Journal of the Faculty of Environmental Science and Technology,
Okayama University
Vol.9, No.1, pp.53-58, February 2004

Experimental and Computational Water Analysis for Kojima Lake

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(Received December 2, 2003)

We study a wind-generated current in Kojima Lake. We introduce an experimental result which was obtained using a float equipped with a GPS unit. We also present a result of numerical study in which a result from finite element analysis of flow is applied to the momentum equation of the float to simulate its motion.

Key words: *water environment, global positioning system, numerical simulation*

1 INTRODUCTION

Kojima Lake is an artificial lake isolated from Kojima Bay in the Seto Inland Sea. We conducted an experiment of a wind generated flow in Kojima Lake using a float equipped with a GPS unit. We introduce a result given in terms of a trajectory of the GPS-float. We also introduce a result of numerical simulation for its motion. The flow is analyzed numerically with application of a finite element method to the governing equations. Then we discuss the significance in similarity and difference between the experimental result and the numerical result.

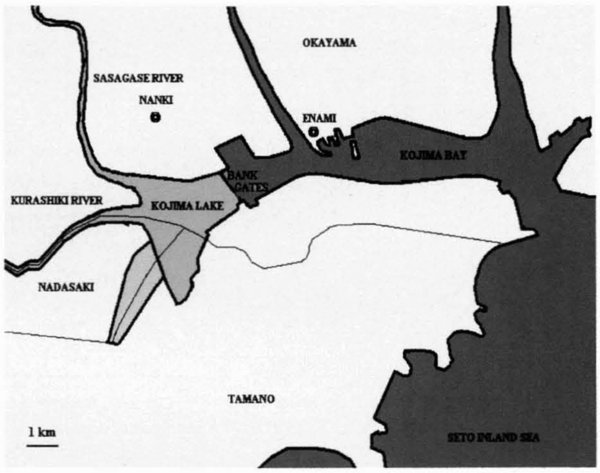

Figure 1: Kojima Lake and its vicinity.

2 GPS-FLOAT EXPERIMENT OF WIND-GENERATED FLOW

Seto Inland Sea separates Shikoku Island from the main island of Japan. Okayama prefecture lies in the main island and faces Seto Inland Sea. Kojima Lake is an artificial lake made by isolating a part of Kojima Bay with a bank (*cf.* Figure 1).

We conducted experiments using a float equipped with a GPS-unit, which we call the GPS-float, to flow generated in Kojima Lake. The GPS-float is designed to travel over the surface receiving the fluid resistance on a pair of rectangular plates attached underneath the surface (*cf.* Figure 2). Here we introduce results

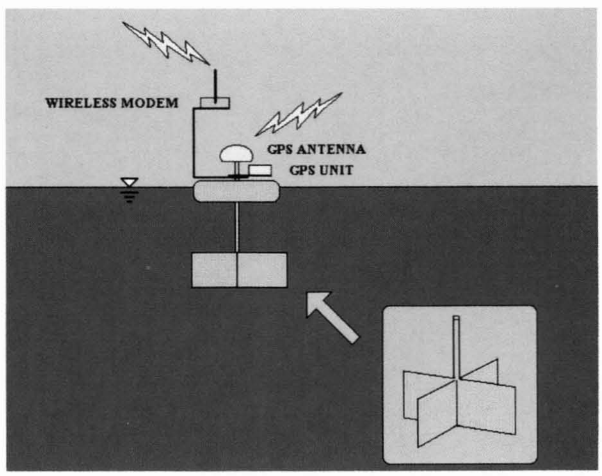

Figure 2: Illustration of the GPS-float.

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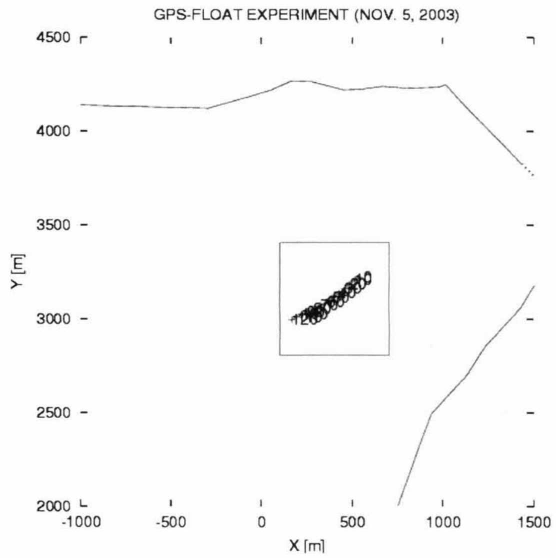

Figure 3: An area where the experiments were conducted.

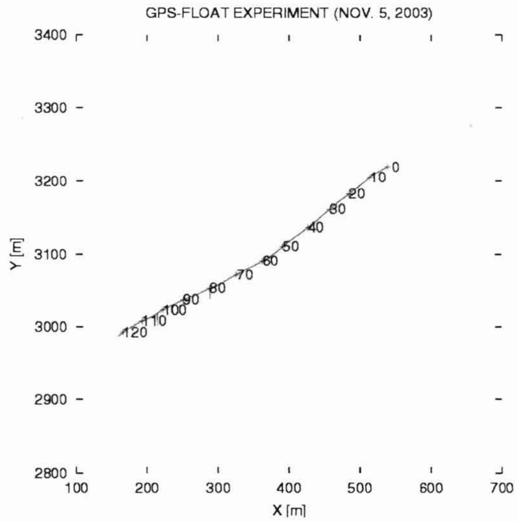

Figure 4: The part enclosed by the box of Figure 3.

of experiment conducted November 5, 2003. The result is given in terms of a trajectory of the GPS-float recorded from 4:31:34 GMT to 6:34:32 GMT. The numbers on the trajectory indicate the elapsed time (minute). The experimental result is shown in Figures 3 and 4. The part enclosed by the box in Figure 3 is shown in Figure 4.

In analysis of flow in the water environment, it often becomes necessary to obtain data concerning the depth of the water. We conducted measurement of water depth in Kojima Lake using a system that consisted of a GPS unit and a supersonic depth measuring unit (*cf.* Figure 5). respectively. New information is introduced to update the existing data. Figures 6 and 7 show the contours of the bottom topography before and after update, respectively.

3 FINITE ELEMENT ANALYSIS OF THE WIND-GENERATED FLOW

We solved the following system consisting of momentum equations and a continuity equation applying a finite element method.

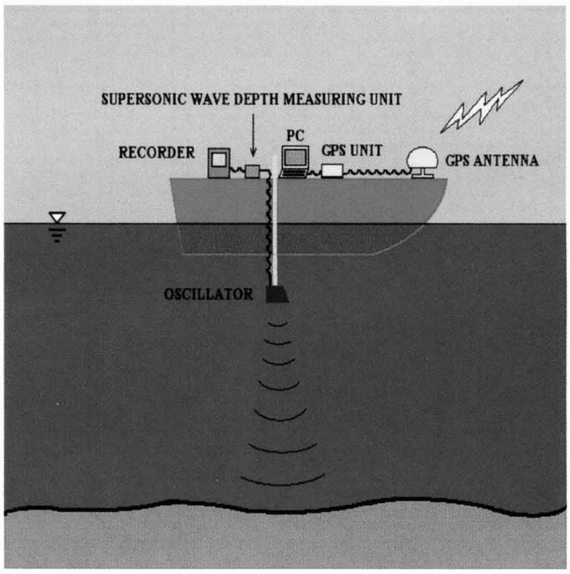

Figure 5: An area where the experiments were conducted.

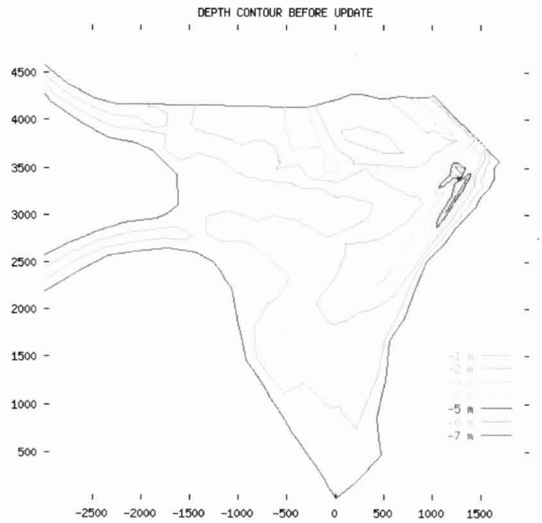

Figure 6: An area where the experiments were conducted.

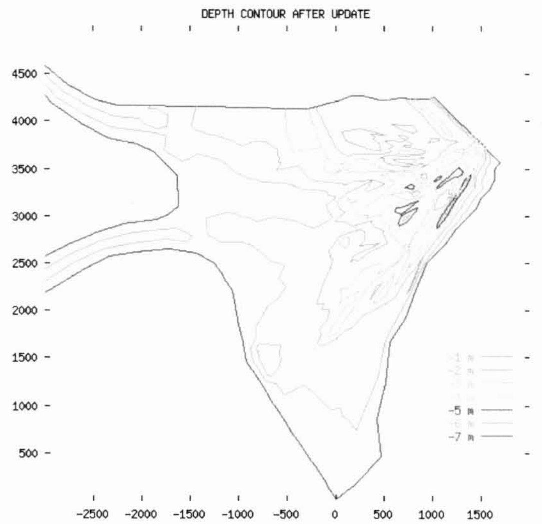

Figure 7: An area where the experiments were conducted.

Table 1: Wind velocity on November 5, 2003.

	Enami		Nanki	
JST	WD	WS m/s	WD	WS m/s
1	π	1.5	$9/8\pi$	2.5
2	π	2.2	$9/8\pi$	3.6
3	π	1.5	π	1.2
4	$9/8\pi$	1.5	$9/8\pi$	1.4
5	$5/4\pi$	0.7	$9/8\pi$	1.5
6	$9/8\pi$	1.4	$9/8\pi$	2.5
7	$3/2\pi$	2.5	$3/2\pi$	2.5
8	π	1.3	$7/8\pi$	1.6
9	π	0.9	$9/8\pi$	1
10	$7/8\pi$	1.7	$9/8\pi$	3.5
11	$9/8\pi$	2.1	π	2.7
12	$7/8\pi$	1.4	$9/8\pi$	3
13	$5/4\pi$	1.4	$11/8\pi$	1.7
14	$7/8\pi$	1.8	$9/8\pi$	1.6
15	π	2	π	1.8
16	π	1.6	$9/8\pi$	2.5
17	$3/2\pi$	0.9	$11/8\pi$	1.5
18	$11/8\pi$	0.5	$3/2\pi$	1.9

$$\begin{aligned} \frac{\partial M}{\partial t} &= -g(h+\zeta)\frac{\partial \zeta}{\partial x} + A\left(\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right) - \frac{\tau_x}{\rho_0}, \\ &\quad + C_f \frac{\rho_a}{\rho_w} W^2 \cos\theta \\ \frac{\partial N}{\partial t} &= -g(h+\zeta)\frac{\partial \zeta}{\partial y} + A\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) - \frac{\tau_y}{\rho_0}, \\ &\quad + C_f \frac{\rho_a}{\rho_w} W^2 \sin\theta \\ \frac{\partial \zeta}{\partial t} &= -\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}. \end{aligned} \tag{1}$$

Here, $z = \zeta$ and $z = -h$ correspond to the water surface and the bottom of the lake, respectively. Let u and v denote x-component and y-component of the velocity, respectively. M and N are defined by

$$M = \int_{-h}^{\zeta} u\,dz, \quad N = \int_{-h}^{\zeta} v\,dz.$$

ρ_0 is a constant that represents the density. τ_x and τ_y are given by

$$\tau_x = \frac{\rho_0 \gamma^2 \sqrt{M^2+N^2}}{(h+\zeta^2)} M, \quad \tau_y = \frac{\rho_0 \gamma^2 \sqrt{M^2+N^2}}{(h+\zeta^2)} N.$$

In order to solve the system of partial differential equations (1) numerically, we set $g = 9.81$, $A = 0.01$. Table shows the velocity of the wind at Enami and Nanki (*cf.* Figure 1).

Figures 8 and 9 show the division of the region into triangular elements. There are 2064 elements and

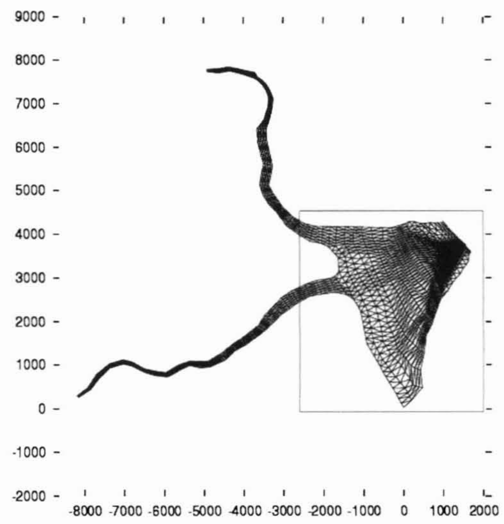

Figure 8: Finite elements in the entire region.

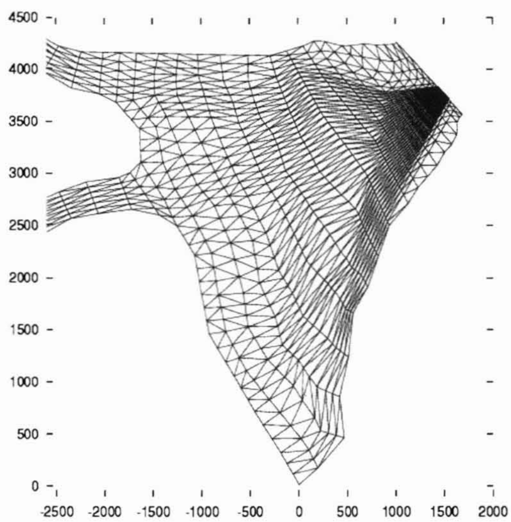

Figure 9: Finite elements enclosed by the box of Figure 8.

1147 nodes in the division. We analyzed the system for 4:00:00 JST to 16:00:00 JST. The part enclosed by the box of Figure 8 is shown in Figure 9. Figures 10 and 11 show the distribution of velocity vectors at 5:40:00 GMT with $\gamma^2 = 0.05$ and $\gamma^2 = 0.02$, respectively.

4 NUMERICAL SIMULATION OF THE GPS-FLOAT MOTION

The fluid resistance exerted on the plates attached to the GPS-float can be evaluated in terms of the fluid velocity. Thus, once the velocity of flow is obtained, the motion of the GPS-float can be simulated by solv-

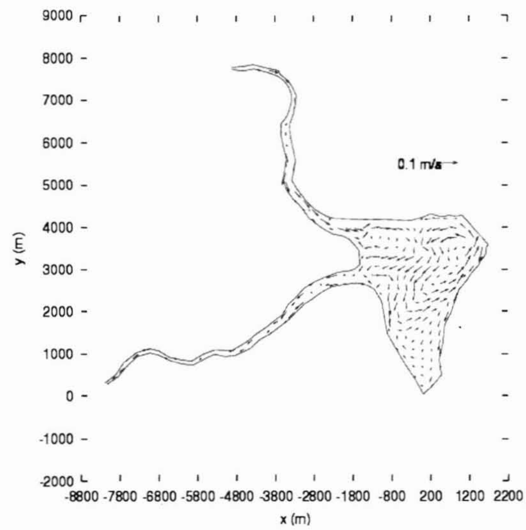

Figure 10: Distribution of velocity vectors at 5:40:00 GMT with $\gamma^2 = 0.05$.

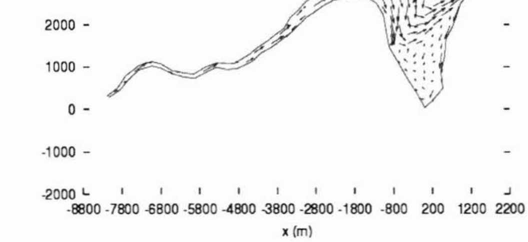

Figure 11: Distribution of velocity vectors at 5:40:00 GMT with $\gamma^2 = 0.02$.

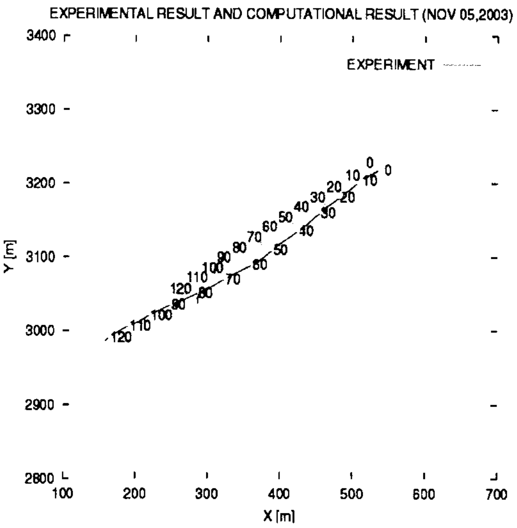

Figure 12: Numerical simulation of the motion of the GPS-float with $\gamma^2 = 0.05$.

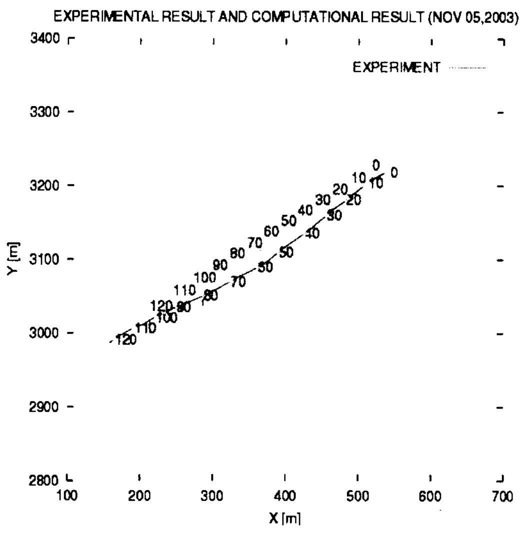

Figure 13: Numerical simulation of the motion of the GPS-float with $\gamma^2 = 0.02$.

ing its momentum equation:

$$M\ddot{x} = \frac{C_D S \rho_0}{2}\sqrt{(u-\dot{x})^2 + (v-\dot{y})^2}\,(u-\dot{x})$$

$$M\ddot{y} = \frac{C_D S \rho_0}{2}\sqrt{(u-\dot{x})^2 + (v-\dot{y})^2}\,(v-\dot{y})$$

We solved the momentum equation to simulate the motion of the GPS-float. Figures 12 and 13 show the numerical results with $\gamma^2 = 0.05$ and $\gamma^2 = 0.02$, respectively. Those figures show simulated trajectories of the GPS-float together with the actual trajectory.

5 CONCLUSION

The GPS-float has been developed to study flow in the water environment experimentally (Watanabe 1999, 2000 (2), 2002 (1), (2), Watanabe *et al.* 2001, Watanabe *et al.* 2003 (1), (2)). It can also be studied numerically by analyzing the equation governing its dynamics (Watanabe 1999, 2000 (1), (2), 2002 (1), (2), Watanabe, et al. 2001, Watanabe *et al.* 2003 (1), (2)). One can simulate the motion of the GPS-float by solving its momentum equation provided the fluid resistance is specified, and, the fluid resistance can be given in terms of the velocity of the flow (Watanabe 1999, 2000 (2), 2002 (1), (2), Watanabe, et al. 2001, Watanabe *et al.* 2003 (1), (2)).

We introduced an experimental result in which a wind-generated current was captured. We presented a numerical result in which the motion of the GPS-float is simulated.

ACKNOWLEDGMENTS

We would like to thank people in the Kojima Bay Central Administration Office, the Section of Land Improvement in the Kojima Bay Area for sharing data concerning the water level of the Kojima Lake and the tide level of the Kojima Bay. We would also like to thank people in the Machining Center, the Faculty of Engineering, Okayama University for building the GPS-float, and those people who joined measurement of the water depth in Kojima Lake, when they were students in the Faculty of Environmental Science and Technology, Okayama University. The information for generating the figures concerning Kojima Lake is partially based on maps ©Shobunsha Publications, Inc., ©Nihon Computer Graphic Co., Ltd.

REFERENCES

Watanabe, M. (1999): A numerical simulation of lake flow and a GPS-float experiment, *The Second International Symposium on Water Environment, Okayama University, Journal of the Faculty of Environmental Science and Technology, Okayama University* (Special Edition), pp. 111-116.

Watanabe, M. (2000) (1): A numerical analysis of unsteady flow in Kojima Lake, *Journal of the Faculty of Environmental Science and Technology, Okayama University* **5**(1), pp. 31-34.

Watanabe, M. (2000) (2): On analysis of unsteady flows in lakes and marshes and its experiments , *Lectures supported by Kogakukai, Science and Engineering Reports of Tohokugakuin University*, **34**(2), pp. 49-53.(in Japanese)

Watanabe, M. and Kunisada, S. (2001): An experimental evaluation of lake flow simulation, *Journal of the Faculty of Environmental Science and Technology, Okayama University* **6**(1), pp. 11-15.

Watanabe, M. (2002) (1): A numerical analysis and an experimental study of unsteady flow in Kojima Lake, *Journal of the Faculty of Environmental Science and Technology, Okayama University* **7**(1), pp. 39-44.

Watanabe, M. (2002) (2): Utilization of the global positioning system for analyses of flows in water environments, The 6TH World Multiconference on Systemics, Cybernetics and Informatics, PROCEEDINGS Volume V, Computer Science I (2002), 172-176.

Watanabe, M. and Numaguchi S., (2003) (1): M. Watanabe and S. Numaguchi, An experimental analysis of unsteady flows generated in Kojima Lake, *Journal of the Faculty of Environmental Science and Technology, Okayama University*, **8**(1), pp. 23-26.

Watanabe, M. and Numaguchi S., (2003) (1): M. Watanabe and S. Numaguchi, An experimental analysis of unsteady flows generated in Kojima Lake, *Journal of the Faculty of Environmental Science and Technology, Okayama University*, **8**(1), pp. 23-26.

Watanabe, M. and Numaguchi S., (2003) (2): Introduction of GPS data into computational analysis of flows in the water environment, *GIS & RS in Hydrology, Water Resources and Environment*, **1**, Chen et al. (eds) (Cyber Proceedings of International Conference of GIS and RS in Hydrology, Water Resources and Environment), 8 pages.