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Abstract

It is well known, that high-energy electrons can be used for tumor therapy. The so-called conventionel therapy with 100 through 250keV x· rays causes a great part of the x.rays to be scattered and absorbed in the sane tissue. In spite of the medicamental radiation prophylaxis additional radiation diseaes result by those compton scattered rays. By application of fast electrons and hard x.rays (so called gamma. rays) one tries to diminish those undesired side-effects and at the same time to increase the therapeutical effect of the ray treatment. As radiation source for fast electrons and hard gamma.rays one uses the Betatron, which was developed by NBRST in 1941 after preliminary operation of SLEPIAN, WALTON, WIDEROE and STEENDECK. The following statements are based on the references (1) through (6).

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THE BETATRON AND ITS MEDICAL APPLICATION

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It is well known, that high-energy electrons can be used for tumor therapy. The so-called conventionel therapy with 100 through 250keV x-rays causes a great part of the x-rays to be scattered and absorbed in the sane tissue. In spite of the medicamental radiation prophylaxis additional radiation diseaes result by those compton scattered rays.

By application of fast electrons and hard x-rays (so-called gamma-rays) one tries to diminish those undesired side-effects and at the same time to increase the therapeutical effect of the ray treatment. As radiation source for fast electrons and hard gamma-rays one uses the Betatron, which was developed by NBRST in 1941 after preliminary operation of SLEPIAN, WALTON, WIDERÖE and STEENDECK. The following statements are based on the references (1) through (6).

The Principle of the Betatron.

The Betatron is comparable to an ordinary transformer wherein the high voltage or secondary winding consists of an evacuated doughnut-shaped tube (K) in Fig. 1 in which electrons moving at high vclocity form the secondary circuit. This ring tube is placed between a C-shaped focusing magnet (St). An electron gun G (heated cathode) proiects free electrons into the evacuated tube. These are accelerated by the action of the changing magnetic field and at the same time they are caused to move in a circular orbit along the axis of the tube by virtue of the existence of a magnetic field. An alternating current is sent through the primary coils P_{sp} of the Betatron. With increasing current the magnetic flux

$$\phi(R)=\int_0^R \overline{B}_Z(r)\, 2\pi + dx = B^z(R)R^2\cdot\pi \qquad (1)$$

increases. In equation (1) R is the radius of the circular orbit, $\phi(R)$ the total magnetic flux of the circular disk with radius R, and $\overline{B_Z}(R)$ is the mean value of the magnetic induction between $r=0$ and R. According to Farraday's law of electromagnetic induction an electromotive force is induced at R, whose momentary value is proportional to the time rate of

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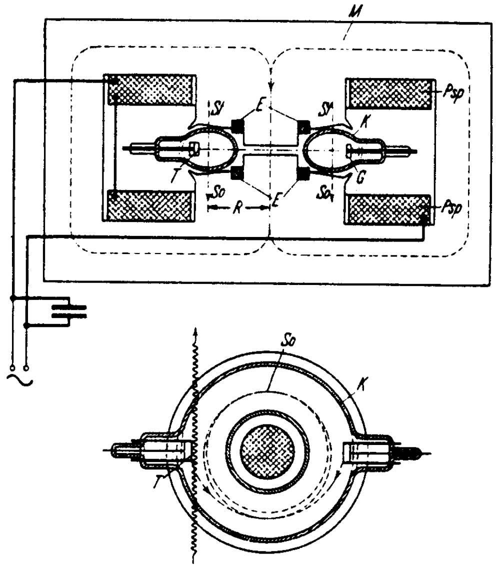


Figure 1 Schematic View of the Betatron.

change of the magnetic flux in the circular disk of radius R.

$$U_{ind}(R) = -\frac{d\phi(R)}{dt} = -R^2\pi\frac{d\overline{B_Z}}{dt} \qquad (2)$$

The electromotive force builds up an electric field whose value is related to the electromotive force $U_{ind}(R)$ by equation (3)

$$|\vec{\varepsilon}(R)| = -\frac{U_{ind}(R)}{2\pi R} = \frac{U(R)}{2\pi R} \qquad (3)$$

An electron with charge e^0, which moves at the velocity v in a circular orbit of radius R is accelerated tangentially by the electric force $K = e_0 \cdot |\vec{\varepsilon}(R)|$ in direction of the electric field. The energy dE picked up by the electron having moved a distance ds in direction of the field during an increase $d\overline{B_Z}$ of the magnetic induction is given by equation (4).

$$dE = kds = e_0|\varepsilon(\vec{R})|ds = e_0\frac{R}{2}\frac{ds}{dt}d\overline{B_Z}(R) = e_0\frac{R}{2}vd\overline{B_Z}(R) \qquad (4)$$

If one makes the electron stay on the circular orbit of R, while the induction increases from 0 to $\overline{B_Z}$, it gains the energy

$$E = e_0\frac{R}{2}\int_0^{\overline{B}z} v\, d\overline{B_Z}(R)$$

To keep the electron on the circular orbit the centrifugal force must

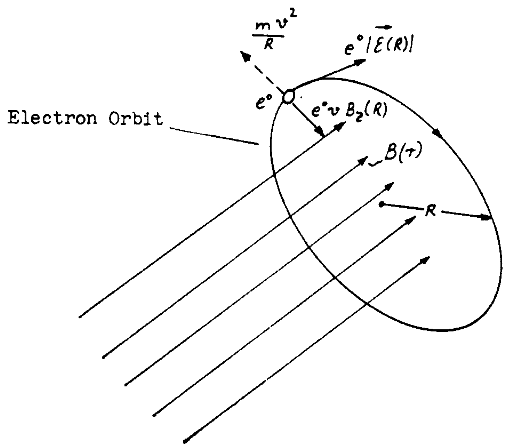
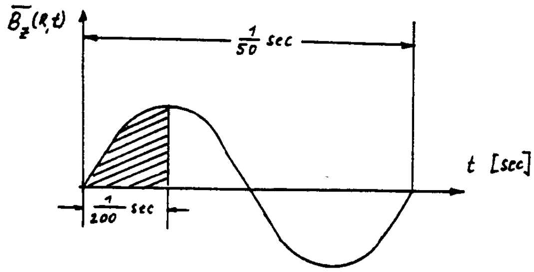


Figure 2 The Magnetic Field and the Interacting Forces



Time Dependence of the Magnetic Induction $\bar{B}_Z$ (R)

be compensated by the Lorentz force (Fig. 2).

In case of the magnetic field from Fig. 2 the Lorentz force is given by the relation (5)

$$K_L = e_0 \, v \, B_Z(R) \qquad (5)$$

By an appropriate form of the focusing magnet St one can give a value to the local induction $B_Z(R)$ at the circular orbit, that causes an equilibrium between the Lorentz force and the centrifugal force. The value of $B_Z(R)$ follows from the equilibrium condition (6)

$$K_L = e_0 v \, B_Z(R) = K_{fugal} = \frac{m \cdot v^2}{R} \qquad (6)$$

and one gets the relation for $B_Z(R)$:

$$B_Z(R) = \frac{m \cdot v}{e_0 \cdot R} \qquad (7)$$

For equilibrium conditions it follows from (7), that $B_z(R)$ must increase linearly with v.

According to Newton's law the time derivative of the electron momentum in tangential direction must be equal to the force from the electric field.

$$\frac{d}{d}t(m\cdot v)=K_{tangential}=e_0\,|\,\varepsilon^{\rightarrow}(R)\,|=\frac{e_0R}{2}\ \frac{d\overline{B_z}}{dt}$$

After integration over the time t one gets the relation (8).

$$m\cdot v=\frac{e_0R}{2}\ \overline{B_z}(R) \qquad (8)$$

From (7) and (8) we get a relation between the mean value of the magnetic induction $\overline{B_z}(R)$ from 0 through R and the local value $B_z(R)$ of the magnetic induction at the electron orbit.

$$R_z(R)=\tfrac{1}{2}\overline{B_z}(R) \qquad (9)$$

Equation (9) is known as so-called Wideröe condition. It says, that for equilibrium conditions between the Lorentz force and centrifugal force the local value $B_z(R)$ must have the half value of the mean value $\overline{B_z}(R)$ of the induction from 0 through R.

The resulting electron-speeds in the Betatron make it necessary to consider the relativistic mass increase of the electron. From the relativistic point of view the kinetic energy $e^0\cdot U$ of the electron follows from the energy difference between the energy of the moving and the resting mass. Therefore, we get relation(10).

$$E=mc^2-m_0c^2=m_0\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}-1\right)c^2=e_0U \qquad (10)$$

with m_0 = electron restmass

c = velocity of light

The solution for v has the form :

$$v=c\frac{\sqrt{U^2+2U\frac{m_0c^2}{e_0}}}{U+\frac{m_0c^2}{e_0}} \qquad (11)$$

In (4) v is substituted by (11) and one gets the relation between the promoting tension U and the mean induction $\overline{B_z}(R)$.

$$\frac{dU\left(U+\frac{m_0e^2}{c_2}\right)}{\sqrt{U^2+2\frac{m_0c^2}{e_0}U}}=\frac{B}{2}c\,dB_z$$

After integration one gets equation (12).

$$U=\sqrt{\left(\frac{m_0c^2}{e_0}\right)^2+\left(\frac{cR}{2}\overline{B_z}\right)^2}-\frac{m_0c^2}{e_0} \qquad (12)$$

For large values of B_z (12) can be approximately given by (13).

$$U\approx\frac{cR}{2}\overline{B_z}(R) \qquad (13)$$

For instance for $\overline{B_z}=1\ Vs/m^2=10000$ Gauss and $R=0.2m$ one gets a promotive tension of 3.10^7V.

A shost time after zero passage of the magnetic field the electrons with a speed according to equation (8) are injected tangentially to the circular orbit. If an alternating field is used, the electron is accelerated only in the first quarter of the period, since the inductivity increases only in this range (Fig. 2).

For a Betatron operated with 50 cycles altermating tension the acceleration takes place every 1/50 sec during 1/200 sec. If one has an energy gain for the electron of 35 ev/circulation the electron has to circulate 1 millton times, if it shall have a final energy of 35 MeV. The way, which the electron moves in 1/200 sec, is about 1500 km.

To keep the electron on the orbit during these circulations one forms a field that way, that little errors of the electron orbit are corrected by an increase or decrease of the centripetal force for greater resp. smaller orbits. By an appropriate form of the focusing magnetic field one makes the electrons oscillate around the radius of the orbit. From the stability conditions of "Steenbeck" follows a relation for the field dependence at stable conditions of electron movement.

$$B_z(r)=B_z(R)\frac{R^n}{r_n} \qquad 0<n<1$$

$B_z(R)$ is the induction at the orbit. At the end of the acceleration, that means at the peak of the magnetic field (Fig. 2) the magnetic forces are changed by switching on of an expansion coil for a short time, thus expanding the circular orbit to a spiral path of the electron. The electrons can hit an anticathode T, where they generate hard x-rays (Bremsstrahlung) while slowing down. The anticathode is a small heavy metal foil.

Small but very intensive Bremsstrahlung-beam leaves the tube and can be used for medical purposes. It has a wide continuum-spectrum from $E=0$ up to the energy of the electrons.

On the other hand, the electrons can be guided directly out of the tube with special devices. They cross a 30 μ thick steel foil, which is necessary for sealing the tube. Compared with the Bremsstrahlung the outcoming electrons are monoenergetic, i. e. they have all about the same kinetic energy.

By changing the point of electron injection but also by variation of the switch-on point of the expansion coils the energy of the electrons or of the Bremsstrahlung can be adjusted within some limits. The efficiency of conversion from kinetic energy of the electrons to Bremsstrahlung isvery good for high energetic electrons as those as those from the Betatron. It is so good, that no cooling device for the anticathode is necessary. Compared with a 200 keV x-ray tube where only ca 1 % of the electron energy is converted to x-ray-energy the efficiency of the 35 MeV Betatron is 30%. The highest ß energy which can be reached with the Betatron is around 200 MeV, since the circulating (accelerated) electrons emit electromagnetic waves with a frequency, which corresponds to their circulating frequency. The energy loss from these oscillations increases with E^4, if E is the ß-energy. Above ca. 100 MeV it is very difficult to account for these energy losses. Since the weight of the magnetic coils of the Betatron increases also with E^2 one builds the Betatrons today only to accelerate up to 45 MeV.

Betatron Application in Medicine.

As already mentioned in the introduction Betatrons can be used for rediation therapy either as strong gamma sources or as strong source of monoenergetic high speed electrons.

Gamma-Therapy with the Betatron.

Fig. 3 shows curves of the depth doses of gamma rays of differeut energies, the relative intensities are plotted over the depth (100 % corresponds to maximum dose value). From the set of curves one sees the advantage of irradiation with high gamma energies. These are maximum depth effect, small effects at the surface, not too large exit doses. The conventional irradiation with 200 keV x-rays has only a small depth effect, since the absorption of x-rays in the tissue by the photoeffect is very strong. The 200 keV-x-rays have their maximum effect directly under the skin surface. The curve of the depth dose from Co-60 is somewhat more favourable. The maximum effect of these 1.16 and 1.30 MeV gamma-rays lies ca. 0.6 cm under the skin-surface. The best depth effect results from the Betatran-Bremsstrahlung. With increasing radiation energy the maximum

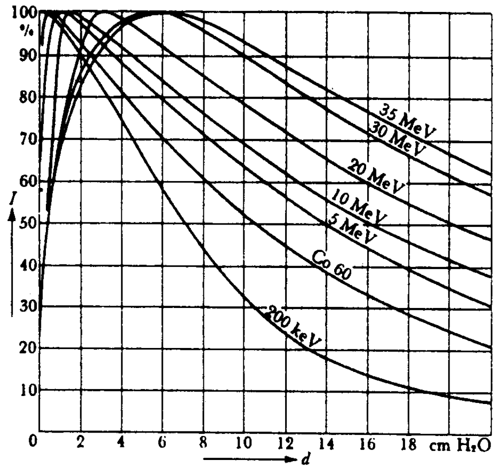


Figure 3 Curves of the Depth Doses for Gamma-Radiation in Water.

effect lies deeper. The reasons why the maximum dose lies under the surface are ionization effects, which arise by Compton-scattering and pair-production effects.

Since the exit-dose increases with increasing energy, one must not use too high energies. The intensity maximum is getting more and more flat with increasing gamma-energy. At 35 MeV the intensity maximum lies in a depth of 6.3 cm, but 3 cm in front of and 4 cm behind this point one has still 90 % of this maximum value.

The Bremsstrahl-spectrum of the 35 MeV Betatron has a range from 0 to 35 MeV with a mean value of 11 through 12 MeV. With increasing energy the gamma-beams are marked off sharply at their sides (Eig. 4),

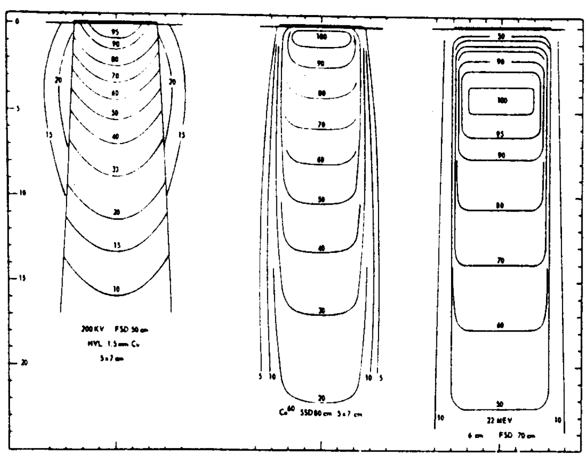


Figure 4 Co.nparison of isodole corves for 200 kv, Cobalt and Betatron radiation

that is because the side scattering is significant only for small gamma energies. This is very important, since one wants to spare same tissue. The beam can be homogenized by the help of an equalizing compound, so that the irradiated area has an even dose per unit area.

Betatherapy with the Betatron.

Fig. 5 shows depth dose curves for ß-radiation of different energies. One can see that the maximum value of the depth dose is reached immedi-

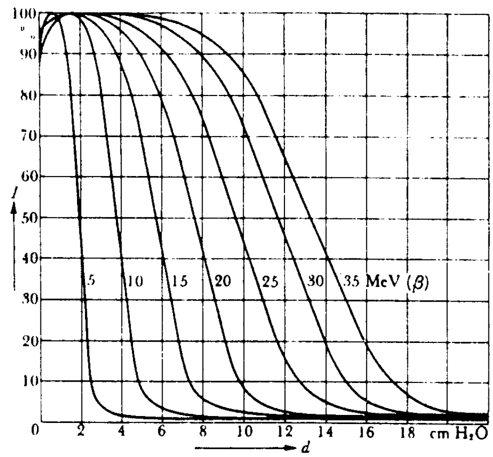


Figure 5 Curves of the Depth Doses for Beta-Radiation in Water

ately after entrance in the tissue. The impact ionization gets effective in deeper regions with increasing energy of the ß-rays, but in a characteristical depth the inteusity decreases rapidly. Beta-radiation is therefore indicated, if essential organs need to be protected. Contrary to gamma radiation, the reach of ß-particles is very well defined. The figure value of the electron reach in cm corresponds to the half value of the initial electron energy in MeV for human tissue. For ß-therapy in great depths (8—12 cm) the tissue above the irradiated tumor is hightly damaged. By secondary ionization the radiation beam also spreads a little in the deeper regions. -But the isodose-curves from Fig. 6 show, that one gets only 5 % of the initial dose in a depth of 12 cm 3 cm besides the initial beam from 35 MeV ß-radiation. Compared with 250 keV x-rays the ß-radiation from side scattering in the tissue is very small because of the small reach of the electrons.

With the help of a magnetic lens the two mentioned difficulties of beam-spreading and damage of the same tissue above the tumor can be partially eliminated. The magnetic lens makes use of the Lorentz force

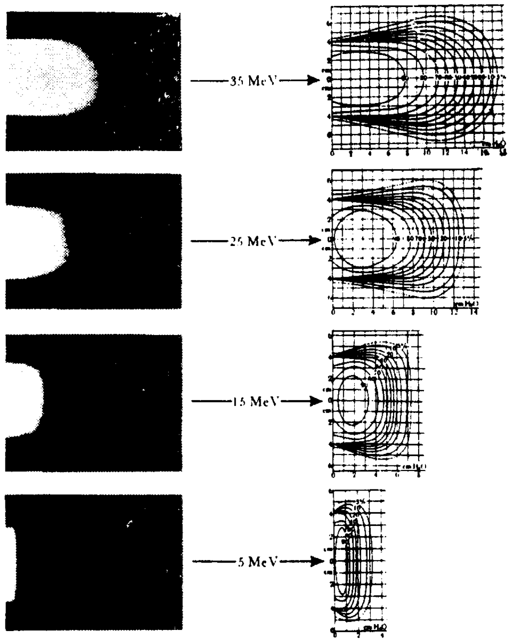


Figure 6 Comparison of Isodose Curves in Water for Different Beta-Energies

which deflects electrical charged particles and for the case of a homogene field perpendicular to their path makes them move on a circle. With this lens the ß-particles can be focused directly on the tumor. Its principle is shown in Fig. 7. The slitted disk right to the axis of the diverging beam lets only pass a part of the electrons, which converges with the help of the rotating permanent magnet directly on the tumor. The point of intersection between initial beam axis and the deflected beam axis lies in a depth that depends on the ß-energy and on the magnetic field strength of the permanent magnet. If the point of intersection lies directly in the tumor,

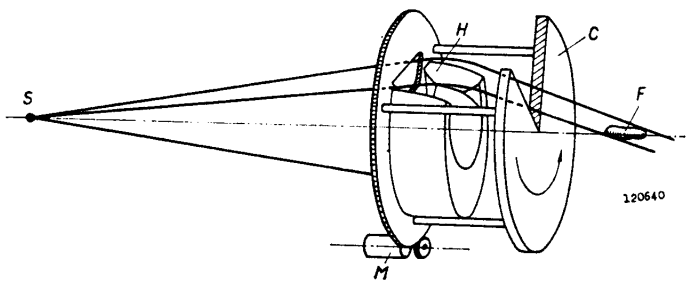


Figure 7 Schematic View of a Magnetic Lense

the specific dose in the tumor gets much higher than the entrance dose which is spread over a large area. Fig. 8 shows the isodose curves of a focused electronbeam in H_2O. The disavantage of this lens is hat only small ß-intensities can be uset, because a great part of the electrond are collimated. Therefore one needs long irradiation times.

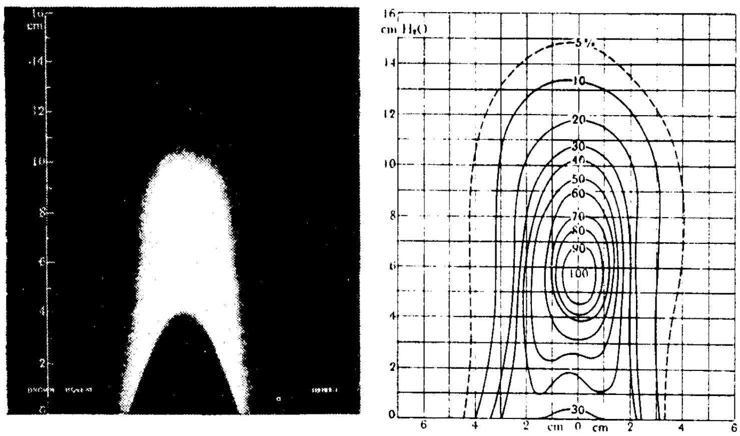


Figure 8 Isodose Curve from 30-MeV Beta-Radiation Focussed with a Magnetic Lense

The intensities available from the Betatron for ß-therapy are much higher than those for gamma therapy. The gamma rays of the 35 MeV Betatron have an intensity of about 100 r/min. For the same operating conditions the ß-rays have an intensity of 1000 r/min. The depth for the maximum effect of the irradiation can be changed by variation of the ß-enirgy only in the case of ß-irradiation, sence the available intensities increase with the third power of the ß-energy. Irradiation with Bremsstrahlung below the maximum energy would not be very effective since the gamma intensities have only 10 % of the value of the ß-intensity. But Betatrons which operate at a higher frequency than 50 cycles (Allis-Chalmers 25 MeV-Betatron) have higher Bremsstrahlintensities.

By irradiation with high dose rates (some 100 r/min) the therapy time gets very small (some minutes). From the medical point of view the ß-radiation is more tissue-compatible, because radiation diseases of the skin do not raise so often. If there are some radiation diseases on the skin they heal much faster than those from x-rays.

Survey of the Advantages for the Application of a Betatron.

Betatrons can be used for gamma-and fer beta-irradiation. Deeper lieing tumors can be treated optimally since the particle energies are very high. (The rate of survival and the healing prospects are better than those from other radiation therapies). Within some limits one can choose the depth for the maximum-effect of the ß-irradiation by variation of the particle energies. By usage of a collimating device and an equalizing compound the electrons from the Betatron can be collimated and their insensity can be homogenized. By this one gets very good local irratiation effects.

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