CORE
Metadata, citation and similar papers at core.ac.uk
Provided by Infoscience - École polytechnique fédérale de Lausanne

19th INTERNATIONAL CONGRESS ON ACOUSTICS
MADRID, 2-7 SEPTEMBER 2007

Efficient and Stable Acoustic Tomography Using Sparse Reconstruction Methods

PACS: 43.30.Pc

Jovanović, Ivana[1]; Hormati, Ali[1]; Sbaiz, Luciano[1]; Vetterli, Matrin[1,2]
[1]School of Computer and Communication Science, Ecole Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland; ivana.jovanovic@epfl.ch, ali.hormati@epfl.ch, luciano.sbaiz@epfl.ch, martin.vetterli@epfl.ch,
[2]Department of Electrical Engineering and Computer Science, University of California at Berkley, Berkley, CA 94720, USA

ABSTRACT

We study an acoustic tomography problem and propose a new inversion technique based on sparsity. Acoustic tomography observes the parameters of the medium that influence the speed of sound propagation. In the human body, the parameters that mostly influence the sound speed are temperature and density, in the ocean - temperature and current, in the atmosphere - temperature and wind. In this study, we focus on estimating temperature in the atmosphere using the information on the average sound speed along the propagation path. The latter is practically obtained from travel time measurements. We propose a reconstruction algorithm that exploits the concept of sparsity. Namely, the temperature is assumed to be a linear combination of some functions (e.g. bases or set of different bases) where many of the coefficients are known to be zero. The goal is to find the non-zero coefficients. To this end, we apply an algorithm based on linear programming that under some constrains finds the solution with minimum l_0 norm. This is actually equivalent to the fact that many of the unknown coefficients are zeros. Finally, we perform numerical simulations to assess the effectiveness of our approach. The simulation results confirm the applicability of the method and demonstrate high reconstruction quality and robustness to noise.

1. INTRODUCTION

Tomography methods use the information from the emitted signals that have crossed the field to be characterized. Their great success stems from their non-invasive nature and the fact that a significantly larger amount of data can be obtained compared to the classical one-sensor one-measurement setup. However, the methods by which the tomographic data are converted to the field values, referred to as the inversion methods, are computationally intensive and often ill-conditioned. A considerable improvement in this area is still needed. In this work, we propose a new inversion technique for some problems in acoustic tomography.

Acoustic tomography observes the parameters of the medium that influence the speed of sound propagation. In the human body, the most important parameters that influence the sound speed are temperature and density, in the ocean - temperature and current, in the atmosphere - temperature and wind. In this study, we focus on estimating temperature in the atmosphere using the information on the average sound speed along the propagation path. The latter is practically obtained from the travel time measurements [1,2]. Generalization to the estimation of other parameters would follow similarly.

The important element in the estimation is the choice of inversion method. In general, the inversion method comprises the modelling part and the inversion part. In the modelling part we devise a mapping W that models the relationship between the parameters θ of the unknown field and the measured data D, i.e. $D = W\theta$. In the second part, we construct an inverse mapping that finds θ which fits *best* the model, where best relates to the minimum of the predefined cost function. The importance of the inversion in many areas resulted in many inversion techniques, among which the iterative reconstruction techniques [3] and the stochastic inversions [4,5] are mostly applied in acoustic tomography in the atmosphere.

In this work, we propose a reconstruction algorithm that uses the concept of sparse signal representation. Namely, we assume that the temperature can be represented as a linear combination of some functions (e.g. bases or set of different bases) where many of the coefficients are zero. This assumption covers the cases where, for example, the temperature is known to be localized in space, in which case it is sparse in signal domain, or other more complicated temperature distributions for which we have a sparse representation in some transform domain - for example, the Fourier or wavelet domains. In all cases, the fact that we search for a sparse vector of parameters θ will allow to find the solution even when having fewer measurements than the total number of unknowns. This concept is known as *Compressed Sensing* (CS) [6], [7], since to observe a sparse vector θ of size N we need $M < N$ measurements. Note that the ideas of CS have been recently applied in the tomographic problem of the Magnetic Resonance Imaging [8].

To assess the effectiveness of the method, we perform numerical simulations. The simulation results confirm the applicability of the method and demonstrate high reconstruction quality and robustness to noise.

The paper is organized as follows. Section 2 introduces the problem of estimating the 2-D temperature field from the measurements obtained by acoustic tomography. Section 3 discusses the different concepts of sparsity and describes the reconstruction algorithm. In Section 4 we show the simulation results and analyze the reconstruction error.

2. TEMPERATURE ESTIMATION IN ACOUSTIC TOMOGRAPHY

In dry air, the temperature T can be inferred from the sound speed, through the following relation

$$c = \sqrt{\gamma R T}, \quad \text{(Eq. 1)}$$

where R is the gas constant and $\gamma = 1.4$. If there is no motion of the medium, the speed of sound can be computed from the time taken by a sound wave to propagate from a transmitter to a receiver, hereafter referred as *travel time*. Namely, the travel time is equal to

$$\tau = \int_{\Gamma} \frac{1}{c} ds, \quad \text{(Eq. 2)}$$

where Γ is the ray along which the sound travels from the transmitter to the receiver. For the field with a small temperature gradient we can assume that the rays are straight. The speed of sound can be further represented as the sum $c = c_0 + \Delta c$ where c_0 is the spatial average of the speed and Δc is the speed fluctuation. Since in the atmosphere the absolute values of the sound speed fluctuations are much smaller than their spatial average, equation (Eq. 2) can be linearized to the first order of the fluctuations:

$$\tau \cong \int_{\Gamma} \frac{1}{c_0} ds - \frac{1}{c_0^2} \int_{\Gamma} \Delta c \, ds = \int_{\Gamma} \frac{1}{c_0} ds - \frac{\gamma R}{c_0^3} \int_{\Gamma} \Delta T \, ds. \quad \text{(Eq. 3)}$$

Clearly, by measuring the variation in the travel time, we obtain the information on the temperature variation

$$d = \frac{c_0^3}{\gamma R} \left(\int_{\Gamma} \frac{1}{c_0} ds - \tau \right) = \int_{\Gamma} \Delta T \, ds. \quad \text{(Eq. 4)}$$

In general, taking into account the physical properties of the unknown field always improves the reconstruction quality. For this specific problem we can use the fact that the change of temperature over time is governed by the heat equation. From this equation, we know that a concentrated deposit of heat diffuses away in a Gaussian manner, as described by the 2-D heat kernel:

$$h(x,y,t)=\frac{1}{4\pi Dt}e^{-\frac{x^2+y^2}{4Dt}},\qquad \text{(Eq. 5)}$$

where D is a diffusion constant. If there are no active heat sources, the temperature at time t can be computed as a convolution of the initial heat at time t_0 with the heat kernel:

$$T_t(x,y)=\left[T_{t_0}*h\right](x,y).\qquad \text{(Eq. 6)}$$

Keeping the same setup (number of emitters and receivers) we can take the measurements over a series of time instants. This will results in a larger number of measurements and therefore in a more accurate estimation.

3. INVERSE METHOD

In the most general approach, the problem of finding ΔT from its line integrals d, or so called projections, is solved by applying the inverse Radon transform. However, this is not a very practical method since it requires a large number of projections, with special setup geometry. An entirely different approach for tomographic reconstruction consists of assuming a parametric model for the unknown temperature field and setting up a system of equations for the unknowns in terms of the measured data. To estimate N unknown parameters employing the classical methods would require at least N measurements. However, if we choose an appropriate model in which most of the parameters are zeros or of small amplitude compared to the rest, then using nonlinear methods, related to CS, it is possible to reconstruct all N parameters having $M<N$ measurements. In Section 3.1 we introduce the basic idea behind CS, and in section 3.2 and section 3.3 we show how this can be practically applied in our temperature estimation problem.

3.1. Compressed sensing

To have a general idea of CS, consider a signal x and suppose that the basis $\Psi=[\psi_1,\psi_2,\ldots,\psi_N]$ provides a K-sparse representation of the signal x, that is, x can be written as a linear combination of K elements of Ψ:

$$x=\sum_{n=1}^{N}\theta_n\psi_n=\sum_{l=1}^{K}\theta_{n_l}\psi_{n_l}.$$

where n_l denotes the position of K nonzero entries of the vector θ. Alternatively, in matrix notation we have

$$x=\Psi\theta,$$

where θ is an $N\times 1$ column vector and has K nonzero elements. Unlike in the traditional approach where we need N measurements, the compressed sensing approach suggests $M<N$ measurements. The measurements are obtained as projections of the sparse signal x onto a second set of basis $\Phi=[\phi_1,\phi_2,\ldots,\phi_M]$, that is

$$y_m=\langle\phi_m,x\rangle=\sum_{n=1}^{N}\langle\phi_m,\psi_n\rangle\theta_n$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product. To simplify the notation we can write in matrix form

$$y=W\theta$$

where $W_{mn}=\langle\phi_m,\psi_n\rangle$. Since $M<N$, the inversion from the measurement vector y back to the signal x is ill-posed. However, it has been shown that for the perfect reconstruction of K-sparse signals of dimension N, we need only $M=O(K\log N)<<N$ measurements [6,7]. In this case, the signal can be recovered by solving an l_1 minimization problem

$$\hat{\theta}=\arg\min\|\theta\|_1 \text{ s.t. } y=W\theta.\qquad \text{(Eq. 8)}$$

This optimization problem, also known as Basis Pursuit [9], can be solved using standard linear programming. The solution exists when the bases Φ and Ψ are incoherent [7], or equivalently, when $\{\phi_m\}$ basis set does not provide a sparse representation of the elements $\{\psi_m\}$.

In the case of noisy measurements, it is possible to adapt the optimization algorithm to incorporate the noise [9]. The new optimization procedure can be stated as:

$$\hat{\theta} = \arg\min(\|y - W\theta\|_2^2 + \lambda\|\theta\|_1), \qquad \text{(Eq. 9)}$$

where λ controls the trade off between the sparsity of the solution and the residual in the reconstruction.

3.2 Sparsity in signal domain

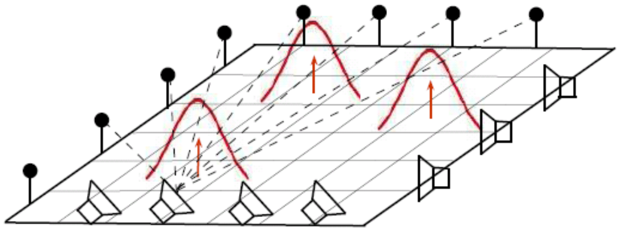

Fig. 1. Acoustic tomography setup with 7 emitters and 8 receivers placed around the region of interest. The temperature distribution is sparse in the signal domain, as it originates from 3 local sorces.

Consider the tomographic problem in which the goal is to reconstruct the temperature field produced by K localized sources inside the region of interest (see Fig.1). In this setup, we define a N -node grid encompassing the tomographic region and assume that the sources are placed on the grid. The temperature field can be seen as a set of 2D Diracs convolved with some normalized kernel $\Lambda(x,y)$.

Assume that there are p possible candidates for the kernels. Since for each of N nodes on the grid we can choose any of the p possible kernels, there are pN total unknowns. As long as there are K active sources, only K of the unknowns are nonzero. To reconstruct the field, one takes M acoustic travel time measurements. The normalized travel time variation along the path Γ can be written as

$$d = \int_\Gamma \Delta T\, ds = \sum_{i=1}^{p}\sum_{j=1}^{N} \theta_{N(i-1)+j} \int_\Gamma \Lambda_i(x(s) - x_j, y(s) - y_j) ds, \qquad \text{(Eq. 10)}$$

where $\theta_{N(i-1)+j}$ is the weight of the kernel i on the node j with the position (x_j, y_j). Putting all M measurements in the matrix form, we get

$$D_{M\times 1} = W_{M\times pN}(\Lambda)\cdot\theta_{pN\times 1}. \qquad \text{(Eq. 11)}$$

The noise of the measurements, the linearization procedure and the model mismatch in the system are the main factors that prevent us from using the exact sparse recovery in (Eq. 8). Therefore we run the noise-enhanced optimization in (Eq. 9)

$$\hat{\theta} = \arg\min(\|D - W\cdot\theta\|_2^2 + \lambda\|\theta\|_1). \qquad \text{(Eq. 12)}$$

The diffusion of the heat in the region can be incorporated into the system to add more data in the reconstruction algorithm. One can predict the change of the kernels in the diffusion process by knowing the physical parameters of the medium. In the diffusion process, the shapes of the kernels change according to (Eq. 6); however, their positions remain the same. By taking measurements over k time instants, one can write

$$\begin{bmatrix} D_{M\times 1}(t_1) \\ D_{M\times 1}(t_2) \\ \vdots \\ D_{M\times 1}(t_k) \end{bmatrix} = \begin{bmatrix} W_{M\times pN}(\Lambda_{t_1}) \\ W_{M\times pN}(\Lambda_{t_2}) \\ \vdots \\ W_{M\times pN}(\Lambda_{t_k}) \end{bmatrix} \cdot \theta_{pN\times 1}\ , \qquad \text{(Eq. 13)}$$

where $W_{M\times pN}(\Lambda_{i_1})$ is found by first computing the new diffused kernel at time t_i .

3.3 Sparsity in transform domain

Consider the case where the temperature field in the region of interest is sparse when transformed into another domain. As an example, assume that the temperature is smooth in the region of interest. It is well known that smooth signals are sparse in the Fourier domain. The idea is to project the temperature field on the subspace spanned by N two dimensional Fourier basis functions and then to try to estimate the sparse coefficients of this transform by using l_1 norm minimization techniques.

Denote the basis elements by $\Psi = [\psi_1, \psi_2, \ldots, \psi_N]$. The normalized travel-time difference along the path Γ can be represented as:

$$d = \int_{\Gamma} \Delta T\, ds = \sum_{n=1}^{N} \theta_n \int_{\Gamma} \varphi_n ds. \qquad \text{(Eq. 14)}$$

By putting all the measurements in one column vector D, one gets

$$D_{M\times 1} = W_{M\times pN}(\Psi)\cdot \theta_{pN\times 1}. \qquad \text{(Eq. 15)}$$

where the element W_{mn} of the measurement matrix W will be the line integral of the basis n along the path of acoustic ray m. Due to the noise in the system, we run the noise-enhanced linear program in (Eq. 9) to find the unknown vector of coefficients.

To make use of the diffusion in the reconstruction, one needs to compute the basis elements after the diffusion. As explained in Section 2, the new basis elements are found by convolving the original elements by the diffusive kernel. In the case of the Fourier domain representation, the convolution is replaced by multiplication by a parameter that changes over time. The way to write the new system of equations is the same as in (Eq. 13).

There is a lot of freedom in choosing the appropriate basis elements to approximate the field of interest in a sparse manner. For example wavelet expansion can be used whenever there are sharp transitions in the field since wavelets can represent sharp transitions in a very efficient manner. As one can see, there is a strong connection between image compression techniques and our tomography inversion method.

4.SIMULATION RESULTS

In this section, we are going to show the simulation results for two different temperature models, one that reflects sparsity in signal domain and the other that reflects sparsity in transform domain. In the simulations, we first compute the travel times for a given temperature distribution and then use them as the input data for our reconstruction algorithm.

In the first scenario, we assume that the temperature is localized in space. The setup is identical to the one explained in Section 3.2. The 2-D Diracs are convolved with the 2-D cubic B-splines:

$$\beta^3\left(\frac{r}{\alpha}\right) = \begin{cases} \frac{2}{3} - \frac{|r|^2}{\alpha^2} + \frac{|r|^3}{2\alpha^3} & 0 \le \frac{|r|}{\alpha} < 1 \\ \dfrac{(2-\frac{|r|}{\alpha})^3}{6} & 1 \le \frac{|r|}{\alpha} < 2 \\ 0 & 2 \le \frac{|r|}{\alpha} \end{cases}$$

where we use $|r| = \sqrt{x^2+y^2}$ as the radial coordinate. The parameter $\alpha > 0$ is the scale of the B-spline that regulates the spline width. In Fig. 2(a) we show the true temperature that is composed of $K = 5$ cubic splines, arbitrary placed on the grid of $12\times 12 = 144$ nodes. Their positions and the weights are randomly chosen. We also assume that the possible spline scales belongs to the set of $p = 4$ elements. In total, we have $pN = 576$ unknowns. There are 8 emitters and 8 receivers placed on the border of the region of interest. The measurements are taken in 2 time slots, resulting in $M = 128$ measurements. To account for the effect of noise and the model mismatch, we add 40dB distortion to D and the coefficients in W (with respect to the energy in D and W). The true temperature distribution is shown on Fig. 2(a). The sparse reconstruction algorithm provides the temperature reconstruction with the error shown in Fig. 2(b). Due to the distortion, the vector $\hat{\theta}$ is not K-sparse but has K large coefficients while the others are close to zero. Choosing the positions of the K largest coefficients as the correct ones, we can recompute the weights using the least squares method. The result is shown in Fig. 2(c). However, it should be noted that the success of the least squares method is directly related to the accuracy of the coefficient positions found in the l_1 optimization.

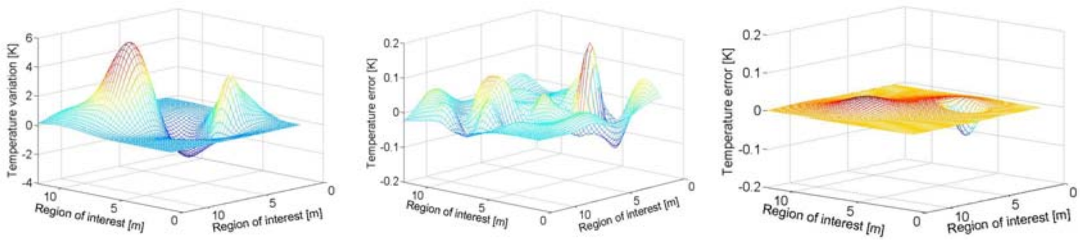

Fig. 2. (a) True temperature distribution- sparsity in signal domain (b) Reconstruction error with l_1 optimization, (c) Reconstruction error after applying $l_1 + l_S$

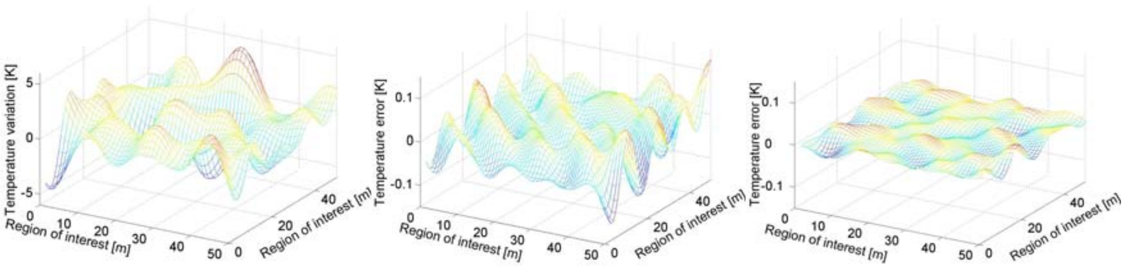

Fig. 3. (a) True temperature distribution- sparsity in transform domain (b) Reconstruction error with l_1 optimization, (c) Reconstruction error after applying $l_1 + l_S$

In the second scenario, we assume that the temperature is sparse in Fourier domain, and it can be represented using a sparse vector of Fourier coefficients. The temperature is assumed to be a sum of $K = 20$ cosine basis functions that are randomly chosen from a set of $N = 484$ functions. In this setup, we take the measurements in 3 time slots, what results in $M = 192$ measurements in total. Again, to model the effect of noise and the model mismatch we add 30dB of distortion, both to D and W. The true temperature and the reconstruction errors are shown in Fig. 3. We can see that the proposed method exhibits good reconstruction accuracy for both scenarios and shows robustness to the noise and model mismatch.

References: [1] I. Jovanovic, L. Sbaiz, and M. Vetterli, : Acoustic Tomography Method for Measuring Temperature and Wind Velocity. IEEE Intern. Conf. on Acoust., Speech, and Signal Proces. (2006)

[2] I. Jovanovic, L. Sbaiz, and M. Vetterli: Acoustic Tomography for Estimating Temperature and Wind Flow. 13-th Intern. Symp. for the Advancement of Boundary Layer Remote Sensing (2006)

[3] A. Ziemann, K. Arnold, and A. Raabe: Acoustic travel time tomography – a method for remote sensing of the atmospheric surface layer. Meteorol. Atmos. Phys., **71** (1999) 43-51

[4] D. K. Wilson and D. W. Thomson, “Acoustic tomographic monitoring of the atmospheric surface layer,” J. Atmos. Ocean. Tech, , **No. 11**,(1994) 751–769

[5] S. N. Vecherin, V. E. Ostachev, G. H. Geodecke, D. K. Wilson, and A. G. Voronovic: Time-dependent stochastic inversion in acoustic travel-time tomography of the atmosphere. J. Acoust. Soc. Am., (2006)

[6] E. J. Candes, J. Romberg, and T. Tao: Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. IEEE Trans. on Information Theory, **52, No. 2**, Feb. (2006)

[7] D. L. Donoho: Compressed sensing. IEEE Trans. on Information Theory, **52, No. 4**, Apr. (2006)

[8] M. Lustig, D. L. Donoho, and J. M. Pauly: Sparse MRI: The application of compressed sensing for rapid MR imaging. Submitted to Magnetic Resonance in Medicine (2007)

[9] S. S. Chen, D. L. Donoho, and M. A. Saunders: Atomic decomposition by basis pursuit. SIAM Journal on Scientific Computing, **21, No. 1**, (1998) 33-61