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An Experimental Evaluation of Lake Flow Simulation

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The global positioning system is utilized in lake flow analysis to facilitate experimental evaluation for numerical results of a lake flow simulation. In our lake flow experiment a float travels on the lake surface receiving the fluid resistance due to lake flow, while a global positioning system receiver keeps track of its position. A momentum equation is derived when the product of the float mass and its acceleration is set equal to the driving force. The temporal change in the position of the float can be simulated when numerical solutions of lake flow equations are given. A comparison of numerical results and experimental data leads to an evaluation of the numerical simulation of lake flow.

Keywords : lake flow, numerical simulation, GPS-float, finite element method

1 INTRODUCTION

In order to investigate actual phenomena that take place in our environment, it is important to evaluate techniques that we apply for their analysis. We introduce an experimental method to analyze the accuracy of numerical techniques for lake flow simulation. Here the global positioning system (GPS) is utilized for lake flow experiments. A GPS receiver calculates its current position analyzing signals transmitted from GPS satellites. The one used in our experiments also analyzes signals from radio beacons to improve its accuracy (differential GPS). Such a GPS receiver is mounted on a float, which we call the GPS-float [2], in order to keep track of its position while it travels on the lake surface receiving the driving force of fluid resistance due to lake flow. The positional information of the GPS-float at each time step is transmitted via a wireless modem to be recorded for analysis.

Such data can be analyzed to evaluate results of a numerical simulation. The driving forth of the float is the fluid resistance exerted on the plates suspended from the float, and such a resistance can be formulated in terms of the relative velocity of the lake flow and the float.

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Then a momentum equation is obtained when the product of the float mass and its acceleration is set equal to the driving forth of the float.

On the other hand, the trajectory of the float based on the momentum equation can be simulated when numerical solutions of equations governing lake flow are given. A simulated trajectory can be compared with the actual trajectory recorded in the experiment, and the comparison of experimental results and numerical results leads to an evaluation of the lake flow analysis.

In section 2, we introduced a result of experiments conducted at the Kojima Lake which lies in the Okayama prefecture, Japan. In the Kojima Lake an unsteady flow is often generated when water is discharged from the lake into the sea to maintain the water level. The experiment was conducted when such an unsteady flow occurred. In section 3, we illustrate how the temporal change in the location of the GPS-float can be simulated. We assume that the driving force is the fluid resistance exerted on the plates suspended from the float. We also assume that such a resistance is parallel to the relative velocity of the lake flow and the GPS-float, and that its magnitude is proportional to the square of the magnitude of the relative velocity. These assumptions leads to a momentum equation of the GPS-float. In section 3, we also introduce an example of lake flow simulation. A finite element analysis leads to numerical solutions of lake flow equations. Then given the numerical solution of lake flow equations, a numerical analysis of the momentum equation leads to a simulated trajectory of the GPS-float.

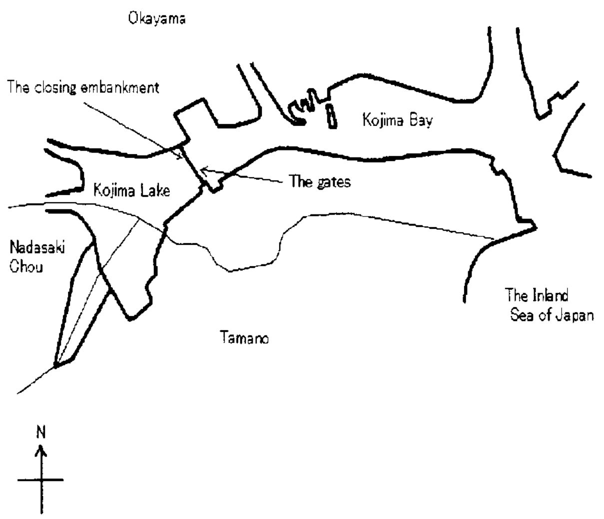

Figure 1: A sketch of the Kojima Lake and its vicinities. The Kojima Lake and its vicinities are shown. The locations of the closing embankment and the gates are indicated.

2 EXPERIMENTAL STUDY

The Kojima Lake is separated from the Kojima Bay with a closing embankment of approximately 1 km long (cf. Figure 1). There are six gates, each of which is 24 m long, installed on the embankment in order to control the water level of the lake. If the water level of the lake needs to be lowered, the gates are often opened when the tide level of the Kojima Bay is lower than the water level of the Kojima Lake. Then the discharge of water from the Kojima Lake into the Kojima Bay generates a flow in the Kojima Lake. On October 25, 2000, the gates were opened approximately from 15:00 to 16:13. Based on the data provided at the Kojima Bay Closing Embankment Central Administration Office, we obtained an approximate temporal change of the tide level in the Kojima Bay, and it is shown in Figure 2.

A GPS-float experiment was conducted approximately from 15:17 to 16:17. In this section we discuss what the experimental result suggests. The experiment was conducted in the area which the rectangle shown in Figure 3 indicates. Figure 4 shows the trajectory of the GPS-float when it traveled on the surface of the Kojima Lake approximately from 15:17 to 16:17. During the experiment, the GPS-float traveled approximately over the distance of 100 m, which leads to an approximate average speed of 0.028 m/s. Figure 4 shows that the GPS-float was headed northwest traveling away from the gates at the beginning of measurement. Then it turned northeast after 5 minutes, and thereafter it travels toward the gates. One speculate that a wind driven flow might have been formed before the discharge of water, and that its effect might

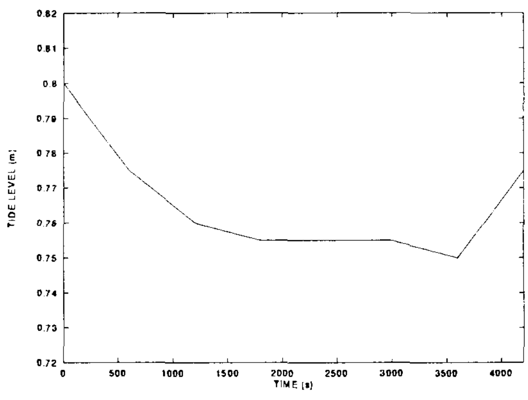

Figure 2: The temporal change in the tide level. The change of the tide level in the Kojima Bay versus the elapsed time from 15:00 is shown.

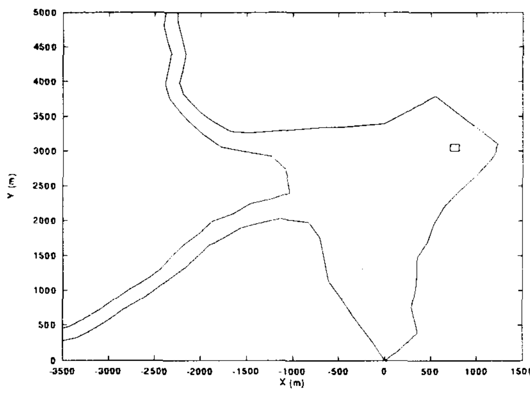

Figure 3: The experimental area. The experiment is conducted in the area which the rectangle indicates.

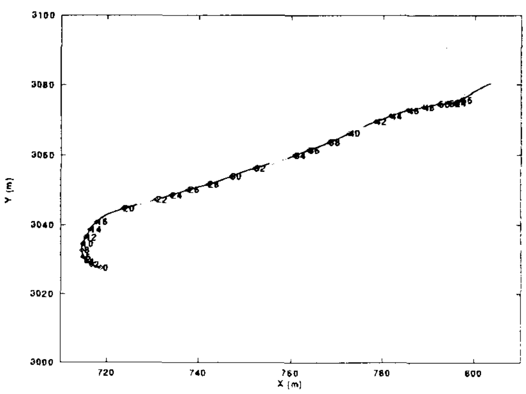

Figure 4: The trajectory of the GPS-float. The trajectory of the GPS-float is shown. Each location of the float at the end of a time interval of 2 minutes are indicated along the trajectory.

have appeared in the movement of the GPS-float at the beginning of measurement.

3 TRAJECTORY ANALYSIS

In this section we show how an equation governing the temporal change in the position of the float can be derived, and how we arrive at a momentum equation of the GPS-float [4]. In our experiment of lake flow, the float receives the fluid resistance of lake flow exerted on the plates suspended from the float. Suppose that a rectangular plate is placed in a uniform flow, and it is perpendicular to the flow. The fluid resistance D exerted on the plate is approximately proportional to the product of the density of the fluid ρ, the square of the fluid velocity U_∞, and the area of the plate A [1]:

$$D = \frac{1}{2} C_D \rho A U_\infty^2.$$

Here, C_D is a constant called the drag coefficient. The value of C_D depends on the ratio of one side of the rectangle to the other, and lies in the approximate range of 1.12 - 2.01. We assume that such an approximation holds for the driving force of the float which is the fluid resistance exerted to the plates suspended from the float.

Suppose that x and y are the coordinates of the float, and that they are functions of the time t. Then the velocity of the float is $(\dot{x}, \dot{y})$, where " $\dot{}$ " denotes the differentiation with respect to t, and its acceleration is $(\ddot{x}, \ddot{y})$. u and v are the x component and y component of the fluid velocity, respectively. We assume that the fluid resistance is parallel to the relative velocity of the flow and the float $(u - \dot{x}, v - \dot{y})$, and that its magnitude is proportional to the product of the density of the fluid ρ, the typical area of a plate suspended underneath the surface A, and the square of the magnitude of the relative velocity of the flow and the float $(u - \dot{x})^2 + (v - \dot{y})^2$. Then the driving force is equated to the product of the mass of the float m and its acceleration to obtain the following momentum equation.

$$\begin{aligned} m\ddot{x} &= \frac{1}{2} C_D \rho A \sqrt{(u-\dot{x})^2 + (v-\dot{y})^2}\,(u-\dot{x}), \\ m\ddot{y} &= \frac{1}{2} C_D \rho A \sqrt{(u-\dot{x})^2 + (v-\dot{y})^2}\,(v-\dot{y}). \end{aligned}$$

When the velocity components are given, these equation can be analyzed to simulate the temporal change in the position of the float.

Given the velocity of the lake flow, the temporal change of the GPS-float can be simulated by analyzing the momentum equation numerically. On the other hand a finite element analysis leads to numerical solutions of equations governing the lake flow. Figures 5 and 6 show the velocity vectors obtained in a finite element analysis of lake flow in the Kojima Lake. Details concerning the method of analysis are found in [2] and [3]. In this analysis we assumed that the gate was opened at 15:00, and that the tide level in the Kojima Bay changed as Figure 2 shows. We also assumed that the initial water level of the Kojima Lake at 15:00 was 0.8 m, and that it was the same level as the tide level in the Kojima Bay was.

The momentum equation is analyzed numerically using the numerical solutions of the lake flow equations. Figure 7 shows the simulated trajectory of the GPS-float. For comparison, the actual trajectory measured in the experiment is also shown in the figure. In this analysis we set $m = 13$ kg, $C_D = 1.166$, $\rho = 1000$ kg, and $A = 0.2\ \mathrm{m}^2$. Our measurement started at 15:17:04, which corresponds to $t = 1024$ s in our analysis.

4 CONCLUSION

Figure 7 shows an agreement between the experiment and the simulation. We believe that we have obtained an acceptable result in evaluation of the numerical simulation of lake flow. We also believe that the method to evaluate the numerical simulation should be found useful in other situations such as studies concerning steady flows.

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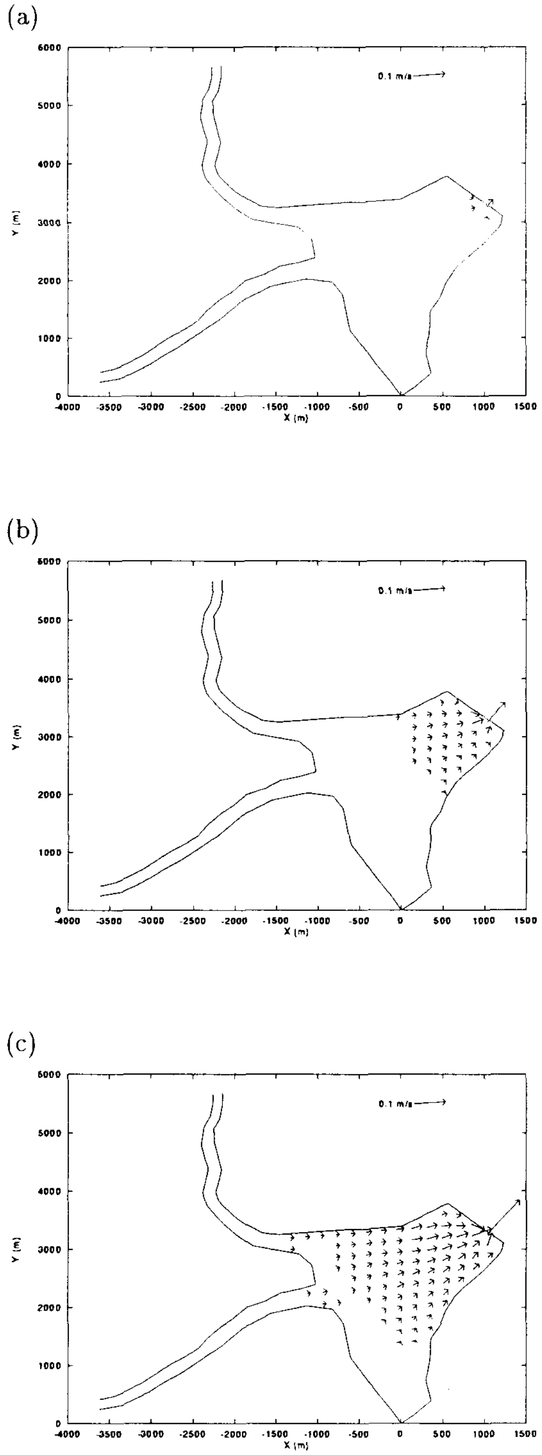

Figure 5: The velocity vectors at t = 600, 1200, 1800. (a), (b), and (c) show the velocity vectors of the unsteady flow 600 s, 1200 s, and 1800 s, respectively after the gates were opened approximately at 15:00.

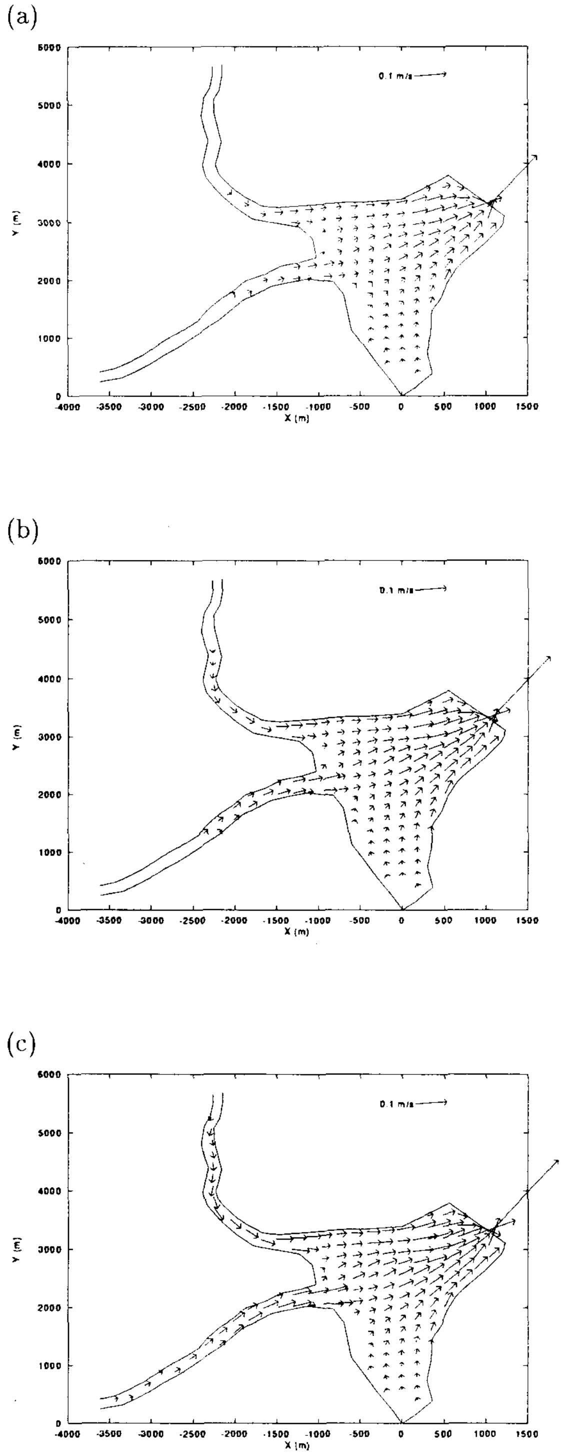

Figure 6: The velocity vectors at $t = 2400, 3000, 3600$. (a), (b), and (c) show the velocity vectors of the unsteady flow 2400 s, 3000 s, and 3600 s, respectively after the gates were opened approximately at 15:00.

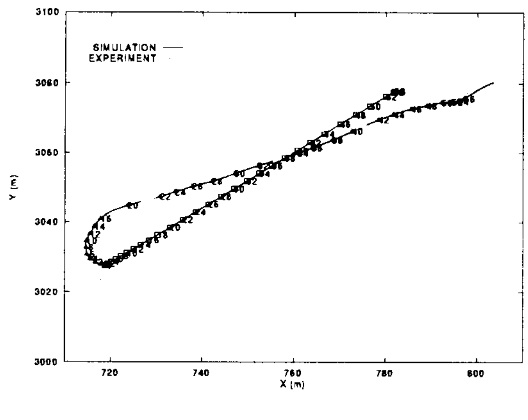

Figure 7: The trajectories of the GPS-float. A simulated trajectory obtained in a numerical analysis of the momentum equation is shown. The actual trajectory is also shown for comparison. Each location of the float at the end of a time interval of 2 minutes are indicated along the trajectories.

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