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Estimation of Ground Resisitivity Distribution Using 3D DRM Charge Simulation Modelling

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Resistivity distribution sounding of the non-homogeneous earth is important for electrical ground system design, geophysical prospecting and survey or monitoring the groundwater flow level. The previous paper presented that the direct inversion of the electric resistivity distribution in a domain is possible from the impedance data measured over the domain boundary using the dual reciprocity boundary element modelling in two-dimentional field [1]. The proposed inversion technique is extended to the distribution in three-dimensional space [2]. This technique is capable of inversion without iteration and meshing of the domain. Electric field with spatially varying conductivity is governed by Laplace equation, which is transformed into a Poisson-type expression with an inhomogeneous term involving the conductivity difference as a source term. Dual reciprocity method (DRM) is a technique for transforming the domain integral associated with the inhomogeneous term in Poisson equation into the boundary integral expression. The resistivity distribution in the field can thus be identified from the data observed over its boundary, for which some examples are demonstrated [2]. In this paper, the examination is extended to the case where only the data measured over the single surface is used for the inversion.

1. Introduction

The BEM is, FEM as well, a powerful tool for numerical engineering analysis. The main attraction of the BEM is that it only requires the surface division of the field under study, while FEM needs the discretization of the whole field into a series of block-like elements. In addition, the field variables under consideration are found simultaneously.

The finding of the resistivity distribution in the non-homogeneous field is an inverse problem in which the resistance distribution is reconstructed from the potential data measured over the field boundary. This kind of problem has attracted many investigators. For instance, Kohn and Vogelius discussed the mathematical foundation of the uniqueness of the impedance distribution determination from the boundary data [3], [4]. Barber and Brown developed a back-projection method through interative process based on the linearization around a constant conductivity [5]. Murai and Kagawa first proposed the use of the finite element model in which a perturbation approach is employed for the solution

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incorporating with the regularization based on Akaike's information criterion to overcome the ill-conditioned nature of the problem [6]. Murai and Kagawa have also proposed boundary element iterative techniques for determining the interface boundary between Laplace and Poisson domains [7].

Many of the approaches of the inversion are based on the minimization of the cost function, or the norm of the boundary values between the calculated and the measured with respect to the conductivity distribution assumed, which requires repeated calculations until the convergence is reached.

The boundary element method has successfully been used for various problems of homogeneous fields. However, for the non-homogeneous field it is difficult or almost impossible to obtain the fundamental solution, and hence in most cases the fundamental solution of the homogeneous problem should be used for the integral equation formulation. In this case, a domain integral arises in the boundary integral equation. For its solution, there are some possibilities [8]; Iterative solution of the boundary integral equation and the application of the dual reciprocity method (DRM) [9], [10].

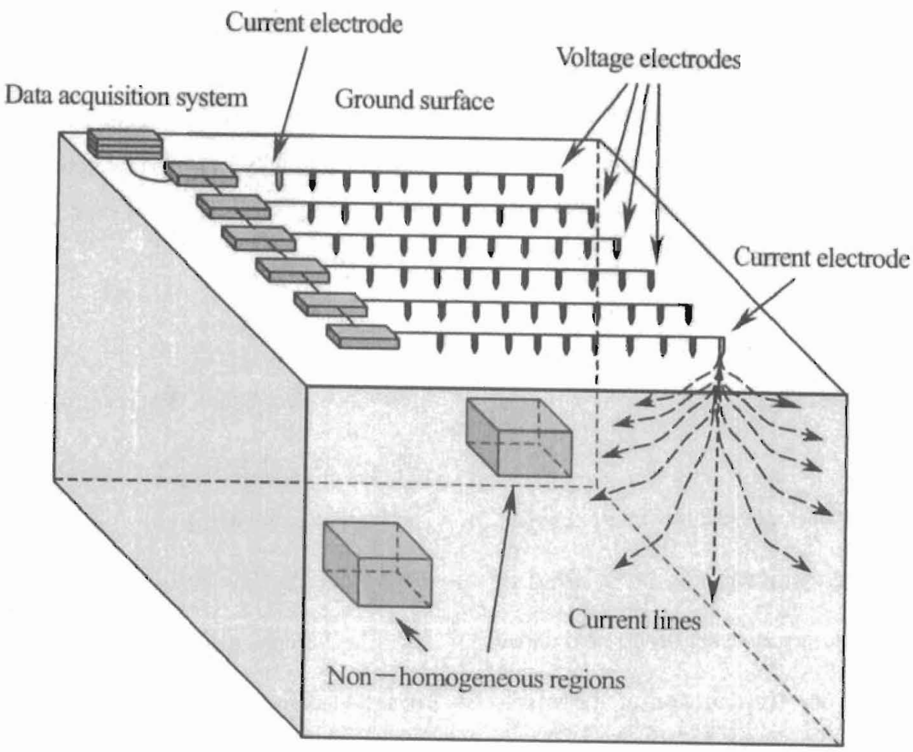
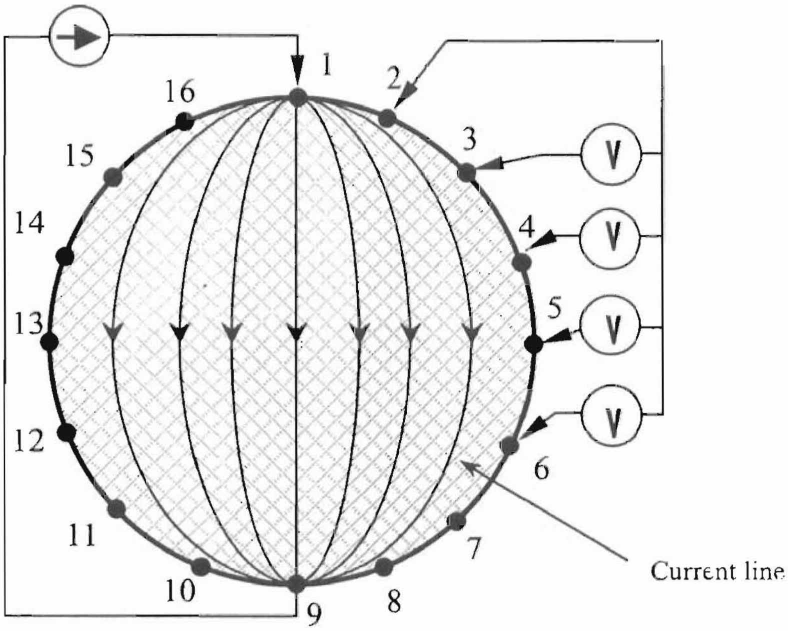
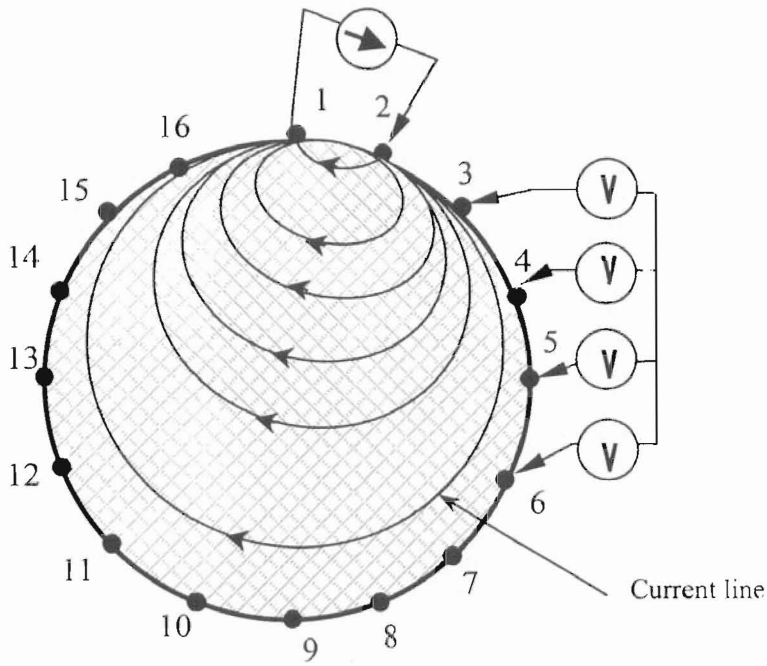


Fig.1. Modelling of the data acquisition system of the three-dimensional electrical prospecting

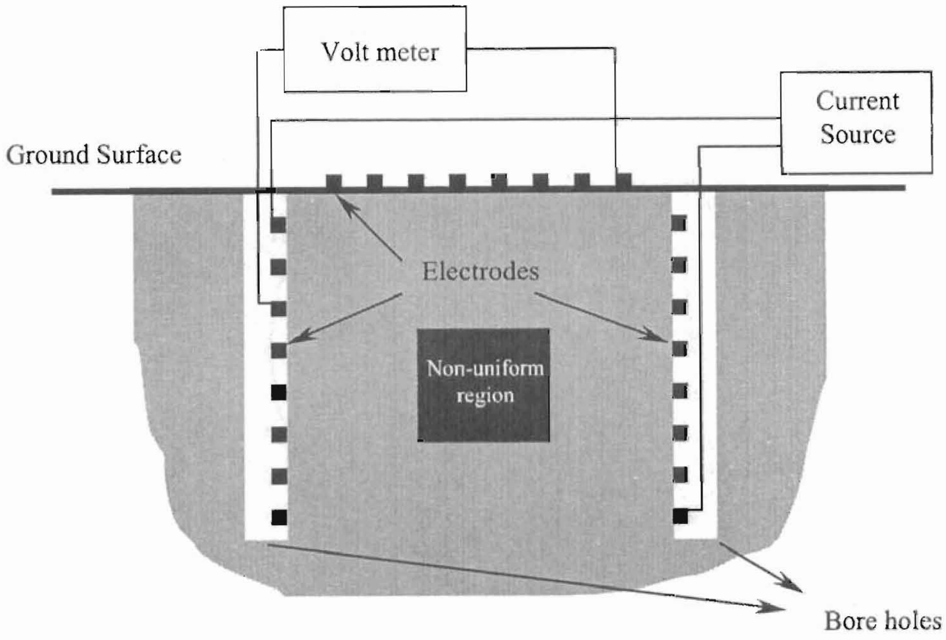


(a). Opposite method

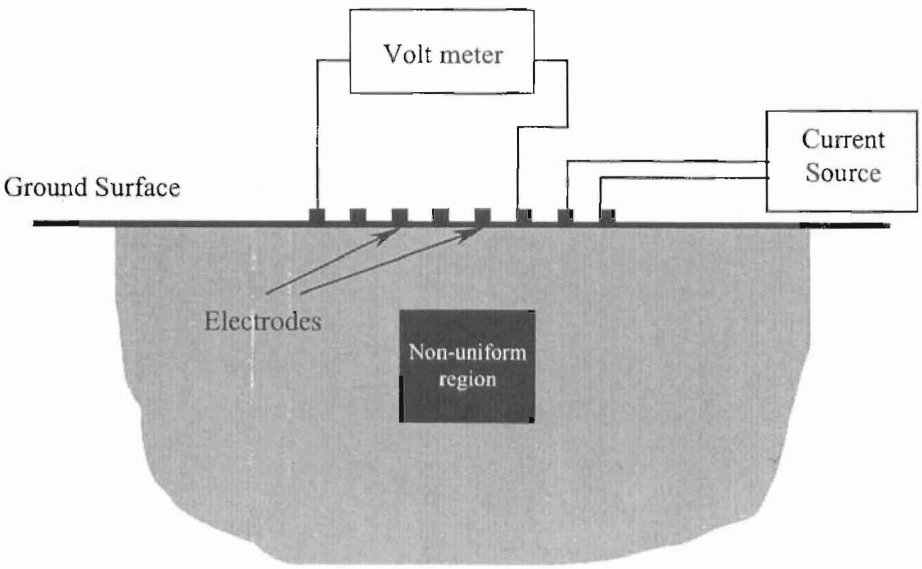


(b). Neighboring method

Fig.2. Methods of the data collection, after the ref.[11]



(a). The case where the electrodes are placed both on the bore holes and the surface.



(b). The case where the data is collected only from the electrodes arrayed on the ground surface

Fig.3. Field and electrode arrangement

In the previous paper [1], a new approach was presented to identify the conductivity distribution without iterative process by using DRM boundary element models for two-dimensional field. The domain integral is evaluated in a meshless manner based on the dual reciprocity method, in which the domain integral is transformed into boundary integrals with simple radial basis functions and particular solutions. The approach was then extended into the three-dimensional case [2]. In the present paper, the data measured only over the single surface of the ground is used to examine the possible inversion.

2. A Model and Data Acquisition

Figure 1 shows a data acquisition system model for electrical ground resistivity distribution prospecting. The data acquisition is to collect the impedance data between the electrodes placed on the region boundary of interest.

There are several methods of the electrode arrangement to obtain the data. One is called the opposite method in which the electric current is diagonally applied to the region in turn to which the potential data are collected at all of the electrodes which is illustrated in Fig.2 (a). The current is injected through two diametrically opposed electrodes. The voltage reference electrode is chosen adjacent to the current driving electrode. For a particular pair of driving electrodes (1-8), the voltages are measured at all the electrodes, except the driving electrodes. To obtain the next set of data, the current is switched to the next pair of opposite electrodes (2-9). Though voltage reference was also changed accordingly, and the voltages are similarly measured with respect to the new reference, and the voltages measured for each pair of current electrodes with respect to the reference electrode. Another method called neighboring method is shown in Fig.2 (b). For the 16 electrodes arrangement, the opposite method has more uniform current density and hence possibly good sensitivity is expected [11].

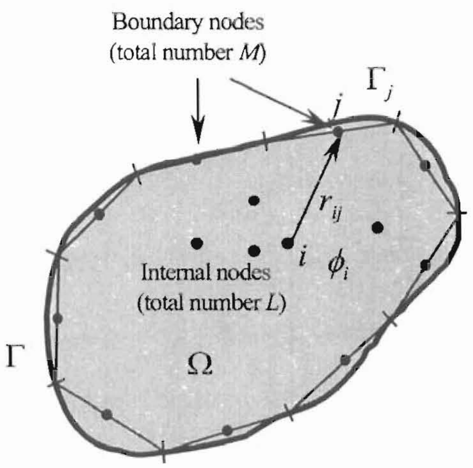


Fig.4. Boundary and nodes

Figure 3 shows the case more realistic situations. Figure 3 (a) is the case when the electrodes are partly placed in the bore holes and partly placed on the ground surface for which the opposite method is used, and Fig.3 (b) is the case when the electrodes are arrayed only on the ground surface, for which the neighboring method is possible. The two-dimensional modelling is not always reasonable as the electric currents are diffusive and may not stay in the cross-sectional plane, so that the two-dimensional modelling is not acceptable except for the field of special configuration. For this case, three-dimensional modelling was attempted [2]. The present paper focuses on the case of Fig3 (b) to which the neighboring method is suitable.

3. Dual Reciprocity Boundary Integral Expression

Poisson equation in three dimensions is written as

$$\nabla^2\phi(x,y,z) = b(x,y,z), \tag{1}$$

where $\phi(x,y,z)$, and $b(x,y,z)$ are the electric potential and charge density defined in the bounded field, and $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplace operator.

The potential $\phi(x,y,z)$ that satisfies equation (1) can be expressed as the sum of the particular solution $\zeta(x,y,z)$ of the inhomogeneous equation and the fundamental solution $\psi(x,y,z)$ of the homogeneous equation, so that one has

$$\phi(x,y,z) = \psi(x,y,z) + \zeta(x,y,z) \tag{2}$$

The boundary is divided into elements $\Gamma_j(j=1,2,\ldots,M)$. M boundary nodes and L internal nodes are also shown in Fig.4. The potential ϕ_i at an arbitrary point i in the field can be expressed as the linear combination of the contribution from the sources taken at the nodes on the boundary and at the interior nodes.

The fundamental solution for the arbitrary point i in the field can be expressed as the linear combination of the fundamental solutions with respect to j

$$\psi_i = \psi(x_i,y_i,z_i) = \sum_{j=1}^{M}\psi^*_{ij}\beta_j = \{\psi^*\}_i^T\{\beta\}, \tag{3}$$

where the fundamental solution of Laplace equation $\nabla^2\psi(x,y,z)=0$ is given by

$$\psi^*_{ij} = \frac{1}{4\pi r_{ij}} \;. \tag{4}$$

r_{ij} is the distance from a source point i to the consideration point j , and β_j is the unknown coefficient.

In the similar manner, the particular solution can be expressed as the linear combination of the particular solutions with respect to l

$$\zeta_i = \zeta(x_i,y_i,z_i) = \sum_{l=1}^{M+L}\zeta^*_{il}\alpha_l = \{\zeta^*\}_i^T\{\alpha\}, \tag{5}$$

where ζ^*_{il} is given by

$$\zeta^*_{il} = \frac{1}{12}\frac{r_{il}^3}{r_{\max}}, \tag{6}$$

and $r_{\max}$ is the maximal length. Then the potential ϕ_i at an arbitrary point i in the field can thus be expressed as

$$\begin{aligned}\phi_i &= \phi(x_i,y_i,z_i)\\ &= \sum_{j=1}^{M}\psi^*_{ij}\beta_j + \sum_{l=1}^{N+L}\zeta^*_{il}\alpha_l = \{\psi^*\}_i^T\{\beta\} + \{\zeta^*\}_i^T\{\alpha\},\end{aligned} \tag{7}$$

where

$$\{\psi^*\}_i = \{\psi^*_{i1}, \psi^*_{i2}, \ldots, \psi^*_{iM}\};$$
$$\{\zeta^*\}_i = \{\zeta^*_{i1}, \zeta^*_{i2}, \ldots, \zeta^*_{i(M+L)}\};$$
$$\{\beta\} = \{\beta_1, \beta_2, \ldots, \beta_M\};$$
$$\{\alpha\} = \{\alpha_1, \alpha_2, \ldots, \alpha_{M+L}\},$$

ψ^*_{ij} is the fundamental solution whose value is evaluated at i for a unit source given at point j on the boundary, and β_j is the unknown coefficients associated with boundary element j which corresponds to the fictitious charges. ζ^*_{il} is the particular solution whose value is evaluated at i for a unit source given at node l and α_l is the unknown coefficients associated with node l which also corresponded to the fictitious charges [12].

It is difficult to find a solution of the above using usual boundary element formulation as it involves the domain integration. In order to avoid the domain integral in the formulation of the boundary integral equation, we use the dual reciprocity method (DRM).

With DRM the following approximate expression for b_i in equation (1) is expanded in such away that

$$b_i = b(x_i, y_i, z_i) \approx \sum_{l=1}^{M+L} \alpha_l f_{il} \tag{8}$$

where α_l are unknown coefficients and f_{il} is approximating functions chosen to express the presence of the source b_i. If there are M boundary nodes and L internal nodes, there will be $M+L$ values for the particular solutions ζ^*_{il}, which is the solution of

$$\nabla^2 \zeta^*_{il} = f_{il}. \tag{9}$$

For the fundamental solution, $\nabla^2 \psi(x,y,z) = 0$ can be discretized into boundary integral expression as follows [9]:

$$[K]\{\psi\} - [G]\left\{\frac{\partial \psi}{\partial \boldsymbol{n}}\right\} = 0 \tag{10}$$

In the process of obtaining boundary element formulation (10), the coefficients $\{\beta\}$ are eliminated.

Based on the equation (10) and with the equation (2), the following discretized expression for the Poisson equation results:

$$\{0\} = [K](\{\phi\} - \{\zeta\}) - [G]\left(\left\{\frac{\partial \phi}{\partial \mathbf{n}}\right\} - \left\{\frac{\partial \zeta}{\partial \mathbf{n}}\right\}\right)$$
$$= [K]\left(\{\phi\} - \sum_{l=1}^{M+L} \zeta^*_{il}\alpha_l\right) - [G]\left(\left\{\frac{\partial \phi}{\partial \mathbf{n}}\right\} - \sum_{l=1}^{M+L} \frac{\partial \zeta^*_{il}}{\partial \mathbf{n}}\alpha_l\right),$$

that is

$$[K]\{\phi\} - [G]\left\{\frac{\partial \phi}{\partial \boldsymbol{n}}\right\} = ([K][H] - [G][Q])\{\alpha\}, \tag{11}$$

where $[K] = [E]([G]^{-1})^T [U]$ and $[E]$ has the components

$$E_{ij} = \frac{1}{2}\sum_{m=1}^{M} \int_{\Gamma_m} \psi^*_{im} \frac{\partial \psi^*_{im}}{\partial \boldsymbol{n}} d\Gamma_m + \frac{1}{2}\sum_{m=1}^{M} \int_{\Gamma_m} \psi^*_{jm} \frac{\partial \psi^*_{jm}}{\partial \boldsymbol{n}} d\Gamma_m \tag{12}$$

The components of other matrices are

$$G_{ij} = \int_{\Gamma_j} \psi_{ij}^{*} d\Gamma_j, \quad H_{il} = \zeta_{il}^{*}, \quad Q_{il} = \frac{\partial \zeta_{il}^{*}}{\partial \boldsymbol{n}} = q_{il}^{*}$$

$$(l = 1,2,\ldots, M+L), \quad (i,j = 1,2,\ldots, M)$$

For constant boundary elements, U_{ij} = (the length of the element j) ($i = j$) and $U_{ij} = 0$ ($i \neq j$).

Referring to the equation (9), the non-homogeneous term b can be expressed as

$$b_i = b(x_i, y_i, z_i) \approx \sum_{j=1}^{M+L} \alpha_l (\nabla^2 \zeta_{il}^{*}) \tag{13}$$

Equation (13) can be substituted into the equation (1) to give the following expression

$$\nabla^2 \phi_i = \sum_{l=1}^{M+L} \alpha_l (\nabla^2 \zeta_{il}^{*}) \cdot \tag{14}$$

The $\{\alpha\}$ in equation (8) is given in matrix form

$$\{b\} = [F]\{\alpha\} \tag{15}$$

which is inverted to obtain

$$\{\alpha\} = [F]^{-1}\{b\}, \tag{16}$$

where $\{b\} = \{b_1, b_2, \ldots, b_{M+L}\}$ and each component of $[F]$ consists of the function f_{il} evaluated at l ($M+L$). Equation (16) is substituted into equation (11) resulting in

$$[K]\{\phi\} - [G]\left\{\frac{\partial \phi}{\partial \boldsymbol{n}}\right\} = [S][F]^{-1}\{b\} \tag{17}$$

and

$$[S] = [K][H] - [G][Q].$$

The accuracy of the solution in dual reciprocity method depends on the choices of the approximating functions f_{il} and the number and positions of the nodes taken. The function recommended is

$$f_{il} = 1 + r_{il} \tag{18}$$

which is the function truncated by the second term for the polynomial

$$f_{il} = 1 + r_{il} + r_{il}^2 + \ldots + r_{il}^m \tag{19}$$

In principle, any combination of the terms may be selected, but the equation (18) is said to be of the simplest but acceptable choice [9].

In this study, we choose the approximating function f_{il} to be

$$f_{il} = \frac{r_{il}}{r_{\max}}, \tag{20}$$

where

$$r_{il} = \sqrt{X^2 + Y^2 + Z^2},$$

$$X = x_i - x_l, \quad Y = y_i - y_l, \quad Z = z_i - z_l.$$

r_{il} is the distance between node i and l, and $r_{\max}$ is the maximum distance.

4. Inverse Solution Procedures

The potential $\phi(x,y,z)$ in the non-homogeneous electric field is governed by the equation

$$\nabla \cdot (\sigma \nabla \phi(x,y,z)) = 0 \tag{21}$$

where σ is the isotropic resisitivity which depends on the position. When the field is homogeneous, equation (21) becomes Laplace equation

$$\nabla^2 \phi(x,y,z) = 0 \tag{22}$$

The equation (21) can be expanded as follows:

$$\nabla^2 \phi(x,y,z) + \frac{1}{\sigma} \nabla \sigma \cdot \nabla \phi(x,y,z) = 0 \cdot \tag{23}$$

The second term is taken as the forcing term, to give

$$\nabla^2 \phi(x,y,z) = \nabla \phi(x,y,z) \cdot \nabla R = b(x,y,z) \tag{24}$$

where $R = -\ln \sigma$.

This alternative expression is to be solved for equation (21), which is the same as the expression (1). The potential distribution in the field can be solved for the boundary condition and the resisitivity distribution prescribed. In the present inverse problem, the resisitivity distribution is to be determined from the measured potential data observed over the boundary.

As equation (24) is a kind of Poisson equation due to the presence of the right hand side term, the DRM boundary element method leads it to the expression (17), which is solved for the boundary conditions or the measured data to find $\{b\}$. The voltages between different pairs of electrodes are measured on the ground surface for the current injected. The left-hand side of equation (24) is now nothing but the forcing term, which is the resisitivity distribution to be identified.

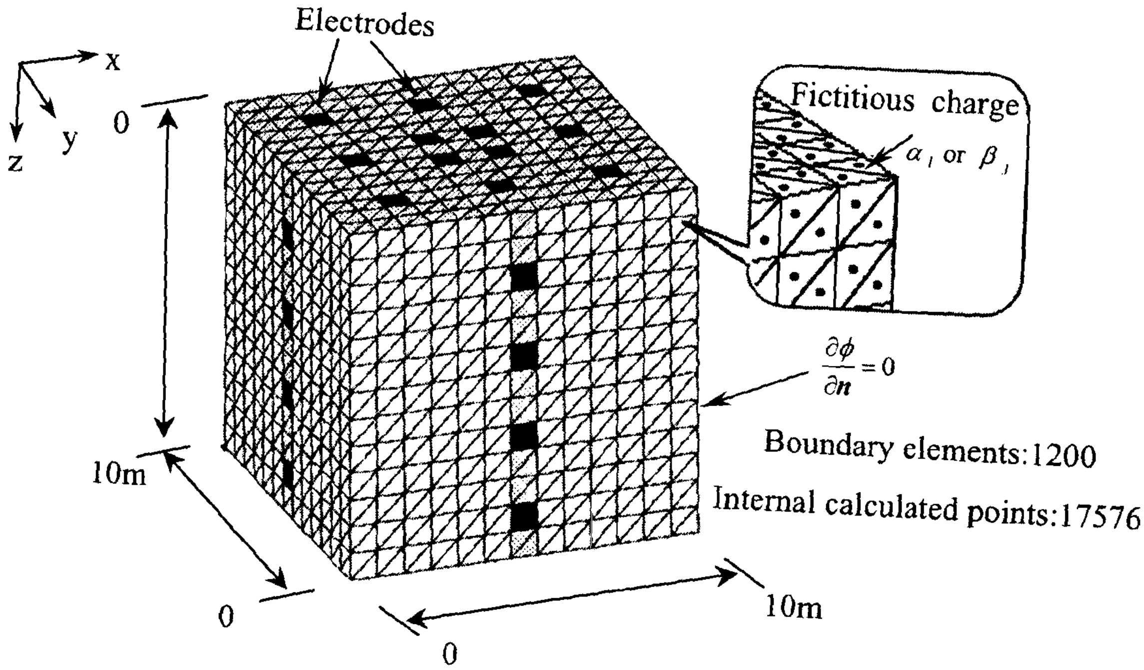
From equation (24), one has the equation with $\{b\}$ known

$$\nabla R_i \cdot \nabla \phi_i = \frac{\partial R_i}{\partial x}\frac{\partial \phi_i}{\partial x} + \frac{\partial R_i}{\partial y}\frac{\partial \phi_i}{\partial y} + \frac{\partial R_i}{\partial z}\frac{\partial \phi_i}{\partial z} = b_i \tag{25}$$

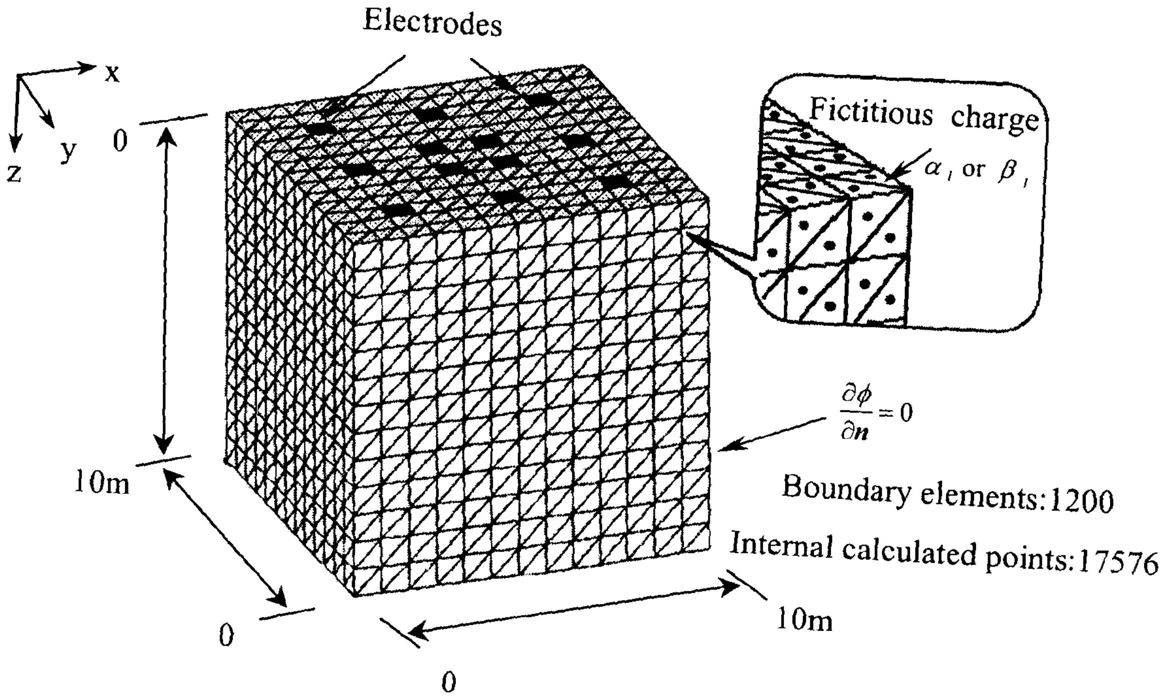
For the k^{th} current injection we can obtain

$$\frac{\partial R_i}{\partial x}\frac{\partial \phi_i^k}{\partial x} + \frac{\partial R_i}{\partial y}\frac{\partial \phi_i^k}{\partial y} + \frac{\partial R_i}{\partial z}\frac{\partial \phi_i^k}{\partial z} = b_i^k \cdot \tag{26}$$

The process of the direct inversion will be given as follows. First, several electrodes are placed on the boundary. A pair of them are chosen as the driving current terminals and the potentials between adjacent electrode pairs are measured to provide the electric potential distribution on the boundary. In the present simulation, the forward solution $\{\phi_0\}$ for the given conductivity distribution is used as the measured values. $\{\beta\}$ in equation (3) is obtained for the solution of the homogeneous field: The potential $\{\phi\}$ of the Laplace problem is given on the boundary, that is



(a). Model for the case A



(b). Model for the case B

Fig.5. Three-dimensional models

$$\{\beta\} = \left(\{\psi^{*}\}_i^T\right)^{-1}\{\phi\}_i \tag{27}$$

Then the difference of the measured potential $\{\phi_0\}$ to the potential as a solution of the Laplace problem provides the information of the conductivity change within the field. Equation (7) is solved for $\{\alpha\}$ with respect to this difference as

$$\{\alpha\} = \left(\{\zeta^{*}\}_i^T\right)^{-1}\left(\{\phi_0\} - \{\phi\}_i\right). \tag{28}$$

Finally, ϕ_i and b_i are evaluated at any point i by equation (15) and equation (7). The potential gradient $\partial\phi_i/\partial x$, $\partial\phi_i/\partial y$ and $\partial\phi_i/\partial z$ are readily evaluated in the region.

Solving the equation (26) for $\partial R_i/\partial x$, $\partial R_i/\partial y$ and $\partial R_i/\partial z$, the logarithm resistivity coefficient R_i at point i are obtained by integrating for a pixel along x, y and z directions. Since there are three unknowns, solution is theoretically obtained for three current injections. The system equation is not symmetric but can generally be solved by the method of least squares.

Resisitivity σ_i is evaluated at the point i by

$$\sigma_i = \exp(-R_i) \tag{29}$$

5. Demonstration Models

The validity of the algorithm has been verified [2]. The present paper considers two models. One corresponds to the case A given in Fig.3 (a) and another is the case B given in Fig.3 (b). The details of the arrangement are shown in Fig.5 with the electrodes locations.

The boundary surface was divided into 1200 triangular elements. The size of the field is considered $10m^3$ and the relative conductivity of the field taken to be $\sigma_0 = 1$.

The boundary condition is assumed to be $\partial\phi/\partial \boldsymbol{n} = 0$ except for the elements chosen as driving electrodes. A pair is chosen to be the driving electrodes to which a unit current is injected. The potential distribution is measured on all of the electrodes. 40 times of injection is made for the first case (case A) and 28 times for the last case (case B).

6. Numerical Results

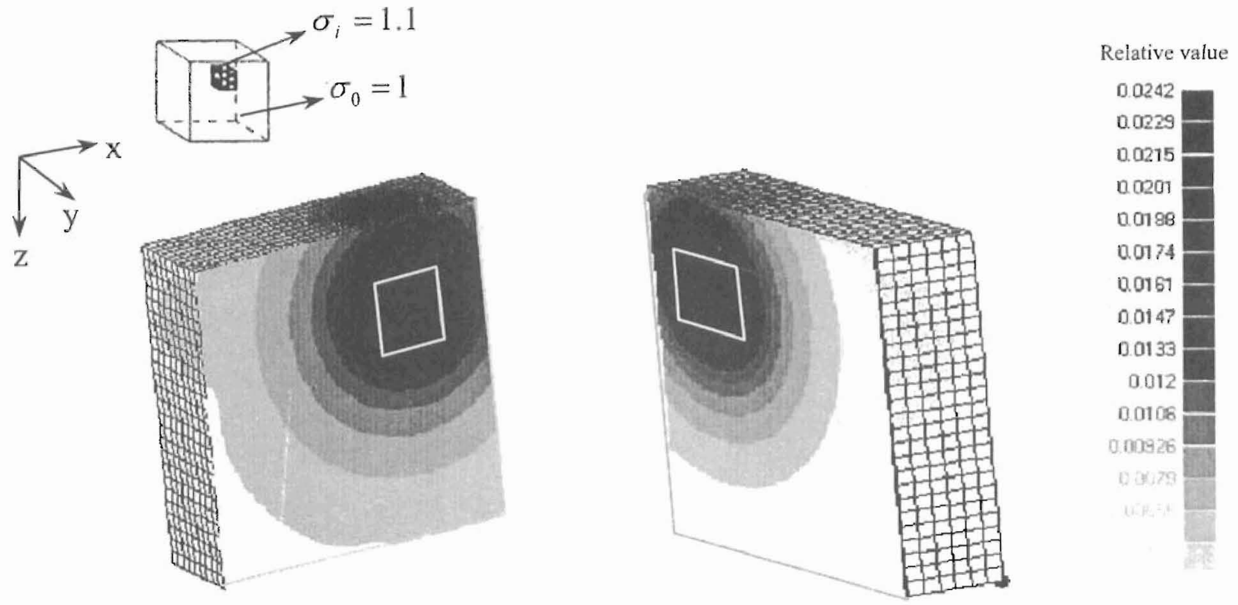
6.1 The case A

Figure 6 shows the result inverted for the case when there is a cube ($2m^3$, $\sigma_i = 1.1$) placed near one of the corners under the ground surface. (a) is the potential change distribution and (b) is the relative conductivity distribution inverted. Figure 7 is the result for the case when there is a cube ($2m^3$, $\sigma_i = 1.1$) placed in the middle of the field.

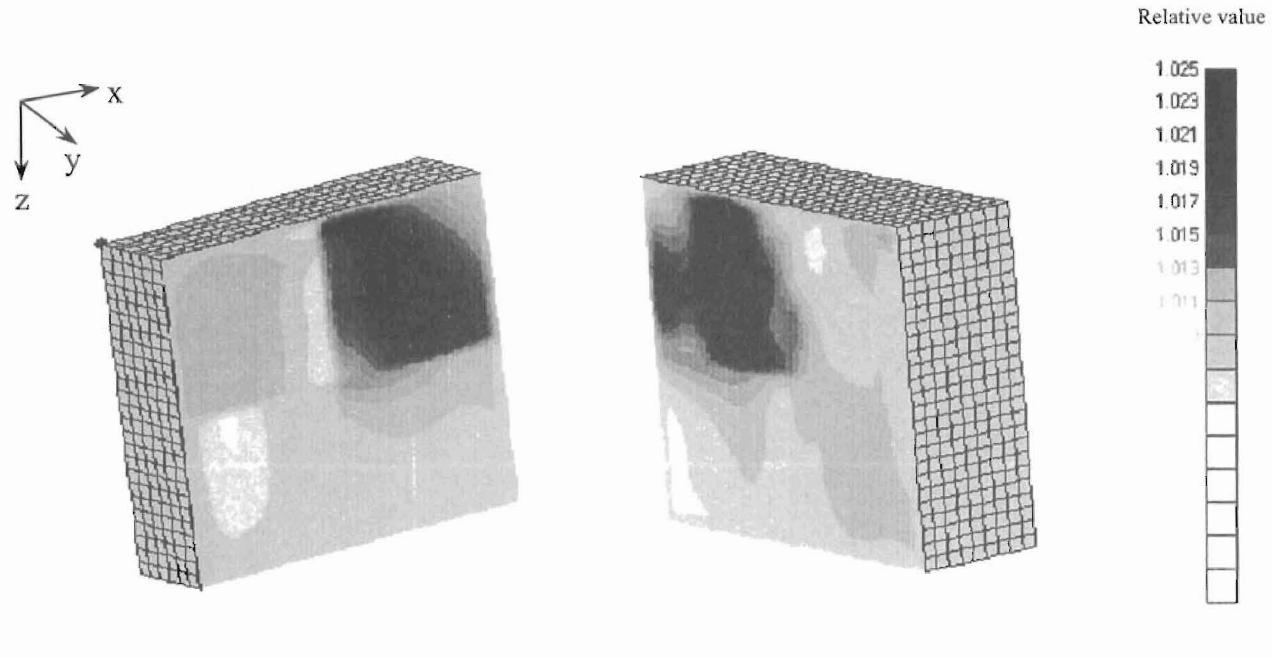
6.2 The case B

Figure 8 shows the result for the case when there is a cube ($2m^3$, $\sigma_i = 1.1$) placed near one of the corners under the surface. Figure 9 shows the result for the case when there is a cube ($2m^3$, $\sigma_i = 1.1$) placed in the middle of the field.

The numerical experiment shows that it is possible to

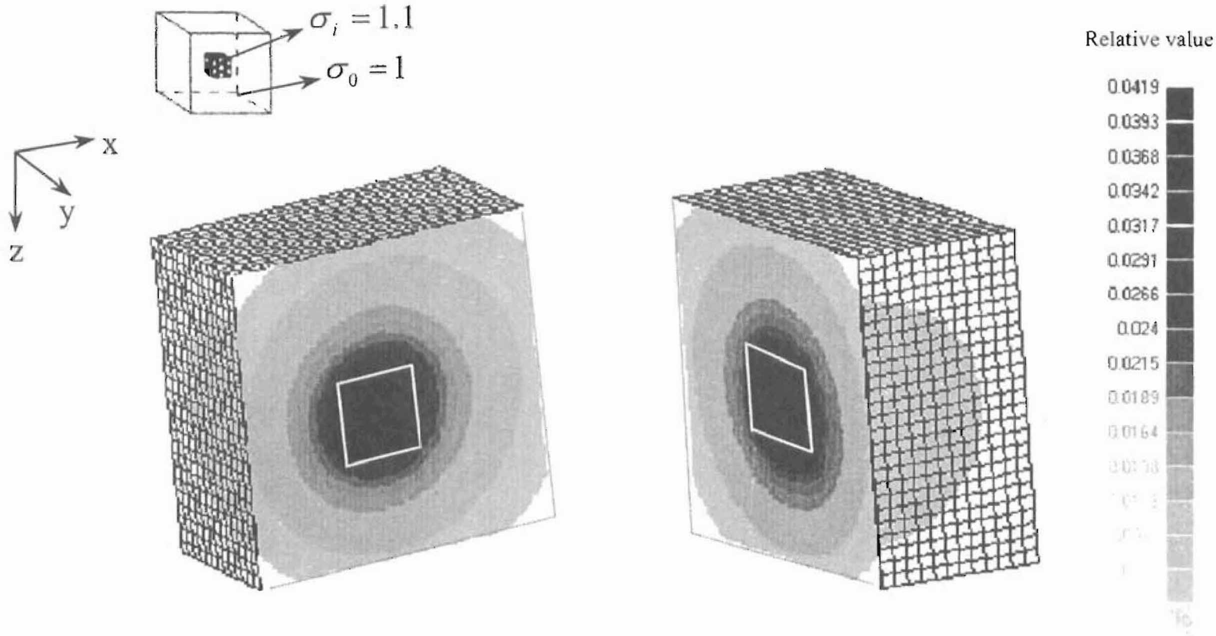


(a). The distribution of potential change (in the planes y=5.0, x=7.0)

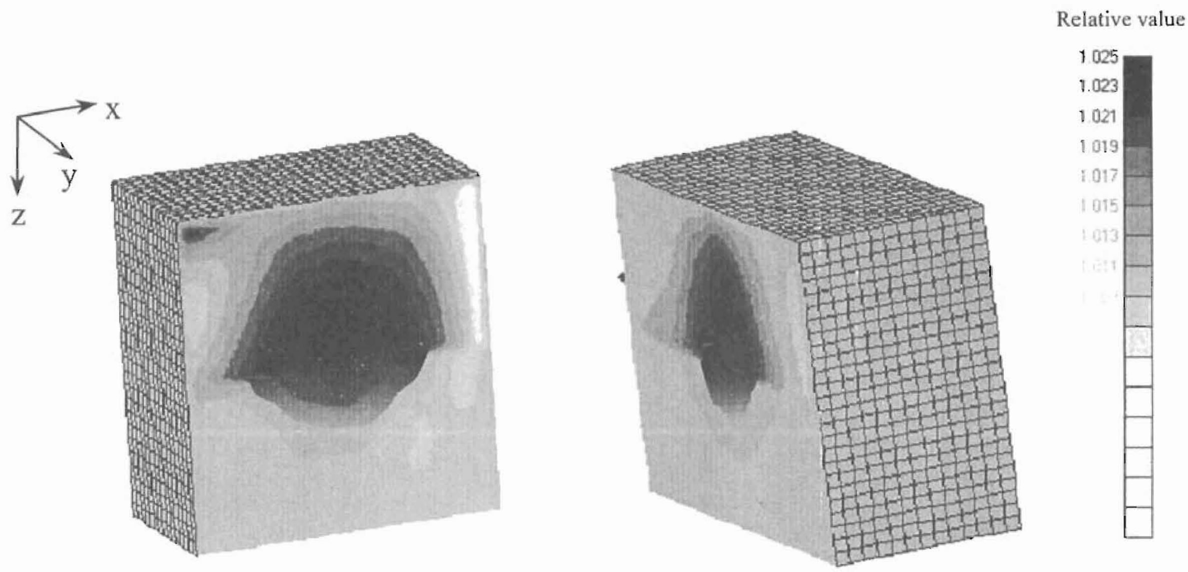


(b). The relative conductivity distribution inverted (in the planes y=5.0, x=5.0)

Fig.6. The case where a non-homogeneous cubic region is placed in one of the corners (with the relative conductivity $\sigma_i = 1.1$)

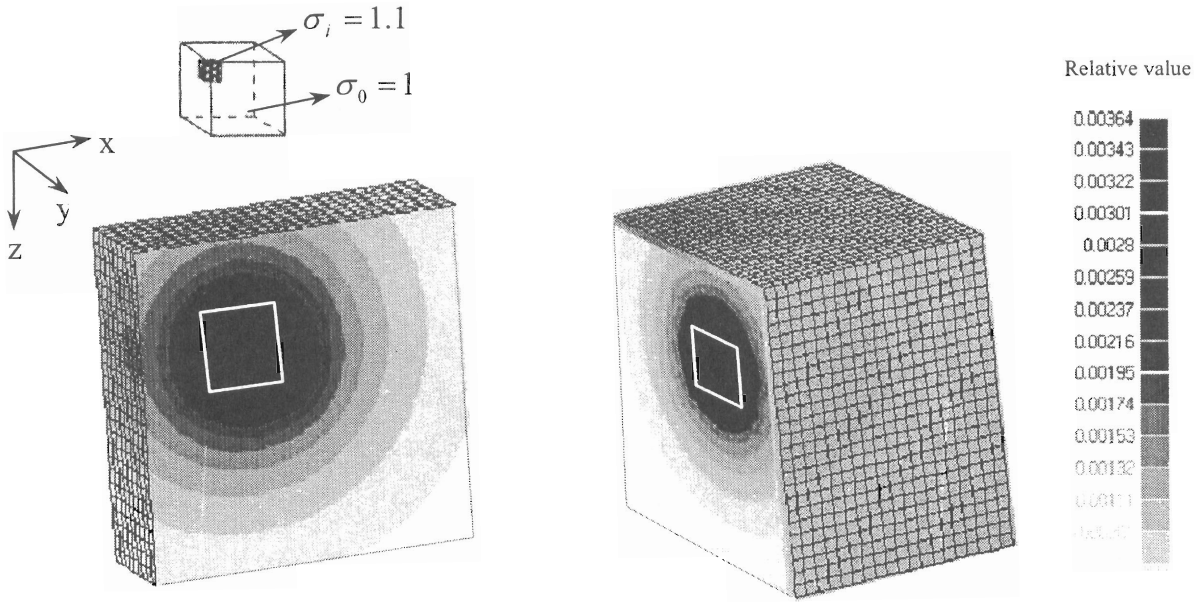


(a). The distribution of potential change (in the planes y=3.0, x=7.0)

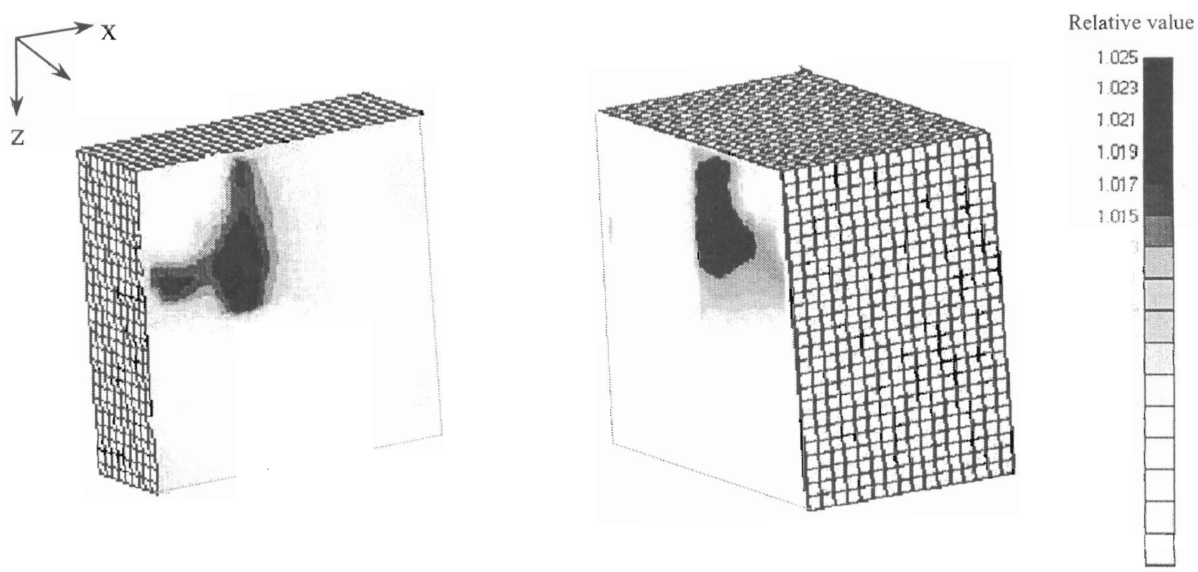


(b). The relative conductivity distribution inverted (in the planes y=3.0, x=7.0)

Fig.7. The case where a non-homogeneous cubic region is placed in the middle (with the relative conductivity $\sigma_i = 1.1$)

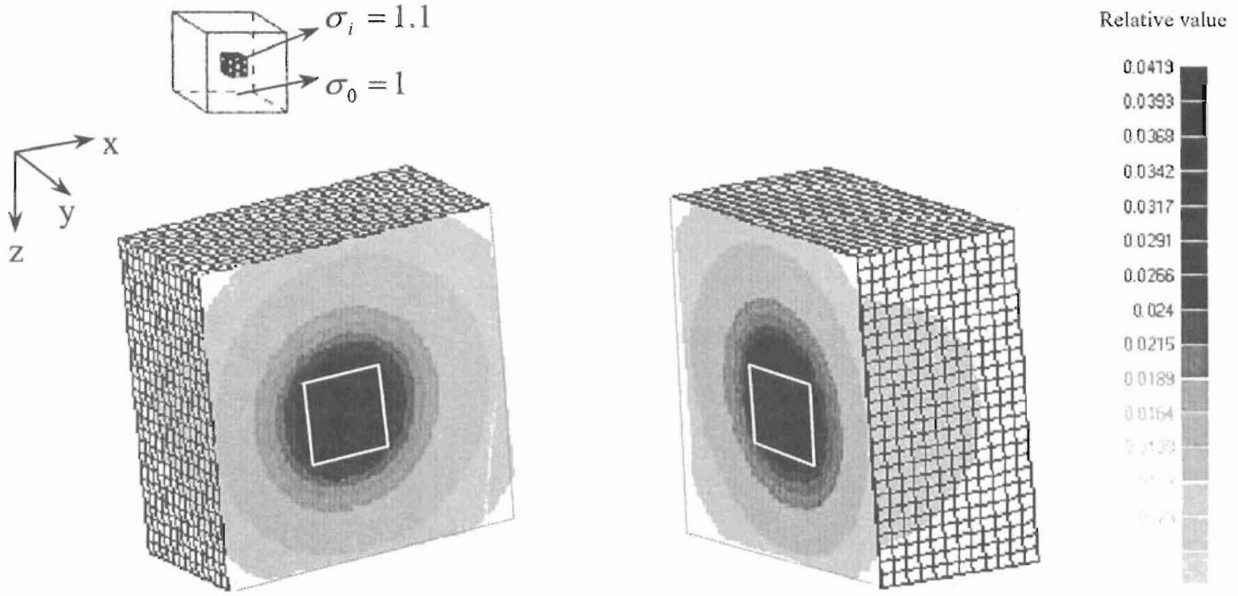


(a). The distribution of potential change (in the planes y=3.0, x=7.0)

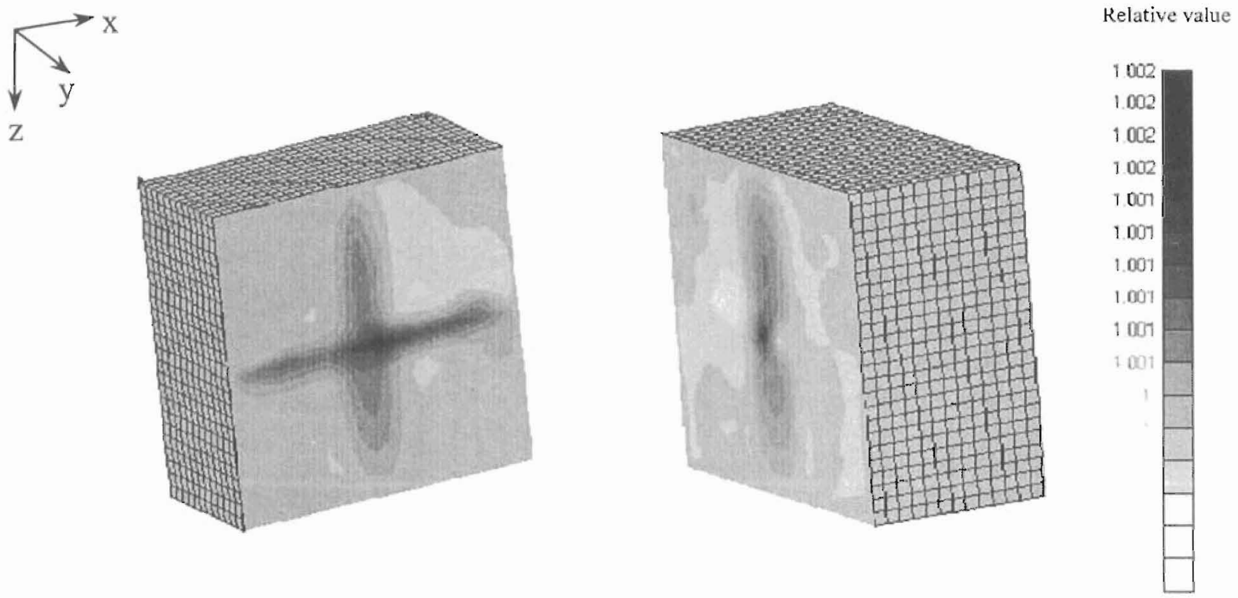


(b). The relative conductivity distribution inverted (in the planes y=3.0, x=7.0)

Fig.8. The case where a non-homogeneous cubic region is placed in one of the corners (with the relative conductivity $\sigma_i = 1.1$)



(a). The distribution of potential change (in the planes y=5.0, x=5.0)



(b). The relative resisitivity distribution inverted (in the planes y=5.0, x=5.0)

Fig.9. The case where a non-homogeneous cubic region is placed in the middle (with the relative conductivity $\sigma_i = 1.1$)

search the position where the conductivity is higher than the surrounding medium, but it is difficult to determine the exact discontinuous boundaries, so that the stepped change of the conductivity is difficult to be discriminated. The value of the conductivity in the cubes is under-estimated and its value is pushed toward the conductivity of the surrounding medium. As expected, the inversion accuracy is better for the case A than the case B.

7. Concluding Remarks

Following the electric impedance imaging technique examined in the previous papers [1] [2], in the present paper, examination was extended to the case where the data is accessible only to the ground surface. The simulation showed that the technique was also applicable to this case. The question is to improve the resolution and accuracy under noisy condition. At the present simulation, noise-free data were used. The inverted solution could at least be a good estimate for the initial data in the optimization type repetition algorithm frequently used. Examples show that the method proposed is successful in identifying conductive objects. The presence of the non-homogeneous regions and their positions are reasonably recognized though their boundaries are blurred.

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