Does the weak coupling limit of the Burden-Tjiang deconstruction of the massless quenched three-dimensional QED vertex agree with perturbation theory?

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We derive constraints on the non-perturbative 3-point fermion-boson transverse vertex in massless QED3 from its perturbative calculation to order α. We also check the transversality condition to two loops and evaluate the fermion propagator to the same order. We compare a conjecture of the non-perturbative vertex by Burden and Tjiang against our results and comment on its drawbacks. Our calculation calls for the need to construct a non-perturbative form for the fermion-boson vertex which agrees with its perturbative limit to $\mathcal{O}(\alpha)$.

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I. INTRODUCTION

QED in 3-dimensions (QED3) is a useful laboratory for studying the strong coupling limit of a gauge theory. The lack of ultraviolet divergences makes it easier to handle than its 4-dimensional counterpart. Moreover, in the quenched approximation, it exhibits confinement which makes it attractive for investigating strong physics. The study of strong coupling gauge theories through the use of Schwinger-Dyson equations requires knowledge of the non-perturbative form of the fundamental fermion-boson interaction. The most commonly used approximation is the bare vertex. However, among other deficiencies, the bare vertex fails to respect a key property of the underlying field theory, namely the gauge invariance of physical observables. An obvious reason is that it fails to respect the Ward-Green-Takahashi identity (WGTI) [1]. Ball and Chiu [2] have proposed an ansatz for what is conventionally called the longitudinal part of the vertex which alone satisfies WGTI. The rest of the vertex, the transverse part, remains undetermined. Problems, like that of dynamical fermion mass generation, have been studied in QED3, both quenched and unquenched, using the bare vertex, as well as an ansatz based on a simple modification of the Ball-Chiu vertex [3,4]. More recently, Burden and Tjiang have constructed a different ansatz for the full vertex to investigate the fermion and photon propagators simultaneously [5], while including an explicit transverse piece. Burden and Tjiang base their *deconstruction* on the assumption that a certain ''transversality condition'' for the fermion propagator holds non-perturbatively for some covariant gauge ξ_0. The bare fermion propagator is then a solution in that gauge. Accordingly, they go on to propose a transverse vertex and use it to study the photon propagator.

The only truncation of the complete set of Schwinger-Dyson equations known so far that incorporates the key properties of a gauge theory at each level of approximation is perturbation theory. Moreover, it is natural to assume that physically meaningful solutions of the Schwinger-Dyson equations must agree with perturbative results in the weak coupling regime. While in QED4 this realization has been of enormous help in constructing a physically acceptable form of the vertex [6–8], need exists to exploit perturbation theory in exploring the non-perturbative form of the vertex in QED3. Following [7], we evaluate the transverse part of the vertex to $\mathcal{O}(\alpha)$. This result is then assumed to be the weak coupling limit for the non-perturbative form of the transverse vertex. We also check the Burden-Tjiang transversality condition to two loops and find that to this order, it is not realized in perturbation theory. We evaluate $F(p^2)$ to $\mathcal{O}(\alpha^2)$ analytically and compare our findings with the conjecture of the vertex proposed by Burden and Tjiang.

II. VERTEX

The full vertex, Fig. 1, $\Gamma^\mu(k,p)$ can be expressed in terms of 12 spin amplitudes formed from the vectors γ^μ, k^μ, p^μ and the scalars $1, \not{k}, \not{p}$ and $\not{k}\not{p}$. It satisfies the Ward-Green-Takahashi identity [1]

$$q_\mu \Gamma^\mu(k,p) = S_F^{-1}(k) - S_F^{-1}(p), \tag{1}$$

where $q = k - p$, and the Ward identity

$$\Gamma^\mu(p,p) = \frac{\partial}{\partial p^\mu} S_F^{-1}(p) \tag{2}$$

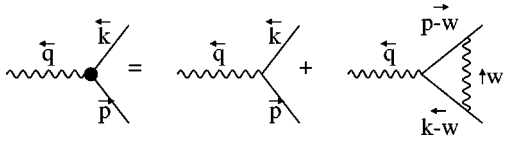

FIG. 1. One loop correction to the vertex.

as the non-singular $k \to p$ limit of Eq. (1). We follow Ball and Chiu and define the longitudinal component of the vertex in terms of the fermion propagator as

$$\Gamma_L^\mu(k,p) = \frac{\gamma^\mu}{2}\left(\frac{1}{F(k^2)} + \frac{1}{F(p^2)}\right) + \frac{1}{2}\frac{(\not{k}+\not{p})(k+p)^\mu}{(k^2-p^2)}\left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right). \quad (3)$$

This Γ_L^μ alone satisfies the Ward-Green-Takahashi identity, Eq. (1), and being free of kinematic singularities the Ward identity, Eq. (2), too. The full vertex can then be written as

$$\Gamma^\mu(k,p) = \Gamma_L^\mu(k,p) + \Gamma_T^\mu(k,p), \quad (4)$$

where the transverse part satisfies

$$q_\mu \Gamma_T^\mu(k,p) = 0 \quad \text{and} \quad \Gamma_T^\mu(p,p) = 0. \quad (5)$$

The Ward-Green-Takahashi identity fixes 4 coefficients of the 12 spin amplitudes in terms of the fermion functions. The transverse component $\Gamma_T^\mu(k,p)$ thus involves 8 vectors, of which the following 4 are sufficient to describe it in the chirally symmetric theory:

$$\Gamma_T^\mu(k,p) = \sum_{i=2,3,6,8} \tau_i(k^2,p^2,q^2) T_i^\mu(k,p), \quad (6)$$

where

$$T_2^\mu(k,p) = [p^\mu(k\cdot q) - k^\mu(p\cdot q)](\not{k}+\not{p})$$

$$T_3^\mu(k,p) = q^2\gamma^\mu - q^\mu \not{q}$$

$$T_6^\mu(k,p) = \gamma^\mu(p^2-k^2) + (p+k)^\mu \not{q}$$

$$T_8^\mu(k,p) = -\gamma^\mu k^\nu p^\lambda \sigma_{\nu\lambda} + k^\mu \not{p} - p^\mu \not{k}$$

$$\text{with} \quad \sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]. \quad (7)$$

The coefficients τ_i are Lorentz scalar functions of k and p, i.e., functions of k^2, p^2, q^2. Solution of the Schwinger-Dyson equations (SDEs) for the fermion and photon propagators requires the knowledge of τ_i in Eq. (6).

Conjecture Proposed by Burden and Tjiang

Burden and Tjiang have recently proposed a non-perturbative deconstruction of the vertex [5] for massless QED3. It involves certain assumptions about the fermion propagator and the 3-point fermion-boson vertex.

Propagator and the transversality condition

In quenched QED, the SDE for the fermion propagator reads (Fig. 2)

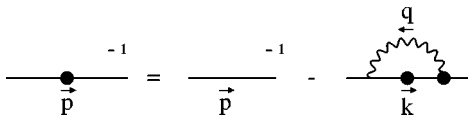

FIG. 2. Schwinger-Dyson equation for fermion propagator in quenched QED.

$$iS_F^{-1}(p) = iS_F^{0\,-1}(p) + e^2 \int \frac{d^3k}{(2\pi)^3} \Gamma^\mu(k,p) S_F(k) \gamma^\nu \Delta_{\mu\nu}^0(q). \quad (8)$$

The photon propagator can be split into the transverse and the longitudinal parts as

$$\Delta_{\mu\nu}^0(q) = \Delta_{\mu\nu}^{0\,T}(q) - \xi \frac{q_\mu q_\nu}{q^4}, \quad (9)$$

where

$$\Delta_{\mu\nu}^{0\,T}(q) = -\frac{1}{q^2}[g_{\mu\nu} - q_\mu q_\nu / q^2]. \quad (10)$$

Burden and Roberts [see Eq. (25) of [9]] have noted that the solution of Eq. (8) is gauge covariant [in the sense of the Landau-Khalatnikov (LK) transformations [10]] if the condition

$$\int \frac{d^3k}{(2\pi)^3} \Gamma^\mu(k,p) S_F(k) \gamma^\nu \Delta_{\mu\nu}^{0\,T}(q) = 0 \quad (11)$$

is simply satisfied. This condition Burden and Tjiang [5] have called the *transversality condition*. It is easy to check that at one loop order this condition is indeed fulfilled and so we are left with

$$iS_F^{-1}(p) = iS_F^{0\,-1}(p) + e^2 \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S_F^0(k) \gamma^\nu \left(-\xi \frac{q_\mu q_\nu}{q^4}\right). \quad (12)$$

Writing $S_F(p) = F(p^2)/\not{p}$, in its most general form, the solution of the above equation is

$$F(p^2) = 1 - \frac{\alpha\xi}{4} \frac{\pi}{\sqrt{-p^2}} + \mathcal{O}(\alpha^2). \quad (13)$$

At this point, it may be useful to compare this result with the implications of the LK transformations and the expression proposed by Burden and Tjiang. Assuming that $F(p^2) = 1$ in the Landau gauge, LK transformations yield the following expression for it in an arbitrary gauge:

$$F(p^2) = 1 - \frac{\alpha\xi}{2\sqrt{-p^2}} \tan^{-1}\left[\frac{2\sqrt{-p^2}}{\alpha\xi}\right]. \quad (14)$$

Using the expansion $\tan^{-1}(1/x) = \pi/2 - x + x^3/3 + \cdots$ for $|x| \ll 1$, we get

$$F(p^2)=1-\frac{\pi\alpha\xi}{4\sqrt{-p^2}}-\frac{\alpha^2\xi^2}{4p^2}+\mathcal{O}(\alpha^3), \tag{15}$$

which is in accordance with the perturbative result to $\mathcal{O}(\alpha)$. Therefore, the LK transformations accompanied by the assumption that $F(p^2)=1$ in the Landau gauge are in accordance with perturbation theory at the one loop level. A similar comparison at the two loop level is discussed in Sec. IV.

Burden and Tjiang [5] propose the following non-perturbative expression for $F(p^2)$:

$$F(p^2)=1-\frac{\alpha(\xi-\xi_0)}{2\sqrt{-p^2}}\tan^{-1}\left[\frac{2\sqrt{-p^2}}{\alpha(\xi-\xi_0)}\right], \tag{16}$$

and they comment that ''Without knowing the transverse contribution to $\overline{\Gamma}^T_\mu$,[1] we are unable to determine the constant ξ_0. The task of determining $\overline{\Gamma}^T_\mu$ is a formidable task, and we have nothing more to say about it in this paper.'' However, it is trivial to see that as the weak coupling limit must agree with the perturbative expansion, $\xi_0=0$.

Burden-Tjiang vertex

Burden *et al.* propose the following deconstruction of the vertex in the Euclidean space:

$$\tau_i^{BT}(k^2,p^2)=\frac{1}{k^2-p^2}\left[\frac{1}{F(k^2)}-\frac{1}{F(p^2)}\right]f_i(k^2,p^2) \tag{17}$$

where

$$f_3(k^2,p^2)=\beta\frac{I(k,p)}{J(k,p)},$$

$$f_6(k^2,p^2)=0, \tag{18}$$

$$\overline{f}(k^2,p^2)=-2(1+\beta)\frac{I(k,p)}{J(k,p)}, \tag{19}$$

with

$$\overline{\tau}(k^2,p^2)=\tau_8(k^2,p^2)-(k^2+p^2)\tau_2(k^2,p^2)$$

then

$$\overline{f}(k^2,p^2)=f_8(k^2,p^2)-(k^2+p^2)f_2(k^2,p^2);$$

$$I(k,p)=\frac{(k^2+p^2)^2}{8kp}\ln\left|\frac{k+p}{k-p}\right|-\frac{1}{4}(k^2+p^2),$$

$$J(k,p)=\frac{(k^2-p^2)^2}{8kp}\ln\left|\frac{k+p}{k-p}\right|-\frac{1}{4}(k^2+p^2).$$

[1] $\gamma_\mu+\overline{\Gamma}^T_\mu$ is the solution of the SDE for the vertex when the fermion propagator is the bare propagator

The superscript BT in Eq. (17) stands for Burden and Tjiang. Note a few sign changes which had to be incorporated to rewrite their ansatz in terms of the basis T_i^μ that we have chosen in our paper. The form $[1/F(k^2)-1/F(p^2)]$ in Eq. (17) has been chosen to ensure that the transverse vertex vanishes in the gauge ξ_0 (note that we have shown that $\xi_0=0$). Their vertex ansatz is also based upon the assumption that the τ_i *have no* ξ *dependence other than a possible implicit dependence through* F. In order to see the validity of this ansatz, the following are some of the important questions to be addressed:

(i) Does the real transverse vertex vanish in the Landau gauge?

(ii) Does perturbation theory allow us to take $\tau_6=0$, a coefficient which plays a vital role in constructing the vertex in QED4?

(iii) Does one loop perturbation theory agree with the non-perturbative τ_i proposed by Burden *et al.*?

(iv) For the selected basis T_i, do the corresponding coefficients have kinematic singularities at the one loop level and beyond, as present in the ansatz of Burden *et al.* when $k\rightarrow p$?

(v) Is β, which appears in the above ansatz independent of the gauge parameter as claimed by Burden *et al.*?

(vi) Does the *transversality condition*, Eq. (11), hold true beyond the one loop order?

We carry out the one loop calculation of the vertex and the two loop calculation of the fermion propagator to answer these questions.

III. PERTURBATIVE CONSTRAINTS ON THE VERTEX

The vertex of Fig. 1 can be expressed as

$$\Gamma^\mu(k,p)=\gamma^\mu+\Lambda^\mu(k,p). \tag{20}$$

Using the Feynman rules, Λ^μ to $O(\alpha)$ is simply given by

$$\begin{aligned}-ie\Lambda^\mu(k,p)=\int_M\frac{d^3w}{(2\pi)^3}&(-ie\gamma^\alpha)\\&\times iS_F^0(p-w)(-ie\gamma^\mu)iS_F^0(k-w)\\&\times(-ie\gamma^\beta)i\Delta^0_{\alpha\beta}(w),\end{aligned} \tag{21}$$

where M denotes the loop integral is to be performed in Minkowski space. The bare quantities are

$$-ie\Gamma^0_\mu=-e\gamma_\mu$$

$$iS_F^0(p)=i\not{p}/p^2$$

$$i\Delta^0_{\mu\nu}(p)=-i[p^2g_{\mu\nu}+(\xi-1)p_\mu p_\nu]/p^4,$$

where e is the usual QED coupling and the parameter ξ specifies the covariant gauge. Following [7], Λ^μ can be re-expressed as

$$\Lambda^{\mu}(k,p) = -\frac{i\alpha}{2\pi^2}\{\gamma^{\alpha}\not{p}\gamma^{\mu}\not{k}\gamma_{\alpha}J^{(0)} - \gamma^{\alpha}(\not{p}\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu}\not{k}) \times \gamma_{\alpha}J^{(1)}_{\nu} + \gamma^{\alpha}\gamma^{\nu}\gamma^{\mu}\gamma^{\lambda}\gamma_{\alpha}J^{(2)}_{\nu\lambda} + (\xi-1) \times [(-\gamma^{\nu}\not{p}\gamma^{\mu} - \gamma^{\mu}\not{k}\gamma^{\nu})J^{(1)}_{\nu} + \gamma^{\mu}K^{(0)} + \gamma^{\nu}\not{p}\gamma^{\mu}\not{k}\gamma^{\lambda}I^{(2)}_{\nu\lambda}]\}, \tag{22}$$

where $J^{(0)}$, $J^{(1)}_{\mu}$, $J^{(2)}_{\mu\nu}$, $K^{(0)}$ and $I^{(2)}_{\mu\nu}$ have been tabulated in the Appendix using the notation $k=\sqrt{-k^2}$, $p=\sqrt{-p^2}$, $q=\sqrt{-q^2}$. The only angular dependence is displayed in $q=\sqrt{k^2+p^2-2kp\cos\theta}$. The expression for the transverse vertex Γ^{μ}_{T} can be obtained by subtracting from Eq. (22), the contribution from the longitudinal part Γ^{μ}_{L} at one loop. Equations (3) and (13) allow us to write

$$\Gamma^{\mu}_{L}(k,p) = \left[1+\frac{\alpha\xi}{4}\eta_1\right]\gamma^{\mu} + \frac{\alpha\xi}{4}\eta_2[k^{\mu}\not{k}+p^{\mu}\not{p}+k^{\mu}\not{p}+p^{\mu}\not{k}], \tag{23}$$

where

$$\eta_1 = \frac{\pi}{2}\left[\frac{k+p}{kp}\right], \quad \eta_2 = \frac{\pi}{2}\left[\frac{1}{kp(k+p)}\right], \tag{24}$$

$$\tau_2(k^2,p^2) = \frac{\alpha\pi}{4}\frac{1}{kp(k+p)(k+p+q)^2}\left[1+(\xi-1)\frac{2k+2p+q}{q}\right], \tag{25}$$

$$\tau_3(k^2,p^2) = \frac{\alpha\pi}{8}\frac{[4kp+3kq+3pq+2q^2+(\xi-1)(2k^2+2p^2+kq+pq)]}{kpq(k+p+q)^2}, \tag{26}$$

$$\tau_6(k^2,p^2) = \frac{\alpha\pi(2-\xi)}{8}\frac{k-p}{kp(k+p+q)^2}, \tag{27}$$

$$\tau_8(k^2,p^2) = \frac{\alpha\pi(2+\xi)}{2}\frac{1}{kp(k+p+q)}. \tag{28}$$

Any non-perturbative vertex ansatz should reproduce Eqs. (25)–(28) in the weak coupling regime. Therefore, these equations should serve as a guide to constructing a non-perturbative vertex in QED3. Note that the τ_i have the required symmetry under the exchange of vectors k and p. Here τ_2, τ_3 and τ_8 are symmetric, whereas τ_6 is antisymmetric. All the τ_i only depend on elementary functions of k and p. This is unlike QED4, where the τ_i involve Spence functions.

Let us now try to answer some of the questions raised in the previous section:

(i) The transverse vertex does not vanish in the Landau gauge.

(ii) The coefficient $\tau_6 \neq 0$. Moreover (as we shall see shortly), in the asymptotic limit $k \gg p$, it contributes dominantly to the transverse vertex along with τ_3.

(iii) None of the τ_i agrees with the form proposed by Burden *et al.* The real τ_i are explicitly functions of q^2. However, we shall later make a comparison with the proposed vertex in the key limit for loop integrals when $k \gg p$ where the real τ_i become independent of q^2, and a direct analogy with the proposed vertex is possible.

(iv) Very importantly, none of the τ_i has kinematic singularity when $k^2 \to p^2$. One should note that *a priori* there was no guarantee that the set of basis vectors T_i which ensure their coefficients be independent of any kinematic singularities in QED4 would achieve the same for QED3. However, we find that these are indeed a correct choice for QED3 as well. As Burden *et al.* realize the logarithmic kinematical singularity in their vertex ansatz is, of course, ruled out by our perturbative calculation.

It is instructive to take the asymptotic limit $k \gg p$ of the transverse vertex:

$$\tau_2(k^2,p^2) \overset{k\gg p}{=} -\frac{\alpha}{16k^4}\frac{\pi}{p}(2-3\xi) + \mathcal{O}(1/k^5) \tag{29}$$

$$\tau_3(k^2,p^2) \overset{k\gg p}{=} \frac{\alpha}{32k^2}\frac{\pi}{p}(2+3\xi) + \mathcal{O}(1/k^3) \tag{30}$$

$$\tau_6(k^2,p^2) \overset{k\gg p}{=} \frac{\alpha}{32k^2}\frac{\pi}{p}(2-\xi) + \mathcal{O}(1/k^3) \tag{31}$$

$$\tau_8(k^2,p^2) \overset{k\gg p}{=} \frac{\alpha}{4k^2}\frac{\pi}{p}(2+\xi) + \mathcal{O}(1/k^3). \tag{32}$$

Note that taking into account the asymptotic limit $k \gg p$ of the corresponding basis vectors, one can easily see that τ_3 and τ_6 provide the dominant contribution to Γ_T in this limit just as in QED4. We are now in a position to compare Eqs. (17)–(19) with Eqs. (29)–(32) in the limit $k \gg p$ to try to extract the value of β. Comparing τ_3, we find

$$\beta = \frac{1}{2\xi} + \frac{3}{4}, \tag{33}$$

which has an explicit dependence on ξ contrary to the assumption of Burden *et al.* Moreover, one could also extract the value of β by comparing $\overline{\tau}$. Such an exercise leads us to

$$\beta = -\frac{5}{4}\left[1 + \frac{2}{\xi}\right], \tag{34}$$

which is inconsistent with the value found earlier. Therefore, the parametrization of the transverse vertex proposed by Burden and Tjiang cannot be correct.

IV. $F(p^2)$ TO TWO LOOPS AND TRANSVERSALITY CONDITION

A. $F(p^2)$ to two loops

We have seen that the transversality condition, i.e. Eq. (11), holds true to the one loop level. Burden and Tjiang [5] have proposed their vertex ansatz assuming this condition to be true non-perturbatively for $\xi = \xi_0$, which we have shown must equal zero. Therefore, a crucial test of the validity of their vertex ansatz is checking the transversality condition to two loop order. This is equivalent to calculating $F(p^2)$ to the same level. We carry out this exercise in this section.

The equation for $F(p^2)$ can be extracted from Eq. (8) by multiplying the equation with $\not{p}$ and taking the trace. On Wick rotating to the Euclidean space and simplifying, this equation can be written as

$$\begin{aligned}\frac{1}{F(p^2)} = 1 - \frac{\alpha}{2\pi^2 p^2}\int \frac{d^3k}{k^2}\frac{F(k^2)}{q^2}\Bigg[& a(k^2,p^2)\frac{2}{q^2}\{(k\cdot p)^2 - (k^2+p^2)k\cdot p + k^2p^2\} + b(k^2,p^2) \\ & \times\left\{(k^2+p^2)k\cdot p + 2k^2p^2 - \frac{1}{q^2}(k^2-p^2)^2 k\cdot p\right\} - \frac{\xi}{F(p^2)}\frac{1}{q^2}\{p^2(k^2 - k\cdot p)\} + \tau_2(k,p)\{-(k^2+p^2)\Delta^2\} \\ & + \tau_3(k,p)2\{-(k\cdot p)^2 + (k^2+p^2)k\cdot p - k^2p^2\} - \tau_6(k,p)2\{(k^2-p^2)k\cdot p\} - \tau_8(k,p)\{\Delta^2\}\Bigg],\end{aligned} \tag{35}$$

where

$$a(k^2,p^2) = \frac{1}{2}\left(\frac{1}{F(k^2)} + \frac{1}{F(p^2)}\right), \quad b(k^2,p^2) = \frac{1}{2}\frac{1}{k^2-p^2}\left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right). \tag{36}$$

Now using the expressions for τ_i, Eqs. (25)–(28), we arrive at

$$\begin{aligned}\frac{1}{F(p^2)} = 1 + \frac{\pi\xi}{4p}\alpha - \frac{\alpha^2}{4p^2}\int_0^\infty dk \frac{1}{2kp(k+p)}\Bigg[& \frac{\xi}{2}(k^2-p^2)\{-(k^2-p^2)^2 I_4 + I_0\} + \frac{\xi}{2}\{(k^2-p^2)^2(k^2+p^2)I_4 - 2(k^2+p^2)^2 I_2 \\ & + (k^2+p^2)I_0\} - \xi^2 p^2(k^2-p^2)\{(k^2-p^2)I_4 + I_2\} + \{(k+p)(2kp(k-p)^2 I_3 - 3(k-p)^2(k+p)I_2 \\ & + (3(k-p)^2 - 2kp)I_1 + 3(k+p)I_0 - 3I_{-1}) + \xi(-kp(k-p)^2 I_2 + (k+p)(k^2+p^2)I_1 + kpI_0 - (k+p)I_{-1})\}\Bigg],\end{aligned}$$

where

(i) The first curly-bracket expression arises from the a–term in Eq. (35), the second one from the b–term, the third from the $\xi/F(p^2)$-term and the fourth from the transverse part of the vertex. On substituting I_4, a–term vanishes identically as it does at one loop level. Note that all the $(k+p+q)$ factors in the τ_i neatly cancel out, leaving us with simpler integrals to be evaluated.

(ii) The I_n are defined as

$$I_n = \int_0^\pi d\theta \frac{\sin\theta}{q^n}$$

with the evaluated expressions given in the Appendix.

Keeping in mind the form of the integrals I_n, we divide the integration region in two parts, $0 \to p$ and $p \to \infty$. For the first region, we make the change of variables $k = px$ and for the

second region, $k=p/x$. On simplification, we arrive at

$$\begin{aligned}\frac{1}{F(p^2)} &= 1+\frac{\pi\xi}{4p}\alpha+\frac{\alpha^2\xi^2}{8p^2}\\ &\times\int_0^1\frac{dx}{x}[2-(1-x)^2\mathcal{L}]-\frac{\alpha^2}{8p^2}\\ &\times\int_0^1\frac{dx}{x^2}(1-x)[2(-2x^2+3x+3)\\ &-3(1-x)(1+x)^2\mathcal{L}]-\frac{\alpha^2\xi}{24p^2}\\ &\times\int_0^1\frac{dx}{x^2}[2(2x^3+5x^2+3x+3)\\ &-3(x^2-x+1)(1+x)^2\mathcal{L}],\end{aligned}\tag{37}$$

where

$$\mathcal{L}=\frac{1}{x}\ln\frac{1+x}{1-x}.\tag{38}$$

The above integrals can be evaluated in a straightforward way. In order to make a direct comparison with Eq. (15), we prefer to write the final expression in Minkowski space by substituting $p\to\sqrt{-p^2}$ and $p^2\to -p^2$:

$$F(p^2)=1-\frac{\pi\,\alpha\xi}{4\sqrt{-p^2}}-\frac{\alpha^2\xi^2}{4p^2}+\frac{3\alpha^2}{4p^2}\left(\frac{7}{3}-\frac{\pi^2}{4}\right)+\mathcal{O}(\alpha^3).\tag{39}$$

One can note various important features of this result:

(i) $F(p^2)\neq 1$ in the Landau gauge. In fact, there is no value of the covariant gauge parameter ξ for which $F(p^2)$ can be 1.

(ii) The existence of constant term at $\mathcal{O}(\alpha^2)$ implies the violation of the transversality condition. We shall elaborate more on this remark in Sec. IV B.

(iii) Equation (14) is derived from the LK transformations based upon the assumption that $F=1$ in the Landau gauge. As we have seen, this assumption is not correct to $\mathcal{O}(\alpha^2)$, and therefore, Eq. (14) is not expected to hold true in general, as is confirmed on comparing Eq. (15) and Eq. (39). However, a comparison between the two results suggests that it contains the correct $\mathcal{O}(\xi^2)$ term at the level $\mathcal{O}(\alpha^2)$, though it does not reproduce other term appearing in the exact perturbative calculation.

B. Burden-Tjiang transversality condition

The perturbative expression for $F(p^2)$ to the two loops shows that the Burden-Tjiang transversality condition does not hold true beyond one loop order. Now we explicitly calculate the left hand side of Eq. (11). In the most general form, it can be expanded as

$$i\int\frac{d^3k}{(2\pi)^3}\Gamma^\mu(k,p)S_F(k)\gamma^\nu\Delta_{\mu\nu}^{0\ T}(q)=A(p^2)+B(p^2)\not{p},\tag{40}$$

where the multiplication with i is only for mathematical convenience. $A(p^2)$ and $B(p^2)$ can be extracted by taking the trace of the above equation, having multiplied by 1 and $\not{p}$ respectively. With the bare fermion being massless, it is easy to see that on doing the trace algebra and contracting the indices, $A(p^2)=0$. Our evaluation of $F(p^2)$ helps us identify $B(p^2)$ from Eq. (39) so that

$$\begin{aligned}i\int\frac{d^3k}{(2\pi)^3}\gamma^\mu S_F(k)\Gamma^\nu(k,p)\Delta_{\mu\nu}^{0\ T}(q)\\ =\left[-\frac{3\alpha}{16\pi p^2}\left(\frac{7}{3}-\frac{\pi^2}{4}\right)+\mathcal{O}(\alpha^2)\right]\not{p}.\end{aligned}\tag{41}$$

Obviously, for $\xi=0$,

$$\begin{aligned}i\int\frac{d^3k}{(2\pi)^3}\gamma^\mu S_F(k)\Gamma^\nu(k,p)\Delta_{\mu\nu}^{0\ T}(q)|_{\xi=0}\\ =\left[-\frac{3\alpha}{16\pi p^2}\left(\frac{7}{3}-\frac{\pi^2}{4}\right)+\mathcal{O}(\alpha^2)\right]\not{p},\end{aligned}\tag{42}$$

which is a violation of the transversality condition at the two loop level.

V. CONCLUSIONS

In this paper, we have presented the one loop calculation of the fermion-boson vertex in QED3 in an arbitrary covariant gauge for massless fermions. In the most general form, the vertex can be written in terms of 12 independent Lorentz vectors. Following the procedure outlined by Ball and Chiu, 4 of the 12 vectors define the longitudinal vertex. It satisfies the Ward-Green-Takahashi identity which relates it to the fermion propagator. The transverse vertex is written in terms of the remaining 8 vectors. For massless fermions, only 4 of these vectors contribute. Subtraction of the longitudinal vertex from the full vertex yields the transverse vertex. We evaluate the coefficients of the basis vectors for the transverse vectors to $\mathcal{O}(\alpha)$. Moreover, using this result, we calculate $F(p^2)$ analytically to $\mathcal{O}(\alpha^2)$ and find that the transversality condition proposed by Burden and Tjiang does not hold true to this order. Therefore, any non-perturbative construction of the transverse vertex based upon this condition cannot be correct.

Knowing the vertex in any covariant gauge may give us an understanding of how the essential gauge dependence of the vertex demanded by its Landau-Khalatnikov transformation [9,10] is satisfied non-perturbatively. Moreover, the perturbative knowledge of the coefficients of the transverse vec-

tors provides a reference for the non-perturbative construction of the vertex as every ansatz should reduce to this perturbative result in the weak coupling regime. In comparison to the transverse vertex obtained by Kızılersü *et al.* [7] for QED4 (which contained Spence functions), an important advantage of QED3 is that the corresponding results contain only basic functions of momenta. This provides us with a realistic possibility of searching for the non-perturbative form of the transverse vertex. The evaluation of $F(p^2)$ to $\mathcal{O}(\alpha^2)$ in an arbitrary covariant gauge should also serve as a useful tool in the hunt for the non-perturbative vertex which is connected to the former through Ward-Green-Takahashi identity nd the Schwinger-Dyson equations. Any vertex ansatz must reproduce Eq. (39) for $F(p^2)$ to $\mathcal{O}(\alpha^2)$ when the coupling is weak, leading to a more reliable non-perturbative truncation of Schwinger-Dyson equations: more reliable than the deconstruction of Burden and Tjiang.

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APPENDIX

Adopting the simplifying notation $k=\sqrt{-k^2}$, $p=\sqrt{-p^2}$ and $q=\sqrt{-q^2}$, the following are some of the integrals used in the calculation presented in the paper:

$$K^{(0)}=\int_M d^3w\frac{1}{(k-w)^2(p-w)^2}=\frac{i\pi^3}{q} \tag{A1}$$

$$J^{(0)}=\int_M d^3w\frac{1}{w^2(p-w)^2(k-w)^2}=\frac{-i\pi^3}{kpq} \tag{A2}$$

$$J^{(1)}_\mu=\int_M d^3w\frac{w_\mu}{w^2(p-w)^2(k-w)^2}=\frac{-i\pi^3}{kpq(k+p+q)}[pk^\mu+kp^\mu] \tag{A3}$$

$$J^{(2)}_{\mu\nu}=\int_M d^3w\frac{w_\mu w_\nu}{w^2(p-w)^2(k-w)^2}=\frac{-i\pi^3}{2kpq(k+p+q)^2}[-g^{\mu\nu}kpq(k+p+q)+k^\mu k^\nu p(k+2p+q)+p^\mu p^\nu k(2k+p+q)+(k^\mu p^\nu+p^\mu k^\nu)kp] \tag{A4}$$

$$I^{(2)}_{\mu\nu}=\int_M d^3w\frac{w_\mu w_\nu}{w^4(p-w)^2(k-w)^2}=\frac{i\pi^3}{2k^3p^3q(k+p+q)^2}[-g^{\mu\nu}k^2p^2q(k+p+q)+k^\mu k^\nu p^3(2k+p+q)+p^\mu p^\nu k^3(k+2p+q)+(k^\mu p^\nu+p^\mu k^\nu)k^2p^2] \tag{A5}$$

$$I_{-1}=\frac{2}{3kp}[p(3k^2+p^2)\theta(k-p)+k(k^2+3p^2)\theta(p-k)] \tag{A6}$$

$$I_0=2 \tag{A7}$$

$$I_1=\left[\frac{2}{k}\theta(k-p)+\frac{2}{p}\theta(p-k)\right] \tag{A8}$$

$$I_2=\frac{1}{2kp}\ln\frac{(k+p)^2}{(k-p)^2} \tag{A9}$$

$$I_3=\frac{2}{kp(k^2-p^2)}[p\,\theta(k-p)-k\,\theta(p-k)] \tag{A10}$$

$$I_4=\frac{2}{(k+p)^2(k-p)^2}. \tag{A11}$$

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