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The Optimal Subsidies to Higher Education, with Reference to Education and Research*

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1 Introduction

In advanced countries, governments intervene in individual educational decisions. This is because education has some characteristics different from other private goods. Governments enforce individuals to receive compulsory education without charging tuition fees. By receiving it, people can achieve at least some minimum cultural standard, and develop integrated–values as citizens, so that the foundation of a democratic state is formed [Friedman (1955)]. Furthermore, for higher education, while people can choose it independently, government affects their decisions by subsidizing their expenditures. Subsidies to higher education are usually said to be necessary to guarantee equal opportunity and to equalize the income distribution. Also, it has been argued from the view point of resource allocation that, since education has external effects, imposing the full cost of education on individuals makes for the under–supply of education [Weisbrod (1960), Pauly (1967)].

It is, however, still controversial whether government intervention in higher education does indeed realize equality of income distribution. In fact, most students receiving higher education belong to higher income classes. Then, subsidies to higher education financed by general tax imply regressive income redistribution [Fernandez and Rogerson (1995)]. Therefore, subsidies should be given directly to qualified students rather than to educational institutes. Alternatively, the students belonging to lower income class should be given an educational loan with zero or low interest.

It is also doubtful whether higher education really has external effects. Several economists have discussed the external effects of higher education. Among them, Welch (1970) argued that educated labor improves the productivity of firms by making information exchange and job allocation in the production process more effective ; and Denison (1974) suggested that educated labor contributes to innovation in new products and technology, and therefore to economic growth. If wages are not paid according to their contribution, there is an external effect. However, this analysis is not decisive. We shall distinguish the external effects of higher education in the production process and insist on the necessity of an educational subsidy, concentrating on the role of educational institutes.

We view educational institutes as constituting the educational sector and producing two educational outputs,

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education and research output. The former contributes to individual human capital formation, and the latter enhances the stock of social knowledge.[1] Both education and research activities are essential objectives of higher institutes. Social knowledge produced by research activity is a public good implying that firms can freely use it to develop new products or raise productivity. On the other hand, the human capital of a worker raises not only his productivity, but also his firm's productivity as a whole. Human capital accumulated in the firm plays an important role in organizing internal organization and in utilizing existing social knowledge. The extent of such effects might depend on the average level of human capital of workers [Lucas (1988)]. We consider that human capital has external effects in the sense that the firm does not take account of such effects in its maximizing behavior.

In this paper, we define the educational sector as one that produces two outputs jointly, education forming human capital and research increasing the stock of social knowledge. Our purposes are to clarify the nature of higher education and analyze the efficient resource allocation in a society consisting of educational and production sectors, considering the external effect of human capital and research activity in a context of an endogenous growth model [Lucas (1988), Romer (1990)]. In section 2 we present the model and in section 3 the optimal resource allocation of educational sector is discussed. Educational policy is proposed in section 4 followed by concluding comments in section 5.

2 The Model

We consider an overlapping–generations model in which individuals live for two periods. They receive education in the first period (as young agents), and work in the second period (as old agents). As for education, we concentrate here on higher education, assuming that individuals have already been given primary and secondary education. We consider a society which consists of two sectors of production; the educational and production sectors. Old agents work in either sector.

2.1 Educational sector

In the educational sector, two types of outputs, education service and research output, are produced jointly by the educational labor.[2] Education service gives an educational achievement to all young agents. We assume that the quality of educational achievement is the same for all young agents. The educational sector plays the role of research sector as showed by Romer (1990), since this sector engages in research activity as well as educational activity. We represent the educational production function in an inverse form:[3]

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1 As for the latest knowledge concepts, see Nahuis (2003).

2 Eicher (1996) constructs a model where the educational process produces research output as by–products.

3 Rothchild and White (1995) define a similar educational production function. While they treat students as one of the inputs,we disregard the input, assuming it to be constant.

$$L_t^E = E(e_t, R_t), \tag{1}$$

where L_t^E, e_t, and R_t are the amount of educational labor, educational achievement, and research output in period t, respectively. Following Rothchild and White (1995), we assume that the technology of the educational sector exhibits constant return to scale with respect to both arguments and that $E_e > 0$, $E_R > 0$, $E_{ee} > 0$, $E_{RR} > 0$, where subscripts represent partial differentiation.

We assume that educational achievement contributes to individual human capital formation, and that human capital is defined by the following formation function:

$$h_{t+1} = h(e_t), \tag{2}$$

where h_{t+1} is human capital in period $t+1$, and $h_e > 0$, $h_{ee} < 0$.

The research output is assumed to increase the social knowledge of the next period. Assuming that the knowledge depreciates at the rate ρ $(0 < \rho < 1)$, the accumulation of social knowledge is

$$H_{t+1} = R_t + (1-\rho)H_t, \tag{3}$$

where H_{t+1} is the social knowledge stock in period $t+1$. This knowledge stock is utilized in the production sector.[4]

2.2 Production sector

Production of consumption goods depends on technology available in the period as well as labor input. The level of technology is determined by how well the firm utilizes social knowledge in the production process. We assume that the absorption of technology into the production process depends on the average level of human capital of employed workers.[5] Then the technology is represented by the following technological function:

$$A_t = A(\overline{h}_t, H_t), \tag{4}$$

where A_t is the level of technology and $\overline{h}_t$ the average human capital of employed workers in period t. We assume that the technology function is strictly increasing and quasi–concave with respect to both arguments. We then define the production function as:

$$f(L_t, A_t), \tag{5}$$

where L_t is the amount of labors in production sector in period t. We assume that the function is also strictly increasing and quasi–concave in both arguments.

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[4] Mansfield (1991) estimates the extent to which technological innovations in various industries have been based on recent academic research, and the time lags between the investment in recent academic research projects and the industrial utilization of their findings.

[5] Eicher (1996) points out that skilled labor is essential in the absorption of innovation into production.

2.3 Labor constraints

Let l_t and l_t^E be the number of workers employed in the production sector and in the educational sector in period t, respectively. For simplicity, we assume that there is no population growth in the economy, and that total population is normalized to one. Then,

$$l_t + l_t^E = 1\ . \tag{6}$$

We assume that each old agent supplies one unit of labor time inelastically, and that the amounts of individual labor supply in an efficiency unit are represented by his labor supply multiplied by his endowed human capital, that is, h_t. Then, the amount of labor in the production sector, L_t, and in the educational sector, L_t^E, are expressed as follows:

$$L_t = h_t l_t, \tag{7}$$

$$L_t^E = h_t l_t^E. \tag{8}$$

Since the total population is assumed to be one, total amount of labor in two sectors is equal to the total human capital of workers, h_t.

3 The Optimization

In each period, there live always two types of agents, young and old agents. We assume that young agents depend on old agents for their subsistence, and consumption of the young is included in that of the old. That is, individuals while at school are assumed to depend completely on their parents.[6] We call old agents working in period t the t–generation. An old agent of the t–generation is assumed to have the following utility function:

$$u_t = u(c_t), \tag{9}$$

where c_t is the amount of consumption of the t–generation, including the consumption of young agents. We assume that the utility function is concave and invariant over generations.

To examine the efficient resource allocation, we consider a social planner who has the following utilitarian social welfare function over present and future generations:

$$\sum_{t=0}^{\infty} u(c_t)(1+\delta)^{-t}, \tag{10}$$

where $\delta\ (> 0)$ is a social rate of time preference.

The planner chooses the allocation of workers between the production and educational sectors as well as the set of educational achievements and research outputs so as to maximize the social welfare function (10) subject

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[6] We suppose the capital market is incomplete. Therefore, young agents cannot borrow to receive education (even higher education).

to the production function and the educational production function.

We define the evaluation function as follows:

$$J(h_t, H_t) \equiv Max\left[u(c_t) + \lambda_t\left(h_t l_t^E - E(e_t, R_t)\right) + (1+\delta)^{-1} J(h_{t+1}, H_{t+1})\right]. \tag{11}$$

Since J is to be maximized we have the following first order conditions:

$$-u' f_{L_t} h_t + \lambda_t h_t + (1+\delta)^{-1}\left(J_{h_{t+1}} \frac{\partial h_{t+1}}{\partial l_t^E} + J_{H_{t+1}} \frac{\partial H_{t+1}}{\partial l_t^E}\right) = 0, \tag{12}$$

$$-\lambda_t E_{e_t} + (1+\delta)^{-1}\left(J_{h_{t+1}} \frac{\partial h_{t+1}}{\partial e_t} + J_{H_{t+1}} \frac{\partial H_{t+1}}{\partial e_t}\right) = 0, \tag{13}$$

$$-\lambda_t E_{Rt} + (1+\delta)^{-1}\left(J_{h_{t+1}} \frac{\partial h_{t+1}}{\partial R_t} + J_{H_{t+1}} \frac{\partial H_{t+1}}{\partial R_t}\right) = 0, \tag{14}$$

$$L_t^E = E(e_t, R_t), \tag{15}$$

where $\partial h_{t+1}/\partial l_t^E = 0$, $\partial H_{t+1}/\partial l_t^E = 0$, $\partial h_{t+1}/\partial e_t = h'_{t+1}$, $\partial H_{t+1}/\partial e_t = 0$, $\partial h_{t+1}/\partial R_t = 0$, and $\partial H_{t+1}/\partial R_t = 1$. Substituting these relations, eqs. (12) – (15) are rewritten as follows:

$$-u' f_{L_t} + \lambda_t = 0, \tag{16}$$

$$-\lambda_t E_{e_t} + (1+\delta)^{-t} J_{h_{t+1}} h'_{t+1} = 0, \tag{17}$$

$$-\lambda_t E_{R_t} + (1+\delta)^{-t} J_{H_{t+1}} = 0, \tag{18}$$

$$h_t l_t^E = E(e_t, R_t). \tag{19}$$

Also, we have the following recursive equations:

$$J_{h_t} = u' f_{L_t}(1 - l_t^E) + f_{A_t} A_{h_t} + \lambda_t l_t^E + (1+\delta)^{-1}\left(J_{h_{t+1}} \frac{\partial h_{t+1}}{\partial h_t} + J_{H_{t+1}} \frac{\partial H_{t+1}}{\partial h_t}\right), \tag{20}$$

$$J_{H_t} = u' f_{A_t} A_{H_t} + (1+\delta)^{-1}\left(J_{h_{t+1}} \frac{\partial h_{t+1}}{\partial H_t} + J_{H_{t+1}} \frac{\partial H_{t+1}}{\partial H_t}\right). \tag{21}$$

Since $\partial h_{t+1}/\partial h_t = 0$, $\partial H_{t+1}/\partial h_t = 0$, $\partial h_{t+1}/\partial H_t = 0$, and $\partial H_{t+1}/\partial H_t = 1 - \rho$, eqs. (20), (21) reduce to

$$J_{h_t} = \lambda_t \left(1 + \frac{f_{A_t}}{f_{L_t}} A_{h_t}\right), \tag{22}$$

$$J_{H_t} = u' f_{A_t} A_{H_t} + (1+\delta)^{-1} J_{H_{t+1}} (1-\rho). \tag{23}$$

We restrict our attention to a steady path, assuming that an optimal choice exists and converges to a steady state. Then the equilibrium equations are

$$-u' f_L + \lambda = 0, \tag{24}$$

$$-\lambda E_e + (1+\delta)^{-1} J_h h' = 0, \tag{25}$$

$$-\lambda E_R + (1+\delta)^{-1} J_H = 0, \tag{26}$$

$$h l^E = E(e, R), \tag{27}$$

$$J_h = \lambda \left(1 + \frac{f_A}{f_L} A_h\right), \tag{28}$$

$$J_H = u' f_A A_H + \frac{1+\delta}{\delta+\rho}. \tag{29}$$

From eqs. (24), (25), (26), (28) and (29), we obtain the following equations:

$$E_e = (1+\delta)^{-1} h' \left(1 + \frac{f_A}{f_L} A_h\right), \tag{30}$$

$$E_R = \frac{f_A}{f_L} A_H \frac{1}{\delta+\rho}. \tag{31}$$

Eqs. (30), (31) can be rewritten as follows:

$$\frac{f_L h'}{f_L E_e} + \frac{f_A A_h h'}{f_L E_e} = 1 + \delta, \tag{32}$$

$$\frac{f_A A_H}{f_L E_R} - \rho = \delta. \tag{33}$$

The first term on the left hand side of eq. (32) is the marginal rate of return to education and the second term is the marginal rate of external return to education through the technological improvement. Eq. (32) shows the sum of both terms is equal to $1+\delta$. Conventionally, it is said that education should be supplied until the marginal rate of return to education is equal to $1+\delta$. Therefore, eq. (32) means that, in the presence of the external effect, the conventional rule produces under−supply of education. Considering external effects, education should be supplied more due to the term $f_A A_h h'/f_L E_e$. And eq. (33) shows that the marginal rate of net return to research should be equal to the social rate of time preference. The problem is how the government evaluates those terms. We examine the tax/subsidy policies which attain the social optimal level of education in the next section.

4 Tax and Subsidy Policies

In this section, we consider the resource allocation achieved by the market mechanism. Since education is a private good in our model, individuals demand education at an appropriate price, while the educational sector supplies education to the market. However, the market machanism for educational outputs fails to attain the social optimum due to the following reasons. First, human capital accumulated by education generates an externality through the absorption of technology in the production process (as shown in the previous section). Second, the educational sector cannot supply research output to a market; it is a public good in our model so that firms may use it free of charge.[7]

In this section, we examine how a subsidy to the educational sector gives an incentive to produce optimal amounts of educational outputs.

4.1 Production sector

In period t, firms competitively demand labor and supply consumption goods so as to maximize profit π_t with the wage rate, technology, and tax given.

$$\text{Max} \quad \pi_t = f(L_t, A_t) - w_t L_t - T_t,$$

where w_t is the wage rate and T_t the lump sum tax in period t. Then the marginal products of labor are equal to the wage rate:

$$f_{L_t} = w_t, \tag{34}$$

which yields the following demand function for labor:

$$L_t^D = L^D(w_t, A_t). \tag{35}$$

4.2 Consumer

For simplicity, we assume that individuals choose the level of children's education so as to maximize their expected net discounted earnings of education with the price of education and the wage rate given.[8] We also assume that parents have static expectation with respect to the children's expected wage rate. Then, individuals demand education to meet the following condition:

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[7] For example, universities can sell research output by patent. But those cases are rare.

$$\frac{w_t h'_{t+1}}{q_t} = 1 + \delta, \tag{36}$$

where q_t is the price of education in period t. Consumption is determined simultaneously from his budget constraint. We assume that firms distribute net profits to individuals through lump sum transfers.[9] We have the following demand functions of each good:

$$e_t^D = e^D(w_t, q_t, h_t, \pi_t), \tag{37}$$

$$c_t^D = c^D(w_t, q_t, h_t, \pi_t). \tag{38}$$

4.3 Educational sector

We assume that the educational sector supplies education and research so as to maximize the surplus given the prices and subsidies. Educational workers are paid the same wage as workers in production sector.

We assume that the educational sector is subsidized to per unit of education and research output. Then educational sector's maximizing problem is formulated as follows:

$$\text{Max} \quad (q_t + s_t)e_t + r_t R_t - w_t L_t^E,$$

$$\text{s. t.} \quad L_t^E = E(e_t, R_t).$$

where s_t, r_t are per unit of subsidies of education and research output in period t, respectively. The first order conditions of this problem are

$$(q_t + s_t) - w_t E_{e_t} = 0, \tag{39}$$

$$r_t - w_t E_{R_t} = 0, \tag{40}$$

$$L_t^E = E(e_t, R_t). \tag{41}$$

These equations yield the profit–maximizing ratio of education to research output, but do not determine the size of the educational sector due to the assumption of the constant return to scale technology. We assume here that the educational sector supplies education equal to individual's demand. Then the amounts of research output supplied and labor demanded are determined by the first order conditions with the wage rate, the price of

[8] We assume away the problem of intergenerational income distribution. We can formulate individual behavior more generally assuming that an individual chooses the level of education of his child so as to maximize his utility over consumption and his child's utility with the prices of both goods and his earnings given. This maximizing problem reduces the same demand function for education at the steady state by some general assumptions.

[9] We suppose that the firm is owned by individuals.

education, subsidies, and the amount of education given.

4.4 Government

Government determines the subsidies to the educational sector and the lump sum tax on production sector to balance the government budget. Ignoring other expenditures, the government budget equation is

$$T_t = s_t e_t + r_t R_t . \tag{42}$$

4.5 Market clearance

In market equilibrium, consumption goods, education, and labor markets should be cleared,

$$c_t^D = f(L_t, A_t), \tag{43}$$

$$e_t^D = e_t^S, \tag{44}$$

$$L_t^{ED} + L_t^D = h_t . \tag{45}$$

where the upper subscript $D\,(S)$ represents demand (supply).

The market mechanism described above determines the equilibrium amounts of education and workers in both sectors, the price of education, and the wage rate at each period given h_t, H_t, r_t, and s_t.

4.6 Dynamics of market equilibrium

The following transit equations determine the initial values of state variables in the next period.

$$H_{t+1} = R_t + (1-\rho)H_t ,$$

$$h_{t+1} = h(e_t).$$

We assume that government adjusts the subsidies to education and research according to the following rules with any initial values given:

$$s_{t+1} = s_t + \theta\left[(\pi_t - \pi_{t-1}) - (1+\delta)s_t(e_{t-1} - e_{t-2})\right], \tag{46}$$

$$r_{t+1} = r_t + \gamma\left[(\pi_t - \pi_{t-1}) - (\delta+\rho)r_t(R_{t-1} - R_{t-2})\right], \tag{47}$$

where θ and γ are positive constants less than one.

In our setting, the market mechanism determines a temporal equilibrium in each period. Subsidy policies, according to the adjustment rules above, make a dynamic process of our economy. Assuming existence of and convergence to a steady state, we examine the nature of the steady state equilibrium and compare it with the steady state of the planned economy in the previous section.

4.7 Steady state equilibrium

At the steady state equilibrium of the market mechanism, we obtain $d\pi/de = (1+\delta)s$ and $d\pi/dR = (\delta+\rho)r$ from the adjustment rules (46), (47), respectively. These reduce to

$$\frac{f_A A_h h'}{1+\delta} = s, \tag{48}$$

$$\frac{f_A A_H}{\delta+\rho} = r. \tag{49}$$

Each variable without subscript t represents the corresponding value in the steady state. And the equilibrium conditions of each agent in the steady state are as follows:

$$q = (1+\delta)^{-1} wh',$$

$$f_L = w,$$

$$h = h(e),$$

$$H = \frac{R}{\rho},$$

$$q + s = wE_e,$$

$$r = wE_R,$$

$$L^E = E(e, R),$$

$$\frac{f_A A_H h'}{1+\delta} = s,$$

$$\frac{f_A A_H}{\delta+\rho} = r.$$

It is easy to show that these equilibrium equations at the steady state meet the efficiency conditions of the planning economy. Thus we may conclude that the market mechanism, which involves publicly supplied education and research, can achieve the optimal resource allocation if government can determine the proper prices of education and research output, according to the adjustment scheme of subsidies like eqs. (46), (47).

5 Concluding Comments

The necessity of subsidy to higher education is recognized widely as an implication of the public goods nature of research activity. On the other hand, if its effects on income distribution or the externality of general education in higher education should be judged to be small or negligible, then government might try to reduce public expenditure on higher education or change the relative weight of subsidies to education and research activity. We have tried to show that a subsidy to higher education is still necessary to efficient allocation by stressing the external effects which have not been paid much attention.

We suppose that higher education affects production through increasing the ability to innovate the technology available in production process, as well as individual human capital. The technology can be utilized in the production process by absorbing social knowledge existing in the period. In this sense, education itself has the external effect in the process of production.

We derived the optimal size of education and research subsidy, in the light of the role of higher education in the production process. Furthermore, we derived the adjustment scheme of subsidies needed to attain an efficient allocation by the market mechanism. Of course, the significance of such external effects of higher education is an empirical question. We must test this problem, which is to be left for our future attention.

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Our purposes are to clarify the nature of higher education and to analyze efficient resource allocation in a two–sector economy, considering the external effects of education and research activities in the context of an endogenous growth model.