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Topography of the Moho discontinuity estimated by column inversion of gravity anomaly data

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We present an inversion method, called "column inversion", to retrieve two-dimensional variation of the depth to the Moho discontinuity from Bouguer gravity anomaly data, assuming that a density jump between the lower crust and upper mantle is regionally invariant. The inversion method is applied to reveal the spatial undulation of the Moho discontinuity in the Chugoku and Shikoku districts, southwest Japan. The result shows that the Moho is deep in the land area and shallow in the sea area. The Moho is anomalously deep in the west regions of Shikoku and Chugoku districts, where the prominent negative gravity anomaly is observed. The two-dimensional variation of the Moho depth is quite similar to that determined by inversion of the first P-arrival time data. The column inversion of the Bouguer gravity anomaly data is proved to be useful for determining the fine structure of the Moho discontinuity undulation.

Keywords: Bouguer Gravity Anomaly, Column Inversion, Moho Discontinuity, Chugoku and Shikoku Districts

1. Introduction

The travel time analysis is a useful method to investigate the structure of the crust and upper mantle. But the seismological method is not applicable in an area where the seismic wave observation is difficult or impossible. In comparison with the seismic wave observations, the gravity measurements have an advantage of being more easily implemented at any place on the ground surface. The earth's structure analysis using the gravity data has an inevitable shortcoming that the density structure cannot be determined uniquely. Other geophysical data to constrain the density structure are needed for overcoming the difficulty. Recent study of the seismic velocity structure under the Chugoku and Shikoku districts, southwest Japan, showed that two-dimensional depth variation of the Moho discontinuity is well correlated with Bouguer gravity anomaly distribution (Oda et al., 2004). This result means that the gravity anomaly is attributable to spatial undulation of the Moho discontinuity interface.

The conventional method to estimate the density structure is based on the Fourier transform of gravity anomaly distribution (e.g., Tsuboi, 1953; Tomoda and Aki, 1955). Application of this method has been limited to retrieval of one-dimensional density structure along a profile line. Kanamori (1963a,b) made first attempt to estimate two-dimensional depth variation of the Moho discontinuity in the whole Japan Islands, inverting the reduced Bouguer gravity anomaly data for the depth to the discontinuity in rectangular parallelepiped columns, into which a volume of the earth under the study area was divided vertically to the ground surface. We call his method "column inversion". However, the gravity data used in the inversion had poor resolution to estimate fine structure of the discontinuity because the data were averaged over the area of $1°\times 1°$ in latitude and longitude. In addition, an approximate formulation was employed to calculate the gravity anomaly arising from a rectangular parallelepiped body. Recently the high-resolution gravity measurements provided the high quality data of Bouguer gravity anomaly in the Chugoku and Shikoku districts (Shichi and Yamamoto, 1994). Moreover, analytic formula to calculate exactly the gravity anomaly was derived (Banerjee and Gupta, 1977). These situations move us to devise a new inversion technique of the gravity anomaly data for estimate of fine structure of the Moho discontinuity.

In this study, we present a column inversion method incorporating the analytic expression of the gravity traction that is caused by a rectangular parallelepiped body. The fine structure of the Moho discontinuity is retrieved from the high-quality gravity anomaly data by making use of the inversion technique. Comparing with the Moho discontinuity undulation estimated by the seismological method, we examine the validity of our estimate of the Moho discontinuity structure.

2. Column inversion method

Imagine a two-layered structure composed of the crust and upper mantle, where constant density is assumed. The boundary between the crust and upper mantle is undulated as shown in Fig. 1. We set a Cartesian coordinate system where the x-y plane is taken at the horizontal ground surface and the z-axis is vertical downward, and divide a volume of the earth under a study area into rectangular parallelepiped columns with edges parallel to x-, y- and

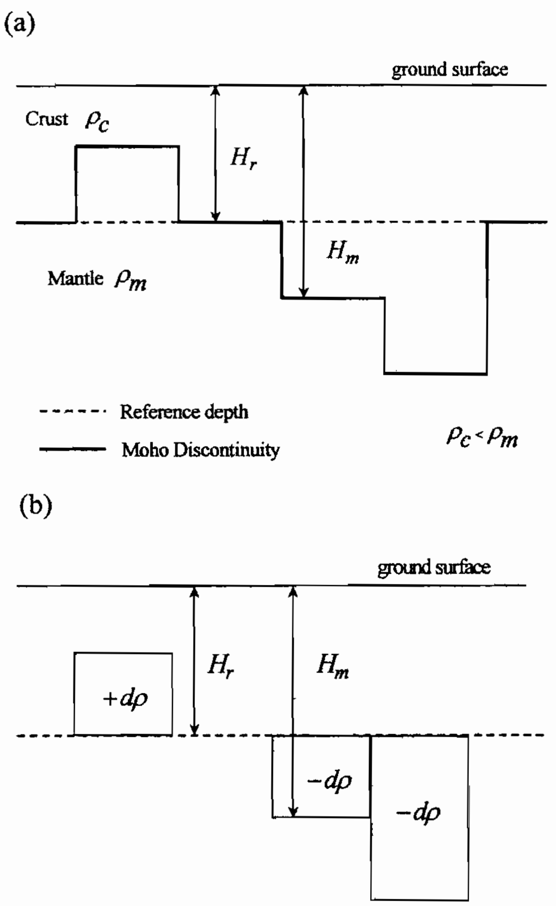

Fig. 1. (a) Schematic illustration of a two-layered structure made of the crust and upper mantle. Thick line represents the Moho discontinuity, and the dashed line indicates the reference depth. (b) The dark and light blocks show negative and positive density anomalies ($d\rho = \rho_m - \rho_c$) due to the Moho discontinuity undulation, respectively.

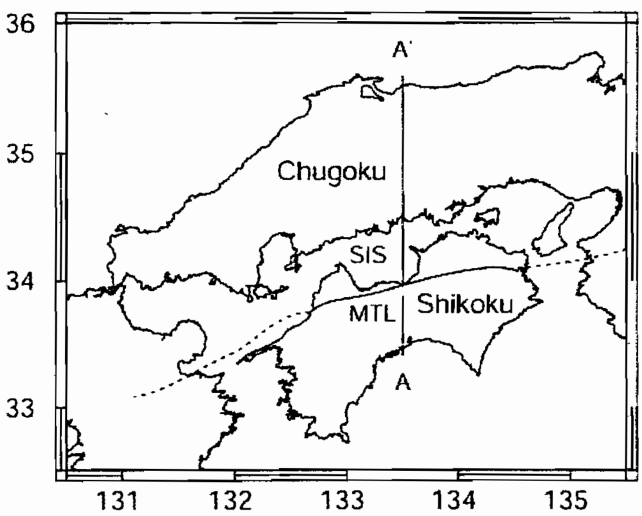

Fig. 3. Map of the study area. SIS and MTL are Seto Inland Sea and Median Tectonic Line, respectively.

z-axes (Fig. 2). The spatial undulation of the Moho discontinuity is represented as lateral variation of the Moho

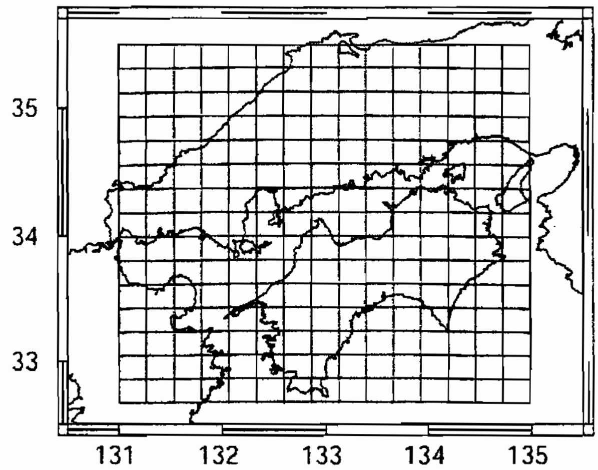

Fig. 2. Column configuration in the study area.

depths in the columns. Here we define a reference depth H_r to the Moho. If a real depth H_m to the Moho in a column is equal to the reference depth H_r, then the contribution of this column to the gravity anomaly disappears because of no apparent density anomaly. On the other hand, if real and reference depths to the Moho are in disagreement, then an apparent density anomaly $d\rho$ occurs within a depth interval between H_m and H_r , and it contributes to the gravity anomaly, that is, the gravity anomaly is regarded as resulting from the apparent density anomaly which takes place between the real and reference depths. Because the upper mantle density ρ_m is larger than the lower crust density ρ_c , when $H_m \ge H_r$ a negative density anomaly of $-(\rho_m - \rho_c)$ takes place in the interval of $H_r \le z < H_m$, and when $H_m \le H_r$ a positive density anomaly of $\rho_m - \rho_c$ occurs in $H_m \le z < H_r$ (see Fig. 1).

The gravity anomaly, $g^c(x,y)$, at a point (x,y) on the ground surface is expressed as sum of vertical gravity tractions which arise from density anomalies $d\rho$ of the columns,

$$g^c(x,y) = \sum_{i=1}^{N} g^{(i)}(x,y) \tag{1}$$

where N denotes the number of columns. The vertical gravity traction $g^{(i)}(x,y)$ due to a rectangular parallelepiped density anomaly of the i-th column is written as (Banerjee and Gupta, 1977)

$$\begin{aligned} g^{(i)}(x,y) &= -G(\rho_m - \rho_c)[(x-x')\log\{(y-y') \\ &+ \sqrt{(x-x')^2 + (y-y')^2 + z'^2}\} \\ &+ (y-y')\log\{(x-x') + \sqrt{(x-x')^2 + (y-y')^2 + z'^2}\} \\ &+ z'\tan^{-1}\{\frac{(x-x')(y-y')}{z'\sqrt{(x-x')^2 + (y-y')^2 + z'^2}}\}] \Big|_{X_l^{(i)}}^{X_u^{(i)}} \Big|_{Y_l^{(i)}}^{Y_u^{(i)}} \Big|_{H_m^{(i)}}^{H_r} \end{aligned} \tag{2}$$

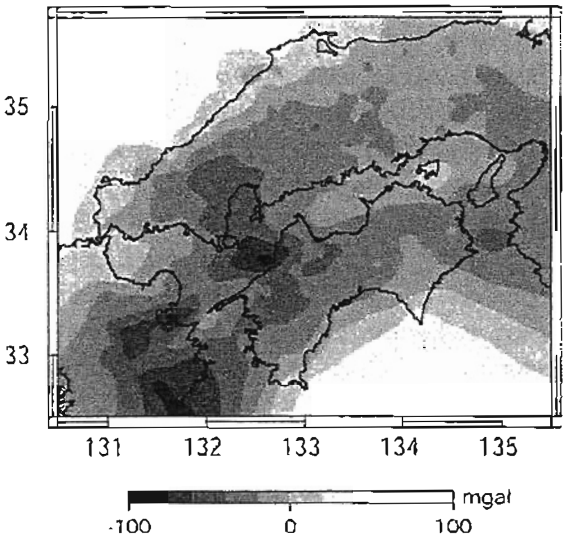

Fig. 4. Bouguer gravity anomaly map corrected for the vertical gravity traction arising from the PHS plate.

where G is the universal constant of gravitation, $X_u^{(i)}$ and $X_l^{(i)}$ are the upper and lower limits of the x-axis of the rectangular parallelepiped, $Y_u^{(i)}$ and $Y_l^{(i)}$ are those of the y-axis, $H_m^{(i)}$ is the Moho depth of the i-th column, and

$$f(x-x')\,\Big|_a^b = f(x-b) - f(x-a).$$

Since the spatial undulation of the Moho discontinuity is represented as lateral variation of the Moho depths in the columns, we must determine the Moho depths optimized so that the theoretical and observed gravity anomalies match well. Because the gravity anomaly is expressed as a nonlinear function of the Moho depths $H_m^{(i)}$, we linearize the differences Δg of the observed gravity anomalies $g^o(x,y)$ from theoretical ones $g^c(x,y)$, which are calculated for a given model of lateral variation of the Moho depth,

$$g^o(x,y) - g^c(x,y) = \sum_{i=1}^{N} \frac{\partial g^{(i)}}{\partial H_m^{(i)}} \delta H_m^{(i)} \qquad (3)$$

where $\delta H_m^{(i)}$ is small correction for $H_m^{(i)}$. Derivative of Eq. (2) with $H_m^{(i)}$ is written as

$$\begin{aligned} \frac{\partial g^{(i)}}{\partial H_m^{(i)}} = -G(\rho_m - \rho_c)[& \frac{(x-x')H_m^{(i)}}{R^{(i)}\{(y-y')+H_m^{(i)}\}} \\ & + \frac{(y-y')H_m^{(i)}}{R^{(i)}\{(x-x')+H_m^{(i)}\}} + \tan^{-1}\{\frac{(x-x')(y-y')}{R^{(i)}H_m^{(i)}}\} \\ & + \frac{(R^{(i)}H_m^{(i)})^2(x-x')(y-y')}{(x-x')^2(y-y')^2+(R^{(i)}H_m^{(i)})^2} \\ & \times\{\frac{1}{R^{(i)}} + \frac{1}{(R^{(i)})^3}\}] \Big|_{X_l^{(i)}}^{X_u^{(i)}} \Big|_{Y_l^{(i)}}^{Y_u^{(i)}} \end{aligned} \qquad (4)$$

with

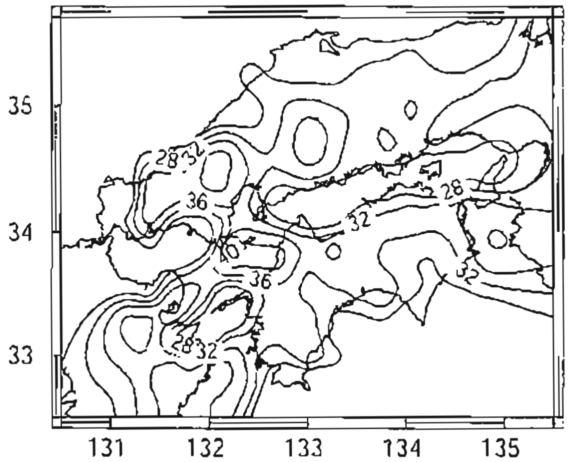

Fig. 5. Contour map of the Moho depth determined from the Bouguer gravity anomaly data.

$$R^{(i)} = \sqrt{(x-x')^2 + (y-y')^2 + (H_m^{(i)})^2}\,.$$

When the gravity anomaly is measured at M points $(x_k, y_k; k = 1,2,3,\ldots,M)$ on the ground surface, the whole set of Eqs. (3) is expressed as

$$\Delta \mathbf{g} = \mathbf{A}\mathbf{h} \qquad (5)$$

where $\mathbf{A}$ is the $M \times N$ matrix made of the elements

$$A_{ik} = \frac{\partial g^{(i)}(x_k, y_k)}{\partial H_m^{(i)}} \qquad (6)$$

and

$$\begin{aligned} \Delta \mathbf{g}^T &= (\Delta g_1, \Delta g_2, \ldots, \Delta g_M) \\ \mathbf{h}^T &= (\delta H_m^{(1)}, \delta H_m^{(2)}, \ldots, \delta H_m^{(N)}) \end{aligned} \qquad (7)$$

The superscript "T" in Eq. (7) denotes transpose of the matrix. The matrix elements of $\mathbf{A}$ and $\Delta\mathbf{g}$ are easily evaluated for a given structure of the Moho discontinuity, and $\mathbf{h}$ is the unknown matrix to be determined. We employ the damped least squares method (Levenberg, 1944) to solve Eq. (5) for $\mathbf{h}$ because the matrix $\mathbf{A}^T\mathbf{A}$ is close to singular. The damped least squares solution of Eq. (5) is written as

$$\tilde{\mathbf{h}} = (\mathbf{A}^T\mathbf{A} + \theta\,\mathbf{I})^{-1}\mathbf{A}^T\Delta\mathbf{g}, \qquad (8)$$

where θ and $\mathbf{I}$ are the damping factor and identity matrix of $N \times N$ dimension, respectively. The Moho depth of the i-th column is determined by adding the solution $\tilde{h}_i$ to the initial depth $H_m^{(i)}$. The error $\Delta\tilde{h}_i$ in the solution is calculated by (cf. Aki and Lee, 1977)

$$\Delta\tilde{h}_i = \sigma\, D_{ii} \qquad (9)$$

where D_{ii} is the diagonal elements of the matrix

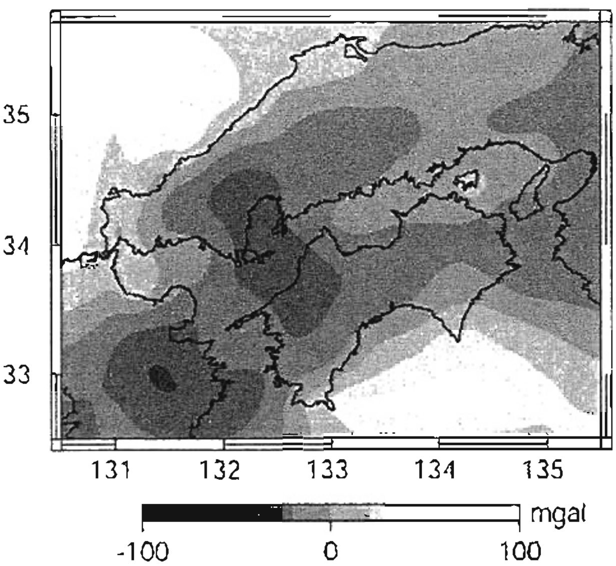

Fig. 6. The Bouguer gravity anomaly map calculated regressively for the two-dimensional variation of the Moho depth.

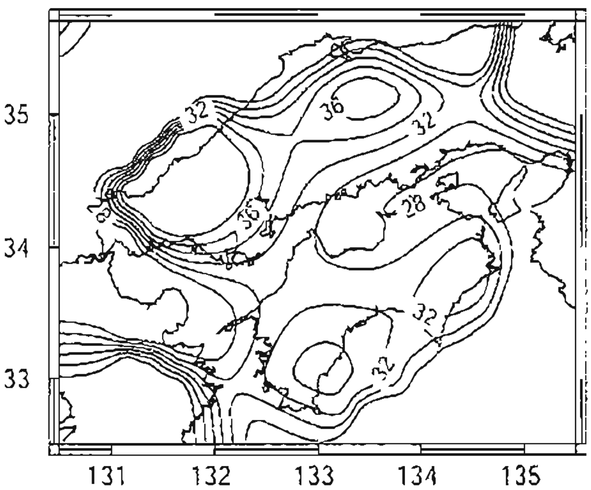

Fig.7. Contour map of the Moho depth, as estimated by the seismological method. (After Oda et al., 2004).

$$\mathbf{D} = (\mathbf{A}^T\mathbf{A} + \theta\,\mathbf{I})^{-1}\mathbf{A}^T\mathbf{A}(\mathbf{A}^T\mathbf{A} + \theta\,\mathbf{I})^{-1}, \tag{10}$$

and σ is the RMS of the gravity anomaly residuals. The best estimate of the Moho depths of the columns is obtained by repeating the calculations of Eq. (1) to Eq. (10) until some convergence criterion is satisfied.

3. Results and discussion

The study area is in the latitude range of 32.0-35.5 °N and the longitude range of 131.5-135.25 °E of the Chugoku and Shikoku districts, southwest Japan (Fig. 3). The density jump across the Moho discontinuity is estimated by using the Birch's law

$$\left(\frac{\partial \rho}{\partial V_p}\right) = 0.302 \text{ (Anderson, 1967).}$$

We analyze the Bouguer gravity anomaly data measured by a research group of Nagoya University (Shichi and Yamamoto, 1994). Since the Philippine Sea (PHS) plate is subducting beneath the Shikoku district, the observed gravity anomaly should be corrected for the vertical gravity traction due to the PHS plate. In order to calculate the gravity traction, we assume that (1) the density of the plate is larger by 0.06 g/cm^3 than the density of the uppermost mantle surrounding the plate, (2) the PHS plate with 20 km thickness horizontally lies below 50 km depth under the Shikoku district, and (3) the leading edge of the PHS plate does not exist under the Chugoku district beyond the Median Tectonic Line (Fig. 3). The calculated gravity traction is smaller than 20 mgal in the study area. Figure 4 depicts the Bouguer gravity anomaly map corrected for the gravity traction due to the PHS plate.

We take the x- and y-axes parallel in the east and north directions, respectively, and divide a volume of the earth under the study area into 225 rectangular columns ($N = 15 \times 15$) (Fig. 2). The number of the gravity anomaly data is 1088. The density jump, $\rho_m - \rho_c$, across the Moho discontinuity is assumed to be 0.33 g/cm^3, which is estimated from P-wave velocity jump of 1.1 km/s across the Moho discontinuity. A damping factor of $\theta = 10^{-8}$ and the reference depth of H_r =30 km, which is nearly equal to the mean depth of the Moho discontinuity estimated from seismological method, are adopted for the column inversion. The RMS of the residuals between observed and theoretical gravity anomalies is 54 mgal for a horizontally flat Moho discontinuity located at 25 km depth, and the initial RMS is reduced to 6 mgal during 6 times of iteration. Figure 5 shows the contour map of the depth variation of the Moho discontinuity. The Moho is located at the depth of 25-40 km with errors less than ±2 km. The Moho depth is small in the Seto Inland Sea area and large in the western Chugoku and Shikoku regions where the pronouncedly negative gravity anomaly is observed. Figure 6 depicts the Bouguer gravity anomaly map predicted regressively for the estimated Moho depth distribution. Comparison between the observed and calculated gravity anomaly maps shows good agreement (see Figs. 4 and 6).

Figure 7 depicts the two-dimensional variation of the Moho depth, which was obtained by the seismological method (Oda et al., 2004). Although details of the Moho depth variations estimated by gravitational and seismological methods are different from each other, both the depth variations reveal a common feature that the Moho is deep in the western Chugoku and Shikoku and shallow in east Seto Inland Sea. Cross section of the Moho discontinuity along A-A' line is compared with that estimated by the seismological method (Fig. 8). Both the cross sections are in good agreement. These results demonstrate the validity of our estimate of the lateral variation of the Moho depth, as retrieved by column

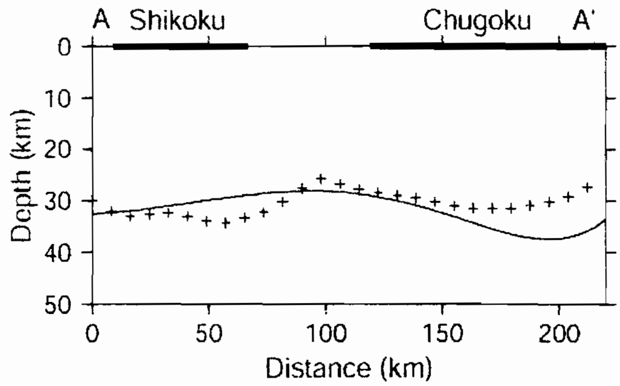

Fig. 8. Cross section along A-A' line of the two-dimensional variation of the Moho depth. Cross and solid curve indicate the Moho depth estimated by the gravitational and seismological methods, respectively. Location of A-A' line is shown in Fig. 3.

inversion of the Bouguer gravity anomaly data. The difference between both the Moho depth variations may come from two plausible origins; one is how to choose the reference Moho depth H_r and the other is that the gravity anomaly is attributed to only density jump across the Moho discontinuity, which is assumed to be regionally invariant.

4. Conclusions

We presented column inversion method to retrieve two-dimensional variation of the depth to the Moho discontinuity from Bouguer gravity anomaly data. In the inversion, we assumed that the gravity anomaly is caused by apparent density anomaly which arises from undulation of the Moho discontinuity interface, across which a constant density jump occurs. The method was applied to determine the fine structure of the Moho discontinuity in the Chugoku and Shikoku districts. The inversion results showed that the Moho is deep in the land area and shallow in the east Seto Inland Sea. The distinct negative gravity anomaly in the west regions of the Chugoku and Shikoku districts was interpreted as being due to the deep Moho. The depth variation was in good agreement with that estimated by the seismological method. The column inversion of the Bouguer gravity anomaly data, as the travel time inversion of seismic waves, was proved to be useful technique for determining the fine structure of the Moho discontinuity undulation.

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