = 0$ as above, then the region of parameter space occupied by the black hole solutions maps into the region $R_{\mathcal{H}} > 0$, $\phi_{\mathcal{H}, \text{rescaled}} > 0$. To investigate the features of the solutions we will take M and Q as the two independent parameters (after rescaling the raw numerical data), as this allows a more direct comparison with known exact solutions than can be obtained using $\phi_{\mathcal{H}}$ and $R_{\mathcal{H}}$.

In Fig. 2 we display a contour plot of the charge ϕ_3 as a function of M and Q , which from (16) (with $\phi_0 = 0$) determines the leading-order asymptotic behavior of the dilaton. For very large $R_{\mathcal{H}}$, which effectively means very large M and small Q , the asymptotic series (10) and (16) are valid right up to the horizon, so that using (11) we find $\phi_3 \approx -aQ^2/(6M\Lambda)^{1/3}$, and thus $\phi_3 = \text{const}$ contours follow curves $Q \propto M^{1/6}$ as is evident in Fig. 2.

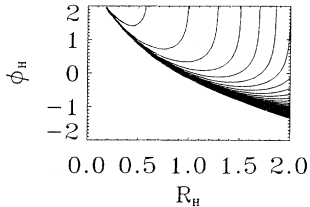


FIG. 1. Contours of constant mass of black holes as a function of the unrescaled value of $\phi_{\mathcal{H}}$ and $R_{\mathcal{H}}$, for solutions with $Q = 1$ and $a = -1$, i.e., magnetic solutions if $g_0 = 1$. For the corresponding electric solutions ($a = 1$), $\phi_{\mathcal{H}} \rightarrow -\phi_{\mathcal{H}}$ in the contour plot.

²All figures are given for the case $D = 4$, $\Lambda = -1$, and $|a| = 1$. Other nonzero values of a (and other negative values of Λ) give results which are qualitatively the same.

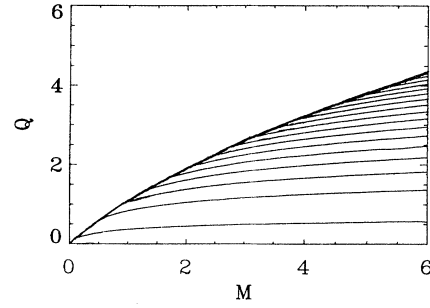


FIG. 2. Contours of constant “scalar charge” ϕ_3 as a function of mass M and charge Q for solutions with $|g_0| = 1$. $\phi_3 = 0$ for the Schwarzschild–anti–de Sitter solution ($Q = 0$). If $a = 1$ ($a = -1$), then $\phi_3 < 0$ ($\phi_3 > 0$), and $|\phi_3|$ increases monotonically with increasing Q for given M .

We have been unable to determine an analytic expression for the relation between Q and M which holds for the extreme solutions, although the general form is clear from Fig. 2. Since the solutions have only one horizon, the extreme black holes correspond to the singular limit $R_{\mathcal{H}} \rightarrow 0$, as in the case of the asymptotically flat black holes with a massless dilaton [1]. In the case of a quadratic dilaton potential, by contrast, black holes which are large with respect to the Compton wavelength of the dilaton have two horizons so that the extreme limit is similar to that of the Reissner–Nordström solution and can be estimated in the regime where the asymptotic expansions hold up to the degenerate horizon [11]. In the present model, however, the extreme solutions have $R_{\mathcal{H}} = 0$ regardless of their mass, and so the extreme limit is closer to the situation of the “small black holes” described in [11]. In particular, for very small black holes with $R_{\mathcal{H}} \ll (-\Lambda)^{-1/2}$ we expect that Q_{ext} is somewhat less than the limit for asymptotically flat dilaton black holes but that it approaches this value $Q_{\text{ext}} \rightarrow \sqrt{1 + a^2 M}$ as $M \rightarrow 0$. This is indeed borne out by the numerical analysis. The only comparison we can make for extreme solutions with large masses is the Reissner–Nordström–anti–de Sitter solution, which has two degenerate horizons with

$$3\sqrt{-2\Lambda} M = \left[\sqrt{1 - 4\Lambda Q_{\text{ext}}^2} - 1 \right]^{1/2} \left[2 + \sqrt{1 - 4\Lambda Q_{\text{ext}}^2} \right]$$

in the extreme limit, so that $Q_{\text{ext}} \approx (-\Lambda)^{-1/6} \left(\frac{3}{2}M\right)^{2/3}$ for large M . Over the range of M shown in Fig. 2 the extreme limit in fact comes very close to this value for larger values of M , though it does in fact eventually become less than this bound.

It is straightforward to determine the thermal properties of the black hole solutions. The entropy is simply given by one-quarter of the area of the horizon, $S = \frac{1}{4}\mathcal{A}_{\mathcal{H}} = \pi R_{\mathcal{H}}^2$. Thus for black holes with large M and small Q , $S \approx \pi \left(\frac{-2M}{3\Lambda}\right)^{2/3}$, while $S = 0$ for the extreme solutions. A contour plot of the adiabats is shown in Fig. 3.

The temperature of the black holes is given by $T = \dot{h}(R_{\mathcal{H}})/(4\pi) = \dot{h}_1/(4\pi)$, where \dot{h}_1 is determined from the

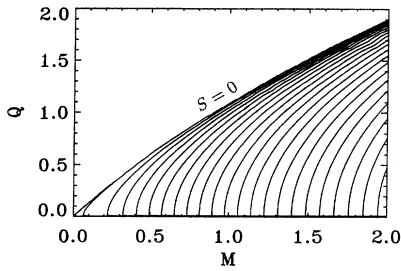


FIG. 3. Contours of constant entropy S as a function of mass M and charge Q for magnetic or electric solutions with $|a| = 1$.

power series solutions near the horizon, giving

$$T = \frac{1}{4\pi R_{\mathcal{H}}^3} \left[R_{\mathcal{H}}^2 - \Lambda R_{\mathcal{H}}^4 - Q^2 e^{2a\phi_{\mathcal{H}}} \right] \quad (17)$$

for $D = 4$. Isotherm contours are plotted in Fig. 4. The $Q = 0$ axis of course corresponds to the Schwarzschild–anti-de Sitter solution — the temperature for this solution diverges at $M = 0$, decreases monotonically to a minimum value $T_{\text{cr}} = \sqrt{-\Lambda}/(2\pi)$ at $M_{\text{cr}} = \frac{2}{3}(-\Lambda)^{-1/2}$, and then rises monotonically with increasing M . The isotherm $T = T_{\text{cr}}$ represents a critical case in Fig. 4 — it has two branches, the right-hand branch with positive specific heat $(\frac{\partial M}{\partial T})_Q$ and the left-hand branch which has negative specific heat for small Q , but for which the specific heat changes sign as Q becomes close to the extremal limit. Isotherms to the right of the right-hand branch have temperatures $T > T_{\text{cr}}$ and the specific heat is strictly positive. Isotherms to the left of the left-hand branch also have temperatures $T > T_{\text{cr}}$ and a behavior similar to the T_{cr} left-hand branch, ultimately approaching the extremal curve. In between the two T_{cr} branches are a class of isotherms with $T < T_{\text{cr}}$ which have positive specific heat for the smaller value of Q for a given M , but which then double back with negative specific heat

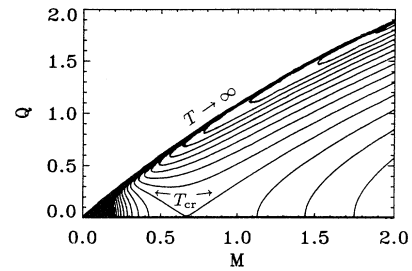


FIG. 4. Contours of constant temperature T as a function of mass M and charge Q for magnetic or electric solutions with $|a| = 1$. The isotherm with a temperature equal to the minimum temperature of the Schwarzschild–anti-de Sitter solution $T_{\text{cr}} = (-\Lambda)^{1/2}/(2\pi)$ is indicated.

close to the extremal curve. The extreme black holes have $T \rightarrow \infty$ for all $a \neq 0$; this is clear from (17) since the extreme case occurs for $R_{\mathcal{H}} \rightarrow 0$ and $a\phi_{\mathcal{H}} \rightarrow -\infty$.

The asymptotically flat black holes with a massless dilaton have the same properties as the solutions here — zero entropy, infinite temperature — only if $|a| > 1$ [1,12]. For those solutions the temperature is zero in the extreme limit if $|a| < 1$, and finite in the intermediate “stringy case” $|a| = 1$ [1]. Of course, an infinite temperature here merely signals the breakdown of the semiclassical limit if one is considering the Hawking evaporation process. As was demonstrated by Holzhey and Wilczek [13], in the case of the $|a| > 1$ Gibbons–Maeda solutions an infinite mass gap develops for quanta with a mass less than that of the black hole so that the Hawking radiation slows down and comes to an end at the extremal limit, despite the infinite temperature. We expect that the situation here would be the same.

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