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Recognition of Surface of 3-dimensional Body and Its Input to Computer - Use of Delaunay Triangulation -

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CAD system is useful to define and input any artificial 3-dimensional configuration into computer, but the input of arbitrary natural 3-dimensional configuration is still serious for computer users. This paper includes a method to input arbitrary 3-dimensional configuration into computer. The method is based on 3-dimensioanl Delaunay triangulation which is a geometrical subdivision of arbitrary 3-dimensional convex domain into tetrahedra, and the author shows a method to find triangles which cover whole surface of 3-dimensional body using the result of Delaunay triangulation.

Key Words : Delaunay Triangulation, 3D Configuration, TIN, CAD, Computational Geometry

1. INTRODUCTION

The method to input the configuration of arbitrary 3-dimensional body has become more and more important according to the development of computers, since the ability of computers allow engineers to solve physical problems in real 3-dimensional domain instead of 2-dimensional problem obtained by the introduction of appropriate 2-dimensional assumption.

3D geometrical configuration is usually treated using CAD system and the shape is input into computers, but there still exist a number of 3D geometries which cann't be treated by CAD. Especially, in the field of civil engineering, effective method to input 3D configuration has been required since civil engineers often encounter to treat 3D domains which are surrounded by natural surfaces like ground surface, surfaces of strata, geological faults and so on. One effective method to prepare these natural surfaces is the use of TIN (i.e., Triangulated Irregular Network). The method can divide whole surface into triangles using nodes on it. But, we find that TIN can treat only the surface itself but cann't treat the volume which is surrounded by natural surfaces.

We find several effective tools which can measure 3-dimensional coordinates of points on the surface of 3D body. The technique in the field of surveying offer us the nodal coordinates on the groundsurface from the two photographs,

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CCD and also from the combination of the bathymeter and GPS(Global Positioning System). It should be mentioned that these methods can offer only 3-dimensional node coordinates instead of its surface. Sometimes, we obtain more informations in addition to node coordinates, i.e., the ordering of nodes. For example, civil engineers often use contour data, in which nodes having the same z coordinate are ordered sequentially. The data from CCD are also ordered sequentially along the axes of pictures. Above consideration suggests us to establish a method to generate surface from 3-dimensional coordinates of nodes on the surface.

In this paper the author first describes theoretical relations between 3-dimensional surface and nodal coordinates, second actual method to obtain 3-dimensional surface which surround a volume, and finally show several examples of the application of the proposed method. In the process of the proposal of the method the author also proposes a modification of Delaunay triangulation which can be applied to non-convex 3D domains.

2. RELATIONS BETWEEN NODES, LINES, SURFACES AND VOLUMES

We denote nodes, lines, surfaces, and volumes by P, L, S and V, respectively. Let ∂ be a boundary operator which determines the boundary.

We find following relations between a volume V and its surface S;

$$\partial V = S$$
$$\partial \partial V = \partial S = \phi$$

Above relations explain that the boundary of a volume is a closed surface, and the surface has no boundary, since it is closed.

If we divide S into a set of small surfaces, i.e., $S = \Sigma s(i)$, (i=1,k), and use above relations, we obtain following relations between small surface s(i) and its surounding L;

$$\partial s(i) = L$$
$$\partial \partial s(i) = \partial L = \phi$$

These relations explain that the boundary of a small area on the surface of a body is its closed surounding loop and the loop has no boundary.

If we express the loop L by a set of line segments, i.e., $L = \Sigma l(j)$, (j=1,m), we obtain following relations between line segements and nodes;

$$\partial l(j) = \{p(a), p(b)\}$$

Above expression indicates that the boundary of a line segment is a pair of nodes.

In Introduction we explained that we can generally obtain a set of nodes on the surface of a body, i.e., P on S. From above relations between nodes, lines, surfaces and a volume, we can obtain line segment from a pair of nodes, small surface like a triangle from at least three line segments, the surface of a body from a set of small surufaces(i.e., triangles) whose line segments appear twice. That is, even if we obtain a set of nodes on S, the procedure to obtain S from P follows all relations mentioned above.

3. DELAUNAY TRIANGULATION [1,2]

The consideration in Section 2 clarifies the relation between nodes and surface and asks a method to obtain a surface from nodes. Delaunay triangulation satisfies this requirement.

Delaunay triangulation is a geometric subdivison of the convex hull of the domain defined by given nodes, and the convex hull is divided into tetrahedra. The method certificates an unique subdivision into tetrahedra which are formed using given set of nodes except the case of degeneracy. That is, each tetraheron has its own circumsphere, and no other nodes locate inside each circumsphere. When several tetrahera have the same circumsphere, the subdivision into tetrahedra is not unique and the case is "degeneracy". Then, except the case of degeneracy, how the domain is divided into tetrahedra is determined only by the location of nodes.

The definition of Delaumay triangulation clarifies that unnecessary domain is also filled by tetrahedra when the domain, i.e., the configuration of the object, is not convex. For example, a hole inside of the object and local concave part locating on its surface are also filled by tetrahedra, since convex hull is divided into tetrahedra. If there exists a method to remove unnecessary tetrahedra outside the object, we can not only express the surface of object using triangular surfaces of tetrahedra but also divide whole volume into tetrahedra.

Above discussion is based on the assumption that Delaunay triangulation can generate tetrahedra, whose triangular surfaces can cover whole surface of the object, using nodes on the surface of the object. Then, the problems to be solved are

(1) whether Delaunay triangulation can surely generate necessary etrahedra, whose triangular surfaces can cover the surface of the object, when nodes are appropriately placed on the surface of the object, and

(2) how to find out triangles, which cover whole surface of the object, among all triangles of generated tetrahedra.

The solution of the first problem is necessary to certificate the use of Delaunay triangulation for present aim. Even if necessary tetrahedra are generated, they form a convex hull and unnecessary tetrahedra must be removed to form original domain of the object in case of non-convex cases. Section 4 and 5 treat the first and second problems, respectively.

4. MODIFIED DELAUNAY TRIANGULATION

We assume thath Delaunay triangulation is already applied to divide a convex hull defined by "n" nodes into tetrahedra, and successively we aim to generate new triangle inside the triangulated domain by introducing additional three nodes, namely, a, b and c. For these nodes we assume that the distance between them are relatively smaller than the distance from them to other n nodes which are already used for triangulation.

The first node, namely a, is introduced and Delaunay triangulation is applied for the node. From the characteristics of the method, a set of tetrahedra whose circumspheres include new node form a polyhedron by removing all common triangles of selected tetrahedra. The surface of the polyhedron is of course covered by triangles, and, then, the volume of the polyhedron is once again divided into tetrahedra using the triangles on its surface and new node. These new tetrahedra and the residuals form Delaunay triangulation for (n+1) nodes.

The introduction of the second node, namely b, for Delaunay triangulation can form new polyhedron by selecting a set of tetrahedra whose circunspheres include new node and by removing their common triangles among selected tetrahedra. Since the distance between two nodes, a and b, is sufficiently smaller than the distances between new node, i.e, b, and other n nodes, the tetrahedron which includes the node, b, must be one of tetrahedra which are formed using the latest node, a. Then, the node, i.e., a, must locate on the surface of the polyhedron, and new node, namely b, must locate inside the polyhedron. Then, the subdivision of the polyhedron into tetrahedra using triangles on its surface and new node gives new Delaunay triangulation for (n+2) nodes, and one edge is generated as to connect two nodes, namely a and b. At this stage we find that the volume close to the edge, a-b, is divided into following three kinds of tetrahedra;

Tet(a...) : tetrahedra whose one edge node is "a"

Tet(b...) : tetrahedra whose one edge node is "b"

Tet(ab..) : tetrahedra whose two edge nodes are "a" and "b"

Next stage is the introduction of the third node, c, and the same procedure generates a polyhedron from a set of tetrahedra whose circumspheres include the node, c. Then, we find following three cases;

Case 1 : "c" is included in Tet(a...) or Tet(b...)

Case 2 : "c" is included in Tet(a...) and also in Tet(b...)

Case 3 : "c" is included in Tet(ab..)

Case 1: The same procedure generates new edge connecting a(or b) and c by the subdivision of the polyhedron into tetrahedra. Then, place on the line between c and b(or a) additional node, x, as to be included in the circumsphere of a tetrahedron whose two edge nodes are c and b(or a), and apply Delaunay triangulation. If the node, x, is also included in the circumsphere of a tetrahedron whose one edge node is a(or b), the subdivision can generate all line segments which connect c and a(or b). Then, the triangle formed by three nodes, a, b and c, is generated. In case that additional node, x, is not included in the circumsphere of a tetrahedron whose edge node is a(or b), the procedure of the addition of new node like x is repeated till the node is included in a cicumsphere of a tetrahedron whose edge node is a(or b).

Case 2: The volume between Tet(a...) and Tet(b...) is filled by a set of Tet(ab..), since the edge a-b locates inside the volume. Then, Case 2 means the node, c, is included in three types of tetrahedra, i.e,, Tet(a...), Tet(b...) and Tet(ab..). Then, the subdivision of the polyhedron obtained from selected tetrahedra necessarily generates a triangle which are formed by three nodes, a, b and c. See Note (1).

Case 3: Tet(ab..) and other tetrahedra,including new node, c, form a polyhedron, and its subdivision into tetrahedra necessarily generates a triangle which is formed by three nodes, a, b and c. See Note (1).

Above discussion shows that a triangle can be generated in a triangulated domain by Delaunay triangulation if the location of three nodes is appropriate. Then, the repetition of the same procedure can prepare in the domain any surface which is formed by a set of triangles. See Note (2). It should be noted again that the distance between three nodes which are used to form a triangle must be shorter than the ones between them and other nodes.

Above discussion is based on original Delaunay triangulation, but the method can generate necessary tetrahedra whose triangular surfaces can form the surface of arbitrary configuration. Then, we call it Modified Delaunay Triangulation.

Note (1): The edge to form a triangle, i.e., a-b, b-c and c-a, should locate on the surface of the polyhedron. If the edge locates inside of the polyhedron, the edge is removed at forming the polyhedron. It means that the location of new node is very important and it should not be too close to the edge, for example a-b. That is, if new node is too close to edge a-b, all Tet(ab..) which can cover all volume adjacent to the edge a-b are included to form the polyhedron. And, as a result the edge, a-b, is included in the volume of the polyhedron, and it is removed at the subdivision of the polyhedron into new tetrahedra.

Note (2): At the successive generation of triangles on a surface, we can distinguish them from other triangles. Then, at the selection of tetrahedra whose circumspheres include new node, namely x, we can remove tetrahedra which locate other side of a triangle which is listed up. That is, only tetrahedra which locate in the same side where new node, x, exists can be selected, and we can form a polyhedron using these tetrahedra. Then, the subdivision of the polyhedron never break the triangles whice are generated to form a surface.

5. DETERMINATION OF SURFACE OF 3D DOMAIN

The discussion in Section 4 clarifies that the convex hull defined by nodes on the surface of arbitrary 3D body can be divided into tetrahedra. At the same time unnecessary volume is also fulfilled by tetrahedra in case of non-convex volume. Then, how to select tetrahedra which fulfill the volume of the object is necessary. The aim of this section is to propose the method. In the following discussion we assume that the surface of the object can be expressed by triangular surfaces of generated tetrahedra.

Nodes used for the triangulation locate on the surface of the object, and, then, each tetrahedron is formed using nodes only on the surface. The examination of triangles of generated tetrahedra can select all triangles appearing on the surface of the convex hull, since they appear only once. It suggests that these triangles show the boundary of the object in cose of object object but some of them do not form actual boundary in case of non-convex cases. That is, in latter cases, tetrahedra which fill non-convex part must be removed. In this section we propose two effective methods to find all triagnles which cover the boundary of arbitrary 3D domain.

METHOD 1 [3]

Assume that the surface the object is smooth and sufficient number of nodes placed on it are used for Delaunay triangulation. After the application of Delaunay triangulation, we select a triangle, which is denoted by L, on the surface and aim to find out a triangle which is adjacent to L and locates on the surface. Fig.1 shows the triangle, L, on the surface, and new triangle, namely R, which is adjacent to L by a common edge.

In general there exist several triangles which are adjacent to the edge of the triangle, L. Since all of them are candidates of the triangle on the surface, appropriate criterion is required to select one triangle which covers the surface. The characteristics of Delaunay triangulation and the assuptions of the location of nodes suggest us that the triangle on the surface is formed using a node whose distance from the edge is relatively short and the intersection angles between two triangles, L and R, which have common edge is also relatively small, since Delaunay triangulation generally forms tetrahedra using neighboring four nodes. Then, we give following criterion for the selection of another node, namely p, to form a triangle R, which has a common edge with the triangle,L:

$$\min [\ \alpha \sin(\theta) + \beta \sin(\phi)\ ]$$

In above expression θ and ϕ are two angles, where the former shows the intersection angle between two triangles and the latter indicates the angle at the node, p, of new triangle. Obviously, these two angles subject to following restriction.

$$-\pi < \theta < \pi$$

$$0 < \phi < \pi$$

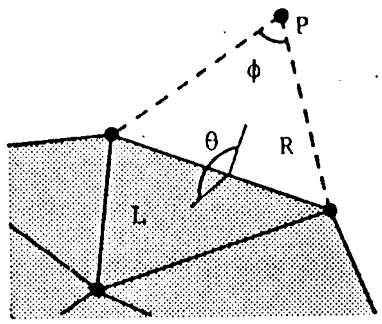


Fig. 1 Selection of New Triangle

α and β are the parameters ($0 < \alpha, \beta < 1$) for each term. If the surface is smooth, the former should have relatively larger value compared to the latter. The values of these parameters depends on the characteristics of the surface and they should be decided by the user.

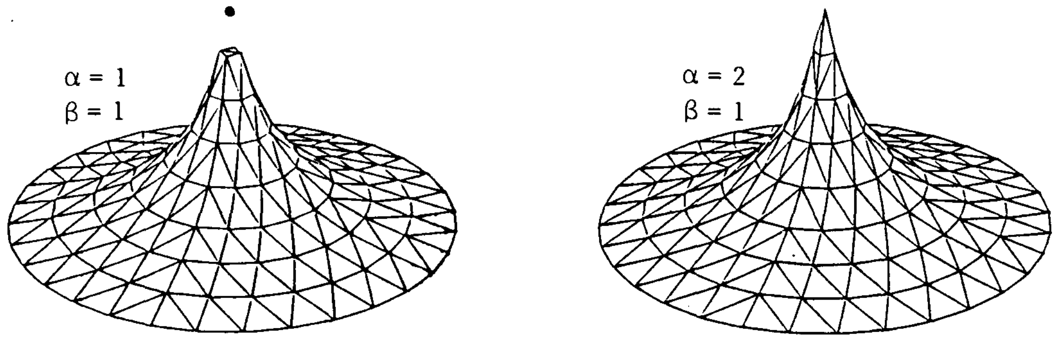
Using above judgement, triangles are successively selected from the initial triangle untill selected triangles can cover whole surface of the object. Whether selected triangles can cover whole surface is examined using following judgement.

If any edge of all triangles appears twice, the set of triangles covers a volume. Otherwise they have hole(s). Even if any edge appears twice, the fact never prove that they cover whole surface of the object. Then, the reader must examine whether the set of selected triangles actually cover the whole surface of the object.

Examples of Method 1

Fig.2 shows a test problem whether proposed method can select appropriate triangles which can cover the surface of the object. Fig.2-1 and 2-2 show the results of selected triangles for $\alpha = \beta = 1$ and $\alpha = 2$, $\beta = 1$, respectively. Comparing them, we can remark the former fails to find out all triangles but the latter can find all triangles. Then, in this case the smoothness is more important than the distance between nodes.

Fig.3 shows another result of the application of the proposed method. In this case the parameters are set as $\alpha = \beta = 1$, and the method could find out all triangles which cover the face of human being.



2-1 Example of Failure 2-2 Example of Success

Fig.2 Application of Method 1

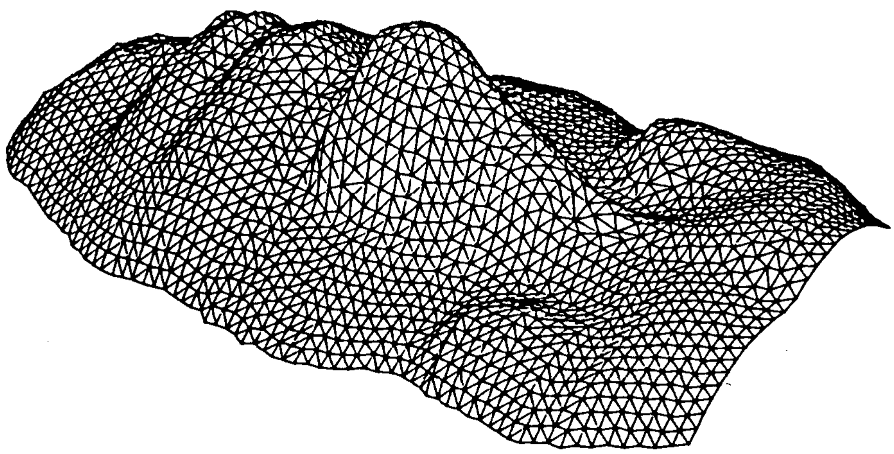


Fig.3 Application of Method 1

METHOD 2

In Method 2 we give additional nodes inside the volume of the object in addition to nodes on the surface and apply Delaunay triangulation for all nodes. Then, the triangulation generates tetrahedra which are formed by (1) nodes only on surface, (2) nodes on surface and inside, and (3) nodes inside the volume. It is obvious tht tetrahedra in (1) and (2) surely locate inside the volume, and tetrahedra in (1) can be judged outside the volume if the location of additional nodes is appropriate.

We assume that the reader of this paper knows how the surface of the object is and where nodes locate, and, therefore, the reader can divide the surface into several parts, if necessary. We denote the node- coordinates by {x(i),y(i),z(i)}, (i=1,n).

The additional nodes, which are placed inside the volume, are generated by copying the coordinates of the original data set and their partial modification (i.e., translation and/or rotation of coordinate system). That is, if the coordinates

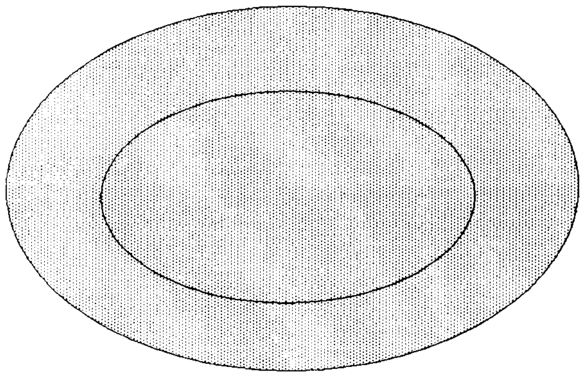


Fig.4 Location of New Nodes for Simple Boundary Geometry

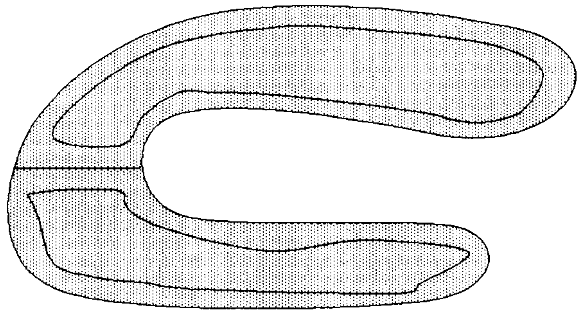


Fig.5 Location of New Nodes for Complex Boundary Geometry

of original nodes are modified as to be placed inside the original volume and these nodes are used for Delaunay triangulation to generate tetrahedra with original nodes, we can easily judge each tetrahedron whether it locates inside or outside of the volume.

How to modify the coordinates of nodes to create additional nodes dependes on the configuration of the object. In some cases they can be created as a whole, and in other cases they can be prepared one by one after dividing the surface into pieces. Fig.4 shows the generation of additional nodes as a whole, and Fig.5 shows the creation of additional nodes after the subdivision of whole domain into several subdomains. In these two figures the author illustrates the objects in 2-dimension but actually in 3-dimension. Fig.4 shows the line inside the domain indicates the location of additional nodes. In Fig.5 the object is at first divided into simple geometries and secondly additional nodes are placed on the lines inside each subdomain. The latter is used in case of very complicated 3D configuration. The most important point of the creation of additional nodes is that they must be appropriately located as to generate tetrahedra, which are formed using additional and also original nodes, that fill all volume adjacent to the surface of the object. One example of the generation of additional nodes is explained.

Example of Generation of Additional Nodes

Assume a set of contour data of topography and we aim to generate its surface using these nodes. Pick up the data set with "n" nodes, and replace only their z(i) values by z(i)-d, where the value "d" is appropriately decided by the reader. Generally speaking, the value "d" is less than the difference of height between adjacent two countour lines. The procedure can prepare 2*n nodes. Then, we apply Delaunay triangulation for them and obtain tetrahedra. Then, generated tetrahedra can be classified into three categories:

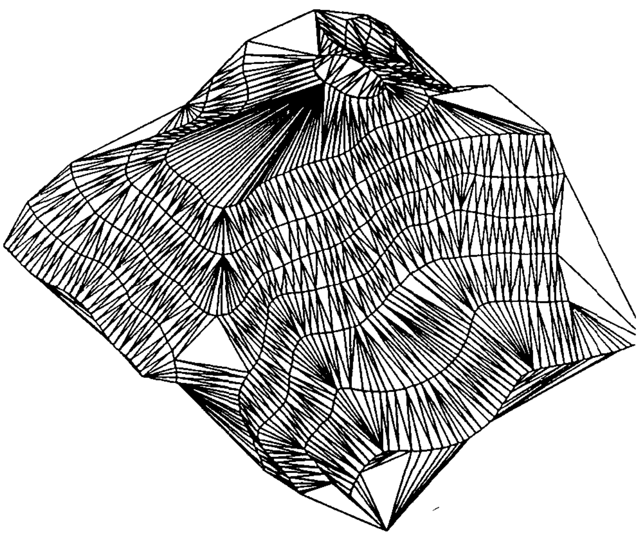
Type 1 : tetrahedra formed by original nodes

Type 2 : tetrahedra formed by original and additional nodes

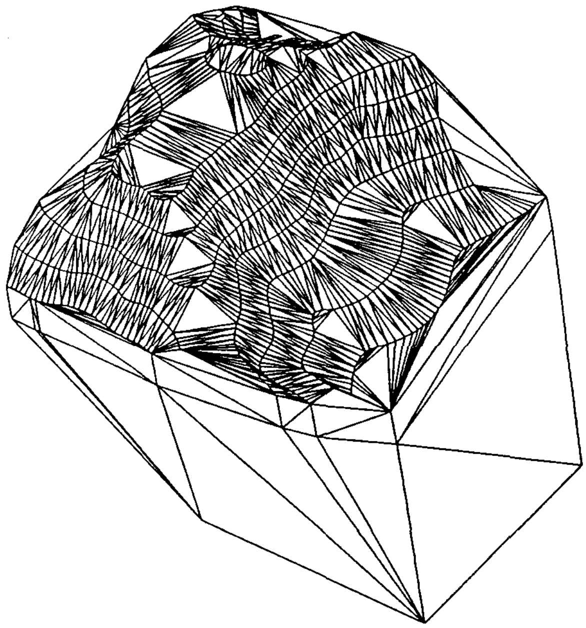
Type 3 : tetrahedra formed by additional nodes

Type 1 must locate outside the surface of the body, and Type 3 locates inside the volume. Type 2 locates between Type 1 and Type 2, and their triangular surfaces formed by only original nodes must cover the surface of the body.

One example of this application is shown in Fig.6. Fig.6-1 shows the result of the application of TIN for nodes of contour lines, and Fig.6-2 shows the application of the proposed method. Comparing these two figures we find the proposed method can work sufficiently to find out all triangles which cover the ground surface. 1 The reader can find out the difference of triangulation between adjacent contours, and the difference comes from that the triangulation of TIN has a function to remove so called paddy field after the original triangulation using nodes on contour lines. The removal of the paddy field is required for the computer graphics of the ground surface, and the original triangulation by TIN is just same as the one in Fig.6-2.



6-1 Surface by TIN



6-2 Surface by Method 2

Fig.6 Application of Method 2

6. CONCLUDING REMARKS

The aims of this investigation were to propose how to recognize the surface of arbitrary 3-dimensional domain and also how to imput it in computer. To achieve these purposed the author theoretically investigated, proposed Modified Delaunay Triangulation which is applicable to non-convex domain and finally proposed two methods which can search all triangles that cover the surface of the object.

The use of Delaunay triangulation necessarily requires the nodes on the surface of the object, and the theoretical investigation in this paper could show that Delaunay triangulation can generate necessary triangles ,which can cover whole surface, using the triangular surfaces of tetrahedra which are generated as the result of the application of Delaunay triangulation, if and only if the location of nodes on the surface is appropriate. It indicates that the tetrahedra generated by Delaunay triangulation cann't express necessary triangles if the location of nodes is unappropriate.

Nodes which we can generaly obtained as input data are only the ones on the surface of object, and additional information is necessary to find out all triangles which cover the surface. The author could find out additional information which originally comes from the characteristic of the geometry of the surface, and a method was proposed using this information. For the cases which have no additional information based on the geometry of the surface, the author could propose a general method which generate new nodes as additional information and also a searching method of triangles which cover whole surface of the object.

The methods shown in this investigation are applicable not only as methods to input 3-dimensional configuration into computers but also as methods of preprocessing for the finite element methods in engineering fields. But, when the reader wishes to use the methods as latter purpose, a method is required to place additional nodes indide the triangulated domain, since generated tetrahedra are still too rough and can give poor computational results. Then, finer meshing of rough triangulation is necessary to obtain reliable computational results. Actual meshing can be done using Modified Delaunay triangulation which was proposed in this investigation.

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