Okayama Economic Review 12 (1980) 77–89

Noncooperative Price-Mediated Exchange

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INTRODUCTION

Theoretical investigations of the market exchange from the game-theoretic point of view has two representative approaches. One is the cooperative market exchange model. In the celebrated game-theoretic formulation of the Edgeworth's conjecture by the work of Debreu and Scarf [1], exchange economy without the price mechanism is considered and in this economy, exchange is understood as a redistribution of resources among the traders in a way, starting from the initial allocation, increases at least one trader's utility without diminishing the other traders' utilities, and competition is regarded as the process of eliminating dominated allocations by contracting and recontracting. In this cooperative market game, the solution concept is the core which is not improved upon by any coalitional redistribution of resources. They does not consider the price-mediated exchange economy and rather they derive theoretical justifications of the Edgeworth's Law in the form : as the number of participants approaches infinity, the core allocation coincides with the one of competitive equilibrium. After their work, cooperative market game models, in most cases, virtually ignore the aspect of price-mediated exchange.

The other is the noncooperative market exchange model which was motivated and suggested by the works of Shapley [3] and Shapley and Shubik [4]. Recently, in succession to these works, several interesting papers are posed, e.g., Schmeidler [2] and Wilson [5] etc. Their contentions commonly focus upon the critique for the theoretical lacuna of the standard Walrasian model : the essential nature of the Walrasian theoretical explanation consists in the treatment of the coordination of activities at the level of market, however, the explanation of the market itself is incomplete, that is, the process or rules by which prices are determined is not explicit. They analyze the market exchange from the noncooperative game-theoretic point of view with explicitly treating the effect of agents' bidding behaviour upon the market price formation.

In this article, we will formulate the noncooperative bidding game in the monetary exchange economy. We will consider a pure monetary exchange economy : we suppose that any trade must be an exchange of money against some other commodity. The j-th market is the one in which the commodity j is exchanged against money. In each market each agent submits his price and net trade offer. Every trade is carried out among the members who bid the same price in a manner that anyone is not forced to trade beyond his wishes. We assume that each agent's characteristics such as his preference and endowment are known to him only and there is no way for any agent to observe his characteristics, however we assume that every agent has a complete market information such as the list of net trade vector and the list of price vector bade by every agent. By these formulation we will examine the properties of the solution concepts of this game, and we give an interpretation of the Walras equilibrium from the viewpoint of a noncooperative game with explicitly treating the effect of each agent's bidding behaviour upon the market price formation.

I. Analytical Framework

Let $\mathscr{E}=\{(Z_i, \succsim_i, \omega_i)_{i\in I}, J\}$ denote an exchange economy with a finite set of consumers $I=\{i|i=1,2,\ldots,n\}$ and a set of commodities $J=\{j|j=0,1,2,\ldots,m\}$. The commodity 0 is called money, which is a medium of exchange and is in itself a consumable commodity. $Z_i\subset R^{m+1}$ denotes consumer i's feasible set of net trade (excess demand) vector, $\succsim_i$ denotes his preference ordering, and ω_i denotes his initial endowment of commodities. Concerning to the characteristics of each consumer we assume the following;

Assumption 1: Z_i is closed, convex and has a lower bound for $\geqq$ and $0\in Z_i$,

Assumption 2: initial endowment vector ω_i is positive, and

Assumption 3: preference ordering $\succsim_i$ is strictly convex, monotone and continuous and it is a complete preordering. Furthurmore the indifference surface is smooth on Z_i.

We will consider the economy which is characterized by the following theoretical hypotheses.

[H-1] We consider a pure monetary exchange economy; we suppose that any trade must be an exchange of money against some other commodity. So there are m markets and the j-th market is the one in which the commodity j is exchanged against the commodity 0 (money).

[H-2] We denote consumer i's strategy vector by s_i and suppose that $s_i = (p^i, z_i)$. $p^i = (p^i_1, 1, p^i_2, 1, \ldots, p^i_m, 1) \in R^{2m}_+$ denotes the price vector bade by the consumer i, where p^i_j is the price of commodity j which is measured in terms of commodity 0. In the j-th market, the consumer i submits the pair of $(p^i_j, 1)$ and $z_i(j) = (z_{ij}, z_{i0}(j))$, where z_{ij} denotes his net trade of commodity j and $z_{i0}(j)$ denotes his net trade of commodity 0 in the j-th market. Concerning to the pair of $(p^i_j, 1)$ and $z_i(j)$ we suppose that the consumer i must satisfy the condition $p^i_j z_{ij} + z_{i0}(j) = 0$. We define $z_i = (z_{i1}, z_{i0}(1), z_{i2}, z_{i0}(2), \ldots, z_{im}, z_{i0}(m)) \in R^{2m}$. Hence the consumer i's strategy set S_i is defined by

$$S_i = \left\{ s_i = (p^i, z_i) \mid (\forall j \neq 0) : (p^i_j \geqq 0), (\forall j \neq 0) : (p^i_j z_{ij} + z_{i0}(j) = 0) \right\}.$$

[H-3] The trading rule in the j-th market is characterized by the mapping f_j ; $f_j : (s_i)_{i \in I} \mapsto (z^a_i(j))_{i \in I}$, where $(s_i)_{i \in I}$ is the list of strategies and $z^a_i(j) = (z^a_{ij}, z^a_{i0}(j))$ is consumer i's realized net trade vector in the j-th market. In the following we denote $f^i_j = z^a_i(j)$. The trading rule which is considered here is summarized by the following characteristics of the mapping f_j : for any consumer k and any j-th market, when the list of strategies $(s_i)_{i \in I} = s$ is given,

[i] if $\# \left\{ i | p^i_j = p^k_j \right\} = 1$, $f^k_j(s) = 0$,

[ii] if $\# \left\{ i | p^i_j = p^k_j \right\} \geqq 2$, we define the set $\mathcal{I}(p^k_j) = \left\{ i | p^i_j = p^k_j \right\}$, and then, $(\forall i \in S \subset \mathcal{I}(p^k_j)) : (f^i_j(s) = z_i(j) = (z_{ij}, z_{i0}(j)))$,

$$(\forall i \in \mathrm{L} \subset \mathcal{I}(p^k_j)) : (f^i_j(s) = -(\textstyle\sum_{i \in S} z_{ij} / \sum_{i \in L} z_{ij})\, z_i(j)),$$

where $S = \left\{ i \in \mathcal{I}(p^k_j) \,|\, z_{ij} \sum_{i \in \mathcal{I}(p^k_j)} z_{ij} \leqq 0 \right\}$ and it denotes the set of consumers who belong to the short side of the j-th market at the price p_j^k, and $L=\left\{i \in \mathscr{I}(p_j^k) \mid z_{ij} \sum_{i \in \mathscr{I}(p^k)} z_{ij} > 0\right\}$ and it denotes the set of consumers who belong to the long side of the j-th market at the price p_j^K. That is, we suppose that in any j-th market every trade is carried out among the members who bid the same price and in which any consumer i who belongs to the short side of the market can carry out his trading offer $z_i(j)$, but any agent who belongs to the long side of the market is forced to trade $z_i^a(j) \neq z_i(j)$. We define the mapping $f=\prod_j f_j$, where $f:(s_i)_{i\in I} \mapsto (z_i^a)_{i\in I}$ and we denote the i-th element of this mapping as $f^i = z_i^a \in R^{2m}$.

[H-4] We suppose that each consumer receives the whole of the market information such as the list of price vectors bade by every agent, the list of net trade offer, and the list of realized net trade of every consumer. Furthermore we suppose that every consumer knows the trading rule. However, we assume that each consumer's characteristics such as his preference and endowment are known to him only and there is no way for any consumer to observe his characteristics. So we suppose that every trading is carried out in the noncooperative game manner.

II. On the trading Rule

In this section we will consider the property of the trading rule f. Before the analysis we will define the following mappings δ, γ for any consumer i, and the mapping π:

mapping $\delta: z_i \mapsto \delta(z_i)$, where $z_i = (z_{i1},\ z_{i0}(1),\ z_{i2},\ z_{i0}(2), \ldots, z_{im},$

$z_{io}(m)) \in R^{2m}$. and

$\delta(z_i) = (z_{i1}, z_{i2}, \ldots, z_{im}, \sum_{j=1}^{m} z_{io}(j)) \in R^{m+1}$,

mapping $\gamma: z_i(j) \mapsto \gamma(z_i(j))$, where $z_i(j) = (z_{ij}, z_{io}(j))$ and

$\gamma(z_i(j)) = (0, 0, \ldots, z_{ij}, 0, \ldots, 0, z_{io}(j)) \in R^{m+1}$, and

mapping $\pi: p \mapsto \pi(p)$, where $p = (p_1, 1, p_2, 1, \ldots, p_m, 1) \in R_+^{2m}$ and

$\pi(p) = (p_1, p_2, \ldots, p_m, 1) \in R_+^{m+1}$.

Definition 1: The trading rule f is called *reasonable* if it satisfies the following conditions : for any consumer i

[i] if $(\forall j \neq 0) : (\gamma(z_i(j)) \in Z_i)$, and, $\delta(z_i) \in Z_i$, then $\delta(z_i^a) \in Z_i$,

[ii] if $(\forall j \neq 0) : (\gamma(z_i(j)) + \omega_i \succsim_i \omega_i)$ and $\delta(z_i) + \omega_i \succsim_i \omega_i$, then $\delta(z_i^a) + \omega_i \succsim_i \omega_i$,

[iii] $\pi(p)\, \delta(z_i^a) = 0$, and

[iv] $\sum_{i \in I} \delta(z_i^a) = 0$.

The condition [i] implies that the outcome given by the trading rule f is feasible, when the consumer submits a feasible net trade. The condition [iii] means the budgetary feasibility of the outcome. The condition [ii] is called an individual rationality, that is, any consumer will not be worse off after the trade when he plans to be better off by the trade. The condition [iv] implies the balance of the outcome. These four conditions may be the natural requirements for the trading rule in the price-mediated exchange economy. Schmeidler's model [2] is not *reasonable* in the sense of *Definition 1*, because it may violate the requirement [i], i.e., the feasibility of the outcome. Hence his model is not a satisfactory description of the price-mediated exchange economy. Here we can show that the our trading rule is *reasonable.*

Proposition 1 : The trading rule f is *reasonable.*

Proof: At first, it is trivial that the trading rule f satisfies the condition [iv]. Next, by noting the definition of the trading rule, it follows that
$(\forall i \in I):(\forall j \neq 0):(z_i^a(j) = (z_{ij}^a, z_{io}^a(j)) = \alpha_j^i (z_{ij}, z_{io}(j))$, where α_j^i is a real number such that $0 \leqq \alpha_j^i \leqq 1$. From the definition of mappings δ and γ, we have $\delta(z_i^a) = \sum_{j=1}^m \alpha_j^i \gamma(z_i(j))$.
When $(\forall j \neq 0):(\alpha_j^i = 0)$ for some consumer i, his outcome $\delta(z_i^a) = 0$. Then it is easily shown that conditions [i], [ii], and [iii] are all satisfied for this consumer i. When $(\exists j \neq 0):(\alpha_j^i \neq 0)$, we define $\hat{\alpha}_j^i = \alpha_j^i / \sum_{j=1}^m \alpha_j^i$. Then it follows that $0 \leqq \hat{\alpha}_j^i \leqq 1$, $\sum_{j=1}^m \hat{\alpha}_j^i = 1$, and $\delta(z_i^a)/\sum_{j=1}^m \alpha_j^i = \sum_{j=1}^m \hat{\alpha}_j^i \gamma(z_i(j))$, that is, $\delta(z_i^a)/\sum_{j=1}^m \alpha_j^i$ is a convex linear combination of $\gamma(z_i(j))$. From this fact and the convexity of the set Z_i and the convexity of the preference $\gtrsim_i$ it is easily verified that the trading rule f satisfies the conditions [i] and [ii]. Finally, from the definition of the strategy set S_i we have $(\forall j \neq 0):(p_j^i z_{ij} + z_{io}(j) = 0)$, hence from the definition of the trading rule we have $(\forall j \neq 0):(p_j^i z_{ij}^a + z_{io}^a(j) = 0)$. So it follows that $\pi(p)\, \delta(z_i^a) = \sum_{j=1}^m (p_j^i z_{ij}^a + z_{io}^a(j)) = 0$. These considerations complete the proof. ||

III. Walras Equilibrium as Nash Equilibrium

In usual the solution concept of the noncooperative game is a Nash equilibrium. In this section we will examine the relation of the equilibrium of our game and a Walras equilibrium.

Definition 2: When the list of strategies $(s_i^*)_{i\in I} = s^*$ is given, $s_k^N(s^*)$ is called a Nash strategy of consumer k, if the following condition is satisfied:

$$(\forall s_k \in S_k) : (\delta(f^k(s_k^N(s^*), s_{|k|}^*)) + \omega_k \gtrsim_k \delta(f^K(s_k, s_{|k|}^*)) + \omega_k)),$$

where $s_{|k|}$ denotes the list of strategies of n-1 consumers other than consumer k.

Definition 3: The list of strategies $(s_i^*)_{i\in I} = s^*$ is called a Nash equilibrium if $(\forall i\in I) : (s_i^N(s^*) = s_i^*)$. We call the list of strategies $(s_i^*)_{i\in I} = s^*$ a nontrivial Nash equilibrium if $(s_i^*)_{i\in I} = s^*$ is a Nash equilibrium and $(\forall j \neq 0) : (\exists i\in I) : (f_j^i(s^*) \neq 0)$.

Definition 4: The list of strategies $(s_i^*)_{i\in I} = s^*$ is called a nontrivial Walras equilibrium if the following conditions are satisfied:

[i] $(\forall i\in I) : (s_i^* = (p, z_i)$ and $\delta(f^i(s^*)) = \tilde{z}_i(\pi(p)))$,

[ii] $(\forall j \neq 0) : (\exists i\in I) : (\tilde{z}_{ij}(\pi(p)) \neq 0)$,

where $\tilde{z}_i(\pi(p))$ is consumer i's Walrasian net trade vector at the price vector $\pi(p) = (p_1, p_2, \ldots, p_m, 1)$, that is, we define the budget set $B_i = \{z_i \in R^{m+1} \mid \pi(p)\bar{z}_i = 0, \bar{z}_i \in Z_i\}$, then $\tilde{z}_i(\pi(p)) \in B_i$ and $(\forall \bar{z}_i \in B_i) : (\tilde{z}_i(\pi(p)) + \omega_i \gtrsim_i \bar{z}_i + \omega_i)$.

Let $\hat{P}(j)$ denote the set of price vectors $p(j) = (p_j, 1)$ which prevail in the j-th market and enable someone to trade. We define the set $\hat{P}$ by $\hat{P} = \{(p(j))_{j\neq 0} \mid (\forall j \neq 0) : (p(j) \in \hat{P}(j))\}$, where $(p(j))_{j\neq 0} = (p_1, 1, p_2, 1, \ldots, p_m, 1) \in R_+^{2m}$.

Definition 5: We call the state of the economy *regular* at $\hat{P}$ if the following conditions are satisfied for any consumer k and in any j-th market : for any $p \in \hat{P}$ which is chosen by the consumer k,

if $\tilde{z}_{kj}(\pi(p)) > 0$, then $|\sum_{i \in \mathscr{I}(p(j))} z_{ij}^{-}| > \tilde{z}_{kj}(\pi(p))$

if $\tilde{z}_{kj}(\pi(p)) < 0$, then $\sum_{i \in \mathscr{I}(p(j))} z_{ij}^{+} > |\tilde{z}_{kj}(\pi(p))|$,

where $\mathscr{I}(p(j))$ denotes the set of consumers who bid the price vector $p(j)$ corresponding to the price vector $p \in \hat{P}$ which is chosen by the consumer k, and $z_{ij}^{-} = \min(z_{ij}, 0)$, $z_{ij}^{+} = \max(z_{ij}, 0)$.

Definition 6: The list of strategies $(s_i^*)_{i \in I} = s^*$ is called a *nontrivial and regular Nash equilibrium*, if s^* is a nontrivial Nash equilibrium and the state of the economy corresponding to s^* is *regular*.

Remark: When the economy consists of a few consumers, in general, the regular property may be violated. The economy which satisfies the regular property may be the one which consists of numerous consumers, each consumer's transaction is sufficiently small relative to the aggregate volume of transaction and there exists a similarity of bidding behaviour between consumers.

Lemma 1: At a *nontrivial and regular Nash equilibrium* there prevails unique price vector.

Proof: Let the list of strategies $(s_i^*)_{i \in I} = s^*$ be a *nontrivial and regular Nash equilibrium.* Then from *Definition 3* we have $(\forall j \neq 0) : (\exists i \in I) : (f_j^i(s^*) \neq 0)$. Hence in each market there prevail price vectors which enable someone to trade. Let $\hat{P}(j)$ be the set of all price vectors $p(j) = (p_j, 1)$ which prevail in the j-th market and enable someone to trade. We define the set $\hat{P}$ by

$\hat{P}=\{(p(j))_{j\neq 0}|(\forall j\neq 0):(p(j)\in\hat{P}(j))\}$. When consumer i bids the price vector $p\in\hat{P}$ and calculates his Walrasian net trade vector $\tilde{z}_i(\pi(p))=(\tilde{z}_{i1}, \tilde{z}_{i2}, \ldots, \tilde{z}_{im}, \tilde{z}_{i0})$, then in any j-th market there exist enough aggregate volume of net trade to achieve $\tilde{z}_{ij}$, because $(s_i^*)_{i\in I}=s^*$ is a *nontrivial and regular Nash equilibrium*. Then, by noting the definition of the trading rule consumer i can select $(z_{ij}, z_{i0}(j))$ whose outcome coincides with $(\tilde{z}_{ij}, \tilde{z}_{i0}(j))$, where $\tilde{z}_{i0}(j)=-p_j\tilde{z}_{ij}$. Hence we have

$(\forall i\in I):(\forall p\in\hat{P}):(\exists s_i=(p, z_i)):(\delta(f^i(s_i, s^*_{\backslash i}))=\tilde{z}_i(\pi(p)))$.

We must note this fact.

We must show that $\#\hat{P}=1$. We prove it by reductio ad absurdum. Let $p\in\hat{P}$ and define the set $\mathcal{I}=\{i|p^i=p\}$. Suppose that $(\exists k\in I):(k\notin\mathcal{I})$. Let $p^k\neq p$ denote the price vector bade by consumer k and $p^k\in\hat{P}$. Then the set $\{\delta(z)|\pi(p)\,\delta(z)=0\}$ is divided by the set $\{\delta(z)|\pi(p^k)\,\delta(z)=0\}$ into

$$\{\delta(z)|\pi(p)\,\delta(z)=0,\ \pi(p^k)\,\delta(z)\geqq 0\}\ \text{and}$$

$$\{\delta(z)|\pi(p)\,\delta(z)=0,\ \pi(p^k)\delta(z)\leqq 0\}.$$

Now $\sum_{i\in I}\delta(f^i(s^*))=0$, because of the fact that $\sum_{i\in I}f^i(s^*)=0$. Then it follows that $(\exists i\in I):(\pi(p^k)\delta(f^i(s^*))\leqq 0)$.

(a) When $\pi(p^k)\delta(f^i(s^*))<0$,

we have $\tilde{z}_i(\pi(p^k))+\omega_i>_i\delta(f^i(s^*))+\omega_i$. However, from the above remark we have

$(\forall i\in I):(\forall p\in\hat{P}):(\exists s_i=(p, z_i)):(\delta(f^i(s_i, s^*_{\backslash i}))=\tilde{z}_i(\pi(p)))$.

This cotradicts the suppositi on of s^*.

(b) When $\pi(p^k)\delta(f^i(s^*))=0$, it follows that

$(\exists z\in R^{m+1}):(\pi(p)z=0,\ \pi(p^k)z<0,\ z+\omega_i\geqq 0)$. Then we can find a scalar $\beta>0$ such that $\pi(p^k)(z+\beta\pi(p))=0$. Hence we have

$$\pi(p)(z+\beta\pi(p))>0 \text{ and } \pi(p^*)(z+\beta\pi(p))=0.$$

From the *Assumption 3* of the preference ordering we have $(\exists t\in(0,1)):(t(z+\beta\pi(p))+(1-t)\delta(f^i(s^*))+\omega_i>_i\delta(f^i(s^*))+\omega_i)$. This result contradicts the supposition that $(s_i^*)_{i\in I}$ is a nontrivial Nash equilibrium. This completes the proof. ||

Theorem 1: In our noncooperative bidding game the *nontrivial and regular Nash equilibrium* coincides with the nontrivial Walras equilibrium.

Proof: (Nash equilibrium→Walras equilibrium); Let the list of strategies $(s_i^*)_{i\in I}=s^*$ be a nontrivial and regular Nash equilibrium. From *Lemma 1*, we know that there prevails unique price vector. Hence we can denote $(\forall i\in I):(s_i^*=(p,\ z_i^*))$
From the definition of nontrivial Nash equilibrium it follows that $(\forall j\neq 0):(\exists i\in I):(f_j^i(s^*)\neq 0)$. Then from the regular property and the definition of the trading rule we have
$(\forall i\in I):(\delta(f^i(s^*))=\tilde{z}_i(\pi(p)))$. We know the fact that $\sum_{i\in I}f^i(s^*)=0$. Hence we have
$\sum_{i\in I}\delta(f^i(s^*))=\sum_{i\in I}\tilde{z}_i(\pi(p))=0$. This result implies that unique prevailing price vector $\pi(p)$ is a Walrasian equilibrium one. Furthermore from the supposition that $(\forall j\neq 0):(\exists i\in I):(f_j^i(s^*)\neq 0)$ it follows directly that $(\forall j\neq 0):(\exists i\in I):(\tilde{z}_j^i(\pi(p))\neq 0)$.
(Walras equilibrium→Nash equilibrium); Let the list of strategies $(s_i^*)_{i\in I}=s^*$ be a nontrivial Walras equilibrium. We denote $s_i^*=(p,\ z_i^*)$ for any consumer i. Then from the definition of nontrivial Walras equilibrium it follows that

$(\forall i \in I) : (\delta(f^i(s^*)) = \tilde{z}_i(\pi(p)))$ and $(\forall j) : (\exists i \in I) : (\tilde{z}_{ij}(\pi(p)) \neq 0)$. We denote consumer i's Nash strategy for s^* by $s_i^N(s^*) = (p^i, z_i^N)$.

From the definition of the trading rule it is trivial that $(\forall i \in I) : (p^i = p)$. Furthermore each consumer achieves his Walrasian net trade, that is, $(\forall i \in I) : (\delta(f^i(s^*)) = \tilde{z}_i(\pi(p)))$. Hence we have $(\forall i \in I) : (z_i^N(s^*) = z_i^*)$.

These results imply that $(\forall i \in I) : (s_i^N(s^*) = s_i^*)$. Finally, from the supposition that $(\forall j) : (\exists i \in I) : (\tilde{z}_{ij}(\pi(p)) \neq 0)$ it follows directly that $(\forall j) : (\exists i \in I) : (f_j^i(s^*) \neq 0)$. This completes the proof. ||

Ⅳ. Concluding Remark

In this article, we have formulated the noncooperative bidding game with explicitly treating the effect of each agent's bidding behaviour upon the market price formation. We considered the monetary exchange economy, which is different from Shapley [3] and Shapley and Shubik [4], and constructed a reasonable trading rule. Then we examined the relation between the noncooperative solution and the Walras equilibrium. By noting the *Remark* the result of *Theorem 1* may support the usual interpretation of the Walras equilibrium. Suppose the *replica economy* which consists of a sufficiently large number of consumers of the same type and suppose that each agent of the same type bids the same price vector (*similarity of the bidding behaviour*). Then this economy satisfies the regular property of *Definition 5* and the *Theorem 1* can be applied.

We must also consider the cases where coalitional behaviours are allowed and the number of market participants is few. This is an interesting investigation but it remains with future one.

* Some of the preliminary work on this was done while the author was a research fellow of the Japan Society for the Promotion of Science: Hitotsubashi University. I gratefully acknowledge the advice and encouragement of Professors Kenjiro Ara (Hitotsubashi University) and Sho-ichiro Kusumoto (University of Tsukuba). Financial assistance from the Japan Society for the Promotion of Science and the Tokyo Center for Economic Research is also acknowledged. Needless to say, any errors are the author's responsibility.

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