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A Statistical Classification on a Mixture Distribution of Intelligence Quotients and Severe Mental Retardation

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Main content of this paper is to classify IQ individuals into two categories of normal and abnormal groups. It is too difficult to divide IQ individuals into two groups of normal IQ group and abnormal group because of sparse number of cases with mental retardation. Therefore, we examined a normality of 1673 IQ individuals, but a significant difference was noted for the IQ data. The lowest three mentally retarded cases of less than or equal to 59 IQ score were excluded, the IQ data then fitted to a normal distribution well. The critical value which minimizes the probability of classification is obtained on the basis on an approximate technique with regard to normality. An approximate probability of misclassification for individuals at random from mixture of two normal populations is 25.5%.

Key words: Compound distribution, normality, Prenatal exposure, IQ, misclassification

1 INTRODUCTION

Consider *k* independent populations, classified according to some factor or treatment whose categories or levels define the different populations. The problem of classification may be considered as problem of "statistical decision function." The issue arises when an investigator wishes to classify an individual into one of several groups on the basis of observations. In this case, it can be assumed that an individual is drawn from a mixed population consisting of the two groups of normal and abnormal individuals in the populations of α and $(1-\alpha)$ respectively, where $0 < \alpha < 1$. If ϕ_1 is the chance of misclassifying an individual into the first abnormal group, and ϕ_2 the corresponding chance for the second normal group, then the probability of misclassification for an individual chosen at random is given by

$$\alpha\phi_1 + (1-\alpha)\phi_2 = \alpha\int_{R_1} f_1(Z)dZ + (1-\alpha)\int_{R_2} f_2(Z)dZ$$

where $f_t(Z)$ is the density of population $\Pi_t(\theta_t, \sigma_t)$ and R_t is a region for misclassifying an observation from Π_t for $t = 1, 2$. The objective is to minimize the probability of misclassification in the case of two normal distributions with five known parameters of $\alpha, \theta_1, \sigma_1, \theta_2$, and σ_2. In this case, it can be assumed that an individual is drawn from a mixed population consisting of the two groups of normal and abnormal individuals in proportions of α and $(1-\alpha)$ respectively, where $0 < \alpha < 1$. Otake in 1972 has classified sum scores into two normal distributions, based on a weighted sum of observations of score in the minute vessels of the fingernail fold, either as normal or abnormal using the distribution of two compound normal curves. He divided the weighted score values into a mixture classification between normal and abnormal findings.

The purpose of this study is to classify an individual into one of two groups of normal and abnormal

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populations, assuming a mixture of IQ observations based on IQ values of mentally retarded cases and normal cases.

2 STUDY MATERIALS

2.1 Mental retardation and intelligence test score samples

The clinical sample used here consists of 1565 prenatally exposed individuals born between 6 (Hiroshima) or 9 (Nagasaki) August 1945 and 31 May 1946. Thirty cases of severe mental retardation have been recognized among these 1565 survivors, all were diagnosed before the age of 17 at the ABCC clinical facility (Otake *et al.* 1987, 1996). The diagnoses of severe mental retardation were based on clinical findings, not on IQ. A child was diagnosed as severely mentally retarded if he or she was "unable to perform simple calculations, to make simple conversation, to care for himself or herself, or if he or she was completely unmanageable or had been institutionalized" (Wood *et al.* 1967). Otake et al. in 1996 shows the distribution of the 1565 individuals including the 30 mentally retarded cases by gestational weeks postovulation and DS86 uterine dose. Almost all of the mentally retarded cases occur among the individuals exposed in the 8th through the 25th week postovulation.

The routine intelligence testing was conducted in 1955-56 in clinical facilities at Atomic Bomb Casualty Commission (ABCC) in Hiroshima and Nagasaki. Although two intelligence tests, the Tanaka-B test and the Koga, were in vogue when these children were tested, both tests were routinely used only in Nagasaki, and not in Hiroshima where the Koga test alone was employed. Accordingly, the prenatally exposed population considered here involves 1673 children. Our analysis focuses on the Koga test, used on 1673 survivors (Schull and Otake 1986, Schull et al. 1988).

Table 1 gives distribution of 1673 children and 12 mentally retarded cases by IQ scores and gestational weeks. Of 12 of 30 mentally retarded cases were involved in this IQ study. Fig. 1 shows the distribution of the 1661 normal individuals and remaining 12 mentally retarded cases by gestational weeks after ovulation and DS86 uterine dose. These 12 mentally retarded cases were listed by sex and head size in Table 2.

2.2 Gestational age of pregnancy

The most important single factor in determining the nature of the insult to the developing cranium and brain from exposure to ionizing radiation is developmental age expressed either in postovulatory weeks. Days of pregnancy are based upon the inferred first day of the last menstrual period, and have been calculated as follows:

Days of pregnancy = 280 - (days between 6 or 9 August 1945 and the date of birth),

where the mean duration of pregnancy is taken to be 280 days. The dates of birth are based on the dates obtained in interviews with the subjects or their mothers and not on the birth reports found in the household registers (koseki). To obtain postovulatory age (weeks after ovulation) (G), 14 days were subtracted from the 'days of pregnancy age at time of bombings (ATB), (Y), and in days was converted to age in weeks by dividing by seven, $G = (Y - 14)/7$, G was presumed to be zero if it was negative.

2.3 Development of the brain

Different functions of the human brain are localized in different structures and the differentiation of these takes place at different stages of development and over different periods of time. The embryonic stage is generally considered to be the period from fertilization through the first 56 days thereafter, i.e., up to and including the 8th week after ovulation. The fetal stage is the period after the 8th week. Most human organs have completed their initial development by the 8th week postovulation but histogenesis may continue much longer. This is particularly true of the brain where development progresses rapidly in the 8-15 week period after fertilization. To reflect the known phases in the normal development of the brain gestational ages in weeks are often grouped. Four age categories, measured from the presumed moment of fertilization, have been used: 0-7, 8-15, 16-25, and 26 weeks or more. These correspond to the following developmental events. In the first period, the precursors of the neurons and neuroglia, the two principal types of cells that make up the cerebrum, have emerged and are mitotically active. In the second period, a rapid increase in the number of neurons occurs; they migrate to the cerebral cortex and lose their capacity to divide, becoming perennial cells (Rakic 1975). In the third period, differentiation in situ accelerates, synaptogenesis that began about the 8th week after fertilization increases, and the definitive cytoarchitecture of the brain unfolds. The fourth period is largely one of continued architectural and cellular differentiation and synaptogenesis; however, in this period a more rapid growth of the cerebellum occurs.

Table 1 Distribution of IQ scores with 12 mentally retarded cases by gestational age group.

Gestational age (weeks)	Item	Total	IQ score values				
			<70	70-89	90-109	110-129	≥130
	Subjects	269	4	37	114	97	17
0-7	Mean	106.1	67.8	83.4	100.9	117.3	134.7
	S.D.	15.03	1.89	4.29	5.39	4.60	4.74
	Subjects	350 (6)	8 (4)	41 (2)	147	112	42
8-15	Mean	107.7	61.6	83.4	101.6	117.6	136.0
	S.D.	17.22	4.98	5.37	5.37	5.37	5.80
	Subjects	480 (4)	8 (3)	40 (1)	194	181	57
16-25	Mean	109.2	61.9	84.5	100.5	117.3	136.7
	S.D.	16.21	5.57	4.18	5.78	5.13	5.36
	Subjects	574 (2)	4 (1)	59 (1)	268	194	49
≥26	Mean	107.3	61.5	82.6	100.6	117.1	138.7
	S.D.	15.67	2.38	5.60	5.63	4.71	7.47
	Subjects	1673 (12)	24 (8)	177 (4)	723	584	165
Total	Mean	107.7	62.7	83.2	100.8	117.3	136.9
	S.D.	16.08	4.85	5.01	5.59	4.95	6.20

Note: The figures in the parenthesis are mentally retarded cases, and SD denotes standard deviation.

3. STATISTICAL CONSIDERATIONS

From the frequency distribution of the IQ scores $Z_1, Z_2, \ldots, Z_n$, it is assumed that these scores are random variables drawn from mixture of two normal distributions with density. First we considered to be designated as compound or mixed distribution based on a mixture of two populations with different two means and two variances, $\theta_1, \sigma_1, \theta_2, \sigma_2$ and α of the compound normal distribution with density (Cohen, 1967). However, only 12 mentally retarded cases were included in the IQ population (Table 1). As shown the IQ distribution with a mixture of mentally retarded cases and other normal cases, it seems to be impossible to classify the IQ variations into two groups.

Accordingly, we attempted the classification of two groups by a simple technique. We examine, first, a normal distribution of entire IQ data. If a distribution is not normal, the case who have the lowest IQ value with mental retardation is excluded from a entire IQ data. We sequentially examine the normality. We repeat such a procedure step by step from cases with the lowest IQ value. It is also possible to do formal tests for normality. We can see if the null hypothesis of a normal distribution is rejected. A commonly used procedure is the Shapiro–Wilk W Statistic (1965). A SAS program gives the value of W and the associated p values for testing that the IQ data came from a normal distribution. Furthermore, we show two statistics of skewness and kurtosis of IQ values are calculated. The W statistics of Shapiro–Wilk procedure is the ratio of the best estimator of variance (based on the square of a linear combination of the order statistics) to the usual corrected sum of squares estimator of the variance. W value must be greater than zero and less than or equal to one, with small values of W leading to rejection of the null hypothesis.

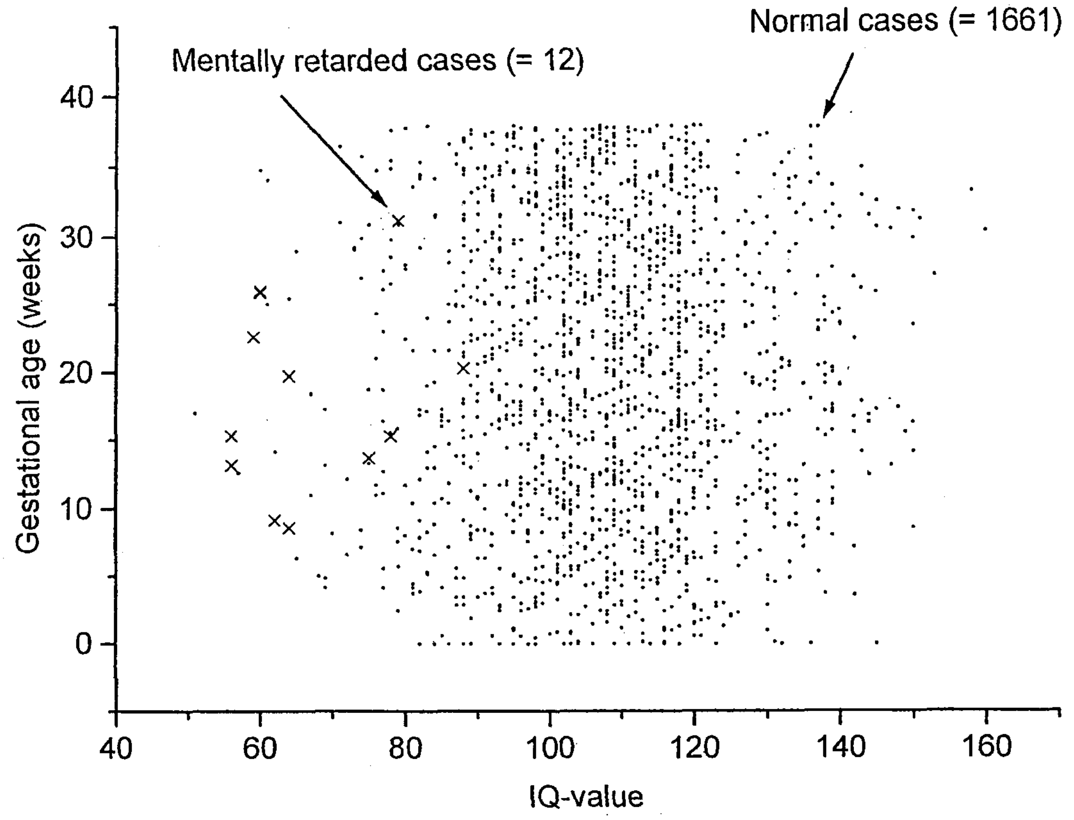

Fig. 1 Distribution of 1661 normal and 12 mentally retarded cases.
Note that two cases looks to overlap with the same 60 IQ value in 25 or 26 gestational week.

4 RESULTS

The distribution of IQ values of 1673 individuals including 12 mentally retarded cases is shown by gestational weeks postovulation in Table 3. Table 3 gives W test values and significant levels of normality based on Shapiro–Wilk statistics by gestational weeks for all individuals with and without 12 mentally retarded cases. As is evident from the table, all individuals with 12 mentally retarded cases and 350 individuals with 2 mentally retarded cases for 8–15 gestational age group are significantly different from normal. All of others are well fitted with a normal distribution. When we excluded the mentally retarded cases, however, data fit a normal distribution well. As is evident from Table 2, small heads are 5 of 12 mentally retarded cases. Their heads are 51.4 cm at 15 years of age and 51.0 cm at 18 or 19 years of age, and they were exposed to high dose of more than 0.9 Gy. The IQ values of 12 mentally retarded cases are all lower than or equal to 88 IQ value (Table 2). However, small heads are evidently different from an occurrence of mental retardation. The normality of an entire data of 1673 individuals was examined when included 12 mentally retarded cases, the fitness to IQ data produces a poor normality with p = 0.039, but when excluded 12 mentally retarded cases from number of IQ data, the distribution gives a good fit for normality with p = 0.215. It is evident that the 12 mentally retarded cases produce a great contribution to the normality. Furthermore, the normality of W-test was examined after we exclude two of the lowest IQ value of 56, we note a suggestive relationship with p = 0.087. The 1670 individuals after excluded the three cases of less than or equal to IQ value of 59 with mental retardation show a normal distribution with p = 0.114 (Fig. 2). As the same situation, the normality of 350 IQ individuals for 8–15 gestational age group was significantly evaluated (p = 0.044), the results gave a good normality with p = 0.175 by excluding two mentally retarded cases to be same as those of whole individuals (Fig. 3).

Table 2 List of IQ values of 12 individuals with severe mental retardation

No.	IQ value	Sex	Head size	Gestational age (Weeks)	SH	DS86 dose in gray (Gy)
1	56	Female	56.6 (18 years old)	15	0	NIC
2	56	Female	49.6 (19 years old)	13	1	1.642
3	59	Male	51.0 (19 years old)	22	1	1.086
4	60	Female	52.7 (17 years old)	25	0	1.770
5	60	Male	55.4 (19 years old)	26	0	NIC
6	62	Male	51.4 (15 years old)	9	1	1.147
7	64	Male	50.7 (18 years old)	8	1	0.868
8	64	Male	53.8 (19.years old)	19	0	1.229
9	75	Male	52.6 (14 years old)	13	0	NIC
10	78	Male	50.9 (19 years old)	15	1	1.462
11	79	Female	52.7 (17 years old)	31	0	0
12	88	Female	53.5 (17 years old)	20	0	0.027

Note: The item in the parenthesis reveals age at the time of examination, and NIC represents those who were not in city. SH denotes small head size defined by two standard deviation or more below the observed specific-age-at measurement means.

The critical value which minimizes the probability of classification is obtained on the basis on an approximate technique with regard to the fitness of normal distribution. Namely three cases of the lowest IQ value of 56 of those who had mental retardation were excluded from 1673 whole individuals. The distribution of 1670 individuals satisfy normality. An approximate probability of misclassification for individuals at random from mixture of two normal populations is 3/12 = 25%.

5 DISCUSSION

Under the assumption of a mixture of two Gaussian distributions from normal IQ individuals and abnormal IQ individuals with mental retardation, we generally planned to estimate two means, two standard deviations, and a proportionality factor of five parameters for a compound normal distribution proposed by Cohen in 1967 and Otake in 1972 applied the technique to quantitate morphological findings for minute vessels of fingernail fold. Otake has estimated reasonable results. However, for the IQ data, it was not estimable for such five parameters of a mixture of two normal distributions because of sparse abnormal cases with mental retardation, as evident from a distribution of Fig.1. We determined, first to examine a normality of distribution for a entire IQ data. If the IQ data did not fit to normal distribution, we investigated the normality of the IQ data after excluding from the lowest IQ value step by step till we obtain a good fit of normal distribution. The results for excluding three cases of the lowest IQ value with mental retardation from 1673 individuals were estimated with a good fit of normality. The approximate misclassification of

Table 3 Test of normality and significant p-value based on Shapiro-Wilk statistic

Item	Total	Gestational age (weeks)			
		0-7	8-15	16-25	≥26
		All individuals			
Cases	1673	269	350	480	574
Mean	107.7	106.1	107.7	109.2	107.3
Standard deviation	16.08	15.03	17.22	16.21	15.67
Skewness	-0.041	-0.146	-0.217	-0.137	0.202
Kurtosis	0.323	-0.164	0.273	0.337	0.556
W-test	0.985	0.981	0.977	0.981	0.981
P-value	0.039*	0.285	0.044*	0.107	0.078
		All individuals without 12 mentally retarded cases			
Cases	1661	269	344	476	572
Mean	108.0	106.1	108.4	109.5	107.4
Standard deviation	15.73	15.03	16.37	15.79	15.53
Skewness	-0.048	-0.146	-0.048	-0.029	0.243
Kurtosis	0.207	-0.164	0.075	0.175	0.522
W-test	0.987	0.981	0.982	0.983	0.981
p-value	0.215	0.285	0.343	0.340	0.088

Note: Significant level is * ($p <0.05$).

the IQ individuals was estimated as 25.5%. Otake in 1972 estimated approximately 79% as normal and 21% as abnormal, using the criterion for a total score. The chance of wrongly classifying an individual as normal is 2.2%, and as abnormal is 3.8%. Therefore, the probability of misclassification for an individual coming at random from a mixture of two normal population is 6.0%. In this study, the misclassification as normal is 0.54%, whereas one as abnormal is 25% for characteristics based on small numbers of mentally retarded cases. On the other hand, the misclassification in 8–15 gestational weeks is 0.87% for normal data of 345 individuals and 40% for sparse abnormal data of 5 mentally retarded cases.

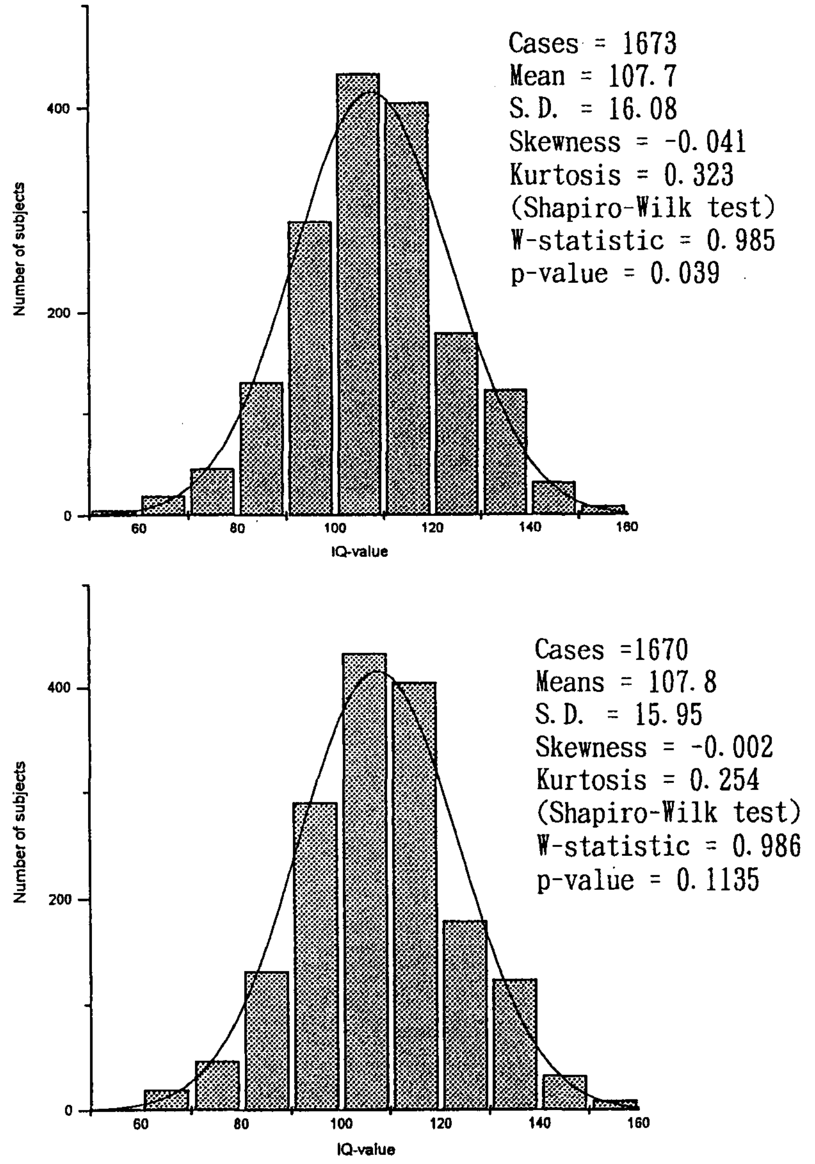

Fig. 2 Normal distribution of all IQ individuals with and without three lowest IQ values for less than 59 IQ score of mental retardation

Four sources of extraneous variation at least, possibly confounded here, could impinge on these findings. They include i) genetic variation, ii) nutritional deprivation, iii) bacterial and viral infections in the course of pregnancy, and iv) since there is substantial evidence to suggest that the cerebrum and its adnexa are especially sensitive to oxygen deprivation, an embryonic or fetal hypoxemia secondary to radiation damage to the hematopoietic system of the mother and (or) her developing child. Assessment of the contribution each of these potential confounders could or did make in the present instance is formidable. There is little direct evidence, and individual speculations are undoubtedly colored by the speculator's preconceptions and interests. Numerous genetic forms of mental retardation are known (Mckusick 1978). Most of these are recessively inherited. They are individually infrequent, but collectively they have a significant impact on the occurrence of this disorder. It is known too that the prevalence (and incidence) of rare, recessively inherited disorders is functionally related to the frequency of consanguineous marriages; at the time these survivors were conceived, such marriages were common in both Hiroshima and Nagasaki, but especially so in the latter (Schull and Neel 1965).

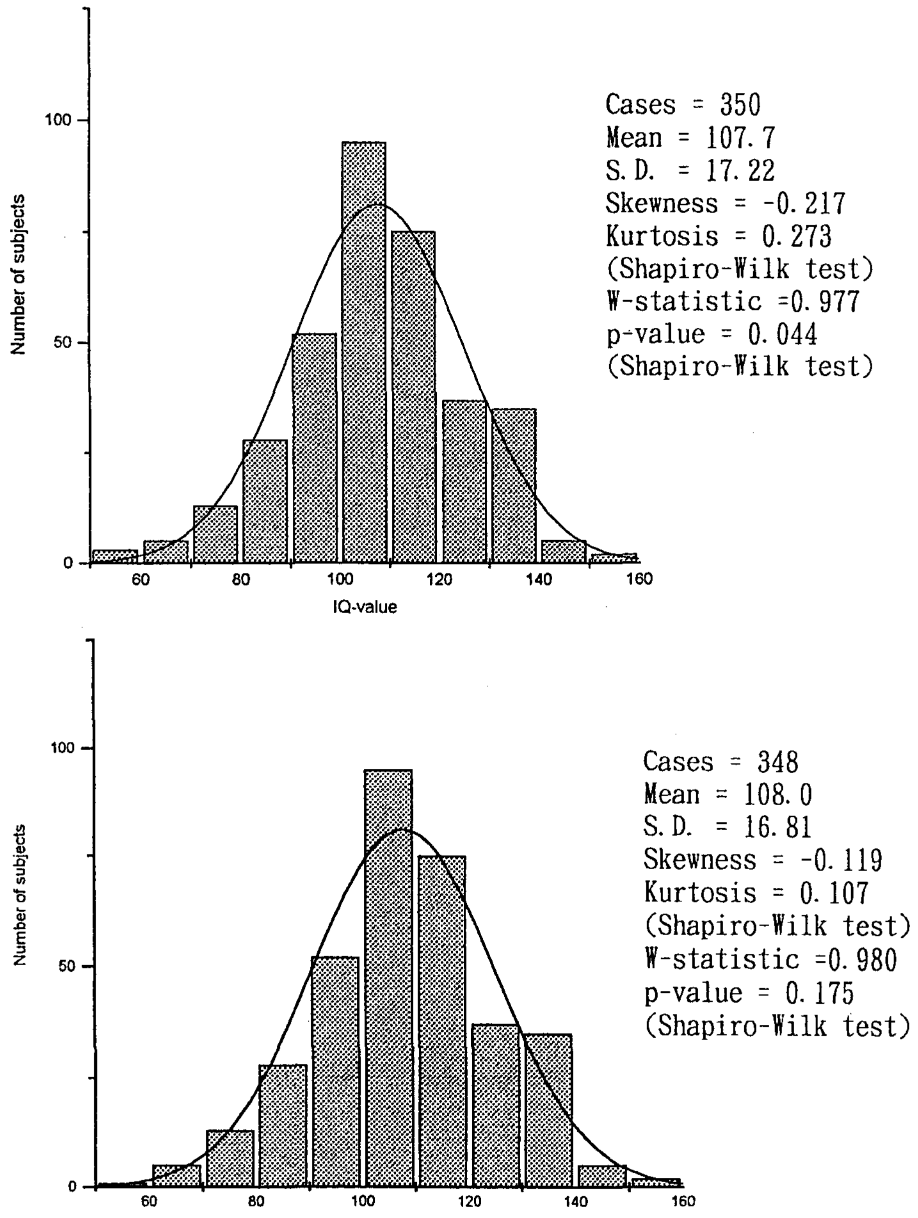

Fig. 3 Normal distribution of all IQ individuals with and without two lowest IQ values of 56 of mental retardation for 8–15 gestational week group

The role of maternal malnutrition on the subsequent mental performance of fetus is controversial, and epidemiologic or experimental evidence on the possible interaction of such malnutrition with radiation damage is virtually nonexistent. There are data showing that maternal malnutrition may affect a child's mental growth by decreasing DNA content or cell number. Marasmic infants, underweight at birth, who die of malnutrition during the first year of life, are known to have brain that are smaller in wet and dry weight than normal; their total RNA, total cholesterol, total phospholipids, and total DNA content are proportionally reduced; DNA synthesis is showed, cell division is curtailed; and a reduced number of brain cell are seen (Pozovski and Winick 1979). Marasmic infants of normal birth weight trend not to show these effects. If the mental impairment seen here stems largely from effects on neuronal number or migration, it seems unlikely that maternal malnutrition would restrict fetal growth as markedly in the second trimester as in the third, supposedly less protected trimester of gestation. Moreover, if malnutrition interferes primarily with the growth and establishment of synaptic connection. The effects of malnutrition will be largely postnatal, since this phase of brain growth is predominantly postnatal (Dobbing and Sands 1973). Under the assumption that the IQ score was obtained from a mixture of two Gaussian distribution for the normal and abnormal groups, we attempted a division to two groups by examining a normality of IQ data including a small number of mentally retarded cases, using an approximate classification technique. The technique has been repeated till the normality is confirmed after the mentally retarded cases with the lowest IQ value were excluded from the model. When three cases of the lowest IQ value of 56 were excluded, the normality was satisfied for IQ data of a entire 1670 sources. We judged a cause of misclassification into normal or abnormal distribution. Mean IQ and its standard deviation (SD) of IQ scores reported by Schull et al. (1988) are 63.8 and 8.5 for the mentally retarded cases with small heads, and 68.9 and 11.9 for the mentally retarded cases without small heads. These values are 96.4 and 19.8 for cases with small heads only. The mean IQ and SD for the overall sample are 107.8 and 16.4, respectively. No significant difference exists between the first two IQ means identified above, but both are significantly smaller than the mean for the individuals with small heads without mental retardation. The mean IQ of individuals with small heads without mental retardation does not differ significantly from the mean for the entire sample. Small head size was determined using a sex-specific criterion of at least 2 standard deviation below the mean observed between ages 9–19 (Otake and Schull 1993).

Epidemiological and experimental evidence testifies to the harmful effects of ionizing radiation exposure on the developing fetal brain. It is known that actively dividing cell are more sensitive to ionizing radiation than cell which have completed division or differentiated cell which seldom undergo cell division. Radiation exerts a great effect in causing microcephaly and mental retardation, but in the case of children with small head size without mental retardation, no evidence other than retardation of growth and development was observed (Wood et al. 1967, Tabuchi et al. 1967). Small Head size is defined as head circumference which is more than two standard deviations below the sex- and age-specific mean head size for the entire clinical study population. Microcephaly is a condition of poor development retardation of the brain in which the cranium and cerebral hemisphere are abnormally small. Since it is generally difficult to determine brain size, the relationship with developmental retardation of the brain was investigated be measuring head size. Craniostenosis due to early cranial accretion is usually considered separately (craniostenosis is a condition in which the head remains small due to the skull not becoming large, and although it externally resembles microcephaly, There is no abnormality in the brain). Besides hereditary microcephaly (true microcephaly), other causes of developmental brain damage include exposure during fetal development to oxygen deprivation (embryonic or fetal hypoxemia), toxication, irradiation, infection disease (e.g. rubella or cytomegalovirus infection), and maternal environmental abnormalities, etc. Microcephaly is frequently accompanied by retardation of mental development and seizures.

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