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R&D, Privatization, Public Monopoly, Mixed Oligopoly, and Productive Efficiency*

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I. Introduction

In a recent paper in *Australian Economic Papers* Nishimori and Ogawa (2002) derive an interesting result that a monopolistic public firm has a greater incentive to make a cost–reducing R&D investment than a public firm in a mixed oligopoly when the firm only makes its investment in the first stage. Although it has been taken for granted that introducing private firms into a monopolistic public market will cause the public firm to conduct more investment, our impression is overthrown admirably by their result.

In order to furthermore investigate cost–reducing R&D behavior of public firms they use a simple model in which products in the mixed oligopoly are perfect substitutes, i.e. homogenous, and the public and private firms are involved in Cournot–type–quantity competition. In this paper we extend the analysis of Nishimori and Ogawa (2002) in three directions: First, we introduce product differentiation into their model, although it is limited to the case without product differentiation; second, we investigate the R&D behavior of public firms; and third, we examine the effects of privatization of public firms. Specifically, public, privatized public, and private firms in a mixed oligopoly with Cournot–type–quantity and Bertrand–type–price competition produce either substitutes or complements. We compare the levels of their cost–reducing R&D investments in differentiated–product oligopoly models in both Cournot–type–quantity and Bertrand–type–price competition. We can observe product differentiation in a lot of industries except for the material industries like iron and steel products, and chemical and allied ones. In particular, its tendency is marked in consumer's goods industries. In contrast, industries producing homogenous goods are rather rare. Public utilities are also no exception, because there exist private firms producing differentiated goods. In addition, firms in oligopolies employ not only quantity–setting strategies but also price–setting strategies. Therefore, there are no reasons to limit the analysis to only oligopolies with quantity competition. It is meaningful to elucidate whether and how product differentiation makes production efficiency in public firms improve. This paper provides a criterion for deciding which policy to choose, e.g. deregulation or not.

In advanced countries public firms have been widely privatized for more than these twenty years for the purpose of improving productive efficiency. At the same time, the effects of privatization on market

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performance such as output, prices, profits and welfare under quantity and price competition have been continued by a lot of literature (see, for example, Merrill and Schneider (1966), De Fraja and Delbone (1988, 1990), White (1996), and Pal and White (1998)). However, we have heard nothing about the theoretical analysis concerning the effects of privatization on R&D investment of public firms yet. Our analysis is the first attempt to investigate the relationship between them, and will contribute to the theory of public firms and industry policy on R&D.

This paper shows the following two main results. One is that the result of Nishimori and Ogawa holds in a Cournot oligopoly with imperfect substitutes, while we have the opposite result to their result in the oligopoly with imperfect complements: That is, the public firm has a less incentive to conduct cost–reducing investment in the public monopoly than in the mixed oligopoly. The other is that their result does not always hold in Bertrand oligopoly. In short, we conclude that their result is proper only in the limited case.

II. Public Monopoly

Let us consider the representative consumer' utility, $U(q_0, q_1)$. His utility is given as follows.

$$U(q_0, q_1) = a(q_0, q_1) - (q_0^2 + 2bq_0q_1 + q_1^2)/2, \quad a > 0 \text{ and } -1 \leq b \leq 1,$$

where q_0 and q_1 are differentiated goods. Then from utility maximization we obtain the following inverse demand functions for the two markets:

$$p_0 = a - q_0 - bq_1 \text{ and } p_1 = a - q_1 - bq_0, \tag{1}$$

where q_i is the output of firm $i(=0, 1)$ and p_i is its output price. The cross–price effects are symmetric. The products are substitutes, complements, and independent according as $0 < b < 1$, $-1 < b < 0$, and $b = 0$. Particularly, they are perfect substitutes (homogenous) if $b = 1$, perfect complements if $b = -1$, and differentiated if $b \neq 1$ and -1. The market 0 is monopolized by the firm 0 if $b = 0$. With these inverse demand functions, the index of product differentiation can be given by b^2.

We fundamentally follow Nishimori and Ogawa (2002) with respect to the use of notation. For the time being, we consider the short–run and long–run decision makings of a publicly owned firm. Namely, in the short run (in the second stage) the public firm determines output so as to maximize social surplus (welfare) SS and in the long run (in the first stage) the level of technology so as to maximize SS. Social surplus is now defined as the sum of producer and consumer surplus, PS and CS. It is assumed that the cost function of the firm is $c_0(q_0) = c_0q_0$. The firm can now reduce marginal cost c_0 by cost–reducing R&D investment in the first stage. In order to reduce marginal cost by c_0 for production the firm must expend $f(c_0)$ on it, where $f'(c_0) < 0$, and

[1] For example, when the expenditure of R&D investment is $f(c_0) = \alpha - \beta c_0 + c_0^2$, $\alpha > 0$, $\beta > 0$, and $c_0 < \beta/2$, this function satisfies the assumption on it.

$f''(c_0) > 0$, where $f(c_0)$ is a convex function of c_0 [1]. Since there exists only one public firm in the market, it follows that the firm 0 faces an inverse demand function such as $p_0 = a - Q$, where $a > c_0$, and Q now stands for the output of the monopolistic public firm for convenience.

We turn to a two–stage optimization problem for the public firm in a monopoly market. We solve this problem by using backward induction. The maximization problem in the second stage for the public firm is given by

$$\max_{Q} \ SS = CS + PS = \int_0^Q p(t)dt - p(Q) \cdot Q + \pi_0^m$$

$$= aQ - \frac{Q^2}{2} - c_0 Q - f(c_0),$$

where $CS = Q^2/2$, and $PS = \pi_0^m = (p_0 - c_0)Q - f(c_0) = (a - c_0)Q - Q^2 - f(c_0)$ is the public firm's production profits net of R&D expenditure in the first stage. Then the first–order condition is obtained:

$$\frac{dSS}{dQ} = a - Q - c_0 = 0. \tag{2}$$

Note that the public firm follows the rule of pricing at marginal cost, i.e. $p_0^m = c_0$. Therefore, the equilibrium output of the monopolistic public firm is given as $Q^m = a - c_0$. Consumer surplus and producer surplus are given by $CS = (a - c_0)^2/2$ and $PS = -f(c_0)$, respectively, so social surplus is $SS = (a - c_0)^2/2 - f(c_0)$. On the other hand, the effect of cost–reducing R&D is reduced to $dQ^m/dc_0 = -1$: Namely, a cost reduction of one unit increases output of one unit. Note that consumer surplus increases but producer surplus decreases with reduced costs.

Next let us turn to the first stage decision of the firm. The maximization problem in the first stage is reduced to maximize social surplus with respect to c_0. Differentiating SS with respect to c_0 yields

$$\frac{dSS}{dc_0} = \frac{\partial SS}{\partial Q}\frac{dQ}{dc_0} + \frac{\partial SS}{\partial c_0} = -\left[a - c_0 + f'(c_0)\right] = 0, \tag{3}$$

where $\partial SS/\partial Q = 0$ from the first–order condition (2)[2]. Then the equilibrium R&D investment c_0^m of the public firm is derived from (3), and $a - c_0^m + f'(c_0^m) = 0$ holds. It turns out that social surplus is $SS^m = (a - c_0^m)^2/2 - f(c_0^m)$. Now R&D investment incurs a deficit and the public monopoly operates under negative profits, $\pi_0^m = -f(c_0^m) < 0$. This implies that even if marginal costs are constant, public firms will not be able to survive in markets as long as they undertake R&D investment to reduce production costs. This is because the firms abide by the rule of pricing at marginal cost.

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[2] The second–order condition in the first stage is satisfied.

III. Mixed Oligopoly under Cournot Competition

We consider a mixed oligopoly. For simplicity, we take up a mixed duopoly composed of a private owned firm and the public firm, different from Nishimori and Ogawa (2002). By this simplicity the results that will be obtained do not undergo a change. The inverse demand functions for the mixed duopoly with product differentiation are given by (1), as demonstrated above. The private firm's cost function for production is given as $c_1(q_1) > \overline{c}q_1$. Moreover, it is assumed that the public firm is less efficient than the private firm, i.e. $c_0 > \overline{c}$ [3]. In addition, it is assumed that $a - \overline{c} > 2c_0$.

In this case the model is turned to a two–stage game model in which both firms noncooperatively choose output in the second stage, but the public firm chooses the level of R&D investment in the first stage. In particular, the private firm determines output so as to maximize its profit in the second stage, while the public firm determines both the level of investment and output so as to maximize social surplus in the first and second stages, respectively. Only the public firm can make an investment in technological innovation to reduce production costs, so that the firm behaves like a Stackelberg leader in the game. Now the profit functions of both firms are represented by $\pi_0 = (p_0 - c_0)q_0 - f(c_0)$ and $\pi_1 = (p_1 - \overline{c})q_1$, respectively. On the other hand, the social surplus is given as follows:

$$SS = U(q_0, q_1) - p_0 q_0 - p_1 q_1 + \pi_0 + \pi_1$$

$$= U(q_0, q_1) - c_0 q_0 - f(c_0) - \overline{c} q_1 .$$

The first–order conditions for the private and public firms in the second stage are[4]

$$\frac{\partial SS}{\partial q_0} = a - c_0 - q_0 - bq_1 = 0 \tag{4}$$

$$\frac{\partial \pi_1}{\partial q_1} = a - \overline{c} - bq_0 - 2q_1 = 0. \tag{5}$$

Solving the conditions (4) and (5), we have the equilibrium outputs such as

$$q_0 = \frac{2(a - c_0) - b(a - \overline{c})}{2 - b^2} \quad \text{and} \quad q_1 = \frac{a - \overline{c} - b(a - c_0)}{2 - b^2}$$

Note that a reduction in c_0 leads to an increase in the output of the public firm and a decrease in the output of the

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[3] This assumption is employed in Nishimori and Ogawa (2002), and others. But this assumption may be inappropriate, as shown by the empirical results of Nelson (1990).

[4] The second–order conditions are satisfied, and the equilibrium in the first stage is locally stable.

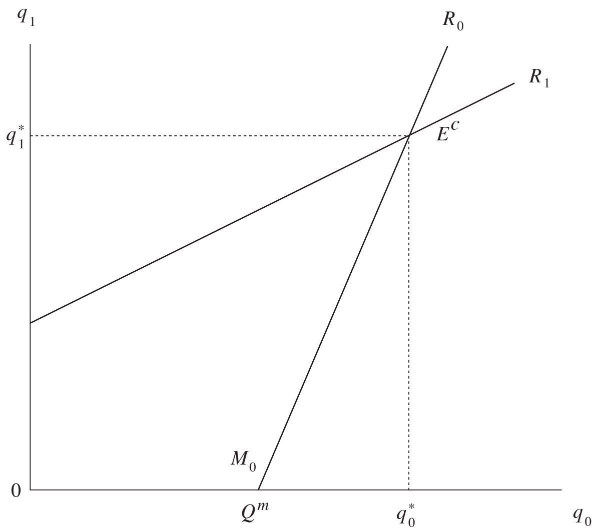

Figure1 Reaction Curves in Cournot Competition with Complements

private firm. In particular, a cost reduction of one unit increases more than one unit of the public firm's output, $dq_0/dc_0 = -2/(2-b^2) \leq -1$. The effect of cost reduction on output of a public firm is greater than the effect on output of a public monopoly. The equilibrium prices of the two markets are derived as follows: $p_0 = c_0$ and $p_1 = \{a + \overline{c} - b(a - c_0) - b^2\overline{c}\}/(2-b^2)$. The price p_1 converges to $p_1 = c_0$ according as the degree of product differentiation gets smaller. In the market of product 0 the rule of pricing at marginal cost is kept, but in the other market it is not, whereas the prices are $p_0 = p_1 = c_0$ as long as the products are homogeneous. From these conditions we obtain the reaction functions of the public and private firms. Both reaction curves slope downwardly (upwardly) as b is positive (negative). In the Figure 1 the upward–sloping reaction curves of the public and private firms are illustrated. The intersection E^C of both curves is Cournot equilibrium in the mixed duopoly. The point M_0 on the horizontal line denotes the output Q^m of the public firm when it monopolizes the market.

In the first stage the public firm determines the level of R&D investment so as to maximize the social surplus SS. Then, when we use the envelop theorem and (4), the first–order condition for this maximization is yielded as[5]:

$$\frac{dSS}{dc_0} = \frac{\partial SS}{\partial q_0}\frac{dq_0}{dc_0} + \frac{\partial SS}{\partial q_1}\frac{dq_1}{dc_0} + \frac{\partial SS}{\partial c_0} = q_1\frac{b}{2-b^2} + \frac{\partial SS}{\partial c_0} = 0, \tag{6}$$

where $\partial SS/\partial c_0 = -q_0 - f'(c_0)$. Hence it follows that the optimal cost–reducing investment of the public firm in the mixed duopoly is obtained as c_0^* at which $bq_1(c_0^*)/(2-b^2) + \partial SS(c_0^*)/\partial c_0 = 0$. Finally, the equilibrium

[5] The second–order condition is assumed to be satisfied, $d^2SS/dc_0^2 = b/(2-b^2)^2 - f''(c_0) < 0$, where $dq_1/dc_0 < 0$. If b is not positive, then the condition is unambiguously satisfied.

outputs and prices are $(q_0^*, q_1^*) = [q_0(c_0^*), q_1(c_1^*)]$ and $(p_0^*, p_1^*) = \left[c_0^*, (a + \overline{c} - b(a - c_0^*) - b^2\overline{c})/(2 - b^2)\right]$. The effects of cost–reducing R&D on the prices depend on the sign of b: Namely, an increase in its R&D leads to a reduction in the price p_0 and to a reduction (rise) in the price p_1 when the products are substitutes (complements). The profits of the public and private firms are $\pi_0^* = -f(c_0^*)$ and $\pi_1^* = \left[(a - \overline{c} - b(a - c_0^*))/(2 - b^2)\right]^2 = (q_1^*)^2$. Similarly, whether an increase in R&D has an advantageous or a disadvantageous effect on the profits of the firms also depend on the sign of b. On the other hand, both consumer surplus and producer surplus are given as $CS = \left[(q_0^*)^2 + 2bq_0^*q_1^* + (q_1^*)^2\right]/2$ and $PS = -f(c_0^*) + (q_1^*)^2$. Note from the results above that producer surplus unambiguously decreases with reduced costs c_0 when the products are substitutes and may or may not increase with them when they are complements.

Let us compare the innovative investments of the public firm in the monopoly and mixed oligopoly markets. Using (3) and (6), and evaluating (6) at the marginal costs of the monopolistic public firm, $c_0 = c_0^m$, we obtain

$$\left.\frac{dSS}{dc_0}\right|_{c_0 = c_0^m} = q_1 \frac{b}{2 - b^2} \lesseqgtr 0 \qquad \text{as } b \lesseqgtr 0. \tag{7}$$

This implies that $c_0^m \lesseqgtr c_0^*$ as $b \lesseqgtr 0$. We have the following proposition.

Proposition 1. When a public and private firms in a mixed oligopoly market are involved in quantity competition, (i) the public firm has a greater incentive to invest in cost–reducing R&D in a public monopoly market than in that market if the products are substitutes, and, furthermore, its incentive reduces according as the degree of product differentiation gets smaller; and (ii) the public firm has a less incentive to invest in cost–reducing R&D in the public monopoly market than in the mixed oligopoly market if the products are complements.

These results fundamentally hold, independent of the number of private firms in the mixed oligopoly, although Nishimori and Ogawa (2002)' result corresponds to the case of homogeneous good. It is of interest that R&D incentives for public firms depend crucially on the feature of products. The intuition behind Proposition 1 is as follows. First, we take the result (i) of the proposition. When the products are substitutes, a reduction in costs c_0 in the public monopoly market leads to an increase in consumer surplus and to a reduction in producer surplus, so social surplus will increase with their reduction. On the other hand, reduced costs in the mixed oligopoly lead to an increase in consumer surplus in the main market 0 for the public firm, and an increase in consumer surplus in the sub market 1 for it, but such an increase is not so large, because the output q_1 decreases with the reduced costs. Moreover, producer surplus is unambiguously decreased by its reduction. It seems that a reduction in production costs due to R&D investment does not cause social surplus to amply increase in the mixed oligopoly, compared with the public monopoly case. It turns out that a public monopoly has a greater incentive to conduct R&D investment than its counterpart in a mixed oligopoly. Next turn to the result (ii). When the products are complements, a reduction in production costs obviously increases consumer surplus, because their reduction increases both outputs. In contrast, since the output q_1 increases, producer surplus is likely to increase, while its surplus in the public monopoly market decreases with reduced costs, as mentioned above. Consequently, social surplus in the mixed oligopoly is amply raised by R&D investment, compared with the public monopoly case. It turns out that less cost–reducing R&D investment is lead in the public monopoly than in the mixed oligopoly.

IV. The Effects of Privatization of a Public Firm

Let us investigate the effects of privatization of a public firm on R&D investment. We first consider the equilibrium outcomes when the public firm is privatized and maximizes profits instead of social surplus. Then we take a duopoly of a privatized public firm and a private profit–maximizing firm, not a mixed duopoly of the public and profit–maximizing firms. The objective functions of the two firms are $\pi_0 = p_0 q_0 - c_0 q_0 - f(c_0)$ and $\pi_1 = p_1 q_1 - \overline{c} q_1$, respectively. In the second stage of the game they choose output so as to maximize their profits. The first–order conditions for both firms are given as follow:

$$\frac{\partial \pi_0}{\partial q_0} = a - c_0 - 2q_0 - bq_1 = 0 \tag{8}$$

$$\frac{\partial \pi_1}{\partial q_1} = a - \overline{c} - bq_0 - 2q_1 = 0. \tag{9}$$

The second–order conditions are satisfied, and the second stage equilibrium is also locally stable. The Cournot equilibrium outputs of the privatized public and private firms are obtained from these conditions as $(q_0, q_1) = \left[\left(2(a - c_0) - b(a - \overline{c})\right)/(4 - b^2), \left(2(a - \overline{c}) - b(a - c_0)\right)/(4 - b^2)\right]$. This shows that consumer surplus is larger in a mixed duopoly than in a pure duopoly after privatization. The effect of cost reduction on the output of the privatized public firm is $dq_0/dc_0 = -2/(4 - b^2)$: Namely, the cost reduction of one unit increases less than its output of one unit, which is, in turn, less than that effect on the public firm. It follows that the equilibrium is an interior solution.

Let us turn to the R&D decision of the privatized public firm in the first stage. The firm determines the level of R&D investment so as to maximize its profit. Then the first–order condition is given as follows:

$$\frac{d\pi_0}{dc_0} = \frac{\partial \pi_0}{\partial q_0}\frac{dq_0}{dc_0} + \frac{\partial \pi_0}{\partial q_1}\frac{dq_1}{dc_0} + \frac{\partial \pi_0}{\partial c_0} = -bq_0 \frac{dq_1}{dc_0} - \left[q_0 + f(c_0)\right] = 0, \tag{10}$$

where $\partial \pi_0 / \partial q_0 = 0$ from (8)[6]. The equilibrium R&D level c_0^* of the firm is obtained from (10), so that the equilibrium outputs and prices are given as $(q_0^{**}, q_1^{**}) = \left[q_0(c_0^{**}), q_1(c_0^{**})\right]$ and $(p_0^{**}, p_1^{**}) =$

[6] The second–order condition is satisfied for any b, $d^2\pi_0/dc_0^2 = -4/(4 - b^2)^2 - f''(c_0) < 0$.

$\left[\left(2a + c_0^{**}(2 - b^2) - b(a - \overline{c})\right)/(4 - b^2), \left(2a + \overline{c}(2 - b^2) - b(a - c_0^{**})\right)/(4 - b^2)\right]$. Given homogenous product, the outputs are reduced to $(q_0^{**}, q_1^{**}) = \left[(a - 2c_0^{**} + \overline{c})/4, (a - 2\overline{c} + c_0^{**})/4\right]$. Note that the equilibrium outputs in the mixed duopoly are larger than those in the duopoly after privatization, i.e., $(q_0^{*}, q_1^{*}) > (q_0^{**}, q_1^{**})$, irrespectively of whether the products are substitutes or complements. Then, it follows that $(p_0^{*}, p_1^{*}) < (p_0^{**}, p_1^{**})$ for both $b > 0$ and $b < 0$. When production costs of the public firm are reduced by R&D investment, the price p_0 decreases and the price p_1 decreases (increases) according as the products are substitutes (complements). Furthermore, we obtain the profits of the two firms as $\pi_0^{**} = (q_0^{**})^2 - f(c_0^{**})$ and $\pi_1^{**} = (q_1^{**})^2$. Finally, consumer surplus and producer surplus are given as $CS = \left[(q_0^{**})^2 + 2bq_0^{**}q_1^{**} + (q_1^{**})^2\right]/2$ and $PS = (q_0^{**})^2 - f(c_0^{**}) + (q_1^{**})^2$, respectively.

We make a comparison between the R&D investment levels of the public and privatized public firms. Using the first–order conditions (6) and (10) in the first stage, we have

$$\frac{dSS}{dc_0} = b\left(\frac{q_1}{2 - b^2} + \frac{bq_0}{4 - b^2}\right) + \frac{d\pi_0}{dc_0} = 0,$$

where $q_1/(2 - b^2) + bq_0/(4 - b^2) = \{(a - \overline{c})(b^2 - 4b + 8) - (a - c_0)b(b^2 - 2b + 4)\}/(4 - b^2)^2(2 - b^2) > 0$. The sign of the first term in the parentheses on the right hand side of the equation above is dependent of that of b. Now, when we evaluate the first–order condition of the public firm at the optimal point of the privatized public firm, dSS/dc_0 is positive as long as the products are substitutes, while it is negative as long as they are complements. The following proposition summarizes this result.

Proposition 2. When a public and private public firms in a mixed oligopoly are involved in quantity competition, (i) the public firm has a greater incentive to undertake cost–reducing R&D than the public firm privatized if the products are substitutes, i.e. $b > 0$, while the public firm has a less incentive to undertake cost–reducing R&D than the privatized public firm if the products are complements, i.e. $b < 0$.

The intuitive explanation behind this result is as follows. Given $b > 0$, a reduction in production costs causes the prices p_0 and p_1 in both oligopolies to decrease, so welfare increases. But their reduction causes the profits of the privatized public firm to decrease, because the expenditure on R&D increases. Consequently, the public firm has a greater incentive to invest in cost–reducing R&D than the privatized one. Given $b < 0$, a reduction in c_0 leads to a decrease in p_0 and an increase in p_1 in both oligopolies, and, in particular, the price p_0 rises more in the mixed oligopoly than in the privatized oilgopoly. This means that reduced c_0 will not increase so much welfare in the former, compared with the latter. In contrast, reduced c_0 increases the profits of the privatized public firm, because it increases its output. It turns out that the privatized public firm has a greater incentive for R&D than its counterpart.

We note from the propositions 1 and 2 that when products are substitutes, incentives for public firms to make an R&D investment in order to reduce production costs become weaker as quantity competition in markets gets keen. Put differently, productive efficiency of the firms decreases with market intensification. This demonstrates that if the purpose of their privatization is to enhance productivity, then such a deregulation will be irrelevant when there is product differentiation among public firms, and if anything, the opposite policy like regulation and nationalization will be preferable for its enhancement. Thus existence of public firms is justified for an improvement in productivity as well as welfare. In contrast, this is not the case when products are complements: Namely, productivity of public firms will be raised by their privatization or deregulation, as commonly adovocated. This implies that the deregulation policy of the government for public firms is not always optimal to increase their productivity by process innovation such as R&D.

V. Mixed Oligopoly under Bertrand Competition

We consider a mixed duopoly of the Bertrand type in which one public firm and one private firm involve in price competition in the second stage, while a public firm only makes an investment in cost–reducing R&D in the first stage.

The following demand functions both products are derived from the inverse ones (1):

$$q_0 = \frac{a}{1+b} - \frac{1}{1-b^2}p_0 + \frac{b}{1-b^2}p_1$$

$$q_1 = \frac{a}{1+b} + \frac{b}{1-b^2}p_0 - \frac{1}{1-b^2}p_1 . \tag{11}$$

When the public and private firms simultaneously choose their outputs, their first–order conditions for maximization in the second stage are obtained, respectively, as follows[7]:

$$\frac{\partial SS}{\partial p_0} = \frac{c_0 - b\overline{c} - p_0 + bp_1}{1-b^2} = 0 \tag{12}$$

$$\frac{\partial \pi_1}{\partial p_1} = \frac{a(1-b) + \overline{c} + bp_0 - 2p_1}{1-b^2} = 0. \tag{13}$$

The reaction curves of the two firms are derived from these conditions. Note that the reaction curve of the private firm is independent of c_0, different from that of the counterpart in the mixed duopoly with quantity competition. The reaction curves slope upwardly or downwardly according as the products are substitutes or complements. From (12) and (13) we have the equilibrium prices such as

[7] The second–order conditions are satisfied, and the equilibrium is locally stable.

$$p_0^{B**} = \frac{ab(1-b) - b\overline{c} + 2c_0}{2-b^2} \quad \text{and} \quad p_1^{B**} = \frac{a(1-b) + (1-b^2)\overline{c} + bc_0}{2-b^2} \tag{14}$$

The public firm's maximization problem in the first stage is to maximize the social surplus. Then the first–order condition is given by

$$\frac{dSS}{dc_0} = \frac{\partial SS}{\partial p_0}\frac{dp_0}{dc_0} + \frac{\partial SS}{\partial p_1}\frac{dp_1}{dc_0} + \frac{\partial SS}{\partial c_0}$$

$$= \frac{b\left[-a(1-b)^2(1+b) + bc_0 - \overline{c}\right]}{(2-b^2)^2(1-b)} + \frac{\partial SS}{\partial c_0} = 0, \tag{15}$$

where $dp_1/dc_0 = b/(2-b^2)$, and $\partial SS/\partial p_0 = 0$ from (12), and the output of the public firm is $q_1^{**} = \left[a(1-b) + bc_0 - \overline{c}\right]/(2-b^2)(1-b^2)$. This condition yields the equilibrium investment level of c_0^{**} of the public firm in the mixed duopoly under Bertrand competition.

In order to make a comparison between R&D investments of the public firm in the public monopoly and mixed duopoly, we evaluate the condition (15) at $c_0 = c_0^m$. Then it follows that

$$\left.\frac{dSS}{dc_0}\right|_{c_0=c_0^m} = \frac{b\left[-a(1-b)^2(1+b) + bc_0 - \overline{c}\right]}{(2-b^2)^2(1-b)}. \tag{16}$$

This implies that the sign of $(dSS/dc_0)_{c_0=c_0^m}$ depends on the sign of the numerator: Namely, whether the public firm invest more or less in the public monopoly than in the mixed duopoly is dependent on the numerator. Then we can classify the possibilities as follows: At $c_0 = c_0^m$, (a) $(dSS/dc_0)_{c_0=c_0^m}$ is positive if b is negative; (b) $(dSS/dc_0)_{c_0=c_0^m}$ is of either sign if b is positive; and (c) $(dSS/dc_0)_{c_0=c_0^m}$ is zero if b is zero. The result (a) implies that if the products are complements, then the firm invests more in the monopoly than in the mixed duopoly. This is the same as that derived in the quantity–setting duopoly with substitutes. On the other hand, in the case in which products are substitutes, we cannot specify whether or not the public firm has a greater incentive to conduct cost–reducing R&D investment in the public monopoly than in the mixed duopoly. However, as far as $b \leq \overline{c}/c_0 < 1$, $(dSS/dc_0)_{c_0=c_0^m} > 0$ holds at $c_0 = c_0^m$ and, otherwise, its sign is ambiguous. We summarize these results as the following proposition.

Proposition 3. Suppose that public and private firms in a mixed oligopoly market are involved in price competition. (i) If the products are complements, then a public firm has a greater incentive to invest in cost–reducing R&D in the public monopoly market than in the mixed oligopoly market; and (ii) if they are substitutes, then the public firm has a less incentive to invest in cost–reducing R&D in the public monopoly market than in the mixed oligopoly market as long as the ratio of the marginal cost of the private firm to that of the public firm is larger than the cross–price effect.

The case with $b = 0$ is reduced to the public monopoly case. As shown above, the possibility that the result that public firms have a less incentive to invest in R&D does not hold is small, but we cannot ignore it. The cause that the possibility is brought about is due to the effect of a change in the price p_1 on social surplus. For example, given $b < 0$, a reduction in the price p_1 through R&D investment increases social surplus, while, given $b > 0$, such a reduction leads to an increase in q_0 and a decrease in q_1, so that the sign of (16) becomes ambiguous, as a result of their composite result. Owing to the results of Propositions 1 and 3, the symmetry of the results derived under both Cournot and Bertrand oligopoly models fails to hold.

VI. Conclusion

This paper extends the paper of Nishimori and Ogawa (2002) to two directions: Namely, we analyze public firms in Bertrand oligopoly model as well as in Cournot oligopoly model, and, furthermore, introduce product differentiation. Then we can get several new results concerning the R&D behavior of public firms.

We note that the results in the price–setting model are different from those in the quantity–setting model, and, moreover, both of them are not symmetric. The public firm has an inclination to make a more cost–reducing investment in the Bertrand price–setting model than in the Cournot quantity–setting model.

In the Bertrand price–setting model with homogenous goods we must pay attention to consideration. In this case, when public and private firms are involved in price competition, the private firm chooses to set its price at slightly less than the marginal cost of the public firm to expel it if the assumption $c_0 > \overline{c}$ is kept[8]. The private firm can get positive profits because its price exceeds its average cost. Even if the public firm conducts cost–reducing R&D investment, the private firm copes with it by reducing its price. This shows that the result of Nishimori and Ogawa (2002) does not hold in Bertrand oligopoly model. This implies that such an assumption adopted in Cournot–type–quantity competition is not justified in Bertrand–type–price competition.

More interestingly, if the purpose of their privatization is to enhance productivity, then such a deregulation will be irrelevant when there is product differentiation among public firms, and if anything, the opposite policy like regulation and nationalization will be preferable for its enhancement. Thus existence of public firms is justified for an improvement in productivity as well as welfare. In contrast, this is not the case when products are complements: Namely, productivity of public firms will rise as a result of their privatization or deregulation. This implies that the relationship between the deregulation policy on public firms and an increase in their productivity are not straightforward, but depends on the feature of products.

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8 Pal (1998) also adopts the same assumption.

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R&D, Privatization, Public Monopoly, Mixed Oligopoly, and Productive Efficiency

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We investigate R&D behavior of public firms by using oligopoly models with Cournot–type–quantity and Bertrand–type–price competition. Then the following results are obtained. When public and private firms in a mixed oligopoly market are involved in quantity competition, (i) the public firm has a greater incentive to invest in cost–reducing R&D in a public monopoly market than in that market if they are substitutes, and (ii) the public firm has a less incentive to invest in R&D in the oligopoly market if they are complements. Furthermore, suppose that public and private firms in a mixed oligopoly market are involved in price competition. (i) If the products are complements, then a public firm has a greater incentive to invest in cost–reducing R&D in the public monopoly market than in the mixed oligopoly market; and (ii) if they are substitutes, then the public firm has a less incentive to invest in cost–reducing R&D in the public monopoly market than in the mixed oligopoly market.