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Parameter Estimation of Unsaturated Soil Hydraulic Properties by Improved Suction Plate Method

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The unsaturated soil hydraulic properties are essential data to predict the seepage behavior in the vadose zone. In this paper, a new experimental methodology of determining unsaturated soil hydraulic properties is proposed. The soil hydraulic properties are assumed to be represented by van Genuchten's closed-form expressions. Unknown parameters of this model are identified by using a optimization techniques. The optimization approach is nonlinear least-squares algorithm in corporating finite element analysis of one-dimensional nonsteady seepage flow. The advantages of the methods are in the possibility of identifying the optimal unsaturated soil hydraulic properties and diminishing experimental time. To evaluate availability of our proposed method, experimental results which are determined by proposed methods and conventional method are compared for decomposed granite soil.

Keywords: *unsaturated hydraulic properties, back analysis, laboratory test, groundwater*

1. INTRODUCTION

Knowledge of the unsaturated soil hydraulic properties is essential requirement for prediction of seepage flow and contaminant transport through the vadose zone. The unsaturated soil hydraulic properties consist of the hydraulic conductivity as a function of pressure head and the soil water retention curve. Usually, the steady state experiment have been performed to measure the soil water retention curve in the laboratory. In these tests, the suction plate method is one-dimensional, vertical equilibrium desorption experiments. This method is widely used because its procedure and apparatus are simple. There are, however, some disadvantages in this method as, (1) time-consuming data collection, (2) difficult to determine the equilibrium state, and (3) difficult to calculate the hydraulic conductivity. We are, therefore, apt to terminate measurements before experiments have reached to the equilibrium state and obtain an mistaken soil water retention information.

Transient experimental methods are inherently faster, and the estimation of soil hydraulic properties using back analysis has become more attractive because more powerful personal computers are available now. In this paper, a new suction plate method is investigated. With this method a soil sample is placed in a cell with a porous ceramic plate at lower end. After the sample is saturated with water, air pressure is applied at its upper boundary. The resultant change of water content in the sample is measured as a function of time.

The numerical feasibility of estimating soil hydraulic properties simultaneously from the suction plate method by a parameter estimation method is evaluated. A one-dimensional nonsteady finite element code simulates the water movement in the saturated-unsaturated soil. The unsaturated soil hydraulic properties are assumed to be represented by

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van Genuchten's closed-form expressions involving five unknown parameters. Unknown three parameters of this model are identified by using two different optimization techniques. This optimization approach is nonlinear least-squares algorithm in corporating finite element analysis of one-dimensional nonsteady seepage flow. To evaluate availability of our proposed method, experimental results which are determined by proposed method and conventional method are compared for decomposed granite soil.

2. IMPROVED SUCTION PLATE METHOD

Schematic of our proposed suction plate method apparatus is shown in Fig.1. Water content data are measured with time by electronic balance (weighing rage 6100g, readability 0.01g) which are installed under the cell directly. The material used for sample is granite soil with grain size ranging from 0.01 mm to 20 mm. The material of 110-mm diameter, and 40-mm length was carefully compacted into the acrylic cell to provide a particular dry density , ρd =1.80 g/cm3 , as homogeneous as possible. A ceramic plate which air-entry value is -2.7 kgf/cm2 was located at the lower end.

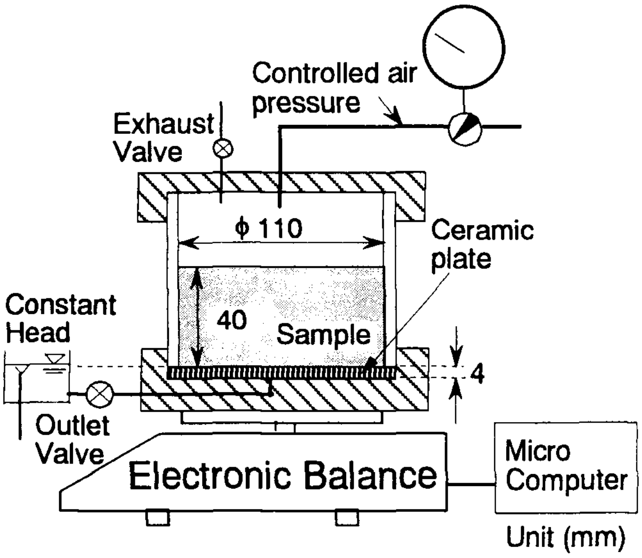

Figure 1 : Schematic of the improved suction plate apparatus

After the sample is saturated with water, air pressure is applied at its upper boundary. The suction plate method was performed for a pneumatic head increment of 100 cm at the top of the sample. The reservoir is located at the same level to the upper edge of the ceramic plate in the cell. At the time point t = 0, the outlet valve was switched over to drainage and at this initial time point the experiments was driven by the gradient between the water-level of the reservoir and applied pneumatic head increment of the sample.

During the experiments the signal of the balance was recorded with time by personal computer and total weight data of sample and cell is converted to water content in sample. The parameter saturated permeability ks and saturated water content θs were independently measured, ks was determined by means of falling head permeability test to 1.14 x 10-4 cm/s , and θs is calculated by dry density to 0.35. The experiments were continued for about 3 days.

3. METHOD

3.1 Formulation of the direct problem

The governing equation for unsaturated one-dimensional, vertical transient flow through the rigid porous medium is described by Richards equation.

$$C(\psi)\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z}\left[k(\psi)\left(\frac{\partial \psi}{\partial z}+1\right)\right] \qquad (1)$$

where ψ is pressure head, C is the soil water capacity, being the slope of the soil water retention curve (= $d\ \theta\ /\ d\ \psi$,

θ is the volumetric water content), k is the hydraulic conductivity, t is the time, and z is the vertical distance taking positive upwards. The appropriate initial and boundary conditions for our proposed experiment are

$$\psi = \psi_0(z) + u_b \quad t = 0 \;, \quad 0 \leqq z \leqq L \qquad (2a)$$

$$\psi = L_c \qquad t > 0 \;, \quad z = 0 \qquad (2b)$$

$$q = 0 \qquad t > 0 \;, \quad z = L \qquad (2c)$$

where $z=0$ is taken at the bottom of the soil sample, $z=L$ is the top of the soil sample, $\psi_0(z)$ is the initial hydrostatic pressure head distribution, q is the outflow or inflow at the top of soil sample, u_b is the applied pneumatic pressure head, and L_c is the thickness of ceramic plate at the bottom of the soil sample lower end. The solution of Eq.(1) and (2) obtained by finite element analysis of nonsteady unsaturated seepage flow.

$$Se = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left[\frac{1}{1+(\alpha\,\psi)^n}\right]^m \qquad (3)$$

$$k(\psi) = ks\, Se^{0.5}\left[1 - (1 - Se^{1/m})^m\right]^2 \qquad (4)$$

$$C(\psi) = \alpha\,(n-1)(\theta_s - \theta_r) Se^{1/m} (1 - Se^{1/m})^m \qquad (5)$$

3.2 Parametric model for the unsaturated soil hydraulic properties

The unsaturated soil hydraulic properties are strongly nonlinear functions of the pressure head. It is assumed that suitable analytical expressions for these functions are available. We assume soil hydraulic properties are described by van Genuchten's closed-form expressions (van Genuchten,1980). These will be referred to hereinafter at the VG model. where $m=1-1/n$, Se is the effective saturation, θs is the saturated water content, θr is the residual water content, ks is the saturated conductivity, and α , n are the soil retention curve shape parameters (empirical parameters). Expressions for $k(\psi)$ and $C(\psi)$ follow from Eq.(3) through Eq.(5). Of the five parameters ks, θs, θr, α , and n in the VG model, the first two have clear physical significance and are independently measured from laboratory tests. The residual water content is defined nominally as the water content at which $k \to 0$ and $\psi \to -\infty$. Literally, it is considered that the residual water content for sandy soils is equal to 0.0.

Respectively, the parameters α and n are inversely related to the air-entry value and width of the pore distribution. From our own data, α generally ranges from 0.02 to 0.1 [1/cm], while n (dimensionless) usually varies from 3 to 10 for sandy soils. In the parameter estimation problem we assume that ks and θs have been measured independently .Values of θr, α and n are sought by the numerical inversion of one-dimensional unsaturated flow problem.

3.3 Parameter estimation procedure

A set of water content measurements W at specific times t_i ($i=1,2,...,N$) are obtained from the suction plate method results. These $W(t_i)$ are employed as input data for the numerical inversion problem. This is formulated as a nonlinear

$$R(b) = \sum_{i=1}^{N}\left[w_i\left(W_m(t_i) - W_c(t_i, b)\right)\right]^2 \qquad (6)$$

optimization problem. Unknown parameters are estimated by minimizing a suitable objective function. In this case measurements is water content values only, the objective function can be written as an ordinary least-square problem.

where b is trial vector of unknown parameter values, the w_i is a weighting function. We consider that w_i takes 1.0 here. W_m is the measured and W_c is the numerically calculated water content. N is the length of the data vector.

To determine $[b]$ we use an optimization algorithm based on the Gauss-Newton method. We wish to investigate the adequacy of water content observed at times $t_1, \ldots, t_N$ to define unique solutions to the inverse problem. The convergence criteria are given by the mean square error and relative mean deviation between measured and computed water content in the sample.

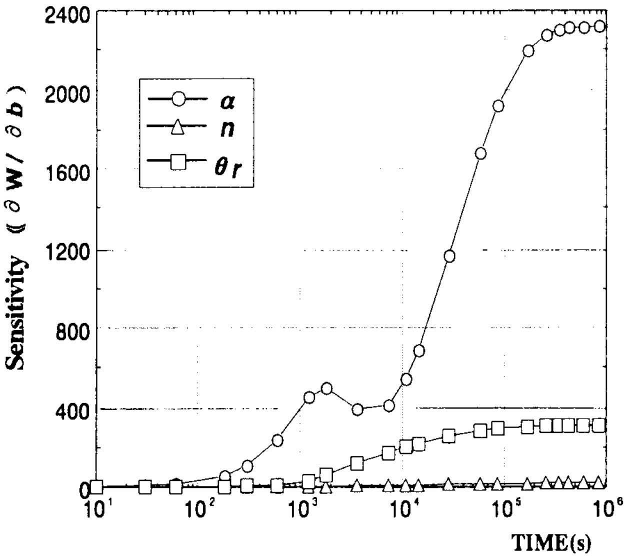

Figure 2 : Sensitivity analysis of the 10kPa experiment

Sensitivity matrix is required to solve Eq.(6). The elements of the sensitivity matrix are the partial derivatives of $W_c(t_i, b)$ with respect to the parameter b. In the experiments proposed here, the sensitivity matrix is approximated simultaneously with the water content Wc, determining the derivatives of Wc with respect to θr, α and n. Figure 2 illustrates the sensitivity for the pneumatic head increment of 10 kPa experiment. This figure shows that sensitivities of water content are found, but the sensitivity to parameter n is low compared to parameter θr, α.

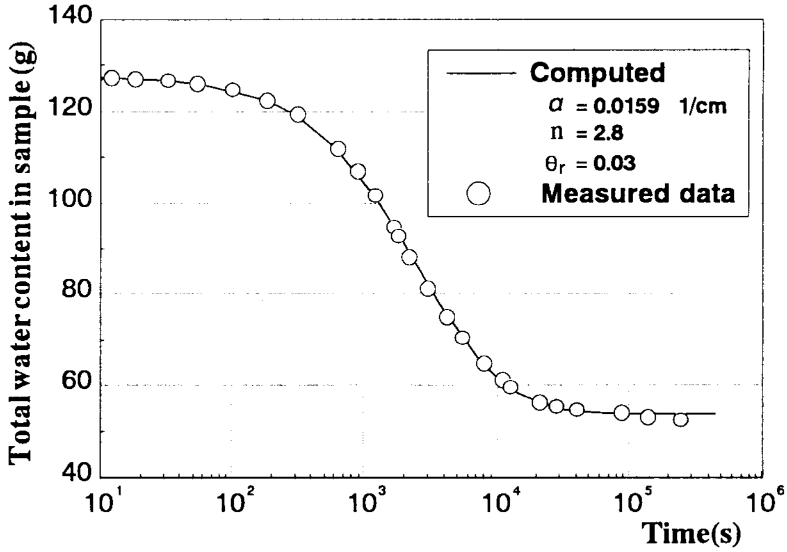

Figure 3 : Measured and estimated water content with time

4. RESULTS AND DISCUSSION

Our proposed procedure was carried out to the suction plate method for Granite soil. Measured water content with time is depicted in Figure 3. From these results, experiment has reached to the equilibrium state for about 2 days.

We employ transient data for the first 24 hours of water content data to perform the Gauss-Newton parameter estimation procedure and the neural network approach. Soil water retention curve which were predicted by VG model was compared with observed data the experiment (Figure 4). As seen in this figure, a reasonable correspondence is obtained between predicted data and measured one.

Figure 3 also shows the comparison of the measured water content data with the data calculated using back-analyzed parameters of unsaturated hydraulic model. The results of the Gauss-Newton was in beautiful agreement with the measured data.

5. CONCLUSIONS

An analysis of the inverse problem have been presented for the determination of unsaturated soil hydraulic properties by the suction plate method. The conclusions obtained in this paper are as follows:

(1) A new suction plate method has been investigated. With this method a soil sample is placed in a cell with a porous ceramic plate at lower end. The constant air pressure is applied at its upper boundary. The resultant change of water content in the sample is measured as a function of time by electronic balance which are installed under the cell directly. This procedure was applied to granite soil.

(2) Soil hydraulic properties are assumed to be represented by van Genuchten's closed-form expressions. When these functions are determined, the cumbersome handling of unsaturated soil hydraulic properties data in models of unsaturated flow can be avoided.

The optimization techniques of estimating unsaturated soil hydraulic properties form the suction plate method is proposed. This optimization approach is nonlinear least-squares algorithm. The advantages of the proposed method are in the possibility of identifying the optimal unsaturated soil hydraulic properties and diminishing experimental time.

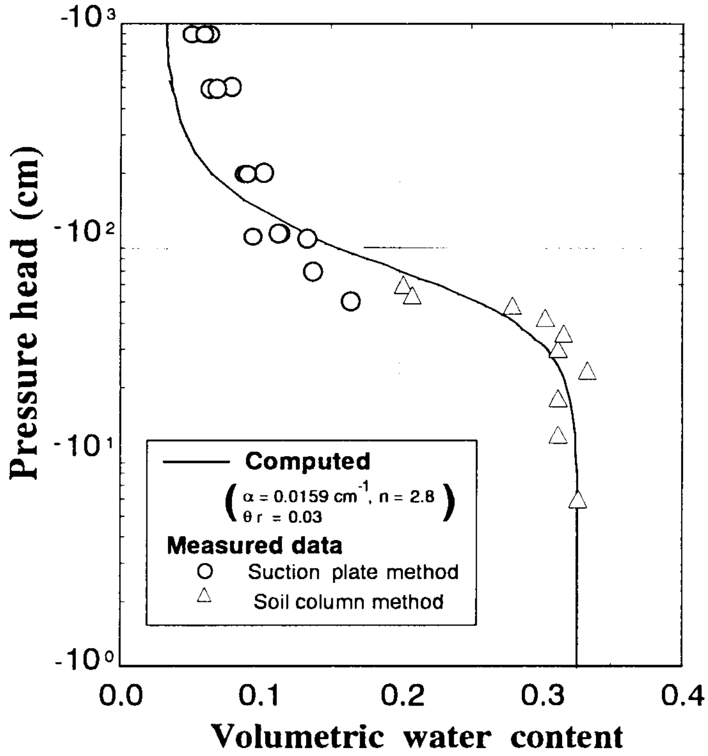

Figure 4 : Measured and estimated soil water retention curve

(3) When solving the inverse problem for the three unknown parameters of VG model simultaneously, the estimated results of the Gauss-Newton was in beautiful agreement with the measured data. These unknown parameters allow great flexibility in the shape of the hydraulic functions. Therefore when these functions are determined, the cumbersome handling of unsaturated soil hydraulic properties data in model of unsaturated flow can be avoided.

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