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Isospin-breaking effects in the extraction of isoscalar and isovector spectral functions from $e^+e^- \rightarrow$ hadrons

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We investigate the problem of the extraction of the isovector and isoscalar spectral functions from data on $e^+e^- \rightarrow$ hadrons, in the presence of non-zero isospin breaking. It is shown that the conventional approach to extracting the isovector spectral function in the ρ resonance region, in which only the isoscalar contribution associated with $\omega \rightarrow \pi\pi$ is subtracted, fails to fully remove the effects of the isoscalar component of the electromagnetic current. The additional subtractions required to extract the pure isovector and isoscalar spectral functions are estimated using results from QCD sum rules. It is shown that the corrections are small ($\sim 2\%$) in the isovector case (though relevant to precision tests of the CVC hypothesis), but very large ($\sim 20\%$) in the case of the ω contribution to the isoscalar spectral function. The reason such a large effect is natural in the isoscalar channel is explained, and implications for other applications, such as the extraction of the sixth order chiral low-energy constant, \mathcal{O} , are discussed. [S0556-2821(98)02113-4]

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I. INTRODUCTION

One of the basic ingredients of the standard model is the conserved vector current (CVC) hypothesis, which postulates that the charged (isovector) weak vector current and the neutral isovector component of the electromagnetic (EM) current are members of the same isovector multiplet. Since the charged current spectral function is now measured rather accurately in τ decay [1–3], the CVC hypothesis can be tested experimentally, provided, that is, one can extract the isovector spectral function from data on $e^+e^- \rightarrow$ hadrons [4–8]. In the absence of isospin breaking, this extraction is straightforward since, for example, for n -pion final states, a state with an even (odd) number of pions has even (odd) G-parity and hence can be produced only through the isovector (isoscalar) component of the hadronic EM current.

Isospin breaking, however, complicates the extraction of both the isovector and isoscalar spectral functions. Before proceeding, it is useful to clarify our notation. We define the standard $SU(3)_F$ octet of vector currents by $J_\mu^a = \bar{q}\gamma_\mu(\lambda^a/2)q$, where λ^a are the usual Gell-Mann matrices. The electromagnetic current is then $J_\mu^{EM} = J_\mu^3 + J_\mu^8/\sqrt{3}$, while the scalar correlators $\Pi^{ab}(q^2)$ (where we will restrict our attention to $a, b = 3, 8$) are defined by

$$i \int d^4x \exp(iqx) \langle 0 | T [J_\mu^a(x) J_\nu^b(0)] | 0 \rangle \\ \equiv (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi^{ab}(q^2). \quad (1)$$

Defining the spectral functions, $\rho^{ab}(q^2)$, corresponding to the $\Pi^{ab}(q^2)$ in the standard manner, $\rho^{ab}(q^2) = (1/\pi) \text{Im} \Pi^{ab}(q^2)$, the EM spectral function is then

$$\rho^{EM} = \rho^{33} + \frac{2}{\sqrt{3}} \rho^{38} + \frac{1}{3} \rho^{88}. \quad (2)$$

ρ^{EM} contains the isovector spectral function, ρ^{33} , which is the isospin rotated version of the corresponding charged current isovector spectral function measured in τ decays. Owing to the relation

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{8\pi^2\alpha^2}{s} \rho^{EM}(s), \quad (3)$$

however, the cross section for $e^+e^- \rightarrow$ hadrons directly measures ρ^{EM} , and not ρ^{33} or ρ^{88} . If isospin were not explicitly broken, this would present no problem since ρ^{38} would necessarily vanish and, as noted above, one could in addition identify the states contributing to the isovector (33) and isoscalar (88) spectral functions by their G-parity. Near threshold it is known, from a study of the mixed isospin vector correlator ($ab=38$) to two loops in chiral perturbation theory (ChPT) [9], that isospin breaking effects in ρ^{EM} are negligible. However, in the region of the ρ and ω resonances, isospin breaking is significant, as signalled by the interference dip in the cross section for the $\pi\pi$ final state [10]. The conventional method [5] for making corrections for this observed isospin breaking, in order to extract the vector component of the EM spectral function, is to first parametrize the amplitude in terms of a sum of ρ and ω Breit-Wigner resonance forms (in general one includes also contributions associated with the higher ρ resonances), and having fitted the parameters to the observed cross sections, remove the ω contribution to the amplitude by hand. The squared modulus of the resulting modified amplitude is then used in place of the squared modulus of the original amplitude to identify that portion of the cross section to be associated with the purely isovector (33) portion of the EM spectral function.

The conventional procedure just described for correcting for isospin breaking, however, does not, in fact, produce the desired 33 component of the vector spectral function. To understand why this is the case, let us first define, for the neutral vector mesons, $V = \rho, \omega, \phi$, the decay constants F_V^a via

$$\langle 0|J_\mu^a|V(k)\rangle = m_V F_V^a \epsilon_\mu(k) \quad (4)$$

where $\epsilon_\mu(k)$ is the vector meson polarization vector, and $a = 3, 8$. $F_\rho^{(3)}$, $F_\rho^{(8)}$, $F_\omega^{(8)}$ and $F_\phi^{(8)}$ are nonzero in the isospin limit, while $F_\rho^{(8)}$, $F_\omega^{(3)}$ and $F_\phi^{(3)}$ vanish in the isospin limit and hence are proportional to the isospin breaking parameter $\delta m = m_d - m_u$. In the presence of isospin breaking, all of the neutral vector mesons, in principle, mix with one another, and so the physical states are of mixed isospin. If we consider the $\pi\pi$ final state mediated by the ω exchange, then, to leading order in isospin breaking, this transition is indeed mediated by the isoscalar component of the EM current (the intermediate ω contribution generated by the isovector current is second order in isospin breaking, one factor from the coupling $F_\omega^{(3)}$, and one from the isospin violating $\omega \rightarrow \pi^+\pi^-$ decay vertex), and hence should be removed if one wishes to extract only the isovector contributions. The remaining ρ exchange contributions are, for a similar reason, however, not purely isovector. The reason is that if one considers the ρ exchange contribution to the EM spectral function, the 38 component is first order in isospin breaking (being proportional to $F_\rho^{(3)}F_\rho^{(8)}$). Thus, the ρ contribution to $e^+e^- \rightarrow \pi^+\pi^-$ necessarily contains a piece first order in isospin breaking, and associated with the 38 part of the EM spectral function, which must be subtracted in order to isolate the purely isovector 33 component. A similar argument shows that there will also be a 38 contribution to the measured cross section for $e^+e^- \rightarrow \omega \rightarrow 3\pi$, which one would have to correct for in order to isolate the purely isoscalar 88 component of ρ^{EM} .

In what follows we will discuss how to perform the corrections associated with first order isospin breaking contributions to the vector meson decay constants. The paper is organized as follows. In Sec. II we first discuss some general issues, which provide useful qualitative guidelines for the subsequent discussion. In addition we discuss an analogous case involving the neutral isoscalar and isovector axial vector currents, which example serves to illustrate the basic features we will meet in the vector current case of interest, but in a context where, unlike the vector case, the isospin breaking decay constants are already known, being fixed by a next-to-leading order analysis using chiral perturbation theory. In Sec. III, we then show how, and with what accuracy, existing QCD sum rule analyses of the mixed isospin (38) vector current correlator can be used to extract the isospin breaking decay constants $F_\rho^{(8)}$, $F_\omega^{(3)}$ and $F_\phi^{(3)}$. In Sec. IV, we use the results of this analysis to evaluate the corrections required to extract the pure isovector contribution associated with the ρ , and pure isoscalar contributions associated with the ω and ϕ from the EM cross-section data, and comment on the effect such corrections would have, for example, on precision tests of the CVC hypothesis and the extraction of chiral low-energy constants (LEC's) via the inverse chiral sum rules method [11].

II. SOME GENERALITIES AND AN ILLUSTRATIVE EXAMPLE

In the Introduction we have explained the reason for the existence of previously neglected isospin breaking correc-

tions in the extraction of the isoscalar and isovector spectral functions from $e^+e^- \rightarrow$ hadrons data. We have, however, not yet demonstrated that such corrections can be expected to be numerically significant. As will be seen below, making numerical estimates for the size of these corrections is non-trivial, and we will be forced to rely on a QCD sum rule analysis of the mixed-isospin vector current correlator to make these estimates. Since it can be difficult to estimate, in a quantitative manner, the errors present in such an analysis, it is useful to consider any qualitative constraints, based on general principles, which one might use to judge the plausibility of the resulting solutions. We will discuss below the existence of such constraints based on the framework of effective chiral Lagrangians. We will also consider, as an illustrative example, an analogous case in which the vector currents of the problem at hand are replaced by axial vector currents. The advantage of this example is that the isospin breaking pseudoscalar decay constants (analogous to the isospin breaking vector meson decay constants to be determined by the QCD sum rule analysis below) can, in this case, be computed with good accuracy using the methods of ChPT. This allows us to show explicitly, in a context where the numerical accuracy of the evaluation of the isospin breaking corrections is not open to question, that such isospin breaking corrections can play a significant role in the correct extraction of flavor-diagonal spectral contributions from the spectral functions of correlators involving mixed-flavor currents. Certain features of the relation of the relative signs and magnitudes of the corrections in the isovector and isoscalar cases, which recur in the vector channel, will also be exposed, again in a context where the accuracy of the numerical estimates is not open to question. From this example we will be able to unambiguously conclude that isospin breaking corrections of the type also present in the vector channel *must* be expected to be numerically significant, especially for observables related to differences of weighted integrals of the isovector and isoscalar spectral functions, for which there will be cancellation between flavor-conserving contributions, but coherence between the isospin breaking corrections.

Let us turn then to the qualitative guidance offered, in the vector channel, by the framework of effective chiral Lagrangians. Certain qualitative features of mixing in the vector meson sector follow immediately from the properties of an effective chiral Lagrangian such as would be obtained by the Callan-Coleman-Wess-Zumino [12,13] construction [14,15]. Note that the lowest order term in quark masses and derivatives which produces isospin mixing in the vector meson propagator matrix involves a single power of the quark mass matrix [where, as per the usual chiral counting, external momenta are counted as $\mathcal{O}(q)$ and quark masses as $\mathcal{O}(q^2)$]. Moreover, at leading order, the isospin violating decay constants associated with the isospin pure states vanish. As a result, at this order, the transformation from the original isospin pure basis to the physical, mixed isospin basis is a rotation, and the isospin breaking decay constants of the physical particles result purely from the mixing in the physical states. If we concentrate on ρ - ω mixing, this would mean that $F_\rho^{(8)}$ and $F_\omega^{(3)}$ were equal in magnitude and opposite in

sign. If we consider effective isospin breaking operators higher order in the quark-mass-derivative expansion, new effects come into play. First, one finds non-vanishing ‘‘direct’’ isospin violating couplings of the external vector currents to the isospin pure states from terms involving both derivatives and one power of the quark mass matrix. Second, terms involving both derivatives and one power of the quark mass matrix can produce off-diagonal mixing elements in the wave function renormalization matrix, a consequence of which is that the transformation from the isospin pure to physical basis is no longer a rotation, but rather the product of a symmetric matrix and a rotation. Third, terms with two powers of the mass matrix will produce modifications in the momentum-independent mixing terms in the vector meson propagator matrix. One would expect the effect of such higher order terms to be manifest in deviations from leading order relations such as $F_\rho^{(8)} = -F_\omega^{(3)}$. The size of such deviations should be typical of next-to-leading order corrections in $SU(3)_L \times SU(3)_R$, and hence might be as large as $\sim 30\%$.

We next turn to the illustrative example mentioned above, in which we replace the vector mesons and vector currents with pseudoscalar mesons and axial vector currents. Not only can several of the qualitative points just made be clearly illustrated in this case, but the actual numerical values of the relevant isospin breaking corrections can, for this example, be calculated to good accuracy using the techniques of ChPT, since all of the relevant decay constants are known at next-to-leading order in the chiral expansion. This is in contrast to the case of the vector current spectral functions, where we will be forced to rely on a QCD sum rule analysis to obtain the isospin breaking decay constants. Let us, therefore, define the axial current combination

$$A_\mu \equiv A_\mu^3 + \frac{1}{\sqrt{3}} A_\mu^8 \quad (5)$$

where $A_\mu^{3,8}$ are the 3, 8 members of the usual axial current octet. We then define the pseudoscalar decay constants, f_P^a , for $P = \pi^0, \eta$ and $a = 3, 8$, via

$$\langle 0 | A_\mu^a | P(k) \rangle = i f_P^a k_\mu. \quad (6)$$

At leading (second) order in the chiral counting the physical π^0, η basis is a pure rotation of the isospin pure octet basis π^3, π^8 ,

$$\pi^0 = \pi^3 + \theta_0, \pi^8, \quad \eta = \pi^8 - \theta_0 \pi^3 \quad (7)$$

where $\theta_0 = \sqrt{3}(m_d - m_u)/4(m_s - \hat{m})$, with $\hat{m} = (m_d + m_u)/2$, is the leading order mixing angle, and the isospin breaking decay constants are produced purely due to the ‘‘wrong-isospin’’ admixture present in the physical π^0, η states,

$$f_\pi^{(8)} = \theta_0 F, \quad f_\eta^{(3)} = -\theta_0 F \quad (8)$$

where F is a second order LEC, identical to both $f_\pi^{(3)} = f_\pi$, the physical pion decay constant, and $f_\eta^{(8)} = f_\eta$, the physical η decay constant, at this order in the chiral expansion. When one considers the full effective Lagrangian at next-to-leading order, however, one finds that (1) there is an infinite renor-

malization required to regularize the mixing of π^3, π^8 even if one works with the versions of the fields renormalized in the isospin limit, (2) there is indeed an off-diagonal element produced in the wave function renormalization matrix and (3) there are indeed ‘‘direct’’ isospin breaking meson-current vertices. (For a detailed exposition of the above features beyond leading order see Ref. [16].) The effect of all these features is to produce the following results for the isospin breaking decay constants [17], written in terms of the next-to-leading order expressions for the π^0 and η decay constants, which differ at this order in the chiral expansion:

$$\begin{aligned} f_\pi^{(8)} &= \epsilon_1 f_\pi \\ f_\eta^{(3)} &= -\epsilon_2 f_\eta. \end{aligned} \quad (9)$$

In Eq. (9), the quantities $\epsilon_{1,2}$ differ by terms which are next-to-leading order in the chiral expansion (the complete expressions can be found in Ref. [17]) and, using the observed experimental ratio of f_K/f_π , the next-to-leading order expressions for the isospin conserving decay constants imply $f_\eta/f_\pi \approx 1.3$ [17].

We are now in a position to consider the analogue of the vector current case of interest. To this end we imagine that we would like to obtain the π^0 and η contributions to the isovector and isoscalar axial spectral functions of the scalar correlators, $\Pi_1^{33}(q^2)$ and $\Pi_1^{88}(q^2)$, defined by

$$\begin{aligned} \Pi_{\mu\nu}^{ab} &= i \int d^4x e^{iqx} \langle 0 | T [A_\mu^a(x) A_\nu^b(0)] | 0 \rangle \\ &\equiv \Pi_1^{ab}(q^2) q_\mu q_\nu + \Pi_2^{ab}(q^2) (q^2 g_{\mu\nu} - q_\mu q_\nu). \end{aligned} \quad (10)$$

(We concentrate on the scalar correlators, Π_1^{ab} , since it is these correlators which contain the pole contributions analogous to those of the vector mesons in the vector current correlators.) It is straightforward, in this case, to simply evaluate the contributions to the spectral functions, ρ_1^{ab} . Keeping only terms up to first order in isospin breaking and to next-to-leading order in the chiral expansion, one finds that

$$\begin{aligned} \rho_1^{33}(q^2) &= [f_\pi^{(3)}]^2 \delta(q^2 - m_\pi^2) \\ \rho_1^{38}(q^2) &= f_\pi^{(3)} f_\pi^{(8)} \delta(q^2 - m_\pi^2) + f_\eta^{(3)} f_\eta^{(8)} \delta(q^2 - m_\eta^2) \\ \rho_1^{88}(q^2) &= [f_\eta^{(8)}]^2 \delta(q^2 - m_\eta^2). \end{aligned} \quad (11)$$

Let us imagine, however, that the way we had to go about extracting these contributions was by analyzing the ‘‘experimental’’ spectral function $\rho_1^A(q^2)$ of the scalar correlator, Π_1^A , appearing in the analogue, $\Pi_{\mu\nu}^A$, of the correlator of the product of two EM currents, i.e.,

$$\begin{aligned} \Pi_{\mu\nu}^A &= i \int d^4x e^{iqx} \langle 0 | T [A_\mu(x) A_\nu(0)] | 0 \rangle \\ &\equiv \Pi_1^A(q^2) q_\mu q_\nu + \Pi_2^A(q^2) (q^2 g_{\mu\nu} - q_\mu q_\nu). \end{aligned} \quad (12)$$

The analogue of the standard vector current extraction of the isovector component would then consist of identifying the

π^0 pole contribution with the isovector ρ_1^{33} component of ρ_1^A and the η pole term with the isoscalar ρ_1^{88} component thereof. We can see, however, that this identification is incorrect. Indeed, the π^0 contribution to ρ_1^A , to first order in isospin breaking, is straightforwardly found to be, not ρ_1^{33} , but

$$\begin{aligned} [\rho_1^A]_\pi &= \rho_1^{33} + \frac{2}{\sqrt{3}}[\rho_1^{38}]_\pi \\ &= \left([f_\pi^{(3)}]^2 + \frac{2}{\sqrt{3}}f_\pi^{(3)}f_\pi^{(8)} \right) \delta(q^2 - m_\pi^2). \end{aligned} \quad (13)$$

Similarly, the η pole contributions to ρ_1^A , to first order in isospin breaking, is not $\frac{1}{3}\rho_1^{88}$ but

$$\begin{aligned} [\rho_1^A]_\eta &= \frac{1}{3}\rho_1^{88} + \frac{2}{\sqrt{3}}[\rho_1^{38}]_\eta \\ &= \left(\frac{1}{3}[f_\eta^{(8)}]^2 + \frac{2}{\sqrt{3}}f_\eta^{(3)}f_\eta^{(8)} \right) \delta(q^2 - m_\eta^2). \end{aligned} \quad (14)$$

Thus, to ‘‘extract’’ the isovector and isoscalar spectral functions from the ‘‘experimental’’ spectral function one would actually have to know the isospin violating decay constants and then form the combinations

$$\begin{aligned} \rho_1^{33} &= [\rho_1^A]_\pi - \frac{2}{\sqrt{3}}f_\pi^{(3)}f_\pi^{(8)} \delta(q^2 - m_\pi^2) \\ \rho_1^{88} &= 3[\rho_1^A]_\eta - 2\sqrt{3}f_\eta^{(3)}f_\eta^{(8)} \delta(q^2 - m_\eta^2). \end{aligned} \quad (15)$$

Note that, because of the structure of the original current, the size of the isospin breaking correction in the isoscalar case is naturally larger by a factor of 3 than that in the isovector case. If we work with the values of $\epsilon_{1,2}$ from Ref. [17] [which correspond to the value of the isospin breaking mass ratio $r = (m_d - m_u)/(m_d + m_u)$ used in the sum rule analysis we will employ below],

$$\epsilon_1 = 1.37 \times 10^{-2}, \quad \epsilon_2 = 1.11 \times 10^{-2}, \quad (16)$$

we find that that the ‘‘experimental’’ pion pole contribution to ρ_1^A is a factor of $1 + 2\epsilon_1/\sqrt{3} = 1.016$ larger than the true isovector spectral function, and that the *nominal* ‘‘experimental’’ isoscalar spectral function, obtained by taking the η pole contribution and multiplying by 3, is smaller than the true isoscalar spectral function by a factor of $1 - 2\sqrt{3}\epsilon_2 = 0.961$. Thus, to extract the true isovector and isoscalar spectral functions, one would have to multiply the nominal ‘‘experimental’’ ones by 0.984 and 1.040, respectively. Note that the corrections go in the opposite direction for the two cases and that the magnitude of the correction is significantly larger in the isoscalar case. The reason for the latter feature of the results has already been explained. The reason for the former is that the leading order result that the π^0 and η contributions to the 38 spectral function differ in sign, but

not in magnitude, is still approximately satisfied by the next-to-leading order expressions, as expected based on the general arguments above. Note also that the effect of these corrections can be greatly enhanced if one considers combinations which would vanish at leading order in the chiral counting. As an example, if we consider the integral over the 88-33 spectral function, the nominal value (without the above corrections) is $0.69f_\pi^2$, while the actual value (after corrections) is $0.77f_\pi^2$. The isospin breaking corrections thus represent a 12% increase for this quantity, much larger than one would guess based on the typical few percent size of familiar isospin breaking corrections.

We will see in what follows that many of the features of the axial current example are recapitulated in the vector current case. In fact, because in the limit in which one considers the vector meson multiplet to be ideally mixed, but takes the decay constants to be otherwise determined by $SU(3)_F$ symmetry, the ω EM decay constant is a factor of 3 smaller than the ρ EM decay constant (rather than just the factor of $\sqrt{3}$ difference between the ‘‘A’’ decay constants of the π^0 and the η in the above example), the discrepancy between the size of the corrections in the isovector and isoscalar channels would be expected to be even greater in the vector case. We will see below that this expectation is indeed borne out.

III. QCD SUM RULE EXTRACTION OF THE ISOSPIN BREAKING VECTOR MESON DECAY CONSTANTS

Since the 3 and 8 components of the vector current octet occur in the standard model only in the combination J_μ^{EM} , it is not possible to directly determine the isospin breaking vector meson decay constants experimentally. One can, however, obtain indirect access to these quantities via a QCD sum rule analysis of the vector current correlator Π^{38} .

The basic idea of such a QCD sum rule analysis [18–20] is straightforward. From the behavior of $\Pi^{38}(q^2)$ as $q^2 \rightarrow \infty$ in QCD, it is known that Π^{38} satisfies an unsubtracted dispersion relation. This dispersion relation allows one to relate the dispersion integral over hadronic spectral function ρ^{38} to the value of Π^{38} at large spacelike value of $q^2 = -Q^2$. The latter can be expressed in terms of vacuum condensates using the operator product expansion (OPE), while the hadronic spectral function depends on the isospin breaking and isospin conserving decay constants of the various vector mesons. The utility of this relation is greatly enhanced, as first noted in Ref. [18], if the original dispersion relation is Borel transformed since, in that case, the higher s portions of the transformed hadronic spectral integral are exponentially suppressed, while the contributions of higher dimensional operators on the OPE side are simultaneously factorially suppressed. In favorable circumstances one is then able to write the dispersion relation in a form in which the parameters of a small number of resonances dominate the hadronic side, and the contributions of a small number of vacuum condensates of low dimension operators (which condensates can be determined from other sum rule analyses) dominate the OPE side. One can then use the known values of the vacuum condensates to extract the (in our case unknown) resonance parameters. Note that it is far preferable, in the case of inter-

est to us, to investigate the isospin breaking decay constants by analyzing an isospin breaking correlator, rather than by looking at the isospin breaking corrections to a correlator which also has an isospin conserving piece. This is because of the fact that, in the latter case, one would have to consider isospin breaking not only in the vector meson decay constants, but also in the continuum thresholds, which would be required in the spectral *Ansätze* in order to model the higher s portions of the hadronic spectral function. Since that portion of the spectral model is a rather crude representation of the actual continuum, this would introduce potentially large, and difficult to control, uncertainties into such an analysis.

Let us turn then to the sum rule analysis of the correlator $\Pi^{38}(q^2)$. We will, in fact, use the results of existing analyses [21–24] of the related correlator, $\Pi^{\rho\omega}(q^2)$, defined by

$$\begin{aligned} \Pi_{\mu\nu}^{\rho\omega}(q) &= i \int d^4x \exp(iq \cdot x) \langle 0 | T J_\mu^\rho(x) J_\nu^\omega(0) | 0 \rangle \\ &\equiv (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi^{\rho\omega}(q^2), \end{aligned} \quad (17)$$

where $J_\mu^\rho = (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d)/2$ and $J_\nu^\omega = (\bar{u} \gamma_\nu u + \bar{d} \gamma_\nu d)/6$. This is possible because, truncating the OPE at $\mathcal{O}(m_q)$, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_{EM})$ and operators of dimension 6 (with either the vacuum saturation hypothesis, or the rescaled version thereof, for the four-quark condensate contributions), the $\bar{s}s$ portion of J_μ^8 does not contribute to the OPE representation of $\Pi^{38}(q^2)$. One then has

$$[\Pi^{38}(q^2)]_{OPE} = \sqrt{3} [\Pi^{\rho\omega}(q^2)]_{OPE}, \quad (18)$$

where, after Borel transformation (indicated by the operator \mathcal{B}) of the truncated OPE expression, one finds [21]

$$\mathcal{B}[\Pi^{\rho\omega}(s)]_{OPE} = \frac{1}{12} \left[c_0 M^2 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{M^4} \right], \quad (19)$$

with M the Borel mass,

$$\begin{aligned} c_0 &= \frac{\alpha_{EM}}{16\pi^3} \\ c_1 &= \mathcal{O}(m_q^2) \sim 0 \\ c_2 &= 4 \left(\frac{m_u - m_d(1 + \gamma)}{2 + \gamma} \right) \langle \bar{q}q \rangle_0 \\ c_3 &= \frac{224\pi}{81} \alpha_s \kappa \langle \bar{q}q \rangle_0^2 - \frac{28\pi}{81} \alpha_e \kappa \langle \bar{q}q \rangle_0^2, \end{aligned} \quad (20)$$

where $\gamma \equiv \langle \bar{d}d \rangle_0 / \langle \bar{u}u \rangle_0 - 1$, $\langle \bar{q}q \rangle_0 \equiv (\langle \bar{u}u \rangle_0 + \langle \bar{d}d \rangle_0)/2$, α_e and α_s are the electromagnetic and strong coupling constants, respectively, and κ is the parameter describing the deviation of the four-quark condensates from their vacuum saturation values.

The hadronic spectral function, ρ^{38} , is parametrized as

$$\rho^{38}(s) = \frac{1}{4\sqrt{3}} [f_\rho \delta_\rho(s) - f_\omega \delta_\omega(s) + f_\phi \delta_\phi(s) + \dots] \quad (21)$$

where

$$\delta_V(s) = \frac{1}{\pi} \frac{m_V \Gamma_V}{(s - m_V^2)^2 + m_V^2 \Gamma_V^2} \quad (22)$$

[which reduces to $\delta(s - m_V^2)$ in the narrow width approximation], and we have followed the notation and sign conventions employed in the literature for the parameters f_V describing the strengths of the various vector meson contributions to the spectral function in question. Note that, in the notation introduced above,

$$\frac{f_V}{4\sqrt{3}} = \pm F_V^{(3)} F_V^{(8)} \quad (23)$$

where the upper sign holds for $V = \rho, \phi, \dots$, and the lower sign for $V = \omega$.

A few comments are in order concerning the form of Eq. (21). The first concerns the presence of the ϕ contribution, which was not included in the earliest sum rule analyses [21,22]. Recall that we expect, as a leading order result, that $f_\rho \simeq f_\omega$ (the absence of a minus sign in this relation is a consequence of the sign convention for the definition of f_ω). This means that, since from far in the spacelike region the ρ and ω masses appear essentially the same, one must expect significant cancellation between the ρ and ω contributions to the original dispersion relation for large values of Q^2 . This is especially true in the narrow width approximation. Based on this observation, it was realized, in Ref. [23], that, although f_ϕ could be expected to be significantly smaller than f_ρ or f_ω (of order 6–7 % if one makes an estimate based on the Particle Data Group evaluation of the deviation of vector meson mixing from ideal mixing [23]), the contribution of the ϕ to the actual sum rule need not be negligible. The sum rule was then re-analyzed, including the ϕ contribution, though still in the narrow width approximation. The results supported the qualitative arguments regarding the importance of including the ϕ contribution, and simultaneously cured a physically unpleasant feature of the earlier analyses, in which contributions from the $\rho' - \omega'$ region of the spectral function were as important, or more important, than those from the $\rho - \omega$ region in determining, through the original dispersion relation, the value of the correlator near $q^2 = 0$. It was, however, then subsequently pointed out [24] [Iqbal, Jin and Leinweber (IJL)], again because of the high degree of cancellation between ρ and ω contributions to the dispersion integral in the far spacelike region, that the use of the narrow width approximation for the ρ might also be a rather poor one. This was borne out by the numerical analysis of Ref. [24]. Of particular note is the fact that, introducing the ρ width into the spectral *Ansatz* for the ρ contribution, one finds that f_ρ and f_ω are lowered in magnitude by a factor of ~ 6 , and f_ϕ by a factor of ~ 2 compared to the values extracted using the narrow width analysis. With the above understanding in mind, we will, therefore, in what follows, employ the spectral *Ansatz* of Eq. (21) in the form arrived at in Ref. [24], i.e.,

with the ω and ϕ treated in the narrow width approximation, but the ρ treated using the Breit-Wigner form given in Eq. (22).

It should be stressed that, in using the results of Ref. [24], the present analysis relies *only* on the extraction of the hadronic spectral function for the mixed-isospin vector current correlator, accomplished in that reference using QCD sum rules, and not on the attempt by the authors of Ref. [24] to interpret this spectral function in terms of off-shell vector meson propagators. The latter interpretation (and that of Ref. [22]) is necessarily incorrect, since off-shell Green functions are well known to be altered by redefinitions of the hadronic fields and, as such, are not capable of being related to physical objects such as the mixed-isospin vector current correlator (an earlier claim by the present author, contained in Ref. [23], that the rescaled versions of the vector fields represented a possible field choice for the vector mesons is also incorrect). The only questions relevant to the use of the spectral function solution of Ref. [24] in the present work are, then, (1) is the sum rule reliable for the scales at which it is employed and (2) is the original *Ansatz* for the spectral function plausible in form? While it is not possible to provide a rigorous proof of the suitability of the analysis of Ref. [24], there is considerable indirect evidence in its favor. First, although a resonance-saturation *Ansatz* involving s -dependent widths, but constant real parts of the resonance self-energies, does not rigorously implement the constraints of analyticity and unitarity, it is well known from phenomenological studies of $e^+e^- \rightarrow$ hadrons that such *Ansätze*, nonetheless, can provide fits of very high numerical accuracy to the experimentally measured spectral function (see Ref. [25] for a recent detailed discussion). This is also true of the resonance-saturation fits to the timelike pion form factor measured in hadronic τ decays mediated by the isovector current (where, for example, the naive form of the resonance contributions involving s dependence only in the widths produces a fit of even slightly better quality than does an *Ansatz* employing the Gounaris-Sakurai form, which correctly implements analyticity and unitarity—see the results of Table 3 of Ref. [1]). Moreover, a recent analysis of the isovector (isospin conserving) vector correlator using a continuous family of finite energy sum rules as a means of implementing the dispersion relation implicit in QCD sum rules shows that fitting a naive form of the resonance saturation *Ansatz* for the hadronic spectral function to the OPE representation (where the widths and decay constants of the resonances are free parameters determined by the fit to the OPE) reproduces the experimentally measured spectral function [1] to an accuracy of a few percent and, moreover, requires a phenomenological value of the ρ width within a few MeV of that obtained by direct fitting of the experimental data using the favored hidden local symmetry (HLS) model in Refs. [25,26]. This indicates that, at least in the isospin conserving case, matching of an *Ansatz* of the type employed in Ref. [24] to the OPE representation of the vector current correlator provides a good fit to the actual vector current spectral function. Note that the matching between the hadronic and OPE sides of the conventional QCD sum rule obtained in Ref. [24] is also considerably better than that obtained in earlier analyses us-

ing the narrow width approximation for the ρ , further indicating the necessity of employing an *Ansatz* of the IJL form. The only improvement in the IJL *Ansatz* one might have hoped for was the use of a variable ρ width, to be fixed by the sum rule analysis. Since, however, in the isospin conserving case, the result of such an exercise is to essentially reproduce the width employed by IJL, it seems unlikely that the analysis would have been significantly altered by such an extension.

In light of the discussion above, we should thus be able to extract the desired isospin breaking vector meson decay constants from the results of Ref. [24]. The results we use below, however, differ somewhat from those quoted in that reference, and the reasons for this difference must first be explained. The first point in need of clarification is related to the fact that there are two sets of results associated with the analysis using the physical ρ width in Ref. [24]. Of these two sets, only the one contained in the column labelled ‘‘physical widths’’ and ‘‘no constraint’’ in Table 1 of that reference should be employed. The reason for rejecting the other set (labelled ‘‘constrained’’) is that these results were obtained assuming $\Pi^{38}(m_\omega^2)$ could be extracted from the $\omega \rightarrow \pi\pi$ contribution to the physical cross section. This assumption, however, as we have seen above, neglects the presence of additional Π^{38} contributions in the ρ exchange contribution to the cross section, and hence is incorrect. The second modification we make in employing the results of Ref. [24] concerns the magnitude of the errors quoted on the extracted values of the parameters f_V . It turns out that, in Ref. [24], an overly conservative error was assumed on the crucial input isospin breaking light quark mass ratio, $r_m = m_u/m_d$. The authors of Ref. [24] employed $r_m = 0.50 \pm 0.25$. The ratio, r_m , however, is actually much better constrained than this from ChPT analyses [27]. As a consequence, the quoted errors in Ref. [24] are unnecessarily inflated. The authors of Ref. [24] have kindly provided unpublished results corresponding to the more realistic input $r_m = 0.54 \pm 0.04$ employed in earlier sum rule analyses of the correlator at hand. We will, therefore, employ, for determining the errors on the extraction of the desired isospin breaking vector meson decay constants, the results of the ‘‘unconstrained’’ fit obtained using the modified input above, $r_m = 0.54 \pm 0.04$. The results, which correspond to a stable Borel regime $1.15 \text{ GeV} < M < 2.45 \text{ GeV}$, are [28]

$$\begin{aligned} f_\rho &= 4\sqrt{3}F_\rho^{(3)}F_\rho^{(8)} = 0.0030 \pm 0.0012 \text{ GeV}^2 \\ f_\omega &= -4\sqrt{3}F_\omega^{(3)}F_\omega^{(8)} = 0.0025 \pm 0.0009 \text{ GeV}^2 \\ f_\phi &= 4\sqrt{3}F_\phi^{(3)}F_\phi^{(8)} = -0.0002 \pm 0.0002 \text{ GeV}^2. \end{aligned} \quad (24)$$

The contributions from the ρ' and ω' are found to be very small, and cannot be reliably extracted from the sum rule in its current form. Note that the reliability of the results is supported by that fact that they display two features which correspond to our general expectations: first, that $f_\rho \approx f_\omega$ and, second, that f_ϕ is of order 6–7 % of f_ρ and f_ω .

We are now in a position to evaluate the isospin breaking decay constants $F_\rho^{(8)}$, $F_\omega^{(3)}$ and $F_\phi^{(3)}$. To do so we employ the results of Eq. (24), together with the relations

$$F_V^{EM} = F_V^3 + \frac{1}{\sqrt{3}} F_V^8, \quad (25)$$

where the physical EM vector meson decay constants, F_V^{EM} , determined experimentally from the partial widths for the decays $V \rightarrow e^+ e^-$, are

$$\begin{aligned} F_\rho^{EM} &= 154 \pm 3.6 \text{ MeV} \\ F_\omega^{EM} &= 45.9 \pm 0.8 \text{ MeV} \\ F_\phi^{EM} &= -79.1 \pm 2.3 \text{ MeV}. \end{aligned} \quad (26)$$

The sign of the ϕ decay constant in Eq. (26) has been chosen to be consistent with expectations from $SU(3)_F$ symmetry and ideal mixing. Note that the relative magnitudes in Eqs. (26) are roughly in line with the expectations of that limit (in which one would expect the ρ , ω and ϕ decay constants to be in the ratios $3:1:-\sqrt{2}$). It is then straightforward to solve for $F_V^{(3)}$ and $F_V^{(8)}$ separately. One finds, for the isospin breaking decay constants,

$$\begin{aligned} F_\rho^{(8)} &= 2.8 \pm 1.1 \text{ MeV} \\ F_\omega^{(3)} &= -4.2 \pm 1.5 \text{ MeV} \\ F_\phi^{(3)} &= 0.21 \pm 0.21 \text{ MeV}, \end{aligned} \quad (27)$$

where the quoted errors are totally dominated by the errors of the sum rule fit values of the products of decay constants. The values of the isospin conserving decay constants then follow immediately from Eqs. (25), (26) and (27). We will quote them in the form of ratios to the relevant experimental values, which form allows one to directly compute the additional corrections required to obtain the pure isovector and isoscalar spectral functions from those obtained conventionally, i.e., via the standard analysis described above. We find

$$\begin{aligned} \left[\frac{F_\rho^{(3)}}{F_\rho^{EM}} \right] - 1 &= -0.011 \pm 0.0043 \\ \left[\frac{F_\omega^{(8)}}{\sqrt{3} F_\omega^{EM}} \right] - 1 &= 0.091 \pm 0.029 \\ \left[\frac{F_\phi^{(8)}}{\sqrt{3} F_\phi^{EM}} \right] - 1 &= 0.0027 \pm 0.0027. \end{aligned} \quad (28)$$

IV. CONSEQUENCES FOR THE ISOVECTOR AND ISOSCALAR SPECTRAL FUNCTIONS

If we drop terms second order in isospin breaking, then the ρ , ω and ϕ resonance contributions to the isovector and isoscalar vector current spectral functions are easily seen to be

$$\begin{aligned} [\rho^{33}(q^2)]_V &= [F_\rho^{(3)}]^2 \delta_\rho(s) \\ [\rho^{88}(q^2)]_V &= [F_\omega^{(8)}]^2 \delta_\omega(s) + [F_\phi^{(8)}]^2 \delta_\phi(s). \end{aligned} \quad (29)$$

The results of the standard extractions, in contrast, are obtained by replacing $F_\rho^{(3)}$ with F_ρ^{EM} , $F_\omega^{(8)}$ with $\sqrt{3} F_\omega^{EM}$ and $F_\phi^{(8)}$ with $\sqrt{3} F_\phi^{EM}$. The corrections to be applied to the standard contributions in order produce the true resonance contributions to the isovector and isoscalar spectral functions are then given by the ratios

$$\begin{aligned} \left[\frac{F_\rho^{(3)}}{F_\rho^{EM}} \right]^2 &= 0.979 \pm 0.0086 \\ \left[\frac{F_\omega^{(8)}}{\sqrt{3} F_\omega^{EM}} \right]^2 &= 1.189 \pm 0.065 \\ \left[\frac{F_\phi^{(8)}}{\sqrt{3} F_\phi^{EM}} \right]^2 &= 1.0054 \pm 0.0054. \end{aligned} \quad (30)$$

From the above results we see that the standard procedure leads to an overestimate of the vector spectral function by $2.1 \pm 0.9\%$. This is still noticeably smaller than the $\sim 5\%$ errors on the $e^+ e^- \rightarrow$ hadrons cross sections in the resonance region. As a result, it is not yet possible, when comparing to τ data, to see the effect of the ρ^{38} contributions to the $e^+ e^- \rightarrow$ hadrons spectral functions extracted using the conventional analysis. (See Fig. 10c of Ref. [1] for a comparison of the spectral functions as extracted from τ and $e^+ e^-$ data. The above correction would lower the $e^+ e^-$ points by ~ 25 nanobarns in the region of the ρ peak.) A correction of this size, however, would certainly become important if one wished to make tests of the CVC hypothesis at the 1% level.

A much greater surprise is the size of the correction required in the case of the ω contribution to the isoscalar spectral function. While a 19% isospin breaking correction might sound unnaturally large, the size of the correction is, in fact, completely natural, and easily understood. The main features of the result follow from considering only the leading order contributions as discussed in Sec. II above. Let us, therefore, consider the approximation in which one considers only the leading $[\mathcal{O}(m_q, q^0)]$ contributions to ρ - ω mixing, neglects the ‘‘direct’’ isospin violating contributions to the vector meson decay constants (which are also higher order), and works in the ideal mixing/ $SU(3)_F$ approximation in which the EM decay constant of the pure isospin 1 component of the ρ is 3 times that of the pure isospin 0 component of the ω . Writing

$$\rho = \rho_I + \epsilon \omega_I, \quad \omega = \omega_I - \epsilon \rho_I, \quad (31)$$

where the subscript I denotes the isospin pure states, and ϵ is $\mathcal{O}(\delta m)$, the physical EM decay constants become

$$\begin{aligned} F_\rho^{EM} &= F_\rho^I + \epsilon F_\omega^I \simeq F_\rho^I \left(1 + \frac{\epsilon}{3} \right) \\ F_\omega^{EM} &= F_\omega^I - \epsilon F_\rho^I \simeq F_\omega^I (1 - 3\epsilon). \end{aligned} \quad (32)$$

The fractional correction in the ω case is thus expected to be ~ 9 times as big as that for the ρ case. That the actual corrections turn out to be exactly a factor of 9 different is a numerical accident, but the large relative size of the corrections is completely natural, and associated with the smallness of the EM ω coupling and the pattern of mixing in the vector meson sector. Note also the fact that the corrections are of opposite signs is exactly what one expects based on the general arguments above.

It is not just the isovector spectral function, with its relation to the CVC hypothesis, for which the corrections obtained above are of phenomenological interest. The difference of the isovector and isoscalar spectral functions also enters a number of interesting sum rules, and these sum rules must, therefore, also be corrected for the effects just discussed. As an example we will consider the extraction of the sixth (chiral) order low energy constant $Q(\mu)$ (in the notation of Refs. [29,9]) from the inverse moment chiral sum rule [29]

$$\int_{4m_\pi^2}^{\infty} ds \frac{(\rho^{33} - \rho^{88})(s)}{s} = \frac{16(m_K^2 - m_\pi^2)}{3F^2} Q(\mu^2) + \frac{1}{48\pi^2} \log\left(\frac{m_K^2}{m_\pi^2}\right) + \left(\frac{L_9^r(\mu^2) + L_{10}^r(\mu^2)}{2\pi^2 F^2}\right) \times \left[m_\pi^2 \log\left(\frac{m_\pi^2}{\mu^2}\right) - m_K^2 \log\left(\frac{m_K^2}{\mu^2}\right) \right]. \quad (33)$$

In Eq. (33), L_k^r are the scale-dependent renormalized fourth order LEC's of Gasser and Letuwyler [17], and μ is the ChPT renormalization scale. The form of this equation relies on the two-loop expressions for the 33 and 88 correlators obtained in Ref. [11]. The difference of the 33 and 88 spectral functions also enters a method of determining the strange current quark mass originally suggested by Narison [30]. In this application, a weighted integral over $\rho^{33}(q^2) - \rho^{88}(q^2)$ is performed, the weight function being that which enters inclusive τ decays. The corrections in this case, and the resulting values of the strange quark mass, will be treated in a separate paper [31].

In the analysis of the sum rule, Eq. (33), performed in Ref. [29], the ρ contribution to the spectral integral was obtained from τ decay data, and hence does not require the correction discussed above. The ω and ϕ contributions, however, are determined from the experimental $V \rightarrow e^+ e^-$ partial widths, and hence contain contributions from ρ^{38} which must be removed. The uncorrected ρ , ω and ϕ contributions to the spectral integral are [29] 0.0374, -0.0103 and -0.0204 , respectively. Implementing the corrections above, one finds that the sum of these three contributions is reduced from

0.0067 to 0.0046 ± 0.0007 , a downward shift of 31%. Including the estimates for the 4π and $\bar{K}K\pi\pi$ contributions as evaluated in Ref. [29], we find that the central value for $Q(m_\rho^2)$ is shifted from 3.7×10^{-5} to 2.2×10^{-5} , a change of 41%. As noted above, because of the cancellations inherent in forming $\rho^{33}(q^2) - \rho^{88}(q^2)$ [the combination vanishes in the $SU(3)_F$ limit], the effect of the isospin breaking corrections is large. A similar effect is found in the case of the strange quark mass analysis.

V. SUMMARY

We have shown that contributions to $e^+ e^- \rightarrow \text{hadrons}$ involving an intermediate state ρ or intermediate state ω or ϕ contain contributions from the isospin violating 38 vector spectral function which are not negligible, and must be removed if one wishes to extract the isovector 33 and isoscalar 88 spectral functions from $e^+ e^- \rightarrow \text{hadrons}$ data. Using the results of a QCD sum rule analysis of the 38 correlator, we have been able to estimate the isospin violating vector meson decay constants required to make these subtractions. We find that the isovector spectral function is $\sim 2\%$ smaller than what one would obtain by assuming it was identical to the full experimental ρ contribution, and that the ω contribution to the isoscalar spectral function is $\sim 19\%$ larger than what one would obtain from experiment without making this correction. We have also explained why it is unavoidable that (1) the isoscalar correction will be much larger than the isovector correction (by roughly an order of magnitude), and (2) the sign of the ρ and ω corrections in the isovector and isoscalar cases, respectively, will be opposite. A consequence of the second point is that all observables related to weighted integrals over the difference of the 33 and 88 spectral functions will receive large isospin breaking corrections, dominated by those which need to be made to correctly obtain the ω contribution to the 88 isoscalar term.

Finally, we note that it might be possible to reduce the errors on the extractions of the isospin breaking decay constants by updating the sum rule analysis of the 38 correlator using recent improved values for the input parameters, and evaluating higher order α_s corrections to the Wilson coefficients appearing in the $D=2,4$ terms of the OPE. This will be the subject of future investigations.

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