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Tetsu Nishimoto*

*Kinki Welfare University

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HOPF ALGEBRA STRUCTURE OF MORAVA K-THEORY OF THE EXCEPTIONAL LIE GROUPS, II

Tetsu NISHIMOTO

1. Introduction

In 1990, D. Ravenel proposed to M. Mimura the problem to determine the Morava K-theory of the classifying spaces of Lie groups at the Adams Memorial Symposium. In order to do it by making use of the Rothenberg-Steenrod spectral sequence one needs to know the Hopf algebra structure of the Morava K-theory of Lie groups. Then, in the paper of the same title [MN], M. Mimura and the author calculated the Hopf (bi) algebra structure of $P(n)$-theory for the exceptional Lie group G when the Atiyah-Hirzebruch spectral sequence (AHSS for short) for $P(n)^*(G)$ collapses except $P(n)^*(E_8)$ for $n \geq 4$ at the prime 2. The result obtained here is already announced in the previous paper. That is, we determine the bi-algebra structure of $P(4)^*(E_8)$ at the prime 2. Thus the bi-algebra structure of the Morava K-theory of E_8 for $n \geq 4$ is given by the isomorphism

$$K(n)^*(E_8) \cong K(n)^* \underset{P(4)^*}{\otimes} P(4)^*(E_8).$$

Especially, according to Theorem 1.1, the Hopf algebra structures of $P(n)^*(E_8)$ and $K(n)^*(E_8)$ are essentially the same as that of $H^*(E_8;\mathbb{Z}/2)$ for $n \geq 5$, that is, there are Hopf algebra isomorphisms

$$P(n)^*(E_8) \cong P(n)^* \otimes H^*(E_8;\mathbb{Z}/2),$$
$$K(n)^*(E_8) \cong K(n)^* \otimes H^*(E_8;\mathbb{Z}/2).$$

In Section 2, we recall the Brown-Peterson cohomology theory and other related cohomology theories, and our method of calculation is to use the commutativity of the Milnor operations and the reduced coproduct, which is entirely similar to that used in [MN]. That is, our main tools are Lemma 2.1 and Proposition 2.2. As is shown by the triviality of AHSS, $P(4)^*(E_8)$ mod $(v_4, v_5, v_6, \cdots)$ is isomorphic to $H^*(E_8;\mathbb{Z}/2)$ as Hopf algebras. So in Section 3, first of all, we determine thoroughly the bi-algebra structure of $P(4)^*(E_8)$ mod $(v_4^2, v_5, v_6, \cdots)$ and actions of the Milnor operations. Then the calculation of $P(4)^*(E_8)$ mod $(v_4^n, v_5, v_6, \cdots)$ for $3 \leq n \leq 5$ is entirely similar and tedious, and hence omitted. For $n \geq 6$, by virtue of Proposition 2.2 we see that there do not appear any terms with v_4^{n-1} in the coproduct. This gives rise to the connective Morava K-theory. As for v_l $(l \geq 5)$,

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we carry out a similar calculation, which turns out to be trivial. Thus we obtain the following theorem.

Theorem 1.1. *Let $p = 2$. Then there is a module isomorphism*

$$P(4)^*(E_8) \cong P(4)^* \otimes H^*(E_8; \mathbb{Z}/3),$$

and the reduced coproduct is given as follows:

$$\begin{aligned}
\bar\psi(x_3) = {} & v_4x_3^{10} \otimes x_3 + v_4x_5^6 \otimes x_3 + v_4x_3^4x_9^2 \otimes x_3 + v_4x_{15}^2 \otimes x_3 + v_4x_5^2x_9^2 \otimes x_5 \\
& + v_4x_3^8 \otimes x_9 + v_4x_3^2x_9^2 \otimes x_9 + v_4x_9^2 \otimes x_{15} + v_4x_3^2x_5^2 \otimes x_{17} \\
& + v_4x_5^2 \otimes x_{23} + v_4x_3^2 \otimes x_{27} + v_4^2x_3^{14}x_9^2 \otimes x_3 + v_4^2x_3^4x_5^6x_9^2 \otimes x_3 \\
& + v_4^2x_3^4x_9^2x_{15}^2 \otimes x_3 + v_4^2x_3^{10}x_5^2x_9^2 \otimes x_5 + v_4^2x_5^2x_9^2x_{15}^2 \otimes x_5 \\
& + v_4^2x_3^8x_5^6 \otimes x_9 + v_4^2x_3^{12}x_9^2 \otimes x_9 + v_4^2x_3^2x_5^6x_9^2 \otimes x_9 + v_4^2x_3^8x_{15}^2 \otimes x_9 \\
& + v_4^2x_3^2x_9^2x_{15}^2 \otimes x_9 + v_4^2x_3^{10}x_9^2 \otimes x_{15} + v_4^2x_5^6x_9^2 \otimes x_{15} + v_4^2x_9^2x_{15}^2 \otimes x_{15} \\
& + v_4^2x_3^6x_5^2x_9^2 \otimes x_{17} + v_4^2x_3^2x_5^2x_{15}^2 \otimes x_{17} + v_4^2x_3^{10}x_5^2 \otimes x_{23} \\
& + v_4^2x_3^4x_5^2x_9^2 \otimes x_{23} + v_4^2x_5^2x_{15}^2 \otimes x_{23} + v_4^2x_3^2x_5^6 \otimes x_{27} + v_4^2x_3^6x_9^2 \otimes x_{27} \\
& + v_4^2x_3^2x_{15}^2 \otimes x_{27} + v_4^2x_3^8x_5^2 \otimes x_{29} + v_4^2x_3^2x_5^2x_9^2 \otimes x_{29} \\
& + v_4^3x_3^{12}x_5^2x_{15}^2 \otimes x_{17} + v_4^3x_3^6x_5^2x_9^2x_{15}^2 \otimes x_{17} + v_4^3x_3^{14}x_5^2x_9^2 \otimes x_{23} \\
& + v_4^3x_3^4x_5^2x_9^2x_{15}^2 \otimes x_{23} + v_4^3x_3^{12}x_5^6 \otimes x_{27} + v_4^3x_3^6x_5^6x_9^2 \otimes x_{27} \\
& + v_4^3x_3^{12}x_{15}^2 \otimes x_{27} + v_4^3x_3^6x_9^2x_{15}^2 \otimes x_{27} + v_4^3x_3^{12}x_5^2x_9^2 \otimes x_{29} \\
& + v_4^3x_3^8x_5^2x_{15}^2 \otimes x_{29} + v_4^3x_3^2x_5^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}$$

$$\begin{aligned}
\bar\psi(x_5) = {} & v_4x_3^4x_5^4 \otimes x_3 + v_4x_3^{10} \otimes x_5 + v_4x_{15}^2 \otimes x_5 + v_4x_3^2x_5^4 \otimes x_9 \\
& + v_4x_5^4 \otimes x_{15} + v_4x_3^6 \otimes x_{17} + v_4x_9^2 \otimes x_{17} + v_4x_3^4 \otimes x_{23} \\
& + v_4x_3^2 \otimes x_{29} + v_4^2x_3^{14}x_5^4 \otimes x_3 + v_4^2x_3^8x_5^4x_9^2 \otimes x_3 + v_4^2x_3^4x_5^4x_{15}^2 \otimes x_3 \\
& + v_4^2x_3^{10}x_5^6 \otimes x_5 + v_4^2x_5^6x_{15}^2 \otimes x_5 + v_4^2x_3^{12}x_5^4 \otimes x_9 + v_4^2x_3^2x_5^4x_{15}^2 \otimes x_9 \\
& + v_4^2x_3^{10}x_5^4 \otimes x_{15} + v_4^2x_5^4x_{15}^2 \otimes x_{15} + v_4^2x_3^6x_{15}^2 \otimes x_{17} + v_4^2x_9^2x_{15}^2 \otimes x_{17} \\
& + v_4^2x_3^{14} \otimes x_{23} + v_4^2x_3^8x_9^2 \otimes x_{23} + v_4^2x_3^4x_{15}^2 \otimes x_{23} + v_4^2x_3^6x_5^4 \otimes x_{27} \\
& + v_4^2x_5^4x_9^2 \otimes x_{27} + v_4^2x_3^{12} \otimes x_{29} + v_4^2x_3^2x_{15}^2 \otimes x_{29} + v_4^3x_3^8x_5^4x_9^2x_{15}^2 \otimes x_3 \\
& + v_4^3x_3^{10}x_5^6x_9^2 \otimes x_{17} + v_4^3x_3^6x_5^6x_{15}^2 \otimes x_{17} + v_4^3x_3^{10}x_9^2x_{15}^2 \otimes x_{17} \\
& + v_4^3x_5^6x_9^2x_{15}^2 \otimes x_{17} + v_4^3x_3^{14}x_5^6 \otimes x_{23} + v_4^3x_3^4x_5^6x_{15}^2 \otimes x_{23} \\
& + v_4^3x_3^8x_9^2x_{15}^2 \otimes x_{23} + v_4^3x_3^6x_5^4x_{15}^2 \otimes x_{27} + v_4^3x_5^4x_9^2x_{15}^2 \otimes x_{27} \\
& + v_4^3x_3^{12}x_5^6 \otimes x_{29} + v_4^3x_3^2x_5^6x_{15}^2 \otimes x_{29} + v_4^4x_3^{10}x_5^6x_9^2x_{15}^2 \otimes x_{17} \\
& + v_4^4x_3^8x_5^6x_9^2x_{15}^2 \otimes x_{23} + v_4^4x_3^{10}x_5^4x_9^2x_{15}^2 \otimes x_{27},
\end{aligned}$$

$$\begin{aligned}
\bar{\psi}(x_9) = {} & v_4x_3^{12}\otimes x_3 + v_4x_3^8x_5^2\otimes x_5 + v_4x_5^6\otimes x_9 + v_4x_{15}^2\otimes x_9 + v_4x_3^8\otimes x_{15} \\
& + v_4x_3^4x_5^2\otimes x_{17} + v_4x_3^4\otimes x_{27} + v_4x_5^2\otimes x_{29} + v_4^2x_3^{12}x_5^6\otimes x_3 \\
& + v_4^2x_3^{12}x_{15}^2\otimes x_3 + v_4^2x_3^8x_5^2x_{15}^2\otimes x_5 + v_4^2x_3^{10}x_5^6\otimes x_9 + v_4^2x_3^{10}x_{15}^2\otimes x_9 \\
& + v_4^2x_3^8x_5^6\otimes x_{15} + v_4^2x_3^8x_{15}^2\otimes x_{15} + v_4^2x_3^4x_5^2x_{15}^2\otimes x_{17} \\
& + v_4^2x_3^{12}x_5^2\otimes x_{23} + v_4^2x_3^4x_5^6\otimes x_{27} + v_4^2x_3^4x_{15}^2\otimes x_{27} + v_4^2x_5^2x_{15}^2\otimes x_{29} \\
& + v_4^3x_3^{14}x_5^2x_{15}^2\otimes x_{17} + v_4^3x_3^{12}x_5^2x_{15}^2\otimes x_{23} + v_4^3x_3^{14}x_5^6\otimes x_{27} \\
& + v_4^3x_3^{14}x_{15}^2\otimes x_{27} + v_4^3x_3^{10}x_5^2x_{15}^2\otimes x_{29}, \\
\bar{\psi}(x_{15}) = {} & x_3^4\otimes x_3 + x_5^2\otimes x_5 + x_3^2\otimes x_9 + v_4x_3^{14}\otimes x_3 + v_4x_3^4x_5^6\otimes x_3 \\
& + v_4x_3^8x_9^2\otimes x_3 + v_4x_3^4x_{15}^2\otimes x_3 + v_4x_3^{10}x_5^2\otimes x_5 + v_4x_5^2x_{15}^2\otimes x_5 \\
& + v_4x_3^{12}\otimes x_9 + v_4x_3^2x_5^6\otimes x_9 + v_4x_3^2x_{15}^2\otimes x_9 + v_4x_3^{10}\otimes x_{15} \\
& + v_4x_5^6\otimes x_{15} + v_4x_{15}^2\otimes x_{15} + v_4x_3^6x_5^2\otimes x_{17} + v_4x_5^2x_9^2\otimes x_{17} \\
& + v_4x_3^4x_5^2\otimes x_{23} + v_4x_3^6\otimes x_{27} + v_4x_9^2\otimes x_{27} + v_4x_3^2x_5^2\otimes x_{29} \\
& + v_4^2x_3^8x_5^6x_9^2\otimes x_3 + v_4^2x_3^8x_9^2x_{15}^2\otimes x_3 + v_4^2x_3^6x_5^2x_{15}^2\otimes x_{17} \\
& + v_4^2x_5^2x_9^2x_{15}^2\otimes x_{17} + v_4^2x_3^{14}x_5^2\otimes x_{23} + v_4^2x_3^8x_5^2x_9^2\otimes x_{23} \\
& + v_4^2x_3^4x_5^2x_{15}^2\otimes x_{23} + v_4^2x_3^6x_5^6\otimes x_{27} + v_4^2x_5^6x_9^2\otimes x_{27} + v_4^2x_3^6x_{15}^2\otimes x_{27} \\
& + v_4^2x_9^2x_{15}^2\otimes x_{27} + v_4^2x_3^{12}x_5^2\otimes x_{29} + v_4^2x_3^2x_5^2x_{15}^2\otimes x_{29} \\
& + v_4^3x_3^{10}x_5^2x_9^2x_{15}^2\otimes x_{17} + v_4^3x_3^8x_5^2x_9^2x_{15}^2\otimes x_{23} + v_4^3x_3^{10}x_5^6x_9^2\otimes x_{27} \\
& + v_4^3x_3^{10}x_9^2x_{15}^2\otimes x_{27}, \\
\bar{\psi}(x_{17}) = {} & v_4x_3^8x_5^4\otimes x_3 + v_4x_3^8x_9^2\otimes x_5 + v_4x_5^4x_9^2\otimes x_9 + v_4x_3^4x_9^2\otimes x_{17} \\
& + v_4x_{15}^2\otimes x_{17} + v_4x_3^8\otimes x_{23} + v_4x_5^4\otimes x_{27} + v_4x_9^2\otimes x_{29} \\
& + v_4^2x_3^{12}x_5^4x_9^2\otimes x_3 + v_4^2x_3^8x_5^4x_{15}^2\otimes x_3 + v_4^2x_3^8x_9^2x_{15}^2\otimes x_5 \\
& + v_4^2x_5^4x_9^2x_{15}^2\otimes x_9 + v_4^2x_3^8x_5^4x_9^2\otimes x_{15} + v_4^2x_3^{10}x_5^6\otimes x_{17} \\
& + v_4^2x_3^{10}x_{15}^2\otimes x_{17} + v_4^2x_5^6x_{15}^2\otimes x_{17} + v_4^2x_3^4x_9^2x_{15}^2\otimes x_{17} \\
& + v_4^2x_3^{12}x_9^2\otimes x_{23} + v_4^2x_3^8x_{15}^2\otimes x_{23} + v_4^2x_3^4x_5^4x_9^2\otimes x_{27} \\
& + v_4^2x_5^4x_{15}^2\otimes x_{27} + v_4^2x_9^2x_{15}^2\otimes x_{29} + v_4^3x_3^{12}x_5^4x_9^2x_{15}^2\otimes x_3 \\
& + v_4^3x_3^8x_5^6x_9^2x_{15}^2\otimes x_5 + v_4^3x_3^{10}x_5^4x_9^2x_{15}^2\otimes x_9 + v_4^3x_3^8x_5^4x_9^2x_{15}^2\otimes x_{15} \\
& + v_4^3x_3^{14}x_5^6x_9^2\otimes x_{17} + v_4^3x_3^{10}x_5^6x_{15}^2\otimes x_{17} + v_4^3x_3^{14}x_9^2x_{15}^2\otimes x_{17} \\
& + v_4^3x_3^4x_5^6x_9^2x_{15}^2\otimes x_{17} + v_4^3x_3^8x_5^6x_{15}^2\otimes x_{23} + v_4^3x_3^{12}x_9^2x_{15}^2\otimes x_{23} \\
& + v_4^3x_3^{10}x_5^4x_{15}^2\otimes x_{27} + v_4^3x_3^4x_5^4x_9^2x_{15}^2\otimes x_{27} + v_4^3x_3^{10}x_5^6x_9^2\otimes x_{29}
\end{aligned}$$

$$
\begin{aligned}
&+ v_4^3x_3^{10}x_9^2x_{15}^2 \otimes x_{29} + v_4^3x_5^6x_9^2x_{15}^2 \otimes x_{29} + v_4^4x_3^{14}x_5^6x_9^2x_{15}^2 \otimes x_{17}\\
&+ v_4^4x_3^{12}x_5^6x_9^2x_{15}^2 \otimes x_{23} + v_4^4x_3^{14}x_5^4x_9^2x_{15}^2 \otimes x_{27} + v_4^4x_3^{10}x_5^6x_9^2x_{15}^2 \otimes x_{29},\\
\bar\psi(x_{23}) = {}& x_5^4 \otimes x_3 + x_9^2 \otimes x_5 + x_3^2 \otimes x_{17} + v_4x_3^{10}x_5^4 \otimes x_3 + v_4x_3^4x_5^4x_9^2 \otimes x_3\\
&+ v_4x_5^4x_{15}^2 \otimes x_3 + v_4x_3^{10}x_9^2 \otimes x_5 + v_4x_9^2x_{15}^2 \otimes x_5 + v_4x_3^8x_5^4 \otimes x_9\\
&+ v_4x_3^2x_5^4x_9^2 \otimes x_9 + v_4x_5^4x_9^2 \otimes x_{15} + v_4x_3^6x_9^2 \otimes x_{17} + v_4x_3^2x_{15}^2 \otimes x_{17}\\
&+ v_4x_3^{10} \otimes x_{23} + v_4x_3^4x_9^2 \otimes x_{23} + v_4x_{15}^2 \otimes x_{23} + v_4x_3^2x_5^4 \otimes x_{27}\\
&+ v_4x_3^8 \otimes x_{29} + v_4x_3^2x_9^2 \otimes x_{29} + v_4^2x_3^{14}x_5^4x_9^2 \otimes x_3\\
&+ v_4^2x_3^4x_5^4x_9^2x_{15}^2 \otimes x_3 + v_4^2x_3^{10}x_5^6x_9^2 \otimes x_5 + v_4^2x_5^6x_9^2x_{15}^2 \otimes x_5\\
&+ v_4^2x_3^{12}x_5^4x_9^2 \otimes x_9 + v_4^2x_3^8x_5^4x_{15}^2 \otimes x_9 + v_4^2x_3^2x_5^4x_9^2x_{15}^2 \otimes x_9\\
&+ v_4^2x_3^{10}x_5^4x_9^2 \otimes x_{15} + v_4^2x_5^4x_9^2x_{15}^2 \otimes x_{15} + v_4^2x_3^{12}x_5^6 \otimes x_{17}\\
&+ v_4^2x_3^{12}x_{15}^2 \otimes x_{17} + v_4^2x_3^2x_5^6x_{15}^2 \otimes x_{17} + v_4^2x_3^6x_9^2x_{15}^2 \otimes x_{17}\\
&+ v_4^2x_3^{10}x_5^6 \otimes x_{23} + v_4^2x_3^{14}x_9^2 \otimes x_{23} + v_4^2x_5^6x_{15}^2 \otimes x_{23}\\
&+ v_4^2x_3^4x_9^2x_{15}^2 \otimes x_{23} + v_4^2x_3^6x_5^4x_9^2 \otimes x_{27} + v_4^2x_3^2x_5^4x_{15}^2 \otimes x_{27}\\
&+ v_4^2x_3^{12}x_9^2 \otimes x_{29} + v_4^2x_3^8x_{15}^2 \otimes x_{29} + v_4^2x_3^2x_9^2x_{15}^2 \otimes x_{29}\\
&+ v_4^3x_3^{12}x_5^6x_{15}^2 \otimes x_{17} + v_4^3x_3^6x_5^6x_9^2x_{15}^2 \otimes x_{17} + v_4^3x_3^{14}x_5^6x_9^2 \otimes x_{23}\\
&+ v_4^3x_3^4x_5^6x_9^2x_{15}^2 \otimes x_{23} + v_4^3x_3^{12}x_5^4x_{15}^2 \otimes x_{27} + v_4^3x_3^6x_5^4x_9^2x_{15}^2 \otimes x_{27}\\
&+ v_4^3x_3^{12}x_5^6x_9^2 \otimes x_{29} + v_4^3x_3^8x_5^6x_{15}^2 \otimes x_{29} + v_4^3x_3^2x_5^6x_9^2x_{15}^2 \otimes x_{29},\\
\bar\psi(x_{27}) = {}& x_3^8 \otimes x_3 + x_9^2 \otimes x_9 + x_5^2 \otimes x_{17} + v_4x_3^8x_5^6 \otimes x_3 + v_4x_3^{12}x_9^2 \otimes x_3\\
&+ v_4x_3^8x_{15}^2 \otimes x_3 + v_4x_3^8x_5^2x_9^2 \otimes x_5 + v_4x_5^6x_9^2 \otimes x_9 + v_4x_9^2x_{15}^2 \otimes x_9\\
&+ v_4x_3^8x_9^2 \otimes x_{15} + v_4x_3^4x_5^2x_9^2 \otimes x_{17} + v_4x_5^2x_{15}^2 \otimes x_{17} + v_4x_3^8x_5^2 \otimes x_{23}\\
&+ v_4x_5^6 \otimes x_{27} + v_4x_3^4x_9^2 \otimes x_{27} + v_4x_{15}^2 \otimes x_{27} + v_4x_5^2x_9^2 \otimes x_{29}\\
&+ v_4^2x_3^{12}x_5^6x_9^2 \otimes x_3 + v_4^2x_3^{12}x_9^2x_{15}^2 \otimes x_3 + v_4^2x_3^8x_5^2x_9^2x_{15}^2 \otimes x_5\\
&+ v_4^2x_3^{10}x_5^6x_9^2 \otimes x_9 + v_4^2x_3^{10}x_9^2x_{15}^2 \otimes x_9 + v_4^2x_3^8x_5^6x_9^2 \otimes x_{15}\\
&+ v_4^2x_3^8x_9^2x_{15}^2 \otimes x_{15} + v_4^2x_3^{10}x_5^2x_{15}^2 \otimes x_{17} + v_4^2x_3^4x_5^2x_9^2x_{15}^2 \otimes x_{17}\\
&+ v_4^2x_3^{12}x_5^2x_9^2 \otimes x_{23} + v_4^2x_3^8x_5^2x_{15}^2 \otimes x_{23} + v_4^2x_3^{10}x_5^6 \otimes x_{27}\\
&+ v_4^2x_3^4x_5^6x_9^2 \otimes x_{27} + v_4^2x_3^{10}x_{15}^2 \otimes x_{27} + v_4^2x_3^4x_9^2x_{15}^2 \otimes x_{27}\\
&+ v_4^2x_5^2x_9^2x_{15}^2 \otimes x_{29} + v_4^3x_3^{14}x_5^2x_9^2x_{15}^2 \otimes x_{17} + v_4^3x_3^{12}x_5^2x_9^2x_{15}^2 \otimes x_{23}\\
&+ v_4^3x_3^{14}x_5^6x_9^2 \otimes x_{27} + v_4^3x_3^{14}x_9^2x_{15}^2 \otimes x_{27} + v_4^3x_3^{10}x_5^2x_9^2x_{15}^2 \otimes x_{29},\\
\bar\psi(x_{29}) = {}& x_3^8 \otimes x_5 + x_5^4 \otimes x_9 + x_3^4 \otimes x_{17} + v_4x_3^{12}x_5^4 \otimes x_3 + v_4x_3^8x_{15}^2 \otimes x_5
\end{aligned}
$$

$$+ v_4x_5^4x_{15}^2 \otimes x_9 + v_4x_3^8x_5^4 \otimes x_{15} + v_4x_3^4x_{15}^2 \otimes x_{17} + v_4x_3^{12} \otimes x_{23}$$
$$+ v_4x_3^4x_5^4 \otimes x_{27} + v_4x_{15}^2 \otimes x_{29} + v_4^2x_3^{12}x_5^4x_{15}^2 \otimes x_3 + v_4^2x_3^8x_5^6x_{15}^2 \otimes x_5$$
$$+ v_4^2x_3^{10}x_5^4x_{15}^2 \otimes x_9 + v_4^2x_3^8x_5^4x_{15}^2 \otimes x_{15} + v_4^2x_3^{14}x_5^6 \otimes x_{17}$$
$$+ v_4^2x_3^{14}x_{15}^2 \otimes x_{17} + v_4^2x_3^4x_5^6x_{15}^2 \otimes x_{17} + v_4^2x_3^{12}x_{15}^2 \otimes x_{23}$$
$$+ v_4^2x_3^4x_5^4x_{15}^2 \otimes x_{27} + v_4^2x_3^{10}x_5^6 \otimes x_{29} + v_4^2x_3^{10}x_{15}^2 \otimes x_{29}$$
$$+ v_4^2x_5^6x_{15}^2 \otimes x_{29} + v_4^3x_3^{14}x_5^6x_{15}^2 \otimes x_{17} + v_4^3x_3^{12}x_5^6x_{15}^2 \otimes x_{23}$$
$$+ v_4^3x_3^{14}x_5^4x_{15}^2 \otimes x_{27} + v_4^3x_3^{10}x_5^6x_{15}^2 \otimes x_{29}.$$

The action of the Milnor operations is given as follows:

$$Q_0x_3 = v_4x_3^8x_5^2 + v_4x_3^2x_5^2x_9^2 + v_4^2x_3^{12}x_5^2x_9^2 + v_4^2x_3^8x_5^2x_{15}^2 + v_4^2x_3^2x_5^2x_9^2x_{15}^2,$$
$$Q_0x_5 = x_3^2 + v_4x_3^{12} + v_4x_3^2x_{15}^2 + v_4^2x_3^{12}x_5^6 + v_4^2x_3^2x_5^6x_{15}^2,$$
$$Q_0x_9 = x_5^2 + v_4x_5^2x_{15}^2 + v_4^2x_3^{10}x_5^2x_{15}^2,$$
$$Q_0x_{15} = x_3^2x_5^2 + v_4x_3^{12}x_5^2 + v_4x_3^2x_5^2x_{15}^2,$$
$$Q_0x_{17} = x_9^2 + v_4x_9^2x_{15}^2 + v_4^2x_3^{10}x_5^6x_9^2 + v_4^2x_3^{10}x_9^2x_{15}^2 + v_4^2x_5^6x_9^2x_{15}^2 + v_4^3x_3^{10}x_5^6x_9^2x_{15}^2,$$
$$Q_0x_{23} = x_3^8 + x_3^2x_9^2 + v_4x_3^{12}x_9^2 + v_4x_3^8x_{15}^2 + v_4x_3^2x_9^2x_{15}^2 + v_4^2x_3^{12}x_5^6x_9^2 + v_4^2x_3^8x_5^6x_{15}^2 + v_4^2x_3^2x_5^6x_9^2x_{15}^2,$$
$$Q_0x_{27} = x_5^2x_9^2 + v_4x_5^2x_9^2x_{15}^2 + v_4^2x_3^{10}x_5^2x_9^2x_{15}^2,$$
$$Q_0x_{29} = x_{15}^2 + v_4x_3^{10}x_5^6 + v_4x_3^{10}x_{15}^2 + v_4x_5^6x_{15}^2 + v_4^2x_3^{10}x_5^6x_{15}^2,$$
$$Q_1x_3 = x_3^2 + v_4x_3^2x_5^6 + v_4x_3^6x_9^2 + v_4x_3^2x_{15}^2 + v_4^2x_3^{12}x_5^6 + v_4^2x_3^6x_5^6x_9^2 + v_4^2x_3^{12}x_{15}^2 + v_4^2x_3^6x_9^2x_{15}^2,$$
$$Q_1x_5 = v_4x_3^6x_5^4 + v_4x_5^4x_9^2 + v_4^2x_3^6x_5^4x_{15}^2 + v_4^2x_5^4x_9^2x_{15}^2 + v_4^3x_3^{10}x_5^4x_9^2x_{15}^2,$$
$$Q_1x_9 = x_3^4 + v_4x_3^4x_5^6 + v_4x_3^4x_{15}^2 + v_4^2x_3^{14}x_5^6 + v_4^2x_3^{14}x_{15}^2,$$
$$Q_1x_{15} = x_9^2 + x_3^6 + v_4x_3^6x_5^6 + v_4x_5^6x_9^2 + v_4x_3^6x_{15}^2 + v_4x_9^2x_{15}^2 + v_4^2x_3^{10}x_5^6x_9^2 + v_4^2x_3^{10}x_9^2x_{15}^2,$$
$$Q_1x_{17} = x_5^4 + v_4x_3^4x_5^4x_9^2 + v_4x_5^4x_{15}^2 + v_4^2x_3^{10}x_5^4x_{15}^2 + v_4^2x_3^4x_5^4x_9^2x_{15}^2 + v_4^3x_3^{14}x_5^4x_9^2x_{15}^2,$$
$$Q_1x_{23} = x_3^2x_5^4 + v_4x_3^6x_5^4x_9^2 + v_4x_3^2x_5^4x_{15}^2 + v_4^2x_3^{12}x_5^4x_{15}^2 + v_4^2x_3^6x_5^4x_9^2x_{15}^2,$$
$$Q_1x_{27} = x_{15}^2 + x_3^4x_9^2 + x_5^6 + v_4x_3^{10}x_5^6 + v_4x_3^4x_5^6x_9^2 + v_4x_3^{10}x_{15}^2 + v_4x_3^4x_9^2x_{15}^2 + v_4^2x_3^{14}x_5^6x_9^2 + v_4^2x_3^{14}x_9^2x_{15}^2,$$

$$
\begin{aligned}
Q_1x_{29} &= x_3^4x_5^4 + v_4x_3^4x_5^4x_{15}^2 + v_4^2x_3^{14}x_5^4x_{15}^2,\\
Q_2x_3 &= x_5^2 + v_4x_3^{10}x_5^2 + v_4x_3^4x_5^2x_9^2 + v_4x_5^2x_{15}^2 + v_4^2x_3^{14}x_5^2x_9^2 + v_4^2x_3^4x_5^2x_9^2x_{15}^2,\\
Q_2x_5 &= x_3^4 + v_4x_3^{14} + v_4x_3^8x_9^2 + v_4x_3^4x_{15}^2 + v_4^2x_3^{14}x_5^6 + v_4^2x_3^4x_5^6x_{15}^2\\
&\quad + v_4^2x_3^8x_9^2x_{15}^2 + v_4^3x_3^8x_5^6x_9^2x_{15}^2,\\
Q_2x_9 &= v_4x_3^{12}x_5^2 + v_4^2x_3^{12}x_5^2x_{15}^2,\\
Q_2x_{15} &= x_3^4x_5^2 + v_4x_3^{14}x_5^2 + v_4x_3^8x_5^2x_9^2 + v_4x_3^4x_5^2x_{15}^2 + v_4^2x_3^8x_5^2x_9^2x_{15}^2,\\
Q_2x_{17} &= x_3^8 + v_4x_3^{12}x_9^2 + v_4x_3^8x_{15}^2 + v_4^2x_3^8x_5^6x_{15}^2 + v_4^2x_3^{12}x_9^2x_{15}^2 + v_4^3x_3^{12}x_5^6x_9^2x_{15}^2,\\
Q_2x_{23} &= x_{15}^2 + x_3^{10} + x_3^4x_9^2 + v_4x_3^{10}x_5^6 + v_4x_3^{14}x_9^2 + v_4x_5^6x_{15}^2 + v_4x_3^4x_9^2x_{15}^2\\
&\quad + v_4^2x_3^{14}x_5^6x_9^2 + v_4^2x_3^4x_5^6x_9^2x_{15}^2,\\
Q_2x_{27} &= x_3^8x_5^2 + v_4x_3^{12}x_5^2x_9^2 + v_4x_3^8x_5^2x_{15}^2 + v_4^2x_3^{12}x_5^2x_9^2x_{15}^2,\\
Q_2x_{29} &= x_3^{12} + v_4x_3^{12}x_{15}^2 + v_4^2x_3^{12}x_5^6x_{15}^2,\\
Q_3x_3 &= x_9^2 + v_4x_3^{10}x_9^2 + v_4x_5^6x_9^2 + v_4x_9^2x_{15}^2,\\
Q_3x_5 &= x_5^4 + v_4x_3^{10}x_5^4 + v_4x_5^4x_{15}^2,\\
Q_3x_9 &= x_3^8 + v_4x_3^8x_5^6 + v_4x_3^8x_{15}^2,\\
Q_3x_{15} &= x_{15}^2 + x_3^{10} + x_5^6,\\
Q_3x_{17} &= v_4x_3^8x_5^4x_9^2 + v_4^2x_3^8x_5^4x_9^2x_{15}^2,\\
Q_3x_{23} &= x_5^4x_9^2 + v_4x_3^{10}x_5^4x_9^2 + v_4x_5^4x_9^2x_{15}^2,\\
Q_3x_{27} &= x_3^8x_9^2 + v_4x_3^8x_5^6x_9^2 + v_4x_3^8x_9^2x_{15}^2,\\
Q_3x_{29} &= x_3^8x_5^4 + v_4x_3^8x_5^4x_{15}^2.
\end{aligned}
$$

The relations are given as follows:

$$x_3^{16} = x_5^8 = x_9^4 = x_{15}^4 = x_{17}^2 = x_{23}^2 = x_{27}^2 = x_{29}^2 = 0$$

and

$$[x_i, x_j] = v_4Q_3x_iQ_3x_j$$

for all i, j.

2. Preliminary

The p-localization of the complex cobordism $MU^*_{(p)}(-)$ has many copies of Brown-Peterson cohomology theory $BP^*(-)$ whose coefficient is

$$BP^* = \mathbb{Z}_{(p)}[v_1, v_2, \cdots],$$

where $|v_n| = -2(p^n - 1)$. The $P(n)$-theory and connective Morava K-theory can be constructed using the Sullivan-Baas construction. Their coefficients are

$$P(n)^* = BP^*/(p, v_1, v_2, \cdots, v_{n-1}) = \mathbb{Z}/p[v_n, v_{n+1}, \cdots],$$
$$k(n)^* = BP^*/(p, v_1, v_2, \cdots, v_{n-1}, v_{n+1}, \cdots) = \mathbb{Z}/p[v_n].$$

For a compact space X, there is a natural number N such that an isomorphism $P(n)^*(X) \cong P(n)^* \otimes H^*(X; \mathbb{Z}/p)$ holds for $n \geq N$. For $P(n)$-theories, we have the following diagram:

$$\begin{array}{ccccccccc} BP^*(X) & \xrightarrow{i_0} & P(1)^*(X) & \to \cdots \to & P(n)^*(X) & \xrightarrow{i_n} & P(n+1)^*(X) & \to \cdots, \\ & {\scriptstyle p}\nwarrow \quad \swarrow{\scriptstyle \delta_0} & & & & {\scriptstyle v_n}\nwarrow \quad \swarrow{\scriptstyle \delta_n} & & \\ & BP^*(X) & & & & P(n)^*(X) & & \end{array}$$

where triangles are exact. It is useful to compute the BP-cohomology of X. The v_n-localization of connective Morava K-theory is the Morava K-theory whose coefficient is

$$K(n)^* = \mathbb{Z}/p[v_n, v_n^{-1}].$$

The Morava K-theory has many good properties, such as Künneth isomorphism, which makes it easy to treat.

The cohomology operation $P(n)^*(P(n))$ is as follows:

$$P(n)^*(P(n)) \cong P(n)^* \underset{BP^*}{\otimes} BP^*(BP) \otimes \Lambda(Q_0, Q_1, \cdots, Q_{n-1}).$$

The cohomology operation Q_i has good properties as follows: The first one is that Q_i is a derivation; The second one is that the following diagram is commutative:

$$\begin{array}{ccc} P(n)^*(X) & \xrightarrow{Q_i} & P(n)^*(X) \\ \downarrow & & \downarrow \\ H^*(X; \mathbb{Z}/p) & \xrightarrow{Q_i} & H^*(X; \mathbb{Z}/p), \end{array}$$

where the vertical arrows are the natural transformations and the lower Q_i is the Milnor operation. So we call $Q_i \in P(n)^*(P(n))$ a Milnor operation. In $K(n)^*(K(n))$, there are cohomology operations Q_i for $0 \leq i \leq n-1$ which are derivations.

If p is an odd prime, then $P(n)^*(X)$ is a commutative algebra for any space X. If $p = 2$, however, $P(n)^*(X)$ is not a commutative algebra in general. The commutator $[x, y]$ is represented by using the Milnor operation Q_{n-1} as follows:

(2.1) $$[x, y] = v_n Q_{n-1} x Q_{n-1} y.$$

If the morphism $i_{n-1} \colon P(n-1)^*(X) \to P(n)^*(X)$ is epimorphic, then $Q_{n-1} = i_{n-1}\delta_{n-1}$ is trivial, so that $P(n)^*(X)$ is commutative. In a similar way, the commutator is represented by the same equation (2.1) in $K(n)^*(X)$ and hence $K(n)^*(X)$ is not commutative for $p = 2$ in general.

Let h be $P(n)$ or $K(n)$ and G a compact Lie group. Suppose that the Künneth isomorphism

$$h^*(G \times G) \cong h^*(G) \underset{h^*}{\otimes} h^*(G)$$

holds. Then the product of Lie group induces a coproduct map

$$\psi\colon h^*(G) \to h^*(G) \underset{h^*}{\otimes} h^*(G).$$

If p is an odd prime, then ψ is an algebra map so that $h^*(G)$ has a Hopf algebra structure. However, if $p = 2$, the following diagram is commutative:

$$\begin{array}{ccccc}
h^*(G) \underset{h^*}{\otimes} h^*(G) & \xrightarrow{\varphi} & h^*(G) & \xrightarrow{\psi} & h^*(G) \underset{h^*}{\otimes} h^*(G) \\
\Big\downarrow{\scriptstyle \psi\otimes\psi} & & & & \Big\uparrow{\scriptstyle \varphi\otimes\varphi} \\
h^*(G) \underset{h^*}{\otimes} h^*(G) \underset{h^*}{\otimes} h^*(G) \underset{h^*}{\otimes} h^*(G) & & \xrightarrow[1\otimes T^*\otimes 1]{} & & h^*(G) \underset{h^*}{\otimes} h^*(G) \underset{h^*}{\otimes} h^*(G) \underset{h^*}{\otimes} h^*(G),
\end{array}$$

where φ is the product and $T^*(x \otimes y) = y \otimes x + v_n Q_{n-1} y \otimes Q_{n-1} x$. Therefore $h^*(G)$ is not a Hopf algebra in general.

We now recall the method of the calculation ([MN]). Let G be a compact Lie group and T its maximal torus. Consider the fibre bundle

$$G \xrightarrow{i} G/T \xrightarrow{\pi} BT.$$

In general, the cohomology of these spaces are given as follows:

$$H^*(BT; \mathbb{Z}/p) = \mathbb{Z}/p[\bar{t}_1, \cdots, \bar{t}_l],$$
$$H^*(G/T; \mathbb{Z}/p) = \mathbb{Z}/p[\bar{t}_1, \cdots, \bar{t}_l, \bar{y}_1, \cdots, \bar{y}_m]/(\bar{f}_1, \cdots, \bar{f}_l, \bar{g}_1, \cdots, \bar{g}_m),$$
$$H^*(G; \mathbb{Z}/p) = \mathbb{Z}/p[\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_m]/(\bar{y}_1^{p^{r_1}}, \bar{y}_2^{p^{r_2}}, \cdots, \bar{y}_m^{p^{r_m}}) \otimes \Delta(\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_l),$$

where Δ denotes a simple system of generators, the relations $\bar{f}_i$ are generated by $\bar{t}_1, \cdots, \bar{t}_l$ and $|\bar{t}_i| = 2$, $|\bar{y}_i|$ are even and $|\bar{x}_i|$ are odd. AHSS for $P(n)^*(G/T)$ and $P(n)^*(BT)$ collapse

$$P(n)^*(BT) = P(n)^*[[t_1, \cdots, t_l]],$$
$$P(n)^*(G/T) = P(n)^*[t_1, \cdots, t_l, y_1, \cdots, y_m]/(f_1, \cdots, f_l, g_1, \cdots, g_m).$$

If AHSS for $P(n)^*(G)$ collapses, then the relations f_i are generated by $t_1, \cdots, t_l$. So we may regard f_i as an element in $P(n)^*(BT)$. Let J be a sequence $(j_1, \cdots, j_m)$ such that $0 \leq j_i < p^{r_i}$ and let $y^J = y_1^{j_1} y_2^{j_2} \cdots y_m^{j_m}$.

Let $T_G^* = \delta^{-1}(\pi_1^* P(n)^{*+1}(BT, *))$ and $x_i = \delta^{-1}\pi_1^*(f_i)$:

$$\begin{array}{ccccccc} P(n)^*(G/T) & \xrightarrow{i^*} & P(n)^*(G) & \xrightarrow{\delta} & P(n)^*(G/T, G) & \xrightarrow{j^*} & P(n)^*(G/T). \\ & & & & \uparrow{\scriptstyle \pi_1^*} & & \\ & & & & P(n)^*(BT, *) & & \end{array}$$

If AHSS for $P(n)^*(G)$ collapses, then T_G^* is the free $P(n)^*$-module generated by x_i and y^J which is the i^*-image of $y^J \in P(n)^*(G/T)$. Using these elements, we have

$$P(n)^*(G) = P(n)^*[y_1, \cdots, y_m]/(g_1, \cdots, g_m) \otimes \Delta(x_1, \cdots, x_l).$$

The following lemma is useful for our calculation.

Lemma 2.1 ([MN]). *T_G^* is closed under cohomology operations.*

The following proposition is also important for our calculation.

Proposition 2.2 ([MN]). *Suppose that AHSS for $P(n)^*(G)$ collapses. Then, for each $x \in \widetilde{P(n)}^*(G)$, the following conditions are equivalent*:

(1) $x \in T_G^*$;
(2) $\mu^*(x) - x \otimes 1 \in \operatorname{Im} i^* \otimes P(n)^*(G)$;
(3) $\mu^*(x) - x \otimes 1 \in \operatorname{Im} i^* \otimes T_G^*$.

Especially, in the case $p = 2$, it is remarkable that $x^2 \in T_G^{\text{even}}$ for any $x \in T_G^{\text{odd}}$.

3. Calculation

In this section, we determine the bi-algebra structure of $P(4)^*(E_8)$. First let us recall the mod 2 ordinary cohomology of E_8.

Proposition 3.1 ([KMS], [KK, Proposition 7.1]). *The algebra structure of* mod 2 *cohomology of E_8 is given by*

$$H^*(E_8; \mathbb{Z}/2) \cong \mathbb{Z}/2[\bar{x}_3, \bar{x}_5, \bar{x}_9, \bar{x}_{15}]/(\bar{x}_3^{16}, \bar{x}_5^8, \bar{x}_9^4, \bar{x}_{15}^4) \otimes \Lambda(\bar{x}_{17}, \bar{x}_{23}, \bar{x}_{27}, \bar{x}_{29}).$$

The reduced coproduct is given as follows:

$$\bar{\psi}(\bar{x}_3) = \bar{\psi}(\bar{x}_5) = \bar{\psi}(\bar{x}_9) = \bar{\psi}(\bar{x}_{17}) = 0,$$
$$\bar{\psi}(\bar{x}_{15}) = \bar{x}_3^4 \otimes \bar{x}_3 + \bar{x}_5^2 \otimes \bar{x}_5 + \bar{x}_3^2 \otimes \bar{x}_9,$$
$$\bar{\psi}(\bar{x}_{23}) = \bar{x}_5^4 \otimes \bar{x}_3 + \bar{x}_9^2 \otimes \bar{x}_5 + \bar{x}_3^2 \otimes \bar{x}_{17},$$
$$\bar{\psi}(\bar{x}_{27}) = \bar{x}_3^8 \otimes \bar{x}_3 + \bar{x}_9^2 \otimes \bar{x}_9 + \bar{x}_5^2 \otimes \bar{x}_{17},$$
$$\bar{\psi}(\bar{x}_{29}) = \bar{x}_3^8 \otimes \bar{x}_5 + \bar{x}_5^4 \otimes \bar{x}_9 + \bar{x}_3^4 \otimes \bar{x}_{17}.$$

The action of the cohomology operations is as follows:

$$
\begin{array}{lll}
Sq^1\bar{x}_3 = 0, & Sq^1\bar{x}_5 = \bar{x}_3^2, & Sq^1\bar{x}_9 = \bar{x}_5^2, \\
Sq^1\bar{x}_{15} = \bar{x}_3^2\bar{x}_5^2, & Sq^1\bar{x}_{17} = \bar{x}_9^2, & Sq^1\bar{x}_{23} = \bar{x}_3^8 + \bar{x}_3^2\bar{x}_9^2, \\
Sq^1\bar{x}_{27} = \bar{x}_5^2\bar{x}_9^2, & Sq^1\bar{x}_{29} = \bar{x}_{15}^2, & \\
Sq^2\bar{x}_3 = \bar{x}_5, & Sq^2\bar{x}_5 = 0, & Sq^2\bar{x}_9 = 0, \\
Sq^2\bar{x}_{15} = \bar{x}_{17}, & Sq^2\bar{x}_{17} = 0, & Sq^2\bar{x}_{23} = 0, \\
Sq^2\bar{x}_{27} = \bar{x}_{29}, & Sq^2\bar{x}_{29} = 0, & \\
Sq^4\bar{x}_3 = 0, & Sq^4\bar{x}_5 = \bar{x}_9, & Sq^4\bar{x}_9 = 0, \\
Sq^4\bar{x}_{15} = 0, & Sq^4\bar{x}_{17} = 0, & Sq^4\bar{x}_{23} = \bar{x}_{27}, \\
Sq^4\bar{x}_{27} = 0, & Sq^4\bar{x}_{29} = 0, & \\
Sq^8\bar{x}_3 = 0, & Sq^8\bar{x}_5 = 0, & Sq^8\bar{x}_9 = \bar{x}_{17}, \\
Sq^8\bar{x}_{15} = \bar{x}_{23}, & Sq^8\bar{x}_{17} = 0, & Sq^8\bar{x}_{23} = 0, \\
Sq^8\bar{x}_{27} = 0, & Sq^8\bar{x}_{29} = 0; & \\
Q_1\bar{x}_3 = \bar{x}_3^2, & Q_1\bar{x}_5 = 0, & Q_1\bar{x}_9 = \bar{x}_3^4, \\
Q_1\bar{x}_{15} = \bar{x}_9^2 + \bar{x}_3^6, & Q_1\bar{x}_{17} = \bar{x}_5^4, & Q_1\bar{x}_{23} = \bar{x}_3^2\bar{x}_5^4, \\
Q_1\bar{x}_{27} = \bar{x}_{15}^2 + \bar{x}_3^4\bar{x}_9^2 + \bar{x}_5^6, & Q_1\bar{x}_{29} = \bar{x}_3^4\bar{x}_5^4, & \\
Q_2\bar{x}_3 = \bar{x}_5^2, & Q_2\bar{x}_5 = \bar{x}_3^4, & Q_2\bar{x}_9 = 0, \\
Q_2\bar{x}_{15} = \bar{x}_3^4\bar{x}_5^2, & Q_2\bar{x}_{17} = \bar{x}_3^8, & Q_2\bar{x}_{23} = \bar{x}_{15}^2 + \bar{x}_3^{10} + \bar{x}_3^4\bar{x}_9^2, \\
Q_2\bar{x}_{27} = \bar{x}_3^8\bar{x}_5^2, & Q_2\bar{x}_{29} = \bar{x}_3^{12}, & \\
Q_3\bar{x}_3 = \bar{x}_9^2, & Q_3\bar{x}_5 = \bar{x}_5^4, & Q_3\bar{x}_9 = \bar{x}_3^8, \\
Q_3\bar{x}_{15} = \bar{x}_{15}^2 + \bar{x}_3^{10} + \bar{x}_5^6, & Q_3\bar{x}_{17} = 0, & Q_3\bar{x}_{23} = \bar{x}_5^4\bar{x}_9^2, \\
Q_3\bar{x}_{27} = \bar{x}_3^8\bar{x}_9^2, & Q_3\bar{x}_{29} = \bar{x}_3^8\bar{x}_5^4. &
\end{array}
$$

Before we start calculation we recall the following notation:

$$
\begin{aligned}
I(k,0,0,\cdots) &= (v_4^{k+1}, v_5, v_6, \cdots), \\
I(k,j,0,0,\cdots) &= (v_4^{k+1}v_5^j, v_5^{j+1}, v_6, v_7, \cdots), \\
I(0,0,1,0,0,\cdots) &= (v_4v_6, v_5v_6, v_6^2, v_7, v_8, \cdots).
\end{aligned}
$$

According to [HMNS], if (G,p) is $(E_8,2)$, then AHSS for $P(n)^*(E_8)$ collapses for $n \geq 4$. Then there holds a module isomorphism

$$P(n)^*(E_8) \cong P(n)^* \otimes H^*(E_8;\mathbb{Z}/2).$$

In order to calculate the algebra structure, we need to determine the action of the Milnor operation Q_3 as well as eight relations which are associated with the eight relations

$$\bar{x}_3^{16} = \bar{x}_5^8 = \bar{x}_9^4 = \bar{x}_{15}^4 = \bar{x}_{17}^2 = \bar{x}_{23}^2 = \bar{x}_{27}^2 = \bar{x}_{29}^2 = 0$$

in the ordinary mod 2 cohomology. We will determine the relations, the action of the Milnor operations and the coproducts at the same time.

Taking into account the ordinary cohomology, Lemma 2.1 and Proposition 2.2, we can put

$$\begin{aligned}
\bar{\psi}(x_3) \equiv{}& a_1 v_4 x_3^{10} \otimes x_3 + a_2 v_4 x_5^6 \otimes x_3 + a_3 v_4 x_3^4 x_9^2 \otimes x_3 + a_4 v_4 x_{15}^2 \otimes x_3 \\
&+ a_5 v_4 x_3^6 x_5^2 \otimes x_5 + a_6 v_4 x_5^2 x_9^2 \otimes x_5 + a_7 v_4 x_3^8 \otimes x_9 + a_8 v_4 x_3^2 x_9^2 \otimes x_9 \\
&+ a_9 v_4 x_3^6 \otimes x_{15} + a_{10} v_4 x_9^2 \otimes x_{15} + a_{11} v_4 x_3^2 x_5^2 \otimes x_{17} + a_{12} v_4 x_5^2 \otimes x_{23} \\
&+ a_{13} v_4 x_3^2 \otimes x_{27}, \\
\bar{\psi}(x_5) \equiv{}& a_{14} v_4 x_3^4 x_5^4 \otimes x_3 + a_{15} v_4 x_3^{10} \otimes x_5 + a_{16} v_4 x_5^6 \otimes x_5 + a_{17} v_4 x_3^4 x_9^2 \otimes x_5 \\
&+ a_{18} v_4 x_{15}^2 \otimes x_5 + a_{19} v_4 x_3^2 x_5^4 \otimes x_9 + a_{20} v_4 x_5^4 \otimes x_{15} + a_{21} v_4 x_3^6 \otimes x_{17} \\
&+ a_{22} v_4 x_9^2 \otimes x_{17} + a_{23} v_4 x_3^4 \otimes x_{23} + a_{24} v_4 x_3^2 \otimes x_{29}, \\
\bar{\psi}(x_9) \equiv{}& a_{25} v_4 x_3^{12} \otimes x_3 + a_{26} v_4 x_3^2 x_5^6 \otimes x_3 + a_{27} v_4 x_3^6 x_9^2 \otimes x_3 \\
&+ a_{28} v_4 x_3^2 x_{15}^2 \otimes x_3 + a_{29} v_4 x_3^8 x_5^2 \otimes x_5 + a_{30} v_4 x_3^2 x_5^2 x_9^2 \otimes x_5 \\
&+ a_{31} v_4 x_3^{10} \otimes x_9 + a_{32} v_4 x_5^6 \otimes x_9 + a_{33} v_4 x_3^4 x_9^2 \otimes x_9 + a_{34} v_4 x_{15}^2 \otimes x_9 \\
&+ a_{35} v_4 x_3^8 \otimes x_{15} + a_{36} v_4 x_3^2 x_9^2 \otimes x_{15} + a_{37} v_4 x_3^4 x_5^2 \otimes x_{17} \\
&+ a_{38} v_4 x_3^2 x_5^2 \otimes x_{23} + a_{39} v_4 x_3^4 \otimes x_{27} + a_{40} v_4 x_5^2 \otimes x_{29}, \\
\bar{\psi}(x_{15}) \equiv{}& x_3^4 \otimes x_3 + x_5^2 \otimes x_5 + x_3^2 \otimes x_9 + a_{41} v_4 x_3^{14} \otimes x_3 + a_{42} v_4 x_3^4 x_5^6 \otimes x_3 \\
&+ a_{43} v_4 x_3^8 x_9^2 \otimes x_3 + a_{44} v_4 x_3^4 x_{15}^2 \otimes x_3 + a_{45} v_4 x_3^{10} x_5^2 \otimes x_5 \\
&+ a_{46} v_4 x_3^4 x_5^2 x_9^2 \otimes x_5 + a_{47} v_4 x_5^2 x_{15}^2 \otimes x_5 + a_{48} v_4 x_3^{12} \otimes x_9 \\
&+ a_{49} v_4 x_3^2 x_5^6 \otimes x_9 + a_{50} v_4 x_3^6 x_9^2 \otimes x_9 + a_{51} v_4 x_3^2 x_{15}^2 \otimes x_9 \\
&+ a_{52} v_4 x_3^{10} \otimes x_{15} + a_{53} v_4 x_5^6 \otimes x_{15} + a_{54} v_4 x_3^4 x_9^2 \otimes x_{15} \\
&+ a_{55} v_4 x_{15}^2 \otimes x_{15} + a_{56} v_4 x_3^6 x_5^2 \otimes x_{17} + a_{57} v_4 x_5^2 x_9^2 \otimes x_{17} \\
&+ a_{58} v_4 x_3^4 x_5^2 \otimes x_{23} + a_{59} v_4 x_3^6 \otimes x_{27} + a_{60} v_4 x_9^2 \otimes x_{27} \\
&+ a_{61} v_4 x_3^2 x_5^2 \otimes x_{29}, \\
\bar{\psi}(x_{17}) \equiv{}& a_{62} v_4 x_3^8 x_5^4 \otimes x_3 + a_{63} v_4 x_3^2 x_5^4 x_9^2 \otimes x_3 + a_{64} v_4 x_3^{14} \otimes x_5 \\
&+ a_{65} v_4 x_3^4 x_5^6 \otimes x_5 + a_{66} v_4 x_3^8 x_9^2 \otimes x_5 + a_{67} v_4 x_3^4 x_{15}^2 \otimes x_5 \\
&+ a_{68} v_4 x_3^6 x_5^4 \otimes x_9 + a_{69} v_4 x_5^4 x_9^2 \otimes x_9 + a_{70} v_4 x_3^4 x_5^4 \otimes x_{15}
\end{aligned}$$

$$
\begin{aligned}
&+ a_{71}v_4x_3^{10} \otimes x_{17} + a_{72}v_4x_5^6 \otimes x_{17} + a_{73}v_4x_3^4x_9^2 \otimes x_{17}\\
&+ a_{74}v_4x_{15}^2 \otimes x_{17} + a_{75}v_4x_3^8 \otimes x_{23} + a_{76}v_4x_3^2x_9^2 \otimes x_{23}\\
&+ a_{77}v_4x_5^4 \otimes x_{27} + a_{78}v_4x_3^6 \otimes x_{29} + a_{79}v_4x_9^2 \otimes x_{29},\\
\bar\psi(x_{23}) \equiv{}& x_5^4 \otimes x_3 + x_9^2 \otimes x_5 + x_3^2 \otimes x_{17} + a_{80}v_4x_3^{10}x_5^4 \otimes x_3\\
&+ a_{81}v_4x_3^4x_5^4x_9^2 \otimes x_3 + a_{82}v_4x_5^4x_{15}^2 \otimes x_3 + a_{83}v_4x_3^6x_5^6 \otimes x_5\\
&+ a_{84}v_4x_3^{10}x_9^2 \otimes x_5 + a_{85}v_4x_5^6x_9^2 \otimes x_5 + a_{86}v_4x_3^6x_{15}^2 \otimes x_5\\
&+ a_{87}v_4x_9^2x_{15}^2 \otimes x_5 + a_{88}v_4x_3^8x_5^4 \otimes x_9 + a_{89}v_4x_3^2x_5^4x_9^2 \otimes x_9\\
&+ a_{90}v_4x_3^6x_5^4 \otimes x_{15} + a_{91}v_4x_5^4x_9^2 \otimes x_{15} + a_{92}v_4x_3^{12} \otimes x_{17}\\
&+ a_{93}v_4x_3^2x_5^6 \otimes x_{17} + a_{94}v_4x_3^6x_9^2 \otimes x_{17} + a_{95}v_4x_3^2x_{15}^2 \otimes x_{17}\\
&+ a_{96}v_4x_3^{10} \otimes x_{23} + a_{97}v_4x_5^6 \otimes x_{23} + a_{98}v_4x_3^4x_9^2 \otimes x_{23}\\
&+ a_{99}v_4x_{15}^2 \otimes x_{23} + a_{100}v_4x_3^2x_5^4 \otimes x_{27} + a_{101}v_4x_3^8 \otimes x_{29}\\
&+ a_{102}v_4x_3^2x_9^2 \otimes x_{29},\\
\bar\psi(x_{27}) \equiv{}& x_3^8 \otimes x_3 + x_9^2 \otimes x_9 + x_5^2 \otimes x_{17} + a_{103}v_4x_3^8x_5^6 \otimes x_3\\
&+ a_{104}v_4x_3^{12}x_9^2 \otimes x_3 + a_{105}v_4x_3^2x_5^6x_9^2 \otimes x_3 + a_{106}v_4x_3^8x_{15}^2 \otimes x_3\\
&+ a_{107}v_4x_3^2x_9^2x_{15}^2 \otimes x_3 + a_{108}v_4x_3^{14}x_5^2 \otimes x_5 + a_{109}v_4x_3^8x_5^2x_9^2 \otimes x_5\\
&+ a_{110}v_4x_3^4x_5^2x_{15}^2 \otimes x_5 + a_{111}v_4x_3^6x_5^6 \otimes x_9 + a_{112}v_4x_3^{10}x_9^2 \otimes x_9\\
&+ a_{113}v_4x_5^6x_9^2 \otimes x_9 + a_{114}v_4x_3^6x_{15}^2 \otimes x_9 + a_{115}v_4x_9^2x_{15}^2 \otimes x_9\\
&+ a_{116}v_4x_3^{14} \otimes x_{15} + a_{117}v_4x_3^4x_5^6 \otimes x_{15} + a_{118}v_4x_3^8x_9^2 \otimes x_{15}\\
&+ a_{119}v_4x_3^4x_{15}^2 \otimes x_{15} + a_{120}v_4x_3^{10}x_5^2 \otimes x_{17} + a_{121}v_4x_3^4x_5^2x_9^2 \otimes x_{17}\\
&+ a_{122}v_4x_5^2x_{15}^2 \otimes x_{17} + a_{123}v_4x_3^8x_5^2 \otimes x_{23} + a_{124}v_4x_3^2x_5^2x_9^2 \otimes x_{23}\\
&+ a_{125}v_4x_3^{10} \otimes x_{27} + a_{126}v_4x_5^6 \otimes x_{27} + a_{127}v_4x_3^4x_9^2 \otimes x_{27}\\
&+ a_{128}v_4x_{15}^2 \otimes x_{27} + a_{129}v_4x_3^6x_5^2 \otimes x_{29} + a_{130}v_4x_5^2x_9^2 \otimes x_{29},\\
\bar\psi(x_{29}) \equiv{}& x_3^8 \otimes x_5 + x_5^4 \otimes x_9 + x_3^4 \otimes x_{17} + a_{131}v_4x_3^{12}x_5^4 \otimes x_3\\
&+ a_{132}v_4x_3^6x_5^4x_9^2 \otimes x_3 + a_{133}v_4x_3^2x_5^4x_{15}^2 \otimes x_3 + a_{134}v_4x_3^8x_5^6 \otimes x_5\\
&+ a_{135}v_4x_3^{12}x_9^2 \otimes x_5 + a_{136}v_4x_3^2x_5^6x_9^2 \otimes x_5 + a_{137}v_4x_3^8x_{15}^2 \otimes x_5\\
&+ a_{138}v_4x_3^2x_9^2x_{15}^2 \otimes x_5 + a_{139}v_4x_3^{10}x_5^4 \otimes x_9 + a_{140}v_4x_3^4x_5^4x_9^2 \otimes x_9\\
&+ a_{141}v_4x_5^4x_{15}^2 \otimes x_9 + a_{142}v_4x_3^8x_5^4 \otimes x_{15} + a_{143}v_4x_3^2x_5^4x_9^2 \otimes x_{15}\\
&+ a_{144}v_4x_3^{14} \otimes x_{17} + a_{145}v_4x_3^4x_5^6 \otimes x_{17} + a_{146}v_4x_3^8x_9^2 \otimes x_{17}\\
&+ a_{147}v_4x_3^4x_{15}^2 \otimes x_{17} + a_{148}v_4x_3^{12} \otimes x_{23} + a_{149}v_4x_3^2x_5^6 \otimes x_{23}
\end{aligned}
$$

$$+ a_{150}v_4x_3^6x_9^2 \otimes x_{23} + a_{151}v_4x_3^2x_{15}^2 \otimes x_{23} + a_{152}v_4x_3^4x_5^4 \otimes x_{27} + a_{153}v_4x_3^{10} \otimes x_{29} + a_{154}v_4x_5^6 \otimes x_{29} + a_{155}v_4x_3^4x_9^2 \otimes x_{29} + a_{156}v_4x_{15}^2 \otimes x_{29};$$

$$Q_0x_3 \equiv b_1v_4x_3^8x_5^2 + b_2v_4x_3^2x_5^2x_9^2,$$
$$Q_0x_5 \equiv x_3^2 + b_3v_4x_3^{12} + b_4v_4x_3^2x_5^6 + b_5v_4x_3^6x_9^2 + b_6v_4x_3^2x_{15}^2,$$
$$Q_0x_9 \equiv x_5^2 + b_7v_4x_3^{10}x_5^2 + b_8v_4x_3^4x_5^2x_9^2 + b_9v_4x_5^2x_{15}^2,$$
$$Q_0x_{15} \equiv x_3^2x_5^2 + b_{10}v_4x_3^{12}x_5^2 + b_{11}v_4x_3^6x_5^2x_9^2 + b_{12}v_4x_3^2x_5^2x_{15}^2,$$
$$Q_0x_{17} \equiv x_9^2 + b_{13}v_4x_3^6x_5^6 + b_{14}v_4x_3^{10}x_9^2 + b_{15}v_4x_5^6x_9^2 + b_{16}v_4x_3^6x_{15}^2 + b_{17}v_4x_9^2x_{15}^2,$$
$$Q_0x_{23} \equiv x_3^8 + x_3^2x_9^2 + b_{18}v_4x_3^8x_5^6 + b_{19}v_4x_3^{12}x_9^2 + b_{20}v_4x_3^2x_5^6x_9^2 + b_{21}v_4x_3^8x_{15}^2 + b_{22}v_4x_3^2x_9^2x_{15}^2,$$
$$Q_0x_{27} \equiv x_5^2x_9^2 + b_{23}v_4x_3^{10}x_5^2x_9^2 + b_{24}v_4x_3^6x_5^2x_{15}^2 + b_{25}v_4x_5^2x_9^2x_{15}^2,$$
$$Q_0x_{29} \equiv x_{15}^2 + b_{26}v_4x_3^{10}x_5^6 + b_{27}v_4x_3^{14}x_9^2 + b_{28}v_4x_3^4x_5^6x_9^2 + b_{29}v_4x_3^{10}x_{15}^2 + b_{30}v_4x_5^6x_{15}^2 + b_{31}v_4x_3^4x_9^2x_{15}^2,$$
$$Q_1x_3 \equiv x_3^2 + b_{32}v_4x_3^{12} + b_{33}v_4x_3^2x_5^6 + b_{34}v_4x_3^6x_9^2 + b_{35}v_4x_3^2x_{15}^2,$$
$$Q_1x_5 \equiv b_{36}v_4x_3^6x_5^4 + b_{37}v_4x_5^4x_9^2,$$
$$Q_1x_9 \equiv x_3^4 + b_{38}v_4x_3^{14} + b_{39}v_4x_3^4x_5^6 + b_{40}v_4x_3^8x_9^2 + b_{41}v_4x_3^4x_{15}^2,$$
$$Q_1x_{15} \equiv x_9^2 + x_3^6 + b_{42}v_4x_3^6x_5^6 + b_{43}v_4x_3^{10}x_9^2 + b_{44}v_4x_5^6x_9^2 + b_{45}v_4x_3^6x_{15}^2 + b_{46}v_4x_9^2x_{15}^2,$$
$$Q_1x_{17} \equiv x_5^4 + b_{47}v_4x_3^{10}x_5^4 + b_{48}v_4x_3^4x_5^4x_9^2 + b_{49}v_4x_5^4x_{15}^2,$$
$$Q_1x_{23} \equiv x_3^2x_5^4 + b_{50}v_4x_3^{12}x_5^4 + b_{51}v_4x_3^6x_5^4x_9^2 + b_{52}v_4x_3^2x_5^4x_{15}^2,$$
$$Q_1x_{27} \equiv x_{15}^2 + x_3^4x_9^2 + x_5^6 + b_{53}v_4x_3^{10}x_5^6 + b_{54}v_4x_3^{14}x_9^2 + b_{55}v_4x_3^4x_5^6x_9^2 + b_{56}v_4x_3^{10}x_{15}^2 + b_{57}v_4x_5^6x_{15}^2 + b_{58}v_4x_3^4x_9^2x_{15}^2,$$
$$Q_1x_{29} \equiv x_3^4x_5^4 + b_{59}v_4x_3^{14}x_5^4 + b_{60}v_4x_3^8x_5^4x_9^2 + b_{61}v_4x_3^4x_5^4x_{15}^2,$$
$$Q_2x_3 \equiv x_5^2 + b_{62}v_4x_3^{10}x_5^2 + b_{63}v_4x_3^4x_5^2x_9^2 + b_{64}v_4x_5^2x_{15}^2,$$
$$Q_2x_5 \equiv x_3^4 + b_{65}v_4x_3^{14} + b_{66}v_4x_3^4x_5^6 + b_{67}v_4x_3^8x_9^2 + b_{68}v_4x_3^4x_{15}^2,$$
$$Q_2x_9 \equiv b_{69}v_4x_3^{12}x_5^2 + b_{70}v_4x_3^6x_5^2x_9^2 + b_{71}v_4x_3^2x_5^2x_{15}^2,$$
$$Q_2x_{15} \equiv x_3^4x_5^2 + b_{72}v_4x_3^{14}x_5^2 + b_{73}v_4x_3^8x_5^2x_9^2 + b_{74}v_4x_3^4x_5^2x_{15}^2,$$
$$Q_2x_{17} \equiv x_3^8 + b_{75}v_4x_3^8x_5^6 + b_{76}v_4x_3^{12}x_9^2 + b_{77}v_4x_3^2x_5^6x_9^2 + b_{78}v_4x_3^8x_{15}^2$$

$$
\begin{aligned}
&+ b_{79} v_4 x_3^2 x_9^2 x_{15}^2,\\
Q_2 x_{23} &\equiv x_{15}^2 + x_3^{10} + x_3^4 x_9^2 + b_{80} v_4 x_3^{10} x_5^6 + b_{81} v_4 x_3^{14} x_9^2 + b_{82} v_4 x_3^4 x_5^6 x_9^2\\
&\quad + b_{83} v_4 x_3^{10} x_{15}^2 + b_{84} v_4 x_5^6 x_{15}^2 + b_{85} v_4 x_3^4 x_9^2 x_{15}^2,\\
Q_2 x_{27} &\equiv x_3^8 x_5^2 + b_{86} v_4 x_3^{12} x_5^2 x_9^2 + b_{87} v_4 x_3^8 x_5^2 x_{15}^2 + b_{88} v_4 x_3^2 x_5^2 x_9^2 x_{15}^2,\\
Q_2 x_{29} &\equiv x_3^{12} + b_{89} v_4 x_3^{12} x_5^6 + b_{90} v_4 x_3^6 x_5^6 x_9^2 + b_{91} v_4 x_3^{12} x_{15}^2 + b_{92} v_4 x_3^2 x_5^6 x_{15}^2\\
&\quad + b_{93} v_4 x_3^6 x_9^2 x_{15}^2,\\
Q_3 x_3 &\equiv x_9^2 + b_{94} v_4 x_3^6 x_5^6 + b_{95} v_4 x_3^{10} x_9^2 + b_{96} v_4 x_5^6 x_9^2 + b_{97} v_4 x_3^6 x_{15}^2\\
&\quad + b_{98} v_4 x_9^2 x_{15}^2,\\
Q_3 x_5 &\equiv x_5^4 + b_{99} v_4 x_3^{10} x_5^4 + b_{100} v_4 x_3^4 x_5^4 x_9^2 + b_{101} v_4 x_5^4 x_{15}^2,\\
Q_3 x_9 &\equiv x_3^8 + b_{102} v_4 x_3^8 x_5^6 + b_{103} v_4 x_3^{12} x_9^2 + b_{104} v_4 x_3^2 x_5^6 x_9^2 + b_{105} v_4 x_3^8 x_{15}^2\\
&\quad + b_{106} v_4 x_3^2 x_9^2 x_{15}^2,\\
Q_3 x_{15} &\equiv x_{15}^2 + x_3^{10} + x_5^6 + b_{107} v_4 x_3^{10} x_5^6 + b_{108} v_4 x_3^{14} x_9^2 + b_{109} v_4 x_3^4 x_5^6 x_9^2\\
&\quad + b_{110} v_4 x_3^{10} x_{15}^2 + b_{111} v_4 x_5^6 x_{15}^2 + b_{112} v_4 x_3^4 x_9^2 x_{15}^2,\\
Q_3 x_{17} &\equiv b_{113} v_4 x_3^{14} x_5^4 + b_{114} v_4 x_3^8 x_5^4 x_9^2 + b_{115} v_4 x_3^4 x_5^4 x_{15}^2,\\
Q_3 x_{23} &\equiv x_5^4 x_9^2 + b_{116} v_4 x_3^{10} x_5^4 x_9^2 + b_{117} v_4 x_3^6 x_5^4 x_{15}^2 + b_{118} v_4 x_5^4 x_9^2 x_{15}^2,\\
Q_3 x_{27} &\equiv x_3^8 x_9^2 + b_{119} v_4 x_3^{14} x_5^6 + b_{120} v_4 x_3^8 x_5^6 x_9^2 + b_{121} v_4 x_3^{14} x_{15}^2 + b_{122} v_4 x_3^4 x_5^6 x_{15}^2\\
&\quad + b_{123} v_4 x_3^8 x_9^2 x_{15}^2,\\
Q_3 x_{29} &\equiv x_3^8 x_5^4 + b_{124} v_4 x_3^{12} x_5^4 x_9^2 + b_{125} v_4 x_3^8 x_5^4 x_{15}^2 + b_{126} v_4 x_3^2 x_5^4 x_9^2 x_{15}^2;
\end{aligned}
$$

$$
\begin{aligned}
x_3^{16} &\equiv c_1 v_4 x_3^{10} x_5^6 x_9^2 + c_2 v_4 x_3^6 x_5^6 x_{15}^2 + c_3 v_4 x_3^{10} x_9^2 x_{15}^2 + c_4 v_4 x_5^6 x_9^2 x_{15}^2,\\
x_5^8 &\equiv c_5 v_4 x_3^{14} x_5^2 x_9^2 + c_6 v_4 x_3^{10} x_5^2 x_{15}^2 + c_7 v_4 x_3^4 x_5^2 x_9^2 x_{15}^2,\\
x_9^4 &\equiv c_8 v_4 x_3^{12} x_5^6 + c_9 v_4 x_3^6 x_5^6 x_9^2 + c_{10} v_4 x_3^{12} x_{15}^2 + c_{11} v_4 x_3^2 x_5^6 x_{15}^2\\
&\quad + c_{12} v_4 x_3^6 x_9^2 x_{15}^2,\\
x_{15}^4 &\equiv c_{13} v_4 x_3^{14} x_5^6 x_9^2 + c_{14} v_4 x_3^{10} x_5^6 x_{15}^2 + c_{15} v_4 x_3^{14} x_9^2 x_{15}^2 + c_{16} v_4 x_3^4 x_5^6 x_9^2 x_{15}^2,\\
x_{17}^2 &\equiv c_{17} v_4 x_3^{12} x_5^2 x_9^2 + c_{18} v_4 x_3^8 x_5^2 x_{15}^2 + c_{19} v_4 x_3^2 x_5^2 x_9^2 x_{15}^2,\\
x_{23}^2 &\equiv c_{20} v_4 x_3^{12} x_5^2 x_{15}^2 + c_{21} v_4 x_3^6 x_5^2 x_9^2 x_{15}^2,\\
x_{27}^2 &\equiv c_{22} v_4 x_3^{12} x_5^6 x_9^2 + c_{23} v_4 x_3^8 x_5^6 x_{15}^2 + c_{24} v_4 x_3^{12} x_9^2 x_{15}^2 + c_{25} v_4 x_3^2 x_5^6 x_9^2 x_{15}^2,\\
x_{29}^2 &\equiv c_{26} v_4 x_3^{10} x_5^2 x_9^2 x_{15}^2,
\end{aligned}
$$

where $\equiv$ is mod $I(1,0,0,\cdots)$ and $a_i, b_i, c_i = 0, 1$.

It is easy to calculate that

$$\bar\psi(x_3^2) \equiv v_4 x_9^2 \otimes x_9^2, \quad \bar\psi(x_3^4) \equiv 0, \quad \bar\psi(x_5^2) \equiv v_4 x_5^4 \otimes x_5^4, \quad \bar\psi(x_5^4) \equiv 0,$$

$$\bar\psi(x_9^2) \equiv v_4 x_3^8 \otimes x_3^8,$$

$$\begin{aligned}\bar\psi(x_{15}^2) \equiv\ & x_3^8 \otimes x_3^2 + x_5^4 \otimes x_5^2 + x_3^4 \otimes x_9^2 \\ & + v_4(x_{15}^2 + x_3^{10} + x_5^6) \otimes (x_{15}^2 + x_3^{10} + x_5^6) \\ & + v_4 x_3^4(x_{15}^2 + x_3^{10} + x_5^6) \otimes x_9^2 + v_4 x_5^2(x_{15}^2 + x_3^{10}) \otimes x_5^4 \\ & + v_4 x_3^2(x_{15}^2 + x_3^{10} + x_5^6) \otimes x_3^8 + v_4 x_3^4 x_5^2 \otimes x_5^4 x_9^2 \\ & + v_4 x_3^6 \otimes x_3^8 x_9^2 + v_4 x_3^2 x_5^2 \otimes x_3^8 x_5^4 + v_4 x_3^4 \otimes x_9^2(x_{15}^2 + x_3^{10} + x_5^6) \\ & + v_4 x_5^2 \otimes x_5^4(x_{15}^2 + x_3^{10}) + v_4 x_3^2 \otimes x_3^8(x_{15}^2 + x_5^6),\end{aligned}$$

where $\equiv$ is mod $I(1,0,0,\cdots)$. Comparing the coefficient of $Q_i\bar\psi(x_j)$ and $\bar\psi(Q_i x_j)$, we can determine all the coefficients a_i and b_i. First we compare the coefficient of $Q_1\bar\psi(x_3)$ and $\bar\psi(Q_1 x_3)$:

$$\begin{aligned}\bar\psi(Q_1 x_3) \equiv\ & v_4 x_9^2 \otimes x_9^2 + b_{32} v_4 x_3^8 \otimes x_3^4 + b_{32} v_4 x_3^4 \otimes x_3^8 + b_{33} v_4 x_3^2 \otimes x_5^6 \\ & + b_{33} v_4 x_5^6 \otimes x_3^2 + b_{33} v_4 x_3^2 x_5^2 \otimes x_5^4 + b_{33} v_4 x_3^2 x_5^4 \otimes x_5^2 \\ & + b_{33} v_4 x_5^2 \otimes x_3^2 x_5^4 + b_{33} v_4 x_5^4 \otimes x_3^2 x_5^2 + b_{34} v_4 x_3^6 \otimes x_9^2 \\ & + b_{34} v_4 x_9^2 \otimes x_3^6 + b_{34} v_4 x_3^2 x_9^2 \otimes x_3^4 + b_{34} v_4 x_3^4 x_9^2 \otimes x_3^2 \\ & + b_{34} v_4 x_3^2 \otimes x_3^4 x_9^2 + b_{34} v_4 x_3^4 \otimes x_3^2 x_9^2 + b_{35} v_4 x_3^2 \otimes x_{15}^2 \\ & + b_{35} v_4 x_{15}^2 \otimes x_3^2 + b_{35} v_4 x_3^{10} \otimes x_3^2 + b_{35} v_4 x_3^8 \otimes x_3^4 \\ & + b_{35} v_4 x_3^2 x_5^4 \otimes x_5^2 + b_{35} v_4 x_5^4 \otimes x_3^2 x_5^2 + b_{35} v_4 x_3^6 \otimes x_9^2 \\ & + b_{35} v_4 x_3^4 \otimes x_3^2 x_9^2,\end{aligned}$$

$$\begin{aligned}Q_1\bar\psi(x_3) \equiv\ & a_1 v_4 x_3^{10} \otimes x_3^2 + a_2 v_4 x_5^6 \otimes x_3^2 + a_3 v_4 x_3^4 x_9^2 \otimes x_3^2 + a_4 v_4 x_{15}^2 \otimes x_3^2 \\ & + a_7 v_4 x_3^8 \otimes x_3^4 + a_8 v_4 x_3^2 x_9^2 \otimes x_3^4 + a_9 v_4 x_3^6 \otimes x_9^2 + a_9 v_4 x_3^6 \otimes x_3^6 \\ & + a_{10} v_4 x_9^2 \otimes x_9^2 + a_{10} v_4 x_9^2 \otimes x_3^6 + a_{11} v_4 x_3^2 x_5^2 \otimes x_5^4 \\ & + a_{12} v_4 x_5^2 \otimes x_3^2 x_5^4 + a_{13} v_4 x_3^2 \otimes x_{15}^2 + a_{13} v_4 x_3^2 \otimes x_3^4 x_9^2 \\ & + a_{13} v_4 x_3^2 \otimes x_5^6.\end{aligned}$$

Then we obtain $b_{32} = a_9 = 0$ and $b_{33} = b_{34} = b_{35} = a_1 = a_2 = a_3 = a_4 = a_7 = a_8 = a_{10} = a_{11} = a_{12} = a_{13} = 1$. We compare the coefficient of $Q_0\bar\psi(x_3)$ and $\bar\psi(Q_0 x_3)$:

$$\begin{aligned}\bar\psi(Q_0 x_3) \equiv\ & b_1 v_4 x_3^8 \otimes x_5^2 + b_1 v_4 x_5^2 \otimes x_3^8 + b_2 v_4 x_3^2 x_5^2 \otimes x_9^2 + b_2 v_4 x_3^2 x_9^2 \otimes x_5^2 \\ & + b_2 v_4 x_3^2 \otimes x_5^2 x_9^2 + b_2 v_4 x_5^2 x_9^2 \otimes x_3^2 + b_2 v_4 x_5^2 \otimes x_3^2 x_9^2 \\ & + b_2 v_4 x_9^2 \otimes x_3^2 x_5^2,\end{aligned}$$

$$\begin{aligned}Q_0\bar{\psi}(x_3) \equiv\ & a_5v_4x_3^6x_5^2\otimes x_3^2 + a_6v_4x_5^2x_9^2\otimes x_3^2 + v_4x_3^8\otimes x_5^2 + v_4x_3^2x_9^2\otimes x_5^2\\ & + v_4x_9^2\otimes x_3^2x_5^2 + v_4x_3^2x_5^2\otimes x_9^2 + v_4x_5^2\otimes x_3^8 + v_4x_5^2\otimes x_3^2x_9^2\\ & + v_4x_3^2\otimes x_5^2x_9^2.\end{aligned}$$

Then we obtain $a_5 = 0$ and $b_1 = b_2 = a_6 = 1$. We compare the coefficient of $Q_2\bar{\psi}(x_3)$ and $\bar{\psi}(Q_2x_3)$:

$$\begin{aligned}\bar{\psi}(Q_2x_3) \equiv\ & v_4x_5^4\otimes x_5^4 + b_{62}v_4x_3^{10}\otimes x_5^2 + b_{62}v_4x_5^2\otimes x_3^{10} + b_{62}v_4x_3^2x_5^2\otimes x_3^8\\ & + b_{62}v_4x_3^2\otimes x_3^8x_5^2 + b_{62}v_4x_3^8x_5^2\otimes x_3^2 + b_{62}v_4x_3^8\otimes x_3^2x_5^2\\ & + b_{63}v_4x_3^4x_5^2\otimes x_9^2 + b_{63}v_4x_3^4x_9^2\otimes x_5^2 + b_{63}v_4x_3^4\otimes x_5^2x_9^2\\ & + b_{63}v_4x_5^2x_9^2\otimes x_3^4 + b_{63}v_4x_5^2\otimes x_3^4x_9^2 + b_{63}v_4x_9^2\otimes x_3^4x_5^2\\ & + b_{64}v_4x_5^2\otimes x_{15}^2 + b_{64}v_4x_{15}^2\otimes x_5^2 + b_{64}v_4x_3^8x_5^2\otimes x_3^2\\ & + b_{64}v_4x_3^8\otimes x_3^2x_5^2 + b_{64}v_4x_5^6\otimes x_5^2 + b_{64}v_4x_5^4\otimes x_5^4\\ & + b_{64}v_4x_3^4x_5^2\otimes x_9^2 + b_{64}v_4x_3^4\otimes x_5^2x_9^2,\end{aligned}$$

$$\begin{aligned}Q_2\bar{\psi}(x_3) \equiv\ & v_4x_3^{10}\otimes x_5^2 + v_4x_5^6\otimes x_5^2 + v_4x_3^4x_9^2\otimes x_5^2 + v_4x_{15}^2\otimes x_5^2\\ & + v_4x_5^2x_9^2\otimes x_3^4 + v_4x_9^2\otimes x_3^4x_5^2 + v_4x_3^2x_5^2\otimes x_3^8 + v_4x_5^2\otimes x_{15}^2\\ & + v_4x_5^2\otimes x_3^{10} + v_4x_5^2\otimes x_3^4x_9^2 + v_4x_3^2\otimes x_3^8x_5^2.\end{aligned}$$

Then we obtain $b_{62} = b_{63} = b_{64} = 1$. We compare the coefficient of $Q_3\bar{\psi}(x_3)$ and $\bar{\psi}(Q_3x_3)$:

$$\begin{aligned}\bar{\psi}(Q_3x_3) \equiv\ & v_4x_3^8\otimes x_3^8 + b_{94}v_4x_3^6\otimes x_5^6 + b_{94}v_4x_5^6\otimes x_3^6 + b_{94}v_4x_3^6x_5^2\otimes x_5^4\\ & + b_{94}v_4x_3^6x_5^4\otimes x_5^2 + b_{94}v_4x_3^2x_5^6\otimes x_3^4 + b_{94}v_4x_3^4x_5^6\otimes x_3^2\\ & + b_{94}v_4x_3^2x_5^2\otimes x_3^4x_5^4 + b_{94}v_4x_3^2x_5^4\otimes x_3^4x_5^2 + b_{94}v_4x_3^4x_5^2\otimes x_3^2x_5^4\\ & + b_{94}v_4x_3^4x_5^4\otimes x_3^2x_5^2 + b_{94}v_4x_3^2\otimes x_3^4x_5^6 + b_{94}v_4x_3^4\otimes x_3^2x_5^6\\ & + b_{94}v_4x_5^2\otimes x_3^6x_5^4 + b_{94}v_4x_5^4\otimes x_3^6x_5^2 + b_{95}v_4x_3^{10}\otimes x_9^2\\ & + b_{95}v_4x_9^2\otimes x_3^{10} + b_{95}v_4x_3^2x_9^2\otimes x_3^8 + b_{95}v_4x_3^8x_9^2\otimes x_3^2\\ & + b_{95}v_4x_3^2\otimes x_3^8x_9^2 + b_{95}v_4x_3^8\otimes x_3^2x_9^2 + b_{96}v_4x_5^6\otimes x_9^2\\ & + b_{96}v_4x_9^2\otimes x_5^6 + b_{96}v_4x_5^2x_9^2\otimes x_5^4 + b_{96}v_4x_5^4x_9^2\otimes x_5^2\\ & + b_{96}v_4x_5^2\otimes x_5^4x_9^2 + b_{96}v_4x_5^4\otimes x_5^2x_9^2 + b_{97}v_4x_3^6\otimes x_{15}^2\\ & + b_{97}v_4x_{15}^2\otimes x_3^6 + b_{97}v_4x_3^2x_{15}^2\otimes x_3^4 + b_{97}v_4x_3^4x_{15}^2\otimes x_3^2\\ & + b_{97}v_4x_3^2\otimes x_3^4x_{15}^2 + b_{97}v_4x_3^4\otimes x_3^2x_{15}^2 + b_{97}v_4x_3^{14}\otimes x_3^2\\ & + b_{97}v_4x_3^{12}\otimes x_3^4 + b_{97}v_4x_3^{10}\otimes x_3^6 + b_{97}v_4x_3^8\otimes x_3^8\\ & + b_{97}v_4x_3^6x_5^4\otimes x_5^2 + b_{97}v_4x_3^4x_5^4\otimes x_3^2x_5^2 + b_{97}v_4x_3^2x_5^4\otimes x_3^4x_5^2\end{aligned}$$

$$
\begin{aligned}
&+ b_{97}v_4x_5^4 \otimes x_3^6x_5^2 + b_{97}v_4x_3^{10} \otimes x_9^2 + b_{97}v_4x_3^8 \otimes x_3^2x_9^2 \\
&+ b_{97}v_4x_3^6 \otimes x_3^4x_9^2 + b_{97}v_4x_3^4 \otimes x_3^6x_9^2 + b_{98}v_4x_9^2 \otimes x_{15}^2 \\
&+ b_{98}v_4x_{15}^2 \otimes x_9^2 + b_{98}v_4x_3^8x_9^2 \otimes x_3^2 + b_{98}v_4x_3^8 \otimes x_3^2x_9^2 \\
&+ b_{98}v_4x_5^4x_9^2 \otimes x_5^2 + b_{98}v_4x_5^4 \otimes x_5^2x_9^2 + b_{98}v_4x_3^4x_9^2 \otimes x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_3\bar{\psi}(x_3) \equiv\ & v_4x_3^{10} \otimes x_9^2 + v_4x_5^6 \otimes x_9^2 + v_4x_3^4x_9^2 \otimes x_9^2 + v_4x_{15}^2 \otimes x_9^2 \\
&+ v_4x_5^2x_9^2 \otimes x_5^4 + v_4x_3^8 \otimes x_3^8 + v_4x_3^2x_9^2 \otimes x_3^8 + v_4x_9^2 \otimes x_{15}^2 \\
&+ v_4x_9^2 \otimes x_3^{10} + v_4x_9^2 \otimes x_5^6 + v_4x_5^2 \otimes x_5^4x_9^2 + v_4x_3^2 \otimes x_3^8x_9^2.
\end{aligned}
$$

Then we obtain $b_{94} = b_{97} = 0$ and $b_{95} = b_{96} = b_{98} = 1$. We compare the coefficient of $Q_0\bar{\psi}(x_5)$ and $\bar{\psi}(Q_0x_5)$:

$$
\begin{aligned}
\bar{\psi}(Q_0x_5) \equiv\ & v_4x_9^2 \otimes x_9^2 + b_3v_4x_3^8 \otimes x_3^4 + b_3v_4x_3^4 \otimes x_3^8 + b_4v_4x_3^2 \otimes x_5^6 \\
&+ b_4v_4x_5^6 \otimes x_3^2 + b_4v_4x_3^2x_5^2 \otimes x_5^4 + b_4v_4x_3^2x_5^4 \otimes x_5^2 + b_4v_4x_5^2 \otimes x_3^2x_5^4 \\
&+ b_4v_4x_5^4 \otimes x_3^2x_5^2 + b_5v_4x_3^6 \otimes x_9^2 + b_5v_4x_9^2 \otimes x_3^6 + b_5v_4x_3^2x_9^2 \otimes x_3^4 \\
&+ b_5v_4x_3^4x_9^2 \otimes x_3^2 + b_5v_4x_3^2 \otimes x_3^4x_9^2 + b_5v_4x_3^4 \otimes x_3^2x_9^2 \\
&+ b_6v_4x_3^2 \otimes x_{15}^2 + b_6v_4x_{15}^2 \otimes x_3^2 + b_6v_4x_3^{10} \otimes x_3^2 + b_6v_4x_3^8 \otimes x_3^4 \\
&+ b_6v_4x_3^2x_5^4 \otimes x_5^2 + b_6v_4x_5^4 \otimes x_3^2x_5^2 + b_6v_4x_3^6 \otimes x_9^2 + b_6v_4x_3^4 \otimes x_3^2x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_0\bar{\psi}(x_5) \equiv\ & a_{15}v_4x_3^{10} \otimes x_3^2 + a_{16}v_4x_5^6 \otimes x_3^2 + a_{17}v_4x_3^4x_9^2 \otimes x_3^2 + a_{18}v_4x_{15}^2 \otimes x_3^2 \\
&+ a_{19}v_4x_3^2x_5^4 \otimes x_5^2 + a_{20}v_4x_5^4 \otimes x_3^2x_5^2 + a_{21}v_4x_3^6 \otimes x_9^2 \\
&+ a_{22}v_4x_9^2 \otimes x_9^2 + a_{23}v_4x_3^4 \otimes x_3^8 + a_{23}v_4x_3^4 \otimes x_3^2x_9^2 \\
&+ a_{24}v_4x_3^2 \otimes x_{15}^2.
\end{aligned}
$$

Then we obtain $b_4 = b_5 = a_{16} = a_{17} = 0$, $a_{22} = 1$ and $b_3 = b_6 = a_{15} = a_{18} = a_{19} = a_{20} = a_{21} = a_{23} = a_{24}$. We compare the coefficient of $Q_1\bar{\psi}(x_5)$ and $\bar{\psi}(Q_1x_5)$:

$$
\begin{aligned}
\bar{\psi}(Q_1x_5) \equiv\ & b_{36}v_4x_3^6 \otimes x_5^4 + b_{36}v_4x_5^4 \otimes x_3^6 + b_{36}v_4x_3^2x_5^4 \otimes x_3^4 + b_{36}v_4x_3^4x_5^4 \otimes x_3^2 \\
&+ b_{36}v_4x_3^2 \otimes x_3^4x_5^4 + b_{36}v_4x_3^4 \otimes x_3^2x_5^4 + b_{37}v_4x_5^4 \otimes x_9^2 \\
&+ b_{37}v_4x_9^2 \otimes x_5^4,
\end{aligned}
$$

$$
\begin{aligned}
Q_1\bar{\psi}(x_5) \equiv\ & a_{14}v_4x_3^4x_5^4 \otimes x_3^2 + a_{19}v_4x_3^2x_5^4 \otimes x_3^4 + a_{20}v_4x_5^4 \otimes x_9^2 + a_{20}v_4x_5^4 \otimes x_3^6 \\
&+ a_{21}v_4x_3^6 \otimes x_5^4 + v_4x_9^2 \otimes x_5^4 + a_{23}v_4x_3^4 \otimes x_3^2x_5^4 + a_{24}v_4x_3^2 \otimes x_3^4x_5^4.
\end{aligned}
$$

Then we obtain $b_3 = b_6 = b_{36} = b_{37} = a_{14} = a_{15} = a_{18} = a_{19} = a_{20} = a_{21} = a_{23} = a_{24} = 1$. We compare the coefficient of $Q_2\bar{\psi}(x_5)$ and $\bar{\psi}(Q_2x_5)$:

$$
\begin{aligned}
\bar{\psi}(Q_2x_5) \equiv\ & b_{65}v_4x_3^2 \otimes x_3^{12} + b_{65}v_4x_3^4 \otimes x_3^{10} + b_{65}v_4x_3^6 \otimes x_3^8 + b_{65}v_4x_3^8 \otimes x_3^6 \\
&+ b_{65}v_4x_3^{10} \otimes x_3^4 + b_{65}v_4x_3^{12} \otimes x_3^2 + b_{66}v_4x_3^4 \otimes x_5^6 + b_{66}v_4x_5^6 \otimes x_3^4
\end{aligned}
$$

$$
\begin{aligned}
&+ b_{66}v_4x_3^4x_5^2 \otimes x_5^4 + b_{66}v_4x_3^4x_5^4 \otimes x_5^2 + b_{66}v_4x_5^2 \otimes x_3^4x_5^4 \\
&+ b_{66}v_4x_5^4 \otimes x_3^4x_5^2 + b_{67}v_4x_3^8 \otimes x_9^2 + b_{67}v_4x_9^2 \otimes x_3^8 + b_{68}v_4x_3^4 \otimes x_{15}^2 \\
&+ b_{68}v_4x_{15}^2 \otimes x_3^4 + b_{68}v_4x_3^{12} \otimes x_3^2 + b_{68}v_4x_3^8 \otimes x_3^6 \\
&+ b_{68}v_4x_3^4x_5^4 \otimes x_5^2 + b_{68}v_4x_5^4 \otimes x_3^4x_5^2 + b_{68}v_4x_3^8 \otimes x_9^2 \\
&+ b_{68}v_4x_3^4 \otimes x_3^4x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_2\bar{\psi}(x_5) \equiv\ & v_4x_3^4x_5^4 \otimes x_5^2 + v_4x_3^{10} \otimes x_3^4 + v_4x_{15}^2 \otimes x_3^4 + v_4x_5^4 \otimes x_3^4x_5^2 \\
&+ v_4x_3^6 \otimes x_3^8 + v_4x_9^2 \otimes x_3^8 + v_4x_3^4 \otimes x_{15}^2 + v_4x_3^4 \otimes x_3^{10} \\
&+ v_4x_3^4 \otimes x_3^4x_9^2 + v_4x_3^2 \otimes x_3^{12}.
\end{aligned}
$$

Then we obtain $b_{66} = 0$ and $b_{65} = b_{67} = b_{68} = 1$. We compare the coefficient of $Q_3\bar{\psi}(x_5)$ and $\bar{\psi}(Q_3x_5)$:

$$
\begin{aligned}
\bar{\psi}(Q_3x_5) \equiv\ & b_{99}v_4x_3^{10} \otimes x_5^4 + b_{99}v_4x_5^4 \otimes x_3^{10} + b_{99}v_4x_3^2x_5^4 \otimes x_3^8 \\
&+ b_{99}v_4x_3^8x_5^4 \otimes x_3^2 + b_{99}v_4x_3^2 \otimes x_3^8x_5^4 + b_{99}v_4x_3^8 \otimes x_3^2x_5^4 \\
&+ b_{100}v_4x_3^4x_5^4 \otimes x_9^2 + b_{100}v_4x_3^4x_9^2 \otimes x_5^4 + b_{100}v_4x_3^4 \otimes x_5^4x_9^2 \\
&+ b_{100}v_4x_5^4x_9^2 \otimes x_3^4 + b_{100}v_4x_5^4 \otimes x_3^4x_9^2 + b_{100}v_4x_9^2 \otimes x_3^4x_5^4 \\
&+ b_{101}v_4x_5^4 \otimes x_{15}^2 + b_{101}v_4x_{15}^2 \otimes x_5^4 + b_{101}v_4x_3^8x_5^4 \otimes x_3^2 \\
&+ b_{101}v_4x_3^8 \otimes x_3^2x_5^4 + b_{101}v_4x_5^4 \otimes x_5^6 + b_{101}v_4x_3^4x_5^4 \otimes x_9^2 \\
&+ b_{101}v_4x_3^4 \otimes x_5^4x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_3\bar{\psi}(x_5) \equiv\ & v_4x_3^4x_5^4 \otimes x_9^2 + v_4x_3^{10} \otimes x_5^4 + v_4x_{15}^2 \otimes x_5^4 + v_4x_3^2x_5^4 \otimes x_3^8 \\
&+ v_4x_5^4 \otimes x_{15}^2 + v_4x_5^4 \otimes x_3^{10} + v_4x_5^4 \otimes x_5^6 + v_4x_3^4 \otimes x_5^4x_9^2 \\
&+ v_4x_3^2 \otimes x_3^8x_5^4.
\end{aligned}
$$

Then we obtain $b_{100} = 0$ and $b_{99} = b_{101} = 1$. We compare the coefficient of $Q_0\bar{\psi}(x_9)$ and $\bar{\psi}(Q_0x_9)$:

$$
\begin{aligned}
\bar{\psi}(Q_0x_9) \equiv\ & v_4x_5^4 \otimes x_5^4 + b_7v_4x_3^{10} \otimes x_5^2 + b_7v_4x_5^2 \otimes x_3^{10} + b_7v_4x_3^2x_5^2 \otimes x_3^8 \\
&+ b_7v_4x_3^8x_5^2 \otimes x_3^2 + b_7v_4x_3^2 \otimes x_3^8x_5^2 + b_7v_4x_3^8 \otimes x_3^2x_5^2 \\
&+ b_8v_4x_3^4x_5^2 \otimes x_9^2 + b_8v_4x_3^4x_9^2 \otimes x_5^2 + b_8v_4x_3^4 \otimes x_5^2x_9^2 \\
&+ b_8v_4x_5^2x_9^2 \otimes x_3^4 + b_8v_4x_5^2 \otimes x_3^4x_9^2 + b_8v_4x_9^2 \otimes x_3^4x_5^2 \\
&+ b_9v_4x_5^2 \otimes x_{15}^2 + b_9v_4x_{15}^2 \otimes x_5^2 + b_9v_4x_3^8x_5^2 \otimes x_3^2 \\
&+ b_9v_4x_3^8 \otimes x_3^2x_5^2 + b_9v_4x_5^6 \otimes x_5^2 + b_9v_4x_5^4 \otimes x_5^4 \\
&+ b_9v_4x_3^4x_5^2 \otimes x_9^2 + b_9v_4x_3^4 \otimes x_5^2x_9^2,
\end{aligned}
$$

$$
Q_0\bar{\psi}(x_9) \equiv a_{29}v_4x_3^8x_5^2 \otimes x_3^2 + a_{30}v_4x_3^2x_5^2x_9^2 \otimes x_3^2 + a_{31}v_4x_3^{10} \otimes x_5^2
$$

$$+ a_{32}v_4x_5^6 \otimes x_5^2 + a_{33}v_4x_3^4x_9^2 \otimes x_5^2 + a_{34}v_4x_{15}^2 \otimes x_5^2$$
$$+ a_{35}v_4x_3^8 \otimes x_3^2x_5^2 + a_{36}v_4x_3^2x_9^2 \otimes x_3^2x_5^2 + a_{37}v_4x_3^4x_5^2 \otimes x_9^2$$
$$+ a_{38}v_4x_3^2x_5^2 \otimes x_3^8 + a_{38}v_4x_3^2x_5^2 \otimes x_3^2x_9^2 + a_{39}v_4x_3^4 \otimes x_5^2x_9^2$$
$$+ a_{40}v_4x_5^2 \otimes x_{15}^2.$$

Then we obtain $b_7 = b_8 = a_{30} = a_{31} = a_{33} = a_{36} = a_{38} = 0$ and $b_9 = a_{29} = a_{32} = a_{34} = a_{35} = a_{37} = a_{39} = a_{40} = 1$. We compare the coefficient of $Q_1\bar{\psi}(x_9)$ and $\bar{\psi}(Q_1x_9)$:

$$\begin{aligned}\bar{\psi}(Q_1x_9) \equiv{}& b_{38}v_4x_3^{12} \otimes x_3^2 + b_{38}v_4x_3^{10} \otimes x_3^4 + b_{38}v_4x_3^8 \otimes x_3^6 + b_{38}v_4x_3^6 \otimes x_3^8\\
&+ b_{38}v_4x_3^4 \otimes x_3^{10} + b_{38}v_4x_3^2 \otimes x_3^{12} + b_{39}v_4x_3^4 \otimes x_5^6 + b_{39}v_4x_5^6 \otimes x_3^4\\
&+ b_{39}v_4x_3^4x_5^2 \otimes x_5^4 + b_{39}v_4x_3^4x_5^4 \otimes x_5^2 + b_{39}v_4x_5^2 \otimes x_3^4x_5^4\\
&+ b_{39}v_4x_5^4 \otimes x_3^4x_5^2 + b_{40}v_4x_3^8 \otimes x_9^2 + b_{40}v_4x_9^2 \otimes x_3^8 + b_{41}v_4x_3^4 \otimes x_{15}^2\\
&+ b_{41}v_4x_{15}^2 \otimes x_3^4 + b_{41}v_4x_3^{12} \otimes x_3^2 + b_{41}v_4x_3^8 \otimes x_3^6\\
&+ b_{41}v_4x_3^4x_5^4 \otimes x_5^2 + b_{41}v_4x_5^4 \otimes x_3^4x_5^2 + b_{41}v_4x_3^8 \otimes x_9^2\\
&+ b_{41}v_4x_3^4 \otimes x_3^4x_9^2,\end{aligned}$$

$$\begin{aligned}Q_1\bar{\psi}(x_9) \equiv{}& a_{25}v_4x_3^{12} \otimes x_3^2 + a_{26}v_4x_3^2x_5^6 \otimes x_3^2 + a_{27}v_4x_3^6x_9^2 \otimes x_3^2\\
&+ a_{28}v_4x_3^2x_{15}^2 \otimes x_3^2 + v_4x_5^6 \otimes x_3^4 + v_4x_{15}^2 \otimes x_3^4 + v_4x_3^8 \otimes x_9^2\\
&+ v_4x_3^8 \otimes x_3^6 + v_4x_3^4x_5^2 \otimes x_5^4 + v_4x_3^4 \otimes x_{15}^2 + v_4x_3^4 \otimes x_3^4x_9^2\\
&+ v_4x_3^4 \otimes x_5^6 + v_4x_5^2 \otimes x_3^4x_5^4.\end{aligned}$$

Then we obtain $b_{38} = b_{40} = a_{26} = a_{27} = a_{28} = 0$ and $b_{39} = b_{41} = a_{25} = 1$. We compare the coefficient of $Q_2\bar{\psi}(x_9)$ and $\bar{\psi}(Q_2x_9)$:

$$\begin{aligned}\bar{\psi}(Q_2x_9) \equiv{}& b_{69}v_4x_3^{12} \otimes x_5^2 + b_{69}v_4x_5^2 \otimes x_3^{12} + b_{69}v_4x_3^4x_5^2 \otimes x_3^8\\
&+ b_{69}v_4x_3^8x_5^2 \otimes x_3^4 + b_{69}v_4x_3^4 \otimes x_3^8x_5^2 + b_{69}v_4x_3^8 \otimes x_3^4x_5^2\\
&+ b_{70}v_4x_3^6x_5^2 \otimes x_9^2 + b_{70}v_4x_3^6x_9^2 \otimes x_5^2 + b_{70}v_4x_3^6 \otimes x_5^2x_9^2\\
&+ b_{70}v_4x_5^2x_9^2 \otimes x_3^6 + b_{70}v_4x_5^2 \otimes x_3^6x_9^2 + b_{70}v_4x_9^2 \otimes x_3^6x_5^2\\
&+ b_{70}v_4x_3^2x_5^2x_9^2 \otimes x_3^4 + b_{70}v_4x_3^4x_5^2x_9^2 \otimes x_3^2 + b_{70}v_4x_3^2x_5^2 \otimes x_3^4x_9^2\\
&+ b_{70}v_4x_3^2x_9^2 \otimes x_3^4x_5^2 + b_{70}v_4x_3^4x_5^2 \otimes x_3^2x_9^2 + b_{70}v_4x_3^4x_9^2 \otimes x_3^2x_5^2\\
&+ b_{70}v_4x_3^2 \otimes x_3^4x_5^2x_9^2 + b_{70}v_4x_3^4 \otimes x_3^2x_5^2x_9^2 + b_{71}v_4x_3^2x_5^2 \otimes x_{15}^2\\
&+ b_{71}v_4x_3^2x_{15}^2 \otimes x_5^2 + b_{71}v_4x_3^2 \otimes x_5^2x_{15}^2 + b_{71}v_4x_5^2x_{15}^2 \otimes x_3^2\\
&+ b_{71}v_4x_5^2 \otimes x_3^2x_{15}^2 + b_{71}v_4x_{15}^2 \otimes x_3^2x_5^2 + b_{71}v_4x_3^{10}x_5^2 \otimes x_3^2\\
&+ b_{71}v_4x_3^{10} \otimes x_3^2x_5^2 + b_{71}v_4x_3^8x_5^2 \otimes x_3^4 + b_{71}v_4x_3^8 \otimes x_3^4x_5^2\end{aligned}$$

$$
\begin{aligned}
&+ b_{71}v_4x_3^2x_5^6 \otimes x_5^2 + b_{71}v_4x_3^2x_5^4 \otimes x_5^4 + b_{71}v_4x_5^6 \otimes x_3^2x_5^2\\
&+ b_{71}v_4x_5^4 \otimes x_3^2x_5^4 + b_{71}v_4x_3^6x_5^2 \otimes x_9^2 + b_{71}v_4x_3^6 \otimes x_5^2x_9^2\\
&+ b_{71}v_4x_3^4x_5^2 \otimes x_3^2x_9^2 + b_{71}v_4x_3^4 \otimes x_3^2x_5^2x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_2\bar{\psi}(x_9) \equiv{}& v_4x_3^{12} \otimes x_5^2 + v_4x_3^8x_5^2 \otimes x_3^4 + v_4x_3^8 \otimes x_3^4x_5^2 + v_4x_3^4x_5^2 \otimes x_3^8\\
&+ v_4x_3^4 \otimes x_3^8x_5^2 + v_4x_5^2 \otimes x_3^{12}.
\end{aligned}
$$

Then we obtain $b_{70} = b_{71} = 0$ and $b_{69} = 1$. We compare the coefficient of $Q_3\bar{\psi}(x_9)$ and $\bar{\psi}(Q_3x_9)$:

$$
\begin{aligned}
\bar{\psi}(Q_3x_9) \equiv{}& b_{102}v_4x_3^8 \otimes x_5^6 + b_{102}v_4x_5^6 \otimes x_3^8 + b_{102}v_4x_3^8x_5^2 \otimes x_5^4\\
&+ b_{102}v_4x_3^8x_5^4 \otimes x_5^2 + b_{102}v_4x_5^2 \otimes x_3^8x_5^4 + b_{102}v_4x_5^4 \otimes x_3^8x_5^2\\
&+ b_{103}v_4x_3^{12} \otimes x_9^2 + b_{103}v_4x_9^2 \otimes x_3^{12} + b_{103}v_4x_3^4x_9^2 \otimes x_3^8\\
&+ b_{103}v_4x_3^8x_9^2 \otimes x_3^4 + b_{103}v_4x_3^4 \otimes x_3^8x_9^2 + b_{103}v_4x_3^8 \otimes x_3^4x_9^2\\
&+ b_{104}v_4x_3^2x_5^6 \otimes x_9^2 + b_{104}v_4x_3^2x_9^2 \otimes x_5^6 + b_{104}v_4x_3^2 \otimes x_5^6x_9^2\\
&+ b_{104}v_4x_5^6x_9^2 \otimes x_3^2 + b_{104}v_4x_5^6 \otimes x_3^2x_9^2 + b_{104}v_4x_9^2 \otimes x_3^2x_5^6\\
&+ b_{104}v_4x_3^2x_5^2x_9^2 \otimes x_5^4 + b_{104}v_4x_3^2x_5^4x_9^2 \otimes x_5^2 + b_{104}v_4x_3^2x_5^2 \otimes x_5^4x_9^2\\
&+ b_{104}v_4x_3^2x_5^4 \otimes x_5^2x_9^2 + b_{104}v_4x_5^2x_9^2 \otimes x_3^2x_5^4 + b_{104}v_4x_5^4x_9^2 \otimes x_3^2x_5^2\\
&+ b_{104}v_4x_5^2 \otimes x_3^2x_5^4x_9^2 + b_{104}v_4x_5^4 \otimes x_3^2x_5^2x_9^2 + b_{105}v_4x_3^8 \otimes x_{15}^2\\
&+ b_{105}v_4x_{15}^2 \otimes x_3^8 + b_{105}v_4x_3^8 \otimes x_3^{10} + b_{105}v_4x_3^8x_5^4 \otimes x_5^2\\
&+ b_{105}v_4x_5^4 \otimes x_3^8x_5^2 + b_{105}v_4x_3^{12} \otimes x_9^2 + b_{105}v_4x_3^4 \otimes x_3^8x_9^2\\
&+ b_{106}v_4x_3^2x_9^2 \otimes x_{15}^2 + b_{106}v_4x_3^2x_{15}^2 \otimes x_9^2 + b_{106}v_4x_3^2 \otimes x_9^2x_{15}^2\\
&+ b_{106}v_4x_9^2x_{15}^2 \otimes x_3^2 + b_{106}v_4x_9^2 \otimes x_3^2x_{15}^2 + b_{106}v_4x_{15}^2 \otimes x_3^2x_9^2\\
&+ b_{106}v_4x_3^{10}x_9^2 \otimes x_3^2 + b_{106}v_4x_3^{10} \otimes x_3^2x_9^2 + b_{106}v_4x_3^8x_9^2 \otimes x_3^4\\
&+ b_{106}v_4x_3^8 \otimes x_3^4x_9^2 + b_{106}v_4x_3^2x_5^4x_9^2 \otimes x_5^2 + b_{106}v_4x_3^2x_5^4 \otimes x_5^2x_9^2\\
&+ b_{106}v_4x_5^4x_9^2 \otimes x_3^2x_5^2 + b_{106}v_4x_5^4 \otimes x_3^2x_5^2x_9^2 + b_{106}v_4x_3^6x_9^2 \otimes x_9^2\\
&+ b_{106}v_4x_3^4x_9^2 \otimes x_9^4,
\end{aligned}
$$

$$
\begin{aligned}
Q_3\bar{\psi}(x_9) \equiv{}& v_4x_3^{12} \otimes x_9^2 + v_4x_3^8x_5^2 \otimes x_5^4 + v_4x_5^6 \otimes x_3^8 + v_4x_{15}^2 \otimes x_3^8 + v_4x_3^8 \otimes x_{15}^2\\
&+ v_4x_3^8 \otimes x_3^{10} + v_4x_3^8 \otimes x_5^6 + v_4x_3^4 \otimes x_3^8x_9^2 + v_4x_5^2 \otimes x_3^8x_5^4.
\end{aligned}
$$

Then we obtain $b_{103} = b_{104} = b_{106} = 0$ and $b_{102} = b_{105} = 1$. We compare the coefficient of $Q_0\bar{\psi}(x_{15})$ and $\bar{\psi}(Q_0x_{15})$:

$$
\begin{aligned}
\bar{\psi}(Q_0x_{15}) \equiv{}& x_3^2 \otimes x_5^2 + x_5^2 \otimes x_3^2 + v_4x_5^2x_9^2 \otimes x_9^2 + v_4x_9^2 \otimes x_5^2x_9^2 + v_4x_3^2x_5^4 \otimes x_5^4\\
&+ v_4x_5^4 \otimes x_3^2x_5^4 + b_{10}v_4x_3^{12} \otimes x_5^2 + b_{10}v_4x_5^2 \otimes x_3^{12}
\end{aligned}
$$

$$
\begin{aligned}
&+ b_{10}v_4x_3^4x_5^2 \otimes x_3^8 + b_{10}v_4x_3^4 \otimes x_3^8x_5^2 + b_{10}v_4x_3^8x_5^2 \otimes x_3^4\\
&+ b_{10}v_4x_3^8 \otimes x_3^4x_5^2 + b_{11}v_4x_3^6x_5^2 \otimes x_9^2 + b_{11}v_4x_3^6x_9^2 \otimes x_5^2\\
&+ b_{11}v_4x_3^6 \otimes x_5^2x_9^2 + b_{11}v_4x_5^2x_9^2 \otimes x_3^6 + b_{11}v_4x_5^2 \otimes x_3^6x_9^2\\
&+ b_{11}v_4x_9^2 \otimes x_3^6x_5^2 + b_{11}v_4x_3^2x_5^2x_9^2 \otimes x_3^4 + b_{11}v_4x_3^2x_5^2 \otimes x_3^4x_9^2\\
&+ b_{11}v_4x_3^2x_9^2 \otimes x_3^4x_5^2 + b_{11}v_4x_3^2 \otimes x_3^4x_5^2x_9^2 + b_{11}v_4x_3^4x_5^2x_9^2 \otimes x_3^2\\
&+ b_{11}v_4x_3^4x_5^2 \otimes x_3^2x_9^2 + b_{11}v_4x_3^4x_9^2 \otimes x_3^2x_5^2 + b_{11}v_4x_3^4 \otimes x_3^2x_5^2x_9^2\\
&+ b_{12}v_4x_3^2x_5^2 \otimes x_{15}^2 + b_{12}v_4x_3^2x_{15}^2 \otimes x_5^2 + b_{12}v_4x_3^2 \otimes x_5^2x_{15}^2\\
&+ b_{12}v_4x_5^2x_{15}^2 \otimes x_3^2 + b_{12}v_4x_5^2 \otimes x_3^2x_{15}^2 + b_{12}v_4x_{15}^2 \otimes x_3^2x_5^2\\
&+ b_{12}v_4x_3^{10}x_5^2 \otimes x_3^2 + b_{12}v_4x_3^{10} \otimes x_3^2x_5^2 + b_{12}v_4x_3^8x_5^2 \otimes x_3^4\\
&+ b_{12}v_4x_3^8 \otimes x_3^4x_5^2 + b_{12}v_4x_3^2x_5^6 \otimes x_5^2 + b_{12}v_4x_3^2x_5^4 \otimes x_5^4\\
&+ b_{12}v_4x_5^6 \otimes x_3^2x_5^2 + b_{12}v_4x_5^4 \otimes x_3^2x_5^4 + b_{12}v_4x_3^6x_5^2 \otimes x_9^2\\
&+ b_{12}v_4x_3^6 \otimes x_5^2x_9^2 + b_{12}v_4x_3^4x_5^2 \otimes x_3^2x_9^2 + b_{12}v_4x_3^4 \otimes x_3^2x_5^2x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_0\bar\psi(x_{15}) \equiv\ & v_4x_3^4 \otimes x_3^8x_5^2 + v_4x_3^4 \otimes x_3^2x_5^2x_9^2 + x_5^2 \otimes x_3^2 + v_4x_5^2 \otimes x_3^{12}\\
&+ v_4x_5^2 \otimes x_3^2x_{15}^2 + x_3^2 \otimes x_5^2 + v_4x_3^2 \otimes x_5^2x_{15}^2 + a_{45}v_4x_3^{10}x_5^2 \otimes x_3^2\\
&+ a_{46}v_4x_3^4x_5^2x_9^2 \otimes x_3^2 + a_{47}v_4x_5^2x_{15}^2 \otimes x_3^2 + a_{48}v_4x_3^{12} \otimes x_5^2\\
&+ a_{49}v_4x_3^2x_5^6 \otimes x_5^2 + a_{50}v_4x_3^6x_9^2 \otimes x_5^2 + a_{51}v_4x_3^2x_{15}^2 \otimes x_5^2\\
&+ a_{52}v_4x_3^{10} \otimes x_3^2x_5^2 + a_{53}v_4x_5^6 \otimes x_3^2x_5^2 + a_{54}v_4x_3^4x_9^2 \otimes x_3^2x_5^2\\
&+ a_{55}v_4x_{15}^2 \otimes x_3^2x_5^2 + a_{56}v_4x_3^6x_5^2 \otimes x_9^2 + a_{57}v_4x_5^2x_9^2 \otimes x_9^2\\
&+ a_{58}v_4x_3^4x_5^2 \otimes x_3^8 + a_{58}v_4x_3^4x_5^2 \otimes x_3^2x_9^2 + a_{59}v_4x_3^6 \otimes x_5^2x_9^2\\
&+ a_{60}v_4x_9^2 \otimes x_5^2x_9^2 + a_{61}v_4x_3^2x_5^2 \otimes x_{15}^2.
\end{aligned}
$$

Then we obtain $b_{11} = a_{46} = a_{50} = a_{54} = 0$ and $b_{10} = b_{12} = a_{45} = a_{47} = a_{48} = a_{49} = a_{51} = a_{52} = a_{53} = a_{55} = a_{56} = a_{57} = a_{58} = a_{59} = a_{60} = a_{61} = 1$. We compare the coefficient of $Q_1\bar\psi(x_{15})$ and $\bar\psi(Q_1x_{15})$:

$$
\begin{aligned}
\bar\psi(Q_1x_{15}) \equiv\ & v_4x_3^8 \otimes x_3^8 + x_3^2 \otimes x_3^4 + x_3^4 \otimes x_3^2 + v_4x_3^4x_9^2 \otimes x_9^2 + v_4x_9^2 \otimes x_3^4x_9^2\\
&+ b_{42}v_4x_3^6 \otimes x_5^6 + b_{42}v_4x_5^6 \otimes x_3^6 + b_{42}v_4x_3^2x_5^6 \otimes x_3^4\\
&+ b_{42}v_4x_3^2 \otimes x_3^4x_5^6 + b_{42}v_4x_3^4x_5^6 \otimes x_3^2 + b_{42}v_4x_3^4 \otimes x_3^2x_5^6\\
&+ b_{42}v_4x_3^6x_5^2 \otimes x_5^4 + b_{42}v_4x_5^2 \otimes x_3^6x_5^4 + b_{42}v_4x_3^6x_5^4 \otimes x_5^2\\
&+ b_{42}v_4x_5^4 \otimes x_3^6x_5^2 + b_{42}v_4x_3^2x_5^2 \otimes x_3^4x_5^4 + b_{42}v_4x_3^2x_5^4 \otimes x_3^4x_5^2\\
&+ b_{42}v_4x_3^4x_5^2 \otimes x_3^2x_5^4 + b_{42}v_4x_3^4x_5^4 \otimes x_3^2x_5^2 + b_{43}v_4x_3^{10} \otimes x_9^2\\
&+ b_{43}v_4x_9^2 \otimes x_3^{10} + b_{43}v_4x_3^2x_9^2 \otimes x_3^8 + b_{43}v_4x_3^2 \otimes x_3^8x_9^2
\end{aligned}
$$

$$
\begin{aligned}
&+ b_{43}v_4x_3^8x_9^2 \otimes x_3^2 + b_{43}v_4x_3^8 \otimes x_3^2x_9^2 + b_{44}v_4x_5^6 \otimes x_9^2\\
&+ b_{44}v_4x_9^2 \otimes x_5^6 + b_{44}v_4x_5^2x_9^2 \otimes x_5^4 + b_{44}v_4x_5^2 \otimes x_5^4x_9^2\\
&+ b_{44}v_4x_5^4x_9^2 \otimes x_5^2 + b_{44}v_4x_5^4 \otimes x_5^2x_9^2 + b_{45}v_4x_3^6 \otimes x_{15}^2\\
&+ b_{45}v_4x_{15}^2 \otimes x_3^6 + b_{45}v_4x_3^2x_{15}^2 \otimes x_3^4 + b_{45}v_4x_3^2 \otimes x_3^4x_{15}^2\\
&+ b_{45}v_4x_3^4x_{15}^2 \otimes x_3^2 + b_{45}v_4x_3^4 \otimes x_3^2x_{15}^2 + b_{45}v_4x_3^{14} \otimes x_3^2\\
&+ b_{45}v_4x_3^{12} \otimes x_3^4 + b_{45}v_4x_3^{10} \otimes x_3^6 + b_{45}v_4x_3^8 \otimes x_3^8\\
&+ b_{45}v_4x_3^6x_5^4 \otimes x_5^2 + b_{45}v_4x_3^4x_5^4 \otimes x_3^2x_5^2 + b_{45}v_4x_3^2x_5^4 \otimes x_3^4x_5^2\\
&+ b_{45}v_4x_5^4 \otimes x_3^6x_5^2 + b_{45}v_4x_3^{10} \otimes x_9^2 + b_{45}v_4x_3^8 \otimes x_3^2x_9^2\\
&+ b_{45}v_4x_3^6 \otimes x_3^4x_9^2 + b_{45}v_4x_3^4 \otimes x_3^6x_9^2 + b_{46}v_4x_9^2 \otimes x_{15}^2\\
&+ b_{46}v_4x_{15}^2 \otimes x_9^2 + b_{46}v_4x_3^8x_9^2 \otimes x_3^2 + b_{46}v_4x_3^8 \otimes x_3^2x_9^2\\
&+ b_{46}v_4x_5^4x_9^2 \otimes x_5^2 + b_{46}v_4x_5^4 \otimes x_5^2x_9^2 + b_{46}v_4x_3^4x_9^2 \otimes x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_1\bar{\psi}(x_{15}) \equiv\ & x_3^4 \otimes x_3^2 + v_4x_3^4 \otimes x_3^2x_5^6 + v_4x_3^4 \otimes x_3^6x_9^2 + v_4x_3^4 \otimes x_3^2x_{15}^2\\
&+ v_4x_5^2 \otimes x_3^6x_5^4 + v_4x_5^2 \otimes x_5^4x_9^2 + x_3^2 \otimes x_3^4 + v_4x_3^2 \otimes x_3^4x_5^6\\
&+ v_4x_3^2 \otimes x_3^4x_{15}^2 + a_{41}v_4x_3^{14} \otimes x_3^2 + a_{42}v_4x_3^4x_5^6 \otimes x_3^2\\
&+ a_{43}v_4x_3^8x_9^2 \otimes x_3^2 + a_{44}v_4x_3^4x_{15}^2 \otimes x_3^2 + v_4x_3^{12} \otimes x_3^4\\
&+ v_4x_3^2x_5^6 \otimes x_3^4 + v_4x_3^2x_{15}^2 \otimes x_3^4 + v_4x_3^{10} \otimes x_9^2 + v_4x_3^{10} \otimes x_3^6\\
&+ v_4x_5^6 \otimes x_9^2 + v_4x_5^6 \otimes x_3^6 + v_4x_{15}^2 \otimes x_9^2 + v_4x_{15}^2 \otimes x_3^6\\
&+ v_4x_3^6x_5^2 \otimes x_5^4 + v_4x_5^2x_9^2 \otimes x_5^4 + v_4x_3^4x_5^2 \otimes x_3^2x_5^4\\
&+ v_4x_3^6 \otimes x_{15}^2 + v_4x_3^6 \otimes x_3^4x_9^2 + v_4x_3^6 \otimes x_5^6 + v_4x_9^2 \otimes x_{15}^2\\
&+ v_4x_9^2 \otimes x_3^4x_9^2 + v_4x_9^2 \otimes x_5^6 + v_4x_3^2x_5^2 \otimes x_3^4x_5^4.
\end{aligned}
$$

Then we obtain $b_{43} = 0$ and $b_{42} = b_{44} = b_{45} = b_{46} = a_{41} = a_{42} = a_{43} = a_{44} = 1$. We compare the coefficient of $Q_2\bar{\psi}(x_{15})$ and $\bar{\psi}(Q_2x_{15})$:

$$
\begin{aligned}
\bar{\psi}(Q_2x_{15}) \equiv\ & x_3^4 \otimes x_5^2 + x_5^2 \otimes x_3^4 + v_4x_3^4x_5^4 \otimes x_5^4 + v_4x_5^4 \otimes x_3^4x_5^4 + b_{72}v_4x_3^{14} \otimes x_5^2\\
&+ b_{72}v_4x_5^2 \otimes x_3^{14} + b_{72}v_4x_3^{12}x_5^2 \otimes x_3^2 + b_{72}v_4x_3^{12} \otimes x_3^2x_5^2\\
&+ b_{72}v_4x_3^{10}x_5^2 \otimes x_3^4 + b_{72}v_4x_3^{10} \otimes x_3^4x_5^2 + b_{72}v_4x_3^8x_5^2 \otimes x_3^6\\
&+ b_{72}v_4x_3^8 \otimes x_3^6x_5^2 + b_{72}v_4x_3^6x_5^2 \otimes x_3^8 + b_{72}v_4x_3^6 \otimes x_3^8x_5^2\\
&+ b_{72}v_4x_3^4x_5^2 \otimes x_3^{10} + b_{72}v_4x_3^4 \otimes x_3^{10}x_5^2 + b_{72}v_4x_3^2x_5^2 \otimes x_3^{12}\\
&+ b_{72}v_4x_3^2 \otimes x_3^{12}x_5^2 + b_{73}v_4x_3^8x_5^2 \otimes x_9^2 + b_{73}v_4x_3^8x_9^2 \otimes x_5^2\\
&+ b_{73}v_4x_3^8 \otimes x_5^2x_9^2 + b_{73}v_4x_5^2x_9^2 \otimes x_3^8 + b_{73}v_4x_5^2 \otimes x_3^8x_9^2\\
&+ b_{73}v_4x_9^2 \otimes x_3^8x_5^2 + b_{74}v_4x_3^4x_5^2 \otimes x_{15}^2 + b_{74}v_4x_3^4x_{15}^2 \otimes x_5^2
\end{aligned}
$$

$$
\begin{aligned}
&+ b_{74}v_4x_3^4 \otimes x_5^2x_{15}^2 + b_{74}v_4x_5^2x_{15}^2 \otimes x_3^4 + b_{74}v_4x_5^2 \otimes x_3^4x_{15}^2 \\
&+ b_{74}v_4x_{15}^2 \otimes x_3^4x_5^2 + b_{74}v_4x_3^{12}x_5^2 \otimes x_3^2 + b_{74}v_4x_3^{12} \otimes x_3^2x_5^2 \\
&+ b_{74}v_4x_3^8x_5^2 \otimes x_3^6 + b_{74}v_4x_3^8 \otimes x_3^6x_5^2 + b_{74}v_4x_3^4x_5^6 \otimes x_5^2 \\
&+ b_{74}v_4x_3^4x_5^4 \otimes x_5^4 + b_{74}v_4x_5^6 \otimes x_3^4x_5^2 + b_{74}v_4x_5^4 \otimes x_3^4x_5^4 \\
&+ b_{74}v_4x_3^8x_5^2 \otimes x_9^2 + b_{74}v_4x_3^8 \otimes x_5^2x_9^2 + b_{74}v_4x_3^4x_5^2 \otimes x_3^4x_9^2 \\
&+ b_{74}v_4x_3^4 \otimes x_3^4x_5^2x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_2\bar{\psi}(x_{15}) \equiv\ & x_3^4 \otimes x_5^2 + v_4x_3^4 \otimes x_3^{10}x_5^2 + v_4x_3^4 \otimes x_3^4x_5^2x_9^2 + v_4x_3^4 \otimes x_5^2x_{15}^2 \\
&+ x_5^2 \otimes x_3^4 + v_4x_5^2 \otimes x_3^{14} + v_4x_5^2 \otimes x_3^8x_9^2 + v_4x_5^2 \otimes x_3^4x_{15}^2 \\
&+ v_4x_3^2 \otimes x_3^{12}x_5^2 + v_4x_3^{14} \otimes x_5^2 + v_4x_3^4x_5^6 \otimes x_5^2 + v_4x_3^8x_9^2 \otimes x_5^2 \\
&+ v_4x_3^4x_{15}^2 \otimes x_5^2 + v_4x_3^{10}x_5^2 \otimes x_3^4 + v_4x_5^2x_{15}^2 \otimes x_3^4 + v_4x_3^{10} \otimes x_3^4x_5^2 \\
&+ v_4x_5^6 \otimes x_3^4x_5^2 + v_4x_{15}^2 \otimes x_3^4x_5^2 + v_4x_3^6x_5^2 \otimes x_3^8 + v_4x_5^2x_9^2 \otimes x_3^8 \\
&+ v_4x_3^4x_5^2 \otimes x_{15}^2 + v_4x_3^4x_5^2 \otimes x_3^{10} + v_4x_3^4x_5^2 \otimes x_3^4x_9^2 + v_4x_3^6 \otimes x_3^8x_5^2 \\
&+ v_4x_9^2 \otimes x_3^8x_5^2 + v_4x_3^2x_5^2 \otimes x_3^{12}.
\end{aligned}
$$

Then we obtain $b_{72} = b_{73} = b_{74} = 1$. We compare the coefficient of $Q_3\bar{\psi}(x_{15})$ and $\bar{\psi}(Q_3x_{15})$:

$$
\begin{aligned}
\bar{\psi}(Q_3x_{15}) \equiv\ & x_3^8 \otimes x_3^2 + x_5^4 \otimes x_5^2 + x_3^4 \otimes x_9^2 + v_4x_{15}^2 \otimes x_{15}^2 + v_4x_{15}^2 \otimes x_3^{10} \\
&+ v_4x_{15}^2 \otimes x_5^6 + v_4x_3^{10} \otimes x_{15}^2 + v_4x_3^{10} \otimes x_3^{10} + v_4x_3^{10} \otimes x_5^6 \\
&+ v_4x_5^6 \otimes x_{15}^2 + v_4x_5^6 \otimes x_3^{10} + v_4x_5^6 \otimes x_5^6 + v_4x_3^4x_{15}^2 \otimes x_9^2 \\
&+ v_4x_3^{14} \otimes x_9^2 + v_4x_3^4x_5^6 \otimes x_9^2 + v_4x_5^2x_{15}^2 \otimes x_5^4 + v_4x_3^{10}x_5^2 \otimes x_5^4 \\
&+ v_4x_3^2x_{15}^2 \otimes x_3^8 + v_4x_3^{12} \otimes x_3^8 + v_4x_3^2x_5^6 \otimes x_3^8 + v_4x_3^4x_5^2 \otimes x_5^4x_9^2 \\
&+ v_4x_3^6 \otimes x_3^8x_9^2 + v_4x_3^2x_5^2 \otimes x_3^8x_5^4 + v_4x_3^4 \otimes x_9^2x_{15}^2 + v_4x_3^4 \otimes x_3^{10}x_9^2 \\
&+ v_4x_3^4 \otimes x_5^6x_9^2 + v_4x_5^2 \otimes x_5^4x_{15}^2 + v_4x_5^2 \otimes x_3^{10}x_5^4 + v_4x_3^2 \otimes x_3^8x_{15}^2 \\
&+ v_4x_3^2 \otimes x_3^8x_5^6 + x_3^8 \otimes x_3^2 + x_3^2 \otimes x_3^8 + v_4x_3^8x_9^2 \otimes x_9^2 \\
&+ v_4x_9^2 \otimes x_3^8x_9^2 + x_5^2 \otimes x_5^4 + x_5^4 \otimes x_5^2 + b_{107}v_4x_3^{10} \otimes x_5^6 \\
&+ b_{107}v_4x_5^6 \otimes x_3^{10} + b_{107}v_4x_3^2x_5^6 \otimes x_3^8 + b_{107}v_4x_3^2 \otimes x_3^8x_5^6 \\
&+ b_{107}v_4x_3^8x_5^6 \otimes x_3^2 + b_{107}v_4x_3^8 \otimes x_3^2x_5^6 + b_{107}v_4x_3^{10}x_5^2 \otimes x_5^4 \\
&+ b_{107}v_4x_5^2 \otimes x_3^{10}x_5^4 + b_{107}v_4x_3^{10}x_5^4 \otimes x_5^2 + b_{107}v_4x_5^4 \otimes x_3^{10}x_5^2 \\
&+ b_{107}v_4x_3^2x_5^2 \otimes x_3^8x_5^4 + b_{107}v_4x_3^2x_5^4 \otimes x_3^8x_5^2 + b_{107}v_4x_3^8x_5^2 \otimes x_3^2x_5^4 \\
&+ b_{107}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 + b_{108}v_4x_3^{14} \otimes x_9^2 + b_{108}v_4x_9^2 \otimes x_3^{14} \\
&+ b_{108}v_4x_3^{12}x_9^2 \otimes x_3^2 + b_{108}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{108}v_4x_3^{10}x_9^2 \otimes x_3^4
\end{aligned}
$$

$$
\begin{aligned}
&+ b_{108}v_4x_3^{10} \otimes x_3^4x_9^2 + b_{108}v_4x_3^8x_9^2 \otimes x_3^6 + b_{108}v_4x_3^8 \otimes x_3^6x_9^2\\
&+ b_{108}v_4x_3^6x_9^2 \otimes x_3^8 + b_{108}v_4x_3^6 \otimes x_3^8x_9^2 + b_{108}v_4x_3^4x_9^2 \otimes x_3^{10}\\
&+ b_{108}v_4x_3^4 \otimes x_3^{10}x_9^2 + b_{108}v_4x_3^2x_9^2 \otimes x_3^{12} + b_{108}v_4x_3^2 \otimes x_3^{12}x_9^2\\
&+ b_{109}v_4x_3^4x_5^6 \otimes x_9^2 + b_{109}v_4x_3^4x_9^2 \otimes x_5^6 + b_{109}v_4x_3^4 \otimes x_5^6x_9^2\\
&+ b_{109}v_4x_5^6x_9^2 \otimes x_3^4 + b_{109}v_4x_5^6 \otimes x_3^4x_9^2 + b_{109}v_4x_9^2 \otimes x_3^4x_5^6\\
&+ b_{109}v_4x_3^4x_5^2x_9^2 \otimes x_5^4 + b_{109}v_4x_3^4x_5^2 \otimes x_5^4x_9^2 + b_{109}v_4x_5^2x_9^2 \otimes x_3^4x_5^4\\
&+ b_{109}v_4x_5^2 \otimes x_3^4x_5^4x_9^2 + b_{109}v_4x_3^4x_5^4x_9^2 \otimes x_5^2 + b_{109}v_4x_3^4x_5^4 \otimes x_5^2x_9^2\\
&+ b_{109}v_4x_5^4x_9^2 \otimes x_3^4x_5^2 + b_{109}v_4x_5^4 \otimes x_3^4x_5^2x_9^2 + b_{110}v_4x_3^{10} \otimes x_{15}^2\\
&+ b_{110}v_4x_{15}^2 \otimes x_3^{10} + b_{110}v_4x_3^2x_{15}^2 \otimes x_3^8 + b_{110}v_4x_3^2 \otimes x_3^8x_{15}^2\\
&+ b_{110}v_4x_3^8x_{15}^2 \otimes x_3^2 + b_{110}v_4x_3^8 \otimes x_3^2x_{15}^2 + b_{110}v_4x_3^{10} \otimes x_3^{10}\\
&+ b_{110}v_4x_3^8 \otimes x_3^{12} + b_{110}v_4x_3^{10}x_5^4 \otimes x_5^2 + b_{110}v_4x_3^8x_5^4 \otimes x_3^2x_5^2\\
&+ b_{110}v_4x_3^2x_5^4 \otimes x_3^8x_5^2 + b_{110}v_4x_5^4 \otimes x_3^{10}x_5^2 + b_{110}v_4x_3^{14} \otimes x_9^2\\
&+ b_{110}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{110}v_4x_3^6 \otimes x_3^8x_9^2 + b_{110}v_4x_3^4 \otimes x_3^{10}x_9^2\\
&+ b_{111}v_4x_5^6 \otimes x_{15}^2 + b_{111}v_4x_{15}^2 \otimes x_5^6 + b_{111}v_4x_5^2x_{15}^2 \otimes x_5^4\\
&+ b_{111}v_4x_5^2 \otimes x_5^4x_{15}^2 + b_{111}v_4x_5^4x_{15}^2 \otimes x_5^2 + b_{111}v_4x_5^4 \otimes x_5^2x_{15}^2\\
&+ b_{111}v_4x_3^8x_5^6 \otimes x_3^2 + b_{111}v_4x_3^8 \otimes x_3^2x_5^6 + b_{111}v_4x_3^8x_5^2 \otimes x_3^2x_5^4\\
&+ b_{111}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 + b_{111}v_4x_5^6 \otimes x_5^6 + b_{111}v_4x_3^4x_5^6 \otimes x_9^2\\
&+ b_{111}v_4x_3^4 \otimes x_5^6x_9^2 + b_{111}v_4x_3^4x_5^2 \otimes x_5^4x_9^2 + b_{111}v_4x_3^4x_5^4 \otimes x_5^2x_9^2\\
&+ b_{112}v_4x_3^4x_9^2 \otimes x_{15}^2 + b_{112}v_4x_3^4x_{15}^2 \otimes x_9^2 + b_{112}v_4x_3^4 \otimes x_9^2x_{15}^2\\
&+ b_{112}v_4x_9^2x_{15}^2 \otimes x_3^4 + b_{112}v_4x_9^2 \otimes x_3^4x_{15}^2 + b_{112}v_4x_{15}^2 \otimes x_3^4x_9^2\\
&+ b_{112}v_4x_3^{12}x_9^2 \otimes x_3^2 + b_{112}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{112}v_4x_3^8x_9^2 \otimes x_3^6\\
&+ b_{112}v_4x_3^8 \otimes x_3^6x_9^2 + b_{112}v_4x_3^4x_5^4x_9^2 \otimes x_5^2 + b_{112}v_4x_3^4x_5^4 \otimes x_5^2x_9^2\\
&+ b_{112}v_4x_5^4x_9^2 \otimes x_3^4x_5^2 + b_{112}v_4x_5^4 \otimes x_3^4x_5^2x_9^2 + b_{112}v_4x_3^8x_9^2 \otimes x_9^2\\
&+ b_{112}v_4x_3^4x_9^2 \otimes x_3^4x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_3\bar{\psi}(x_{15}) \equiv\ & x_3^4 \otimes x_9^2 + v_4x_3^4 \otimes x_3^{10}x_9^2 + v_4x_3^4 \otimes x_5^6x_9^2 + v_4x_3^4 \otimes x_9^2x_{15}^2\\
&+ x_5^2 \otimes x_5^4 + v_4x_5^2 \otimes x_3^{10}x_5^4 + v_4x_5^2 \otimes x_5^4x_{15}^2 + x_3^2 \otimes x_3^8\\
&+ v_4x_3^2 \otimes x_3^8x_5^6 + v_4x_3^2 \otimes x_3^8x_{15}^2 + v_4x_3^{14} \otimes x_9^2 + v_4x_3^4x_5^6 \otimes x_9^2\\
&+ v_4x_3^8x_9^2 \otimes x_9^2 + v_4x_3^4x_{15}^2 \otimes x_9^2 + v_4x_3^{10}x_5^2 \otimes x_5^4 + v_4x_5^2x_{15}^2 \otimes x_5^4\\
&+ v_4x_3^{12} \otimes x_3^8 + v_4x_3^2x_5^6 \otimes x_3^8 + v_4x_3^2x_{15}^2 \otimes x_3^8 + v_4x_3^{10} \otimes x_{15}^2
\end{aligned}
$$

$$
\begin{aligned}
&+ v_4x_3^{10} \otimes x_3^{10} + v_4x_3^{10} \otimes x_5^6 + v_4x_5^6 \otimes x_{15}^2 + v_4x_5^6 \otimes x_3^{10} \\
&+ v_4x_5^6 \otimes x_5^6 + v_4x_{15}^2 \otimes x_{15}^2 + v_4x_{15}^2 \otimes x_3^{10} + v_4x_{15}^2 \otimes x_5^6 \\
&+ v_4x_3^4x_5^2 \otimes x_5^4x_9^2 + v_4x_3^6 \otimes x_3^8x_9^2 + v_4x_9^2 \otimes x_3^8x_9^2 + v_4x_3^2x_5^2 \otimes x_3^8x_5^4.
\end{aligned}
$$

Then we obtain $b_{107} = b_{108} = b_{109} = b_{110} = b_{111} = b_{112} = 0$. We compare the coefficient of $Q_0\bar{\psi}(x_{17})$ and $\bar{\psi}(Q_0x_{17})$:

$$
\begin{aligned}
\bar{\psi}(Q_0x_{17}) \equiv\ & v_4x_3^8 \otimes x_3^8 + b_{13}v_4x_3^6 \otimes x_5^6 + b_{13}v_4x_5^6 \otimes x_3^6 + b_{13}v_4x_3^2x_5^6 \otimes x_3^4 \\
&+ b_{13}v_4x_3^2 \otimes x_3^4x_5^6 + b_{13}v_4x_3^4x_5^6 \otimes x_3^2 + b_{13}v_4x_3^4 \otimes x_3^2x_5^6 \\
&+ b_{13}v_4x_3^6x_5^2 \otimes x_5^4 + b_{13}v_4x_5^2 \otimes x_3^6x_5^4 + b_{13}v_4x_3^6x_5^4 \otimes x_5^2 \\
&+ b_{13}v_4x_5^4 \otimes x_3^6x_5^2 + b_{13}v_4x_3^2x_5^2 \otimes x_3^4x_5^4 + b_{13}v_4x_3^2x_5^4 \otimes x_3^4x_5^2 \\
&+ b_{13}v_4x_3^4x_5^2 \otimes x_3^2x_5^4 + b_{13}v_4x_3^4x_5^4 \otimes x_3^2x_5^2 + b_{14}v_4x_3^{10} \otimes x_9^2 \\
&+ b_{14}v_4x_9^2 \otimes x_3^{10} + b_{14}v_4x_3^2x_9^2 \otimes x_3^8 + b_{14}v_4x_3^2 \otimes x_3^8x_9^2 \\
&+ b_{14}v_4x_3^8x_9^2 \otimes x_3^2 + b_{14}v_4x_3^8 \otimes x_3^2x_9^2 + b_{15}v_4x_5^6 \otimes x_9^2 \\
&+ b_{15}v_4x_9^2 \otimes x_5^6 + b_{15}v_4x_5^2x_9^2 \otimes x_5^4 + b_{15}v_4x_5^2 \otimes x_5^4x_9^2 \\
&+ b_{15}v_4x_5^4x_9^2 \otimes x_5^2 + b_{15}v_4x_5^4 \otimes x_5^2x_9^2 + b_{16}v_4x_3^6 \otimes x_{15}^2 \\
&+ b_{16}v_4x_{15}^2 \otimes x_3^6 + b_{16}v_4x_3^2x_{15}^2 \otimes x_3^4 + b_{16}v_4x_3^2 \otimes x_3^4x_{15}^2 \\
&+ b_{16}v_4x_3^4x_{15}^2 \otimes x_3^2 + b_{16}v_4x_3^4 \otimes x_3^2x_{15}^2 + b_{16}v_4x_3^{14} \otimes x_3^2 \\
&+ b_{16}v_4x_3^{12} \otimes x_3^4 + b_{16}v_4x_3^{10} \otimes x_3^6 + b_{16}v_4x_3^8 \otimes x_3^8 \\
&+ b_{16}v_4x_3^6x_5^4 \otimes x_5^2 + b_{16}v_4x_3^4x_5^4 \otimes x_3^2x_5^2 + b_{16}v_4x_3^2x_5^4 \otimes x_3^4x_5^2 \\
&+ b_{16}v_4x_5^4 \otimes x_3^6x_5^2 + b_{16}v_4x_3^{10} \otimes x_9^2 + b_{16}v_4x_3^8 \otimes x_3^2x_9^2 \\
&+ b_{16}v_4x_3^6 \otimes x_3^4x_9^2 + b_{16}v_4x_3^4 \otimes x_3^6x_9^2 + b_{17}v_4x_9^2 \otimes x_{15}^2 \\
&+ b_{17}v_4x_{15}^2 \otimes x_9^2 + b_{17}v_4x_3^8x_9^2 \otimes x_3^2 + b_{17}v_4x_3^8 \otimes x_3^2x_9^2 \\
&+ b_{17}v_4x_5^4x_9^2 \otimes x_5^2 + b_{17}v_4x_5^4 \otimes x_5^2x_9^2 + b_{17}v_4x_3^4x_9^2 \otimes x_9^2, \\
Q_0\bar{\psi}(x_{17}) \equiv\ & a_{64}v_4x_3^{14} \otimes x_3^2 + a_{65}v_4x_3^4x_5^6 \otimes x_3^2 + a_{66}v_4x_3^8x_9^2 \otimes x_3^2 \\
&+ a_{67}v_4x_3^4x_{15}^2 \otimes x_3^2 + a_{68}v_4x_3^6x_5^4 \otimes x_5^2 + a_{69}v_4x_5^4x_9^2 \otimes x_5^2 \\
&+ a_{70}v_4x_3^4x_5^4 \otimes x_3^2x_5^2 + a_{71}v_4x_3^{10} \otimes x_9^2 + a_{72}v_4x_5^6 \otimes x_9^2 \\
&+ a_{73}v_4x_3^4x_9^2 \otimes x_9^2 + a_{74}v_4x_{15}^2 \otimes x_9^2 + a_{75}v_4x_3^8 \otimes x_3^8 \\
&+ a_{75}v_4x_3^8 \otimes x_3^2x_9^2 + a_{76}v_4x_3^2x_9^2 \otimes x_3^8 + a_{76}v_4x_3^2x_9^2 \otimes x_3^2x_9^2 \\
&+ a_{77}v_4x_5^4 \otimes x_5^2x_9^2 + a_{78}v_4x_3^6 \otimes x_{15}^2 + a_{79}v_4x_9^2 \otimes x_{15}^2.
\end{aligned}
$$

Then we obtain $b_{13} = b_{14} = b_{15} = b_{16} = a_{64} = a_{65} = a_{67} = a_{68} = a_{70} = a_{71} = a_{72} = a_{76} = a_{78} = 0$ and $b_{17} = a_{66} = a_{69} = a_{73} = a_{74} = a_{75} = a_{77} = a_{79} = 1$. We compare the coefficient of $Q_1\bar{\psi}(x_{17})$ and $\bar{\psi}(Q_1x_{17})$:

$$\begin{aligned}\bar{\psi}(Q_1x_{17}) \equiv\ & b_{47}v_4x_3^{10} \otimes x_5^4 + b_{47}v_4x_5^4 \otimes x_3^{10} + b_{47}v_4x_3^2x_5^4 \otimes x_3^8\\ & + b_{47}v_4x_3^2 \otimes x_3^8x_5^4 + b_{47}v_4x_3^8x_5^4 \otimes x_3^2 + b_{47}v_4x_3^8 \otimes x_3^2x_5^4\\ & + b_{48}v_4x_3^4x_5^4 \otimes x_9^2 + b_{48}v_4x_3^4x_9^2 \otimes x_5^4 + b_{48}v_4x_3^4 \otimes x_5^4x_9^2\\ & + b_{48}v_4x_5^4x_9^2 \otimes x_3^4 + b_{48}v_4x_5^4 \otimes x_3^4x_9^2 + b_{48}v_4x_9^2 \otimes x_3^4x_5^4\\ & + b_{49}v_4x_5^4 \otimes x_{15}^2 + b_{49}v_4x_{15}^2 \otimes x_5^4 + b_{49}v_4x_3^8x_5^4 \otimes x_3^2\\ & + b_{49}v_4x_3^8 \otimes x_3^2x_5^4 + b_{49}v_4x_5^4 \otimes x_5^6 + b_{49}v_4x_3^4x_5^4 \otimes x_9^2\\ & + b_{49}v_4x_3^4 \otimes x_5^4x_9^2,\end{aligned}$$

$$\begin{aligned}Q_1\bar{\psi}(x_{17}) \equiv\ & a_{62}v_4x_3^8x_5^4 \otimes x_3^2 + a_{63}v_4x_3^2x_5^4x_9^2 \otimes x_3^2 + v_4x_5^4x_9^2 \otimes x_3^4\\ & + v_4x_3^4x_9^2 \otimes x_5^4 + v_4x_{15}^2 \otimes x_5^4 + v_4x_3^8 \otimes x_3^2x_5^4 + v_4x_5^4 \otimes x_{15}^2\\ & + v_4x_5^4 \otimes x_3^4x_9^2 + v_4x_5^4 \otimes x_5^6 + v_4x_9^2 \otimes x_3^4x_5^4.\end{aligned}$$

Then we obtain $b_{47} = a_{63} = 0$ and $b_{48} = b_{49} = a_{62} = 1$. We compare the coefficient of $Q_2\bar{\psi}(x_{17})$ and $\bar{\psi}(Q_2x_{17})$:

$$\begin{aligned}\bar{\psi}(Q_2x_{17}) \equiv\ & b_{75}v_4x_3^8 \otimes x_5^6 + b_{75}v_4x_5^6 \otimes x_3^8 + b_{75}v_4x_3^8x_5^2 \otimes x_5^4 + b_{75}v_4x_5^2 \otimes x_3^8x_5^4\\ & + b_{75}v_4x_3^8x_5^4 \otimes x_5^2 + b_{75}v_4x_5^4 \otimes x_3^8x_5^2 + b_{76}v_4x_3^{12} \otimes x_9^2\\ & + b_{76}v_4x_9^2 \otimes x_3^{12} + b_{76}v_4x_3^4x_9^2 \otimes x_3^8 + b_{76}v_4x_3^4 \otimes x_3^8x_9^2\\ & + b_{76}v_4x_3^8x_9^2 \otimes x_3^4 + b_{76}v_4x_3^8 \otimes x_3^4x_9^2 + b_{77}v_4x_3^2x_5^6 \otimes x_9^2\\ & + b_{77}v_4x_3^2x_9^2 \otimes x_5^6 + b_{77}v_4x_3^2 \otimes x_5^6x_9^2 + b_{77}v_4x_5^6x_9^2 \otimes x_3^2\\ & + b_{77}v_4x_5^6 \otimes x_3^2x_9^2 + b_{77}v_4x_9^2 \otimes x_3^2x_5^6 + b_{77}v_4x_3^2x_5^2x_9^2 \otimes x_5^4\\ & + b_{77}v_4x_3^2x_5^2 \otimes x_5^4x_9^2 + b_{77}v_4x_5^2x_9^2 \otimes x_3^2x_5^4 + b_{77}v_4x_5^2 \otimes x_3^2x_5^4x_9^2\\ & + b_{77}v_4x_3^2x_5^4x_9^2 \otimes x_5^2 + b_{77}v_4x_3^2x_5^4 \otimes x_5^2x_9^2 + b_{77}v_4x_5^4x_9^2 \otimes x_3^2x_5^2\\ & + b_{77}v_4x_5^4 \otimes x_3^2x_5^2x_9^2 + b_{78}v_4x_3^8 \otimes x_{15}^2 + b_{78}v_4x_{15}^2 \otimes x_3^8\\ & + b_{78}v_4x_3^8 \otimes x_3^{10} + b_{78}v_4x_3^8x_5^4 \otimes x_5^2 + b_{78}v_4x_5^4 \otimes x_3^8x_5^2\\ & + b_{78}v_4x_3^{12} \otimes x_9^2 + b_{78}v_4x_3^4 \otimes x_3^8x_9^2 + b_{79}v_4x_3^2x_9^2 \otimes x_{15}^2\\ & + b_{79}v_4x_3^2x_{15}^2 \otimes x_9^2 + b_{79}v_4x_3^2 \otimes x_9^2x_{15}^2 + b_{79}v_4x_9^2x_{15}^2 \otimes x_3^2\\ & + b_{79}v_4x_9^2 \otimes x_3^2x_{15}^2 + b_{79}v_4x_{15}^2 \otimes x_3^2x_9^2 + b_{79}v_4x_3^{10}x_9^2 \otimes x_3^2\\ & + b_{79}v_4x_3^{10} \otimes x_3^2x_9^2 + b_{79}v_4x_3^8x_9^2 \otimes x_3^4 + b_{79}v_4x_3^8 \otimes x_3^4x_9^2\\ & + b_{79}v_4x_3^2x_5^4x_9^2 \otimes x_5^2 + b_{79}v_4x_3^2x_5^4 \otimes x_5^2x_9^2 + b_{79}v_4x_5^4x_9^2 \otimes x_3^2x_5^2\end{aligned}$$

$$+ b_{79}v_4x_5^4 \otimes x_3^2x_5^2x_9^2 + b_{79}v_4x_3^6x_9^2 \otimes x_9^2 + b_{79}v_4x_3^4x_9^2 \otimes x_3^2x_9^2,$$

$$Q_2\bar{\psi}(x_{17}) \equiv v_4x_3^8x_5^4 \otimes x_5^2 + v_4x_3^8x_9^2 \otimes x_3^4 + v_4x_3^4x_9^2 \otimes x_3^8 + v_4x_{15}^2 \otimes x_3^8 + v_4x_3^8 \otimes x_{15}^2 + v_4x_3^8 \otimes x_3^{10} + v_4x_3^8 \otimes x_3^4x_9^2 + v_4x_5^4 \otimes x_3^8x_5^2 + v_4x_9^2 \otimes x_3^{12}.$$

Then we obtain $b_{75} = b_{77} = b_{79} = 0$ and $b_{76} = b_{78} = 1$. We compare the coefficient of $Q_3\bar{\psi}(x_{17})$ and $\bar{\psi}(Q_3x_{17})$:

$$\begin{aligned}\bar{\psi}(Q_3x_{17}) \equiv{}& b_{113}v_4x_3^{14} \otimes x_5^4 + b_{113}v_4x_5^4 \otimes x_3^{14} + b_{113}v_4x_3^{12}x_5^4 \otimes x_3^2 \\ &+ b_{113}v_4x_3^{12} \otimes x_3^2x_5^4 + b_{113}v_4x_3^{10}x_5^4 \otimes x_3^4 + b_{113}v_4x_3^{10} \otimes x_3^4x_5^4 \\ &+ b_{113}v_4x_3^8x_5^4 \otimes x_3^6 + b_{113}v_4x_3^8 \otimes x_3^6x_5^4 + b_{113}v_4x_3^6x_5^4 \otimes x_3^8 \\ &+ b_{113}v_4x_3^6 \otimes x_3^8x_5^4 + b_{113}v_4x_3^4x_5^4 \otimes x_3^{10} + b_{113}v_4x_3^4 \otimes x_3^{10}x_5^4 \\ &+ b_{113}v_4x_3^2x_5^4 \otimes x_3^{12} + b_{113}v_4x_3^2 \otimes x_3^{12}x_5^4 + b_{114}v_4x_3^8x_5^4 \otimes x_9^2 \\ &+ b_{114}v_4x_3^8x_9^2 \otimes x_5^4 + b_{114}v_4x_3^8 \otimes x_5^4x_9^2 + b_{114}v_4x_5^4x_9^2 \otimes x_3^8 \\ &+ b_{114}v_4x_5^4 \otimes x_3^8x_9^2 + b_{114}v_4x_9^2 \otimes x_3^8x_5^4 + b_{115}v_4x_3^4x_5^4 \otimes x_{15}^2 \\ &+ b_{115}v_4x_3^4x_{15}^2 \otimes x_5^4 + b_{115}v_4x_3^4 \otimes x_5^4x_{15}^2 + b_{115}v_4x_5^4x_{15}^2 \otimes x_3^4 \\ &+ b_{115}v_4x_5^4 \otimes x_3^4x_{15}^2 + b_{115}v_4x_{15}^2 \otimes x_3^4x_5^4 + b_{115}v_4x_3^{12}x_5^4 \otimes x_3^2 \\ &+ b_{115}v_4x_3^{12} \otimes x_3^2x_5^4 + b_{115}v_4x_3^8x_5^4 \otimes x_3^6 + b_{115}v_4x_3^8 \otimes x_3^6x_5^4 \\ &+ b_{115}v_4x_3^4x_5^4 \otimes x_5^6 + b_{115}v_4x_5^4 \otimes x_3^4x_5^6 + b_{115}v_4x_3^8x_5^4 \otimes x_9^2 \\ &+ b_{115}v_4x_3^8 \otimes x_5^4x_9^2 + b_{115}v_4x_3^4x_5^4 \otimes x_3^4x_9^2 + b_{115}v_4x_3^4 \otimes x_3^4x_5^4x_9^2,\end{aligned}$$

$$Q_3\bar{\psi}(x_{17}) \equiv v_4x_3^8x_5^4 \otimes x_9^2 + v_4x_3^8x_9^2 \otimes x_5^4 + v_4x_5^4x_9^2 \otimes x_3^8 + v_4x_3^8 \otimes x_5^4x_9^2 + v_4x_5^4 \otimes x_3^8x_9^2 + v_4x_9^2 \otimes x_3^8x_5^4.$$

Then we obtain $b_{113} = b_{115} = 0$ and $b_{114} = 1$. We compare the coefficient of $Q_0\bar{\psi}(x_{23})$ and $\bar{\psi}(Q_0x_{23})$:

$$\begin{aligned}\bar{\psi}(Q_0x_{23}) \equiv{}& x_3^2 \otimes x_9^2 + x_9^2 \otimes x_3^2 + v_4x_3^{10} \otimes x_3^8 + v_4x_3^8 \otimes x_3^{10} + b_{18}v_4x_3^8 \otimes x_5^6 \\ &+ b_{18}v_4x_5^6 \otimes x_3^8 + b_{18}v_4x_3^8x_5^2 \otimes x_5^4 + b_{18}v_4x_5^2 \otimes x_3^8x_5^4 \\ &+ b_{18}v_4x_3^8x_5^4 \otimes x_5^2 + b_{18}v_4x_5^4 \otimes x_3^8x_5^2 + b_{19}v_4x_3^{12} \otimes x_9^2 \\ &+ b_{19}v_4x_9^2 \otimes x_3^{12} + b_{19}v_4x_3^4x_9^2 \otimes x_3^8 + b_{19}v_4x_3^4 \otimes x_3^8x_9^2 \\ &+ b_{19}v_4x_3^8x_9^2 \otimes x_3^4 + b_{19}v_4x_3^8 \otimes x_3^4x_9^2 + b_{20}v_4x_3^2x_5^6 \otimes x_9^2 \\ &+ b_{20}v_4x_3^2x_9^2 \otimes x_5^6 + b_{20}v_4x_3^2 \otimes x_5^6x_9^2 + b_{20}v_4x_5^6x_9^2 \otimes x_3^2 \\ &+ b_{20}v_4x_5^6 \otimes x_3^2x_9^2 + b_{20}v_4x_9^2 \otimes x_3^2x_5^6 + b_{20}v_4x_3^2x_5^2x_9^2 \otimes x_5^4 \\ &+ b_{20}v_4x_3^2x_5^2 \otimes x_5^4x_9^2 + b_{20}v_4x_5^2x_9^2 \otimes x_3^2x_5^4 + b_{20}v_4x_5^2 \otimes x_3^2x_5^4x_9^2\end{aligned}$$

$$
\begin{aligned}
&+ b_{20}v_4x_3^2x_5^4x_9^2 \otimes x_5^2 + b_{20}v_4x_3^2x_5^4 \otimes x_5^2x_9^2 + b_{20}v_4x_5^4x_9^2 \otimes x_3^2x_5^2\\
&+ b_{20}v_4x_5^4 \otimes x_3^2x_5^2x_9^2 + b_{21}v_4x_3^8 \otimes x_{15}^2 + b_{21}v_4x_{15}^2 \otimes x_3^8\\
&+ b_{21}v_4x_3^8 \otimes x_3^{10} + b_{21}v_4x_3^8x_5^4 \otimes x_5^2 + b_{21}v_4x_5^4 \otimes x_3^8x_5^2\\
&+ b_{21}v_4x_3^{12} \otimes x_9^2 + b_{21}v_4x_3^4 \otimes x_3^8x_9^2 + b_{22}v_4x_3^2x_9^2 \otimes x_{15}^2\\
&+ b_{22}v_4x_3^2x_{15}^2 \otimes x_9^2 + b_{22}v_4x_3^2 \otimes x_9^2x_{15}^2 + b_{22}v_4x_9^2x_{15}^2 \otimes x_3^2\\
&+ b_{22}v_4x_9^2 \otimes x_3^2x_{15}^2 + b_{22}v_4x_{15}^2 \otimes x_3^2x_9^2 + b_{22}v_4x_3^{10}x_9^2 \otimes x_3^2\\
&+ b_{22}v_4x_3^{10} \otimes x_3^2x_9^2 + b_{22}v_4x_3^8x_9^2 \otimes x_3^4 + b_{22}v_4x_3^8 \otimes x_3^4x_9^2\\
&+ b_{22}v_4x_3^2x_5^4x_9^2 \otimes x_5^2 + b_{22}v_4x_3^2x_5^4 \otimes x_5^2x_9^2 + b_{22}v_4x_5^4x_9^2 \otimes x_3^2x_5^2\\
&+ b_{22}v_4x_5^4 \otimes x_3^2x_5^2x_9^2 + b_{22}v_4x_3^6x_9^2 \otimes x_9^2 + b_{22}v_4x_3^4x_9^2 \otimes x_3^2x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_0\bar{\psi}(x_{23}) \equiv\ & v_4x_5^4 \otimes x_3^8x_5^2 + v_4x_5^4 \otimes x_3^2x_5^2x_9^2 + x_9^2 \otimes x_3^2 + v_4x_9^2 \otimes x_3^{12}\\
&+ v_4x_9^2 \otimes x_3^2x_{15}^2 + x_3^2 \otimes x_9^2 + v_4x_3^2 \otimes x_9^2x_{15}^2 + a_{83}v_4x_3^6x_5^6 \otimes x_3^2\\
&+ a_{84}v_4x_3^{10}x_9^2 \otimes x_3^2 + a_{85}v_4x_5^6x_9^2 \otimes x_3^2 + a_{86}v_4x_3^6x_{15}^2 \otimes x_3^2\\
&+ a_{87}v_4x_9^2x_{15}^2 \otimes x_3^2 + a_{88}v_4x_3^8x_5^4 \otimes x_5^2 + a_{89}v_4x_3^2x_5^4x_9^2 \otimes x_5^2\\
&+ a_{90}v_4x_3^6x_5^4 \otimes x_3^2x_5^2 + a_{91}v_4x_5^4x_9^2 \otimes x_3^2x_5^2 + a_{92}v_4x_3^{12} \otimes x_9^2\\
&+ a_{93}v_4x_3^2x_5^6 \otimes x_9^2 + a_{94}v_4x_3^6x_9^2 \otimes x_9^2 + a_{95}v_4x_3^2x_{15}^2 \otimes x_9^2\\
&+ a_{96}v_4x_3^{10} \otimes x_3^8 + a_{96}v_4x_3^{10} \otimes x_3^2x_9^2 + a_{97}v_4x_5^6 \otimes x_3^8\\
&+ a_{97}v_4x_5^6 \otimes x_3^2x_9^2 + a_{98}v_4x_3^4x_9^2 \otimes x_3^8 + a_{98}v_4x_3^4x_9^2 \otimes x_3^2x_9^2\\
&+ a_{99}v_4x_{15}^2 \otimes x_3^8 + a_{99}v_4x_{15}^2 \otimes x_3^2x_9^2 + a_{100}v_4x_3^2x_5^4 \otimes x_5^2x_9^2\\
&+ a_{101}v_4x_3^8 \otimes x_{15}^2 + a_{102}v_4x_3^2x_9^2 \otimes x_{15}^2.
\end{aligned}
$$

Then we obtain $b_{18} = b_{20} = a_{83} = a_{85} = a_{86} = a_{90} = a_{92} = a_{93} = a_{97} = 0$ and $b_{19} = b_{21} = b_{22} = a_{84} = a_{87} = a_{88} = a_{89} = a_{91} = a_{94} = a_{95} = a_{96} = a_{98} = a_{99} = a_{100} = a_{101} = a_{102} = 1$. We compare the coefficient of $Q_1\bar{\psi}(x_{23})$ and $\bar{\psi}(Q_1x_{23})$:

$$
\begin{aligned}
\bar{\psi}(Q_1x_{23}) \equiv\ & x_3^2 \otimes x_5^4 + x_5^4 \otimes x_3^2 + v_4x_5^4x_9^2 \otimes x_9^2 + v_4x_9^2 \otimes x_5^4x_9^2 + b_{50}v_4x_3^{12} \otimes x_5^4\\
&+ b_{50}v_4x_5^4 \otimes x_3^{12} + b_{50}v_4x_3^4x_5^4 \otimes x_3^8 + b_{50}v_4x_3^4 \otimes x_3^8x_5^4\\
&+ b_{50}v_4x_3^8x_5^4 \otimes x_3^4 + b_{50}v_4x_3^8 \otimes x_3^4x_5^4 + b_{51}v_4x_3^6x_5^4 \otimes x_9^2\\
&+ b_{51}v_4x_3^6x_9^2 \otimes x_5^4 + b_{51}v_4x_3^6 \otimes x_5^4x_9^2 + b_{51}v_4x_5^4x_9^2 \otimes x_3^6\\
&+ b_{51}v_4x_5^4 \otimes x_3^6x_9^2 + b_{51}v_4x_9^2 \otimes x_3^6x_5^4 + b_{51}v_4x_3^2x_5^4x_9^2 \otimes x_3^4\\
&+ b_{51}v_4x_3^2x_5^4 \otimes x_3^4x_9^2 + b_{51}v_4x_3^2x_9^2 \otimes x_3^4x_5^4 + b_{51}v_4x_3^2 \otimes x_3^4x_5^4x_9^2\\
&+ b_{51}v_4x_3^4x_5^4x_9^2 \otimes x_3^2 + b_{51}v_4x_3^4x_5^4 \otimes x_3^2x_9^2 + b_{51}v_4x_3^4x_9^2 \otimes x_3^2x_5^4
\end{aligned}
$$

$$
\begin{aligned}
&+ b_{51}v_4x_3^4 \otimes x_3^2x_5^4x_9^2 + b_{52}v_4x_3^2x_5^4 \otimes x_{15}^2 + b_{52}v_4x_3^2x_{15}^2 \otimes x_5^4\\
&+ b_{52}v_4x_3^2 \otimes x_5^4x_{15}^2 + b_{52}v_4x_5^4x_{15}^2 \otimes x_3^2 + b_{52}v_4x_5^4 \otimes x_3^2x_{15}^2\\
&+ b_{52}v_4x_{15}^2 \otimes x_3^2x_5^4 + b_{52}v_4x_3^{10}x_5^4 \otimes x_3^2 + b_{52}v_4x_3^{10} \otimes x_3^2x_5^4\\
&+ b_{52}v_4x_3^8x_5^4 \otimes x_3^4 + b_{52}v_4x_3^8 \otimes x_3^4x_5^4 + b_{52}v_4x_3^2x_5^4 \otimes x_5^6\\
&+ b_{52}v_4x_5^4 \otimes x_3^2x_5^6 + b_{52}v_4x_3^6x_5^4 \otimes x_9^2 + b_{52}v_4x_3^6 \otimes x_5^4x_9^2\\
&+ b_{52}v_4x_3^4x_5^4 \otimes x_3^2x_9^2 + b_{52}v_4x_3^4 \otimes x_3^2x_5^4x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_1\bar{\psi}(x_{23}) \equiv\ & x_5^4 \otimes x_3^2 + v_4x_5^4 \otimes x_3^2x_5^6 + v_4x_5^4 \otimes x_3^6x_9^2 + v_4x_5^4 \otimes x_3^2x_{15}^2\\
&+ v_4x_9^2 \otimes x_3^6x_5^4 + v_4x_9^2 \otimes x_5^4x_9^2 + x_3^2 \otimes x_5^4 + v_4x_3^2 \otimes x_3^4x_5^4x_9^2\\
&+ v_4x_3^2 \otimes x_5^4x_{15}^2 + a_{80}v_4x_3^{10}x_5^4 \otimes x_3^2 + a_{81}v_4x_3^4x_5^4x_9^2 \otimes x_3^2\\
&+ a_{82}v_4x_5^4x_{15}^2 \otimes x_3^2 + v_4x_3^8x_5^4 \otimes x_3^4 + v_4x_3^2x_5^4x_9^2 \otimes x_3^4\\
&+ v_4x_5^4x_9^2 \otimes x_9^2 + v_4x_5^4x_9^2 \otimes x_3^6 + v_4x_3^6x_9^2 \otimes x_5^4 + v_4x_3^2x_{15}^2 \otimes x_5^4\\
&+ v_4x_3^{10} \otimes x_3^2x_5^4 + v_4x_3^4x_9^2 \otimes x_3^2x_5^4 + v_4x_{15}^2 \otimes x_3^2x_5^4 + v_4x_3^2x_5^4 \otimes x_{15}^2\\
&+ v_4x_3^2x_5^4 \otimes x_3^4x_9^2 + v_4x_3^2x_5^4 \otimes x_5^6 + v_4x_3^8 \otimes x_3^4x_5^4 + v_4x_3^2x_9^2 \otimes x_3^4x_5^4.
\end{aligned}
$$

Then we obtain $b_{50} = 0$ and $b_{51} = b_{52} = a_{80} = a_{81} = a_{82} = 1$. We compare the coefficient of $Q_2\bar{\psi}(x_{23})$ and $\bar{\psi}(Q_2x_{23})$:

$$
\begin{aligned}
\bar{\psi}(Q_2x_{23}) \equiv\ & x_3^8 \otimes x_3^2 + x_5^4 \otimes x_5^2 + x_3^4 \otimes x_9^2 + v_4x_{15}^2 \otimes x_{15}^2 + v_4x_{15}^2 \otimes x_3^{10}\\
&+ v_4x_{15}^2 \otimes x_5^6 + v_4x_3^{10} \otimes x_{15}^2 + v_4x_3^{10} \otimes x_3^{10} + v_4x_3^{10} \otimes x_5^6\\
&+ v_4x_5^6 \otimes x_{15}^2 + v_4x_5^6 \otimes x_3^{10} + v_4x_5^6 \otimes x_5^6 + v_4x_3^4x_{15}^2 \otimes x_9^2\\
&+ v_4x_3^{14} \otimes x_9^2 + v_4x_3^4x_5^6 \otimes x_9^2 + v_4x_5^2x_{15}^2 \otimes x_5^4 + v_4x_3^{10}x_5^2 \otimes x_5^4\\
&+ v_4x_3^2x_{15}^2 \otimes x_3^8 + v_4x_3^{12} \otimes x_3^8 + v_4x_3^2x_5^6 \otimes x_3^8 + v_4x_3^4x_5^2 \otimes x_5^4x_9^2\\
&+ v_4x_3^6 \otimes x_3^8x_9^2 + v_4x_3^2x_5^2 \otimes x_3^8x_5^4 + v_4x_3^4 \otimes x_9^2x_{15}^2 + v_4x_3^4 \otimes x_3^{10}x_9^2\\
&+ v_4x_3^4 \otimes x_5^6x_9^2 + v_4x_5^2 \otimes x_5^4x_{15}^2 + v_4x_5^2 \otimes x_3^{10}x_5^4 + v_4x_3^2 \otimes x_3^8x_{15}^2\\
&+ v_4x_3^2 \otimes x_3^8x_5^6 + x_3^2 \otimes x_3^8 + x_3^8 \otimes x_3^2 + v_4x_3^8x_9^2 \otimes x_9^2\\
&+ v_4x_9^2 \otimes x_3^8x_9^2 + x_3^4 \otimes x_9^2 + x_9^2 \otimes x_3^4 + v_4x_3^{12} \otimes x_3^8 + v_4x_3^8 \otimes x_3^{12}\\
&+ b_{80}v_4x_3^{10} \otimes x_5^6 + b_{80}v_4x_5^6 \otimes x_3^{10} + b_{80}v_4x_3^2x_5^6 \otimes x_3^8\\
&+ b_{80}v_4x_3^2 \otimes x_3^8x_5^6 + b_{80}v_4x_3^8x_5^6 \otimes x_3^2 + b_{80}v_4x_3^8 \otimes x_3^2x_5^6\\
&+ b_{80}v_4x_3^{10}x_5^2 \otimes x_5^4 + b_{80}v_4x_5^2 \otimes x_3^{10}x_5^4 + b_{80}v_4x_3^{10}x_5^4 \otimes x_5^2\\
&+ b_{80}v_4x_5^4 \otimes x_3^{10}x_5^2 + b_{80}v_4x_3^2x_5^2 \otimes x_3^8x_5^4 + b_{80}v_4x_3^2x_5^4 \otimes x_3^8x_5^2\\
&+ b_{80}v_4x_3^8x_5^2 \otimes x_3^2x_5^4 + b_{80}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 + b_{81}v_4x_3^{14} \otimes x_9^2\\
&+ b_{81}v_4x_9^2 \otimes x_3^{14} + b_{81}v_4x_3^{12}x_9^2 \otimes x_3^2 + b_{81}v_4x_3^{12} \otimes x_3^2x_9^2
\end{aligned}
$$

$$
\begin{aligned}
&+ b_{81}v_4x_3^{10}x_9^2 \otimes x_3^4 + b_{81}v_4x_3^{10} \otimes x_3^4x_9^2 + b_{81}v_4x_3^8x_9^2 \otimes x_3^6\\
&+ b_{81}v_4x_3^8 \otimes x_3^6x_9^2 + b_{81}v_4x_3^6x_9^2 \otimes x_3^8 + b_{81}v_4x_3^6 \otimes x_3^8x_9^2\\
&+ b_{81}v_4x_3^4x_9^2 \otimes x_3^{10} + b_{81}v_4x_3^4 \otimes x_3^{10}x_9^2 + b_{81}v_4x_3^2x_9^2 \otimes x_3^{12}\\
&+ b_{81}v_4x_3^2 \otimes x_3^{12}x_9^2 + b_{82}v_4x_3^4x_5^6 \otimes x_9^2 + b_{82}v_4x_3^4x_9^2 \otimes x_5^6\\
&+ b_{82}v_4x_3^4 \otimes x_5^6x_9^2 + b_{82}v_4x_5^6x_9^2 \otimes x_3^4 + b_{82}v_4x_5^6 \otimes x_3^4x_9^2\\
&+ b_{82}v_4x_9^2 \otimes x_3^4x_5^6 + b_{82}v_4x_3^4x_5^2x_9^2 \otimes x_5^4 + b_{82}v_4x_3^4x_5^2 \otimes x_5^4x_9^2\\
&+ b_{82}v_4x_5^2x_9^2 \otimes x_3^4x_5^4 + b_{82}v_4x_5^2 \otimes x_3^4x_5^4x_9^2 + b_{82}v_4x_3^4x_5^4x_9^2 \otimes x_5^2\\
&+ b_{82}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 + b_{82}v_4x_5^4x_9^2 \otimes x_3^4x_5^2 + b_{82}v_4x_5^4 \otimes x_3^4x_5^2x_9^2\\
&+ b_{83}v_4x_3^{10} \otimes x_{15}^2 + b_{83}v_4x_{15}^2 \otimes x_3^{10} + b_{83}v_4x_3^2x_{15}^2 \otimes x_3^8\\
&+ b_{83}v_4x_3^2 \otimes x_3^8x_{15}^2 + b_{83}v_4x_3^8x_{15}^2 \otimes x_3^2 + b_{83}v_4x_3^8 \otimes x_3^2x_{15}^2\\
&+ b_{83}v_4x_3^{10} \otimes x_3^{10} + b_{83}v_4x_3^8 \otimes x_3^{12} + b_{83}v_4x_3^{10}x_5^4 \otimes x_5^2\\
&+ b_{83}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 + b_{83}v_4x_3^2x_5^4 \otimes x_3^8x_5^2 + b_{83}v_4x_5^4 \otimes x_3^{10}x_5^2\\
&+ b_{83}v_4x_3^{14} \otimes x_9^2 + b_{83}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{83}v_4x_3^6 \otimes x_3^8x_9^2\\
&+ b_{83}v_4x_3^4 \otimes x_3^{10}x_9^2 + b_{84}v_4x_5^6 \otimes x_{15}^2 + b_{84}v_4x_{15}^2 \otimes x_5^6\\
&+ b_{84}v_4x_5^2x_{15}^2 \otimes x_5^4 + b_{84}v_4x_5^2 \otimes x_5^4x_{15}^2 + b_{84}v_4x_5^4x_{15}^2 \otimes x_5^2\\
&+ b_{84}v_4x_5^4 \otimes x_5^2x_{15}^2 + b_{84}v_4x_3^8x_5^6 \otimes x_3^2 + b_{84}v_4x_3^8x_5^4 \otimes x_3^2x_5^2\\
&+ b_{84}v_4x_3^8x_5^2 \otimes x_3^2x_5^4 + b_{84}v_4x_3^8 \otimes x_3^2x_5^6 + b_{84}v_4x_5^6 \otimes x_5^6\\
&+ b_{84}v_4x_3^4x_5^6 \otimes x_9^2 + b_{84}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 + b_{84}v_4x_3^4x_5^2 \otimes x_5^4x_9^2\\
&+ b_{84}v_4x_3^4 \otimes x_5^6x_9^2 + b_{85}v_4x_3^4x_9^2 \otimes x_{15}^2 + b_{85}v_4x_3^4x_{15}^2 \otimes x_9^2\\
&+ b_{85}v_4x_3^4 \otimes x_9^2x_{15}^2 + b_{85}v_4x_9^2x_{15}^2 \otimes x_3^4 + b_{85}v_4x_9^2 \otimes x_3^4x_{15}^2\\
&+ b_{85}v_4x_{15}^2 \otimes x_3^4x_9^2 + b_{85}v_4x_3^{12}x_9^2 \otimes x_3^2 + b_{85}v_4x_3^{12} \otimes x_3^2x_9^2\\
&+ b_{85}v_4x_3^8x_9^2 \otimes x_3^6 + b_{85}v_4x_3^8 \otimes x_3^6x_9^2 + b_{85}v_4x_3^4x_5^4x_9^2 \otimes x_5^2\\
&+ b_{85}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 + b_{85}v_4x_5^4x_9^2 \otimes x_3^4x_5^2 + b_{85}v_4x_5^4 \otimes x_3^4x_5^2x_9^2\\
&+ b_{85}v_4x_3^8x_9^2 \otimes x_9^2 + b_{85}v_4x_3^4x_9^2 \otimes x_3^4x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_2\bar{\psi}(x_{23}) \equiv\ & x_5^4 \otimes x_5^2 + v_4x_5^4 \otimes x_3^{10}x_5^2 + v_4x_5^4 \otimes x_3^4x_5^2x_9^2 + v_4x_5^4 \otimes x_5^2x_{15}^2\\
&+ x_9^2 \otimes x_3^4 + v_4x_9^2 \otimes x_3^{14} + v_4x_9^2 \otimes x_3^8x_9^2 + v_4x_9^2 \otimes x_3^4x_{15}^2 + x_3^2 \otimes x_3^8\\
&+ v_4x_3^2 \otimes x_3^{12}x_9^2 + v_4x_3^2 \otimes x_3^8x_{15}^2 + v_4x_3^{10}x_5^4 \otimes x_5^2 + v_4x_3^4x_5^4x_9^2 \otimes x_5^2\\
&+ v_4x_5^4x_{15}^2 \otimes x_5^2 + v_4x_3^{10}x_9^2 \otimes x_3^4 + v_4x_9^2x_{15}^2 \otimes x_3^4 + v_4x_5^4x_9^2 \otimes x_3^4x_5^2\\
&+ v_4x_3^6x_9^2 \otimes x_3^8 + v_4x_3^2x_{15}^2 \otimes x_3^8 + v_4x_3^{10} \otimes x_{15}^2 + v_4x_3^{10} \otimes x_3^{10}
\end{aligned}
$$

$$+ v_4x_3^{10} \otimes x_3^4x_9^2 + v_4x_3^4x_9^2 \otimes x_{15}^2 + v_4x_3^4x_9^2 \otimes x_3^{10} + v_4x_3^4x_9^2 \otimes x_3^4x_9^2 + v_4x_{15}^2 \otimes x_{15}^2 + v_4x_{15}^2 \otimes x_3^{10} + v_4x_{15}^2 \otimes x_3^4x_9^2 + v_4x_3^2x_5^4 \otimes x_3^8x_5^2 + v_4x_3^8 \otimes x_3^{12} + v_4x_3^2x_9^2 \otimes x_3^{12}.$$

Then we obtain $b_{82} = b_{83} = 0$ and $b_{80} = b_{81} = b_{84} = b_{85} = 1$. We compare the coefficient of $Q_3\bar{\psi}(x_{23})$ and $\bar{\psi}(Q_3x_{23})$:

$$\begin{aligned}
\bar{\psi}(Q_3x_{23}) \equiv\ & x_5^4 \otimes x_9^2 + x_9^2 \otimes x_5^4 + v_4x_3^8x_5^4 \otimes x_3^8 + v_4x_3^8 \otimes x_3^8x_5^4 \\
&+ b_{116}v_4x_3^{10}x_5^4 \otimes x_9^2 + b_{116}v_4x_3^{10}x_9^2 \otimes x_5^4 + b_{116}v_4x_3^{10} \otimes x_5^4x_9^2 \\
&+ b_{116}v_4x_5^4x_9^2 \otimes x_3^{10} + b_{116}v_4x_5^4 \otimes x_3^{10}x_9^2 + b_{116}v_4x_9^2 \otimes x_3^{10}x_5^4 \\
&+ b_{116}v_4x_3^2x_5^4x_9^2 \otimes x_3^8 + b_{116}v_4x_3^2x_5^4 \otimes x_3^8x_9^2 + b_{116}v_4x_3^2x_9^2 \otimes x_3^8x_5^4 \\
&+ b_{116}v_4x_3^2 \otimes x_3^8x_5^4x_9^2 + b_{116}v_4x_3^8x_5^4x_9^2 \otimes x_3^2 + b_{116}v_4x_3^8x_5^4 \otimes x_3^2x_9^2 \\
&+ b_{116}v_4x_3^8x_9^2 \otimes x_3^2x_5^4 + b_{116}v_4x_3^8 \otimes x_3^2x_5^4x_9^2 + b_{117}v_4x_3^6x_5^4 \otimes x_{15}^2 \\
&+ b_{117}v_4x_3^6x_{15}^2 \otimes x_5^4 + b_{117}v_4x_3^6 \otimes x_5^4x_{15}^2 + b_{117}v_4x_5^4x_{15}^2 \otimes x_3^6 \\
&+ b_{117}v_4x_5^4 \otimes x_3^6x_{15}^2 + b_{117}v_4x_{15}^2 \otimes x_3^6x_5^4 + b_{117}v_4x_3^2x_5^4x_{15}^2 \otimes x_3^4 \\
&+ b_{117}v_4x_3^2x_5^4 \otimes x_3^4x_{15}^2 + b_{117}v_4x_3^2x_{15}^2 \otimes x_3^4x_5^4 \\
&+ b_{117}v_4x_3^2 \otimes x_3^4x_5^4x_{15}^2 + b_{117}v_4x_3^4x_5^4x_{15}^2 \otimes x_3^2 \\
&+ b_{117}v_4x_3^4x_5^4 \otimes x_3^2x_{15}^2 + b_{117}v_4x_3^4x_{15}^2 \otimes x_3^2x_5^4 \\
&+ b_{117}v_4x_3^4 \otimes x_3^2x_5^4x_{15}^2 + b_{117}v_4x_3^{14}x_5^4 \otimes x_3^2 + b_{117}v_4x_3^{14} \otimes x_3^2x_5^4 \\
&+ b_{117}v_4x_3^{12}x_5^4 \otimes x_3^4 + b_{117}v_4x_3^{12} \otimes x_3^4x_5^4 + b_{117}v_4x_3^{10}x_5^4 \otimes x_3^6 \\
&+ b_{117}v_4x_3^{10} \otimes x_3^6x_5^4 + b_{117}v_4x_3^8x_5^4 \otimes x_3^8 + b_{117}v_4x_3^8 \otimes x_3^8x_5^4 \\
&+ b_{117}v_4x_3^6x_5^4 \otimes x_5^6 + b_{117}v_4x_3^4x_5^4 \otimes x_3^2x_5^6 + b_{117}v_4x_3^2x_5^4 \otimes x_3^4x_5^6 \\
&+ b_{117}v_4x_5^4 \otimes x_3^6x_5^6 + b_{117}v_4x_3^{10}x_5^4 \otimes x_9^2 + b_{117}v_4x_3^{10} \otimes x_5^4x_9^2 \\
&+ b_{117}v_4x_3^8x_5^4 \otimes x_3^2x_9^2 + b_{117}v_4x_3^8 \otimes x_3^2x_5^4x_9^2 + b_{117}v_4x_3^6x_5^4 \otimes x_3^4x_9^2 \\
&+ b_{117}v_4x_3^6 \otimes x_3^4x_5^4x_9^2 + b_{117}v_4x_3^4x_5^4 \otimes x_3^6x_9^2 + b_{117}v_4x_3^4 \otimes x_3^6x_5^4x_9^2 \\
&+ b_{118}v_4x_5^4x_9^2 \otimes x_{15}^2 + b_{118}v_4x_5^4x_{15}^2 \otimes x_9^2 + b_{118}v_4x_5^4 \otimes x_9^2x_{15}^2 \\
&+ b_{118}v_4x_9^2x_{15}^2 \otimes x_5^4 + b_{118}v_4x_9^2 \otimes x_5^4x_{15}^2 + b_{118}v_4x_{15}^2 \otimes x_5^4x_9^2 \\
&+ b_{118}v_4x_3^8x_5^4x_9^2 \otimes x_3^2 + b_{118}v_4x_3^8x_5^4 \otimes x_3^2x_9^2 + b_{118}v_4x_3^8x_9^2 \otimes x_3^2x_5^4 \\
&+ b_{118}v_4x_3^8 \otimes x_3^2x_5^4x_9^2 + b_{118}v_4x_5^4x_9^2 \otimes x_5^6 + b_{118}v_4x_5^4 \otimes x_5^6x_9^2 \\
&+ b_{118}v_4x_3^4x_5^4x_9^2 \otimes x_9^2 + b_{118}v_4x_3^4x_9^2 \otimes x_5^4x_9^2,
\end{aligned}$$

$$\begin{aligned}
Q_3\bar{\psi}(x_{23}) \equiv\ & x_5^4 \otimes x_9^2 + v_4x_5^4 \otimes x_3^{10}x_9^2 + v_4x_5^4 \otimes x_5^6x_9^2 + v_4x_5^4 \otimes x_9^2x_{15}^2 \\
&+ x_9^2 \otimes x_5^4 + v_4x_9^2 \otimes x_3^{10}x_5^4 + v_4x_9^2 \otimes x_5^4x_{15}^2 + v_4x_3^2 \otimes x_3^8x_5^4x_9^2
\end{aligned}$$

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$$
\begin{aligned}
&+ v_4x_3^{10}x_5^4 \otimes x_9^2 + v_4x_3^4x_5^4x_9^2 \otimes x_9^2 + v_4x_5^4x_{15}^2 \otimes x_9^2 + v_4x_3^{10}x_9^2 \otimes x_5^4 \\
&+ v_4x_9^2x_{15}^2 \otimes x_5^4 + v_4x_3^8x_5^4 \otimes x_3^8 + v_4x_3^2x_5^4x_9^2 \otimes x_3^8 + v_4x_5^4x_9^2 \otimes x_{15}^2 \\
&+ v_4x_5^4x_9^2 \otimes x_3^{10} + v_4x_5^4x_9^2 \otimes x_5^6 + v_4x_3^{10} \otimes x_5^4x_9^2 + v_4x_3^4x_9^2 \otimes x_5^4x_9^2 \\
&+ v_4x_{15}^2 \otimes x_5^4x_9^2 + v_4x_3^2x_5^4 \otimes x_3^8x_9^2 + v_4x_3^8 \otimes x_3^8x_5^4 \\
&+ v_4x_3^2x_9^2 \otimes x_3^8x_5^4.
\end{aligned}
$$

Then we obtain $b_{117} = 0$ and $b_{116} = b_{118} = 1$. We compare the coefficient of $Q_0\bar{\psi}(x_{27})$ and $\bar{\psi}(Q_0x_{27})$:

$$
\begin{aligned}
\bar{\psi}(Q_0x_{27}) \equiv\ & x_5^2 \otimes x_9^2 + x_9^2 \otimes x_5^2 + v_4x_5^4x_9^2 \otimes x_5^4 + v_4x_5^4 \otimes x_5^4x_9^2 + v_4x_3^8x_5^2 \otimes x_3^8 \\
&+ v_4x_3^8 \otimes x_3^8x_5^2 + b_{23}v_4x_3^{10}x_5^2 \otimes x_9^2 + b_{23}v_4x_3^{10}x_9^2 \otimes x_5^2 \\
&+ b_{23}v_4x_3^{10} \otimes x_5^2x_9^2 + b_{23}v_4x_5^2x_9^2 \otimes x_3^{10} + b_{23}v_4x_5^2 \otimes x_9^2x_3^{10} \\
&+ b_{23}v_4x_9^2 \otimes x_5^2x_3^{10} + b_{23}v_4x_3^2x_5^2x_9^2 \otimes x_3^8 + b_{23}v_4x_3^2x_5^2 \otimes x_3^8x_9^2 \\
&+ b_{23}v_4x_3^2x_9^2 \otimes x_3^8x_5^2 + b_{23}v_4x_3^2 \otimes x_3^8x_5^2x_9^2 + b_{23}v_4x_3^8x_5^2x_9^2 \otimes x_3^2 \\
&+ b_{23}v_4x_3^8x_5^2 \otimes x_3^2x_9^2 + b_{23}v_4x_3^8x_9^2 \otimes x_3^2x_5^2 + b_{23}v_4x_3^8 \otimes x_3^2x_5^2x_9^2 \\
&+ b_{24}v_4x_3^6x_5^2 \otimes x_{15}^2 + b_{24}v_4x_3^6x_{15}^2 \otimes x_5^2 + b_{24}v_4x_3^6 \otimes x_5^2x_{15}^2 \\
&+ b_{24}v_4x_5^2x_{15}^2 \otimes x_3^6 + b_{24}v_4x_5^2 \otimes x_3^6x_{15}^2 + b_{24}v_4x_{15}^2 \otimes x_3^6x_5^2 \\
&+ b_{24}v_4x_3^2x_5^2x_{15}^2 \otimes x_3^4 + b_{24}v_4x_3^2x_5^2 \otimes x_3^4x_{15}^2 + b_{24}v_4x_3^2x_{15}^2 \otimes x_3^4x_5^2 \\
&+ b_{24}v_4x_3^2 \otimes x_3^4x_5^2x_{15}^2 + b_{24}v_4x_3^4x_5^2x_{15}^2 \otimes x_3^2 + b_{24}v_4x_3^4x_5^2 \otimes x_3^2x_{15}^2 \\
&+ b_{24}v_4x_3^4x_{15}^2 \otimes x_3^2x_5^2 + b_{24}v_4x_3^4 \otimes x_3^2x_5^2x_{15}^2 + b_{24}v_4x_3^{14}x_5^2 \otimes x_3^2 \\
&+ b_{24}v_4x_3^{14} \otimes x_3^2x_5^2 + b_{24}v_4x_3^{12}x_5^2 \otimes x_3^4 + b_{24}v_4x_3^{12} \otimes x_3^4x_5^2 \\
&+ b_{24}v_4x_3^{10}x_5^2 \otimes x_3^6 + b_{24}v_4x_3^{10} \otimes x_3^6x_5^2 + b_{24}v_4x_3^8x_5^2 \otimes x_3^8 \\
&+ b_{24}v_4x_3^8 \otimes x_3^8x_5^2 + b_{24}v_4x_3^6x_5^6 \otimes x_5^2 + b_{24}v_4x_3^6x_5^4 \otimes x_5^4 \\
&+ b_{24}v_4x_3^4x_5^6 \otimes x_3^2x_5^2 + b_{24}v_4x_3^4x_5^4 \otimes x_3^2x_5^4 + b_{24}v_4x_3^2x_5^6 \otimes x_3^4x_5^2 \\
&+ b_{24}v_4x_3^2x_5^4 \otimes x_3^4x_5^4 + b_{24}v_4x_5^6 \otimes x_3^6x_5^2 + b_{24}v_4x_5^4 \otimes x_3^6x_5^4 \\
&+ b_{24}v_4x_3^{10}x_5^2 \otimes x_9^2 + b_{24}v_4x_3^{10} \otimes x_5^2x_9^2 + b_{24}v_4x_3^8x_5^2 \otimes x_3^2x_9^2 \\
&+ b_{24}v_4x_3^8 \otimes x_3^2x_5^2x_9^2 + b_{24}v_4x_3^6x_5^2 \otimes x_3^4x_9^2 + b_{24}v_4x_3^6 \otimes x_3^4x_5^2x_9^2 \\
&+ b_{24}v_4x_3^4x_5^2 \otimes x_3^6x_9^2 + b_{24}v_4x_3^4 \otimes x_3^6x_5^2x_9^2 + b_{25}v_4x_5^2x_9^2 \otimes x_{15}^2 \\
&+ b_{25}v_4x_5^2x_{15}^2 \otimes x_9^2 + b_{25}v_4x_5^2 \otimes x_9^2x_{15}^2 + b_{25}v_4x_9^2x_{15}^2 \otimes x_5^2 \\
&+ b_{25}v_4x_9^2 \otimes x_5^2x_{15}^2 + b_{25}v_4x_{15}^2 \otimes x_5^2x_9^2 + b_{25}v_4x_3^8x_5^2x_9^2 \otimes x_3^2 \\
&+ b_{25}v_4x_3^8x_5^2 \otimes x_3^2x_9^2 + b_{25}v_4x_3^8x_9^2 \otimes x_3^2x_5^2 + b_{25}v_4x_3^8 \otimes x_3^2x_5^2x_9^2 \\
&+ b_{25}v_4x_5^6x_9^2 \otimes x_5^2 + b_{25}v_4x_5^6 \otimes x_5^2x_9^2 + b_{25}v_4x_5^4x_9^2 \otimes x_5^4
\end{aligned}
$$

$$+ b_{25}v_4x_5^4 \otimes x_5^4x_9^2 + b_{25}v_4x_3^4x_5^2x_9^2 \otimes x_9^2 + b_{25}v_4x_3^4x_9^2 \otimes x_5^2x_9^2,$$

$$\begin{aligned} Q_0\bar{\psi}(x_{27}) \equiv\ & v_4x_3^8 \otimes x_3^8x_5^2 + v_4x_3^8 \otimes x_3^2x_5^2x_9^2 + x_9^2 \otimes x_5^2 + v_4x_9^2 \otimes x_5^2x_{15}^2 \\ & + x_5^2 \otimes x_9^2 + v_4x_5^2 \otimes x_9^2x_{15}^2 + a_{108}v_4x_3^{14}x_5^2 \otimes x_3^2 \\ & + a_{109}v_4x_3^8x_5^2x_9^2 \otimes x_3^2 + a_{110}v_4x_3^4x_5^2x_{15}^2 \otimes x_3^2 + a_{111}v_4x_3^6x_5^6 \otimes x_5^2 \\ & + a_{112}v_4x_3^{10}x_9^2 \otimes x_5^2 + a_{113}v_4x_5^6x_9^2 \otimes x_5^2 + a_{114}v_4x_3^6x_{15}^2 \otimes x_5^2 \\ & + a_{115}v_4x_9^2x_{15}^2 \otimes x_5^2 + a_{116}v_4x_3^{14} \otimes x_3^2x_5^2 + a_{117}v_4x_3^4x_5^6 \otimes x_3^2x_5^2 \\ & + a_{118}v_4x_3^8x_9^2 \otimes x_3^2x_5^2 + a_{119}v_4x_3^4x_{15}^2 \otimes x_3^2x_5^2 + a_{120}v_4x_3^{10}x_5^2 \otimes x_9^2 \\ & + a_{121}v_4x_3^4x_5^2x_9^2 \otimes x_9^2 + a_{122}v_4x_5^2x_{15}^2 \otimes x_9^2 + a_{123}v_4x_3^8x_5^2 \otimes x_3^8 \\ & + a_{123}v_4x_3^8x_5^2 \otimes x_3^2x_9^2 + a_{124}v_4x_3^2x_5^2x_9^2 \otimes x_3^8 \\ & + a_{124}v_4x_3^2x_5^2x_9^2 \otimes x_3^2x_9^2 + a_{125}v_4x_3^{10} \otimes x_5^2x_9^2 + a_{126}v_4x_5^6 \otimes x_5^2x_9^2 \\ & + a_{127}v_4x_3^4x_9^2 \otimes x_5^2x_9^2 + a_{128}v_4x_{15}^2 \otimes x_5^2x_9^2 + a_{129}v_4x_3^6x_5^2 \otimes x_{15}^2 \\ & + a_{130}v_4x_5^2x_9^2 \otimes x_{15}^2. \end{aligned}$$

Then we obtain $b_{23} = b_{24} = a_{108} = a_{110} = a_{111} = a_{112} = a_{114} = a_{116} = a_{117} = a_{119} = a_{120} = a_{124} = a_{125} = a_{129} = 0$ and $b_{25} = a_{109} = a_{113} = a_{115} = a_{118} = a_{121} = a_{122} = a_{123} = a_{126} = a_{127} = a_{128} = a_{130} = 1$. We compare the coefficient of $Q_1\bar{\psi}(x_{27})$ and $\bar{\psi}(Q_1x_{27})$:

$$\begin{aligned} \bar{\psi}(Q_1x_{27}) \equiv\ & x_3^8 \otimes x_3^2 + x_5^4 \otimes x_5^2 + x_3^4 \otimes x_9^2 + v_4x_{15}^2 \otimes x_{15}^2 + v_4x_{15}^2 \otimes x_3^{10} \\ & + v_4x_{15}^2 \otimes x_5^6 + v_4x_3^{10} \otimes x_{15}^2 + v_4x_3^{10} \otimes x_3^{10} + v_4x_3^{10} \otimes x_5^6 \\ & + v_4x_5^6 \otimes x_{15}^2 + v_4x_5^6 \otimes x_3^{10} + v_4x_5^6 \otimes x_5^6 + v_4x_3^4x_{15}^2 \otimes x_9^2 \\ & + v_4x_3^{14} \otimes x_9^2 + v_4x_3^4x_5^6 \otimes x_9^2 + v_4x_5^2x_{15}^2 \otimes x_5^4 + v_4x_3^{10}x_5^2 \otimes x_5^4 \\ & + v_4x_3^2x_{15}^2 \otimes x_3^8 + v_4x_3^{12} \otimes x_3^8 + v_4x_3^2x_5^6 \otimes x_3^8 + v_4x_3^4x_5^2 \otimes x_5^4x_9^2 \\ & + v_4x_3^6 \otimes x_3^8x_9^2 + v_4x_3^2x_5^2 \otimes x_3^8x_5^4 + v_4x_3^4 \otimes x_9^2x_{15}^2 + v_4x_3^4 \otimes x_3^{10}x_9^2 \\ & + v_4x_3^4 \otimes x_5^6x_9^2 + v_4x_5^2 \otimes x_5^4x_{15}^2 + v_4x_5^2 \otimes x_3^{10}x_5^4 + v_4x_3^2 \otimes x_3^8x_{15}^2 \\ & + v_4x_3^2 \otimes x_3^8x_5^6 + x_3^4 \otimes x_9^2 + x_9^2 \otimes x_3^4 + v_4x_3^{12} \otimes x_3^8 + v_4x_3^8 \otimes x_3^{12} \\ & + x_5^2 \otimes x_5^4 + x_5^4 \otimes x_5^2 + b_{53}v_4x_3^{10} \otimes x_5^6 + b_{53}v_4x_5^6 \otimes x_3^{10} \\ & + b_{53}v_4x_3^2x_5^6 \otimes x_3^8 + b_{53}v_4x_3^2 \otimes x_3^8x_5^6 + b_{53}v_4x_3^8x_5^6 \otimes x_3^2 \\ & + b_{53}v_4x_3^8 \otimes x_3^2x_5^6 + b_{53}v_4x_3^{10}x_5^2 \otimes x_5^4 + b_{53}v_4x_5^2 \otimes x_3^{10}x_5^4 \\ & + b_{53}v_4x_3^{10}x_5^4 \otimes x_5^2 + b_{53}v_4x_5^4 \otimes x_3^{10}x_5^2 + b_{53}v_4x_3^2x_5^2 \otimes x_3^8x_5^4 \\ & + b_{53}v_4x_3^2x_5^4 \otimes x_3^8x_5^2 + b_{53}v_4x_3^8x_5^2 \otimes x_3^2x_5^4 + b_{53}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 \\ & + b_{54}v_4x_3^{14} \otimes x_9^2 + b_{54}v_4x_9^2 \otimes x_3^{14} + b_{54}v_4x_3^2x_9^2 \otimes x_3^{12} \end{aligned}$$

$$
\begin{aligned}
&+ b_{54}v_4x_3^2 \otimes x_3^{12}x_9^2 + b_{54}v_4x_3^4x_9^2 \otimes x_3^{10} + b_{54}v_4x_3^4 \otimes x_3^{10}x_9^2\\
&+ b_{54}v_4x_3^6x_9^2 \otimes x_3^8 + b_{54}v_4x_3^6 \otimes x_3^8x_9^2 + b_{54}v_4x_3^8x_9^2 \otimes x_3^6\\
&+ b_{54}v_4x_3^8 \otimes x_3^6x_9^2 + b_{54}v_4x_3^{10}x_9^2 \otimes x_3^4 + b_{54}v_4x_3^{10} \otimes x_3^4x_9^2\\
&+ b_{54}v_4x_3^{12}x_9^2 \otimes x_3^2 + b_{54}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{55}v_4x_3^4x_5^6 \otimes x_9^2\\
&+ b_{55}v_4x_3^4x_9^2 \otimes x_5^6 + b_{55}v_4x_3^4 \otimes x_5^6x_9^2 + b_{55}v_4x_5^6x_9^2 \otimes x_3^4\\
&+ b_{55}v_4x_5^6 \otimes x_3^4x_9^2 + b_{55}v_4x_9^2 \otimes x_3^4x_5^6 + b_{55}v_4x_3^4x_5^2x_9^2 \otimes x_5^4\\
&+ b_{55}v_4x_3^4x_5^2 \otimes x_5^4x_9^2 + b_{55}v_4x_5^2x_9^2 \otimes x_3^4x_5^4 + b_{55}v_4x_5^2 \otimes x_3^4x_5^4x_9^2\\
&+ b_{55}v_4x_3^4x_5^4x_9^2 \otimes x_5^2 + b_{55}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 + b_{55}v_4x_5^4x_9^2 \otimes x_3^4x_5^2\\
&+ b_{55}v_4x_5^4 \otimes x_3^4x_5^2x_9^2 + b_{56}v_4x_3^{10} \otimes x_{15}^2 + b_{56}v_4x_{15}^2 \otimes x_3^{10}\\
&+ b_{56}v_4x_3^2x_{15}^2 \otimes x_3^8 + b_{56}v_4x_3^2 \otimes x_3^8x_{15}^2 + b_{56}v_4x_3^8x_{15}^2 \otimes x_3^2\\
&+ b_{56}v_4x_3^8 \otimes x_3^2x_{15}^2 + b_{56}v_4x_3^{10} \otimes x_3^{10} + b_{56}v_4x_3^8 \otimes x_3^{12}\\
&+ b_{56}v_4x_3^{10}x_5^4 \otimes x_5^2 + b_{56}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 + b_{56}v_4x_3^2x_5^4 \otimes x_3^8x_5^2\\
&+ b_{56}v_4x_5^4 \otimes x_3^{10}x_5^2 + b_{56}v_4x_3^{14} \otimes x_9^2 + b_{56}v_4x_3^{12} \otimes x_3^2x_9^2\\
&+ b_{56}v_4x_3^6 \otimes x_3^8x_9^2 + b_{56}v_4x_3^4 \otimes x_3^{10}x_9^2 + b_{57}v_4x_5^6 \otimes x_{15}^2\\
&+ b_{57}v_4x_{15}^2 \otimes x_5^6 + b_{57}v_4x_5^2x_{15}^2 \otimes x_5^4 + b_{57}v_4x_5^2 \otimes x_5^4x_{15}^2\\
&+ b_{57}v_4x_5^4x_{15}^2 \otimes x_5^2 + b_{57}v_4x_5^4 \otimes x_5^2x_{15}^2 + b_{57}v_4x_3^8x_5^6 \otimes x_3^2\\
&+ b_{57}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 + b_{57}v_4x_3^8x_5^2 \otimes x_3^2x_5^4 + b_{57}v_4x_3^8 \otimes x_3^2x_5^6\\
&+ b_{57}v_4x_5^6 \otimes x_5^6 + b_{57}v_4x_3^4x_5^6 \otimes x_9^2 + b_{57}v_4x_3^4x_5^4 \otimes x_5^2x_9^2\\
&+ b_{57}v_4x_3^4x_5^2 \otimes x_5^4x_9^2 + b_{57}v_4x_3^4 \otimes x_5^6x_9^2 + b_{58}v_4x_3^4x_9^2 \otimes x_{15}^2\\
&+ b_{58}v_4x_3^4x_{15}^2 \otimes x_9^2 + b_{58}v_4x_3^4 \otimes x_9^2x_{15}^2 + b_{58}v_4x_9^2x_{15}^2 \otimes x_3^4\\
&+ b_{58}v_4x_9^2 \otimes x_3^4x_{15}^2 + b_{58}v_4x_{15}^2 \otimes x_3^4x_9^2 + b_{58}v_4x_3^{12}x_9^2 \otimes x_3^2\\
&+ b_{58}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{58}v_4x_3^8x_9^2 \otimes x_3^6 + b_{58}v_4x_3^8 \otimes x_3^6x_9^2\\
&+ b_{58}v_4x_3^4x_5^4x_9^2 \otimes x_5^2 + b_{58}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 + b_{58}v_4x_5^4x_9^2 \otimes x_3^4x_5^2\\
&+ b_{58}v_4x_5^4 \otimes x_3^4x_5^2x_9^2 + b_{58}v_4x_3^8x_9^2 \otimes x_9^2 + b_{58}v_4x_3^4x_9^2 \otimes x_3^4x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_1\bar{\psi}(x_{27}) \equiv\ & x_3^8 \otimes x_3^2 + v_4x_3^8 \otimes x_3^2x_5^6 + v_4x_3^8 \otimes x_3^6x_9^2 + v_4x_3^8 \otimes x_3^2x_{15}^2 + x_9^2 \otimes x_3^4\\
&+ v_4x_9^2 \otimes x_3^4x_5^6 + v_4x_9^2 \otimes x_3^4x_{15}^2 + x_5^2 \otimes x_5^4 + v_4x_5^2 \otimes x_3^4x_5^4x_9^2\\
&+ v_4x_5^2 \otimes x_5^4x_{15}^2 + a_{103}v_4x_3^8x_5^6 \otimes x_3^2 + a_{104}v_4x_3^{12}x_9^2 \otimes x_3^2\\
&+ a_{105}v_4x_3^2x_5^6x_9^2 \otimes x_3^2 + a_{106}v_4x_3^8x_{15}^2 \otimes x_3^2 + a_{107}v_4x_3^2x_9^2x_{15}^2 \otimes x_3^2\\
&+ v_4x_5^6x_9^2 \otimes x_3^4 + v_4x_9^2x_{15}^2 \otimes x_3^4 + v_4x_3^8x_9^2 \otimes x_9^2 + v_4x_3^8x_9^2 \otimes x_3^6
\end{aligned}
$$

$$+ v_4x_3^4x_5^2x_9^2 \otimes x_5^4 + v_4x_5^2x_{15}^2 \otimes x_5^4 + v_4x_3^8x_5^2 \otimes x_3^2x_5^4 + v_4x_5^6 \otimes x_{15}^2$$
$$+ v_4x_5^6 \otimes x_3^4x_9^2 + v_4x_5^6 \otimes x_5^6 + v_4x_3^4x_9^2 \otimes x_{15}^2 + v_4x_3^4x_9^2 \otimes x_3^4x_9^2$$
$$+ v_4x_3^4x_9^2 \otimes x_5^6 + v_4x_{15}^2 \otimes x_{15}^2 + v_4x_{15}^2 \otimes x_3^4x_9^2 + v_4x_{15}^2 \otimes x_5^6$$
$$+ v_4x_5^2x_9^2 \otimes x_3^4x_5^4.$$

Then we obtain $b_{54} = b_{57} = a_{105} = a_{107} = 0$ and $b_{53} = b_{55} = b_{56} = b_{58} = a_{103} = a_{104} = a_{106} = 1$. We compare the coefficient of $Q_2\bar{\psi}(x_{27})$ and $\bar{\psi}(Q_2x_{27})$:

$$\bar{\psi}(Q_2x_{27}) \equiv x_3^8 \otimes x_5^2 + x_5^2 \otimes x_3^8 + v_4x_3^8x_5^4 \otimes x_5^4 + v_4x_5^4 \otimes x_3^8x_5^4$$
$$+ b_{86}v_4x_3^{12}x_5^2 \otimes x_9^2 + b_{86}v_4x_3^{12}x_9^2 \otimes x_5^2 + b_{86}v_4x_3^{12} \otimes x_5^2x_9^2$$
$$+ b_{86}v_4x_5^2x_9^2 \otimes x_3^{12} + b_{86}v_4x_5^2 \otimes x_3^{12}x_9^2 + b_{86}v_4x_9^2 \otimes x_3^{12}x_5^2$$
$$+ b_{86}v_4x_3^4x_5^2x_9^2 \otimes x_3^8 + b_{86}v_4x_3^4x_5^2 \otimes x_3^8x_9^2 + b_{86}v_4x_3^4x_9^2 \otimes x_3^8x_5^2$$
$$+ b_{86}v_4x_3^4 \otimes x_3^8x_5^2x_9^2 + b_{86}v_4x_3^8x_5^2x_9^2 \otimes x_3^4 + b_{86}v_4x_3^8x_5^2 \otimes x_3^4x_9^2$$
$$+ b_{86}v_4x_3^8x_9^2 \otimes x_3^4x_5^2 + b_{86}v_4x_3^8 \otimes x_3^4x_5^2x_9^2 + b_{87}v_4x_3^8x_5^2 \otimes x_{15}^2$$
$$+ b_{87}v_4x_3^8x_{15}^2 \otimes x_5^2 + b_{87}v_4x_3^8 \otimes x_5^2x_{15}^2 + b_{87}v_4x_5^2x_{15}^2 \otimes x_3^8$$
$$+ b_{87}v_4x_5^2 \otimes x_3^8x_{15}^2 + b_{87}v_4x_{15}^2 \otimes x_3^8x_5^2 + b_{87}v_4x_3^8x_5^2 \otimes x_3^{10}$$
$$+ b_{87}v_4x_3^8 \otimes x_3^{10}x_5^2 + b_{87}v_4x_3^8x_5^6 \otimes x_5^2 + b_{87}v_4x_3^8x_5^4 \otimes x_5^4$$
$$+ b_{87}v_4x_5^6 \otimes x_3^8x_5^2 + b_{87}v_4x_5^4 \otimes x_3^8x_5^4 + b_{87}v_4x_3^{12}x_5^2 \otimes x_9^2$$
$$+ b_{87}v_4x_3^{12} \otimes x_5^2x_9^2 + b_{87}v_4x_3^4x_5^2 \otimes x_3^8x_9^2 + b_{87}v_4x_3^4 \otimes x_3^8x_5^2x_9^2$$
$$+ b_{88}v_4x_3^2x_5^2x_9^2 \otimes x_{15}^2 + b_{88}v_4x_3^2x_5^2x_{15}^2 \otimes x_9^2 + b_{88}v_4x_3^2x_5^2 \otimes x_9^2x_{15}^2$$
$$+ b_{88}v_4x_3^2x_9^2x_{15}^2 \otimes x_5^2 + b_{88}v_4x_3^2x_9^2 \otimes x_5^2x_{15}^2 + b_{88}v_4x_3^2x_{15}^2 \otimes x_5^2x_9^2$$
$$+ b_{88}v_4x_3^2 \otimes x_5^2x_9^2x_{15}^2 + b_{88}v_4x_5^2x_9^2x_{15}^2 \otimes x_3^2 + b_{88}v_4x_5^2x_9^2 \otimes x_3^2x_{15}^2$$
$$+ b_{88}v_4x_5^2x_{15}^2 \otimes x_3^2x_9^2 + b_{88}v_4x_5^2 \otimes x_3^2x_9^2x_{15}^2 + b_{88}v_4x_9^2x_{15}^2 \otimes x_3^2x_5^2$$
$$+ b_{88}v_4x_9^2 \otimes x_3^2x_5^2x_{15}^2 + b_{88}v_4x_{15}^2 \otimes x_3^2x_5^2x_9^2 + b_{88}v_4x_3^{10}x_5^2x_9^2 \otimes x_3^2$$
$$+ b_{88}v_4x_3^{10}x_5^2 \otimes x_3^2x_9^2 + b_{88}v_4x_3^{10}x_9^2 \otimes x_3^2x_5^2 + b_{88}v_4x_3^{10} \otimes x_3^2x_5^2x_9^2$$
$$+ b_{88}v_4x_3^8x_5^2x_9^2 \otimes x_3^4 + b_{88}v_4x_3^8x_5^2 \otimes x_3^4x_9^2 + b_{88}v_4x_3^8x_9^2 \otimes x_3^4x_5^2$$
$$+ b_{88}v_4x_3^8 \otimes x_3^4x_5^2x_9^2 + b_{88}v_4x_3^2x_5^6x_9^2 \otimes x_5^2 + b_{88}v_4x_3^2x_5^6 \otimes x_5^2x_9^2$$
$$+ b_{88}v_4x_3^2x_5^4x_9^2 \otimes x_5^4 + b_{88}v_4x_3^2x_5^4 \otimes x_5^4x_9^2 + b_{88}v_4x_5^6x_9^2 \otimes x_3^2x_5^2$$
$$+ b_{88}v_4x_5^6 \otimes x_3^2x_5^2x_9^2 + b_{88}v_4x_5^4x_9^2 \otimes x_3^2x_5^4 + b_{88}v_4x_5^4 \otimes x_3^2x_5^4x_9^2$$
$$+ b_{88}v_4x_3^6x_5^2x_9^2 \otimes x_9^2 + b_{88}v_4x_3^6x_9^2 \otimes x_5^2x_9^2 + b_{88}v_4x_3^4x_5^2x_9^2 \otimes x_3^2x_9^2$$
$$+ b_{88}v_4x_3^4x_9^2 \otimes x_3^2x_5^2x_9^2,$$

$$\begin{aligned}
Q_2\bar{\psi}(x_{27}) \equiv\ & x_3^8 \otimes x_5^2 + v_4x_3^8 \otimes x_3^{10}x_5^2 + v_4x_3^8 \otimes x_3^4x_5^2x_9^2 + v_4x_3^8 \otimes x_5^2x_{15}^2 \\
& + v_4x_9^2 \otimes x_3^{12}x_5^2 + x_5^2 \otimes x_3^8 + v_4x_5^2 \otimes x_3^{12}x_9^2 + v_4x_5^2 \otimes x_3^8x_{15}^2 \\
& + v_4x_3^8x_5^6 \otimes x_5^2 + v_4x_3^{12}x_9^2 \otimes x_5^2 + v_4x_3^8x_{15}^2 \otimes x_5^2 + v_4x_3^8x_5^2x_9^2 \otimes x_3^4 \\
& + v_4x_3^8x_9^2 \otimes x_3^4x_5^2 + v_4x_3^4x_5^2x_9^2 \otimes x_3^8 + v_4x_5^2x_{15}^2 \otimes x_3^8 \\
& + v_4x_3^8x_5^2 \otimes x_{15}^2 + v_4x_3^8x_5^2 \otimes x_3^{10} + v_4x_3^8x_5^2 \otimes x_3^4x_9^2 + v_4x_5^6 \otimes x_3^8x_5^2 \\
& + v_4x_3^4x_9^2 \otimes x_3^8x_5^2 + v_4x_{15}^2 \otimes x_3^8x_5^2 + v_4x_5^2x_9^2 \otimes x_3^{12}.
\end{aligned}$$

Then we obtain $b_{88} = 0$ and $b_{86} = b_{87} = 1$. We compare the coefficient of $Q_3\bar{\psi}(x_{27})$ and $\bar{\psi}(Q_3x_{27})$:

$$\begin{aligned}
\bar{\psi}(Q_3x_{27}) \equiv\ & x_3^8 \otimes x_9^2 + x_9^2 \otimes x_3^8 + b_{119}v_4x_3^{14} \otimes x_5^6 + b_{119}v_4x_5^6 \otimes x_3^{14} \\
& + b_{119}v_4x_3^2x_5^6 \otimes x_3^{12} + b_{119}v_4x_3^2 \otimes x_3^{12}x_5^6 + b_{119}v_4x_3^4x_5^6 \otimes x_3^{10} \\
& + b_{119}v_4x_3^4 \otimes x_3^{10}x_5^6 + b_{119}v_4x_3^6x_5^6 \otimes x_3^8 + b_{119}v_4x_3^6 \otimes x_3^8x_5^6 \\
& + b_{119}v_4x_3^8x_5^6 \otimes x_3^6 + b_{119}v_4x_3^8 \otimes x_3^6x_5^6 + b_{119}v_4x_3^{10}x_5^6 \otimes x_3^4 \\
& + b_{119}v_4x_3^{10} \otimes x_3^4x_5^6 + b_{119}v_4x_3^{12}x_5^6 \otimes x_3^2 + b_{119}v_4x_3^{12} \otimes x_3^2x_5^6 \\
& + b_{119}v_4x_3^{14}x_5^2 \otimes x_5^4 + b_{119}v_4x_5^2 \otimes x_3^{14}x_5^4 + b_{119}v_4x_3^{14}x_5^4 \otimes x_5^2 \\
& + b_{119}v_4x_5^4 \otimes x_3^{14}x_5^2 + b_{119}v_4x_3^2x_5^2 \otimes x_3^{12}x_5^4 + b_{119}v_4x_3^2x_5^4 \otimes x_3^{12}x_5^2 \\
& + b_{119}v_4x_3^4x_5^2 \otimes x_3^{10}x_5^4 + b_{119}v_4x_3^4x_5^4 \otimes x_3^{10}x_5^2 + b_{119}v_4x_3^6x_5^2 \otimes x_3^8x_5^4 \\
& + b_{119}v_4x_3^6x_5^4 \otimes x_3^8x_5^2 + b_{119}v_4x_3^8x_5^2 \otimes x_3^6x_5^4 + b_{119}v_4x_3^8x_5^4 \otimes x_3^6x_5^2 \\
& + b_{119}v_4x_3^{10}x_5^2 \otimes x_3^4x_5^4 + b_{119}v_4x_3^{10}x_5^4 \otimes x_3^4x_5^2 \\
& + b_{119}v_4x_3^{12}x_5^2 \otimes x_3^2x_5^4 + b_{119}v_4x_3^{12}x_5^4 \otimes x_3^2x_5^2 + b_{120}v_4x_3^8x_5^6 \otimes x_9^2 \\
& + b_{120}v_4x_3^8x_9^2 \otimes x_5^6 + b_{120}v_4x_3^8 \otimes x_5^6x_9^2 + b_{120}v_4x_5^6x_9^2 \otimes x_3^8 \\
& + b_{120}v_4x_5^6 \otimes x_3^8x_9^2 + b_{120}v_4x_9^2 \otimes x_3^8x_5^6 + b_{120}v_4x_3^8x_5^2x_9^2 \otimes x_5^4 \\
& + b_{120}v_4x_3^8x_5^2 \otimes x_5^4x_9^2 + b_{120}v_4x_5^2x_9^2 \otimes x_3^8x_5^4 + b_{120}v_4x_5^2 \otimes x_3^8x_5^4x_9^2 \\
& + b_{120}v_4x_3^8x_5^4x_9^2 \otimes x_5^2 + b_{120}v_4x_3^8x_5^4 \otimes x_5^2x_9^2 + b_{120}v_4x_5^4x_9^2 \otimes x_3^8x_5^2 \\
& + b_{120}v_4x_5^4 \otimes x_3^8x_5^2x_9^2 + b_{121}v_4x_3^{14} \otimes x_{15}^2 + b_{121}v_4x_{15}^2 \otimes x_3^{14} \\
& + b_{121}v_4x_3^2x_{15}^2 \otimes x_3^{12} + b_{121}v_4x_3^2 \otimes x_3^{12}x_{15}^2 + b_{121}v_4x_3^4x_{15}^2 \otimes x_3^{10} \\
& + b_{121}v_4x_3^4 \otimes x_3^{10}x_{15}^2 + b_{121}v_4x_3^6x_{15}^2 \otimes x_3^8 + b_{121}v_4x_3^6 \otimes x_3^8x_{15}^2 \\
& + b_{121}v_4x_3^8x_{15}^2 \otimes x_3^6 + b_{121}v_4x_3^8 \otimes x_3^6x_{15}^2 + b_{121}v_4x_3^{10}x_{15}^2 \otimes x_3^4 \\
& + b_{121}v_4x_3^{10} \otimes x_3^4x_{15}^2 + b_{121}v_4x_3^{12}x_{15}^2 \otimes x_3^2 + b_{121}v_4x_3^{12} \otimes x_3^2x_{15}^2 \\
& + b_{121}v_4x_3^{14} \otimes x_3^{10} + b_{121}v_4x_3^{12} \otimes x_3^{12} + b_{121}v_4x_3^{10} \otimes x_3^{14} \\
& + b_{121}v_4x_3^{14}x_5^4 \otimes x_5^2 + b_{121}v_4x_5^4 \otimes x_3^{14}x_5^2 + b_{121}v_4x_3^2x_5^4 \otimes x_3^{12}x_5^2
\end{aligned}$$

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\begin{aligned}
&+ b_{121}v_4x_3^4x_5^4 \otimes x_3^{10}x_5^2 + b_{121}v_4x_3^6x_5^4 \otimes x_3^8x_5^2 + b_{121}v_4x_3^8x_5^4 \otimes x_3^6x_5^2 \\
&+ b_{121}v_4x_3^{10}x_5^4 \otimes x_3^4x_5^2 + b_{121}v_4x_3^{12}x_5^4 \otimes x_3^2x_5^2 + b_{121}v_4x_3^{14} \otimes x_3^4x_9^2 \\
&+ b_{121}v_4x_3^{12} \otimes x_3^6x_9^2 + b_{121}v_4x_3^{10} \otimes x_3^8x_9^2 + b_{121}v_4x_3^8 \otimes x_3^{10}x_9^2 \\
&+ b_{121}v_4x_3^6 \otimes x_3^{12}x_9^2 + b_{121}v_4x_3^4 \otimes x_3^{14}x_9^2 + b_{122}v_4x_3^4x_5^6 \otimes x_{15}^2 \\
&+ b_{122}v_4x_3^4x_{15}^2 \otimes x_5^6 + b_{122}v_4x_3^4 \otimes x_5^6x_{15}^2 + b_{122}v_4x_5^6x_{15}^2 \otimes x_3^4 \\
&+ b_{122}v_4x_5^6 \otimes x_3^4x_{15}^2 + b_{122}v_4x_{15}^2 \otimes x_3^4x_5^6 + b_{122}v_4x_3^4x_5^2x_{15}^2 \otimes x_5^4 \\
&+ b_{122}v_4x_3^4x_5^2 \otimes x_5^4x_{15}^2 + b_{122}v_4x_5^2x_{15}^2 \otimes x_3^4x_5^4 \\
&+ b_{122}v_4x_5^2 \otimes x_3^4x_5^4x_{15}^2 + b_{122}v_4x_3^4x_5^4x_{15}^2 \otimes x_5^2 \\
&+ b_{122}v_4x_3^4x_5^4 \otimes x_5^2x_{15}^2 + b_{122}v_4x_5^4x_{15}^2 \otimes x_3^4x_5^2 \\
&+ b_{122}v_4x_5^4 \otimes x_3^4x_5^2x_{15}^2 + b_{122}v_4x_3^{12}x_5^6 \otimes x_3^2 + b_{122}v_4x_3^{12} \otimes x_3^2x_5^6 \\
&+ b_{122}v_4x_3^8x_5^6 \otimes x_3^6 + b_{122}v_4x_3^8 \otimes x_3^6x_5^6 + b_{122}v_4x_3^{12}x_5^2 \otimes x_3^2x_5^4 \\
&+ b_{122}v_4x_3^{12}x_5^4 \otimes x_3^2x_5^2 + b_{122}v_4x_3^8x_5^2 \otimes x_3^6x_5^4 + b_{122}v_4x_3^8x_5^4 \otimes x_3^6x_5^2 \\
&+ b_{122}v_4x_3^4x_5^6 \otimes x_5^6 + b_{122}v_4x_5^6 \otimes x_3^4x_5^6 + b_{122}v_4x_3^8x_5^6 \otimes x_9^2 \\
&+ b_{122}v_4x_3^8 \otimes x_5^6x_9^2 + b_{122}v_4x_3^4x_5^6 \otimes x_3^4x_9^2 + b_{122}v_4x_3^4 \otimes x_3^4x_5^6x_9^2 \\
&+ b_{122}v_4x_3^8x_5^2 \otimes x_5^4x_9^2 + b_{122}v_4x_3^8x_5^4 \otimes x_9^2x_5^2 \\
&+ b_{122}v_4x_3^4x_5^2 \otimes x_3^4x_5^4x_9^2 + b_{122}v_4x_3^4x_5^4 \otimes x_3^4x_5^2x_9^2 \\
&+ b_{123}v_4x_3^8x_9^2 \otimes x_{15}^2 + b_{123}v_4x_3^8x_{15}^2 \otimes x_9^2 + b_{123}v_4x_3^8 \otimes x_9^2x_{15}^2 \\
&+ b_{123}v_4x_9^2x_{15}^2 \otimes x_3^8 + b_{123}v_4x_9^2 \otimes x_3^8x_{15}^2 + b_{123}v_4x_{15}^2 \otimes x_3^8x_9^2 \\
&+ b_{123}v_4x_3^8x_9^2 \otimes x_3^{10} + b_{123}v_4x_3^8 \otimes x_3^{10}x_9^2 + b_{123}v_4x_3^8x_5^4x_9^2 \otimes x_5^2 \\
&+ b_{123}v_4x_3^8x_5^4 \otimes x_5^2x_9^2 + b_{123}v_4x_5^4x_9^2 \otimes x_3^8x_5^2 + b_{123}v_4x_5^4 \otimes x_3^8x_5^2x_9^2 \\
&+ b_{123}v_4x_3^{12}x_9^2 \otimes x_9^2 + b_{123}v_4x_3^4x_9^2 \otimes x_3^8x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_3\bar\psi(x_{27}) \equiv\ & x_3^8 \otimes x_9^2 + v_4x_3^8 \otimes x_3^{10}x_9^2 + v_4x_3^8 \otimes x_5^6x_9^2 + v_4x_3^8 \otimes x_9^2x_{15}^2 \\
&+ x_9^2 \otimes x_3^8 + v_4x_9^2 \otimes x_3^8x_5^6 + v_4x_9^2 \otimes x_3^8x_{15}^2 + v_4x_5^2 \otimes x_3^8x_5^4x_9^2 \\
&+ v_4x_3^8x_5^6 \otimes x_9^2 + v_4x_3^{12}x_9^2 \otimes x_9^2 + v_4x_3^8x_{15}^2 \otimes x_9^2 + v_4x_3^8x_5^2x_9^2 \otimes x_5^4 \\
&+ v_4x_5^6x_9^2 \otimes x_3^8 + v_4x_9^2x_{15}^2 \otimes x_3^8 + v_4x_3^8x_9^2 \otimes x_{15}^2 + v_4x_3^8x_9^2 \otimes x_3^{10} \\
&+ v_4x_3^8x_9^2 \otimes x_5^6 + v_4x_3^8x_5^2 \otimes x_5^4x_9^2 + v_4x_5^6 \otimes x_3^8x_9^2 + v_4x_3^4x_9^2 \otimes x_3^8x_9^2 \\
&+ v_4x_{15}^2 \otimes x_3^8x_9^2 + v_4x_5^2x_9^2 \otimes x_3^8x_5^4.
\end{aligned}
$$

Then we obtain $b_{119} = b_{121} = b_{122} = 0$ and $b_{120} = b_{123} = 1$. We compare the coefficient of $Q_0\bar\psi(x_{29})$ and $\bar\psi(Q_0x_{29})$:

$$\bar\psi(Q_0x_{29}) \equiv x_3^8 \otimes x_3^2 + x_5^4 \otimes x_5^2 + x_3^4 \otimes x_9^2 + v_4x_{15}^2 \otimes x_{15}^2 + v_4x_{15}^2 \otimes x_3^{10}$$

$$
\begin{aligned}
&+ v_4x_{15}^2 \otimes x_5^6 + v_4x_3^{10} \otimes x_{15}^2 + v_4x_3^{10} \otimes x_3^{10} + v_4x_3^{10} \otimes x_5^6 \\
&+ v_4x_5^6 \otimes x_{15}^2 + v_4x_5^6 \otimes x_3^{10} + v_4x_5^6 \otimes x_5^6 + v_4x_3^4x_{15}^2 \otimes x_9^2 \\
&+ v_4x_3^{14} \otimes x_9^2 + v_4x_3^4x_5^6 \otimes x_9^2 + v_4x_5^2x_{15}^2 \otimes x_5^4 + v_4x_3^{10}x_5^2 \otimes x_5^4 \\
&+ v_4x_3^2x_{15}^2 \otimes x_3^8 + v_4x_3^{12} \otimes x_3^8 + v_4x_3^2x_5^6 \otimes x_3^8 + v_4x_3^4x_5^2 \otimes x_5^4x_9^2 \\
&+ v_4x_3^6 \otimes x_3^8x_9^2 + v_4x_3^2x_5^2 \otimes x_3^8x_5^4 + v_4x_3^4 \otimes x_9^2x_{15}^2 + v_4x_3^4 \otimes x_3^{10}x_9^2 \\
&+ v_4x_3^4 \otimes x_5^6x_9^2 + v_4x_5^2 \otimes x_5^4x_{15}^2 + v_4x_5^2 \otimes x_3^{10}x_5^4 + v_4x_3^2 \otimes x_3^8x_{15}^2 \\
&+ v_4x_3^2 \otimes x_3^8x_5^6 + b_{26}v_4x_3^{10} \otimes x_5^6 + b_{26}v_4x_5^6 \otimes x_3^{10} \\
&+ b_{26}v_4x_3^2x_5^6 \otimes x_3^8 + b_{26}v_4x_3^2 \otimes x_3^8x_5^6 + b_{26}v_4x_3^8x_5^6 \otimes x_3^2 \\
&+ b_{26}v_4x_3^8 \otimes x_3^2x_5^6 + b_{26}v_4x_3^{10}x_5^2 \otimes x_5^4 + b_{26}v_4x_5^2 \otimes x_3^{10}x_5^4 \\
&+ b_{26}v_4x_3^{10}x_5^4 \otimes x_5^2 + b_{26}v_4x_5^4 \otimes x_3^{10}x_5^2 + b_{26}v_4x_3^2x_5^2 \otimes x_3^8x_5^4 \\
&+ b_{26}v_4x_3^2x_5^4 \otimes x_3^8x_5^2 + b_{26}v_4x_3^8x_5^2 \otimes x_3^2x_5^4 + b_{26}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 \\
&+ b_{27}v_4x_3^{14} \otimes x_9^2 + b_{27}v_4x_9^2 \otimes x_3^{14} + b_{27}v_4x_3^2x_9^2 \otimes x_3^{12} \\
&+ b_{27}v_4x_3^2 \otimes x_3^{12}x_9^2 + b_{27}v_4x_3^4x_9^2 \otimes x_3^{10} + b_{27}v_4x_3^4 \otimes x_3^{10}x_9^2 \\
&+ b_{27}v_4x_3^6x_9^2 \otimes x_3^8 + b_{27}v_4x_3^6 \otimes x_3^8x_9^2 + b_{27}v_4x_3^8x_9^2 \otimes x_3^6 \\
&+ b_{27}v_4x_3^8 \otimes x_3^6x_9^2 + b_{27}v_4x_3^{10}x_9^2 \otimes x_3^4 + b_{27}v_4x_3^{10} \otimes x_3^4x_9^2 \\
&+ b_{27}v_4x_3^{12}x_9^2 \otimes x_3^2 + b_{27}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{28}v_4x_3^4x_5^6 \otimes x_9^2 \\
&+ b_{28}v_4x_3^4x_9^2 \otimes x_5^6 + b_{28}v_4x_3^4 \otimes x_5^6x_9^2 + b_{28}v_4x_5^6x_9^2 \otimes x_3^4 \\
&+ b_{28}v_4x_5^6 \otimes x_3^4x_9^2 + b_{28}v_4x_9^2 \otimes x_3^4x_5^6 + b_{28}v_4x_3^4x_5^2x_9^2 \otimes x_5^4 \\
&+ b_{28}v_4x_3^4x_5^2 \otimes x_5^4x_9^2 + b_{28}v_4x_5^2x_9^2 \otimes x_3^4x_5^4 + b_{28}v_4x_5^2 \otimes x_3^4x_5^4x_9^2 \\
&+ b_{28}v_4x_3^4x_5^4x_9^2 \otimes x_5^2 + b_{28}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 + b_{28}v_4x_5^4x_9^2 \otimes x_3^4x_5^2 \\
&+ b_{28}v_4x_5^4 \otimes x_3^4x_5^2x_9^2 + b_{29}v_4x_3^{10} \otimes x_{15}^2 + b_{29}v_4x_{15}^2 \otimes x_3^{10} \\
&+ b_{29}v_4x_3^2x_{15}^2 \otimes x_3^8 + b_{29}v_4x_3^2 \otimes x_3^8x_{15}^2 + b_{29}v_4x_3^8x_{15}^2 \otimes x_3^2 \\
&+ b_{29}v_4x_3^8 \otimes x_3^2x_{15}^2 + b_{29}v_4x_3^{10} \otimes x_3^{10} + b_{29}v_4x_3^8 \otimes x_3^{12} \\
&+ b_{29}v_4x_5^4 \otimes x_3^{10}x_5^2 + b_{29}v_4x_3^2x_5^4 \otimes x_3^8x_5^2 + b_{29}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 \\
&+ b_{29}v_4x_3^{10}x_5^4 \otimes x_5^2 + b_{29}v_4x_3^4 \otimes x_3^{10}x_9^2 + b_{29}v_4x_3^6 \otimes x_3^8x_9^2 \\
&+ b_{29}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{29}v_4x_3^{14} \otimes x_9^2 + b_{30}v_4x_5^6 \otimes x_{15}^2 \\
&+ b_{30}v_4x_{15}^2 \otimes x_5^6 + b_{30}v_4x_5^2x_{15}^2 \otimes x_5^4 + b_{30}v_4x_5^2 \otimes x_5^4x_{15}^2 \\
&+ b_{30}v_4x_5^4x_{15}^2 \otimes x_5^2 + b_{30}v_4x_5^4 \otimes x_5^2x_{15}^2 + b_{30}v_4x_3^8 \otimes x_3^2x_5^6 \\
&+ b_{30}v_4x_3^8x_5^2 \otimes x_3^2x_5^4 + b_{30}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 + b_{30}v_4x_3^8x_5^6 \otimes x_3^2
\end{aligned}
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\begin{aligned}
&+ b_{30}v_4x_5^6 \otimes x_5^6 + b_{30}v_4x_3^4 \otimes x_5^6x_9^2 + b_{30}v_4x_3^4x_5^2 \otimes x_5^4x_9^2 \\
&+ b_{30}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 + b_{30}v_4x_3^4x_5^6 \otimes x_9^2 + b_{31}v_4x_3^4x_9^2 \otimes x_{15}^2 \\
&+ b_{31}v_4x_3^4x_{15}^2 \otimes x_9^2 + b_{31}v_4x_3^4 \otimes x_9^2x_{15}^2 + b_{31}v_4x_9^2x_{15}^2 \otimes x_3^4 \\
&+ b_{31}v_4x_9^2 \otimes x_3^4x_{15}^2 + b_{31}v_4x_{15}^2 \otimes x_3^4x_9^2 + b_{31}v_4x_3^{12}x_9^2 \otimes x_3^2 \\
&+ b_{31}v_4x_3^{12} \otimes x_3^2x_9^2 + b_{31}v_4x_3^8x_9^2 \otimes x_3^6 + b_{31}v_4x_3^8 \otimes x_3^6x_9^2 \\
&+ b_{31}v_4x_3^4x_5^4x_9^2 \otimes x_5^2 + b_{31}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 + b_{31}v_4x_5^4x_9^2 \otimes x_3^4x_5^2 \\
&+ b_{31}v_4x_5^4 \otimes x_3^4x_5^2x_9^2 + b_{31}v_4x_3^8x_9^2 \otimes x_9^2 + b_{31}v_4x_3^4x_9^2 \otimes x_3^4x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_0\bar{\psi}(x_{29}) \equiv\ & x_3^8 \otimes x_3^2 + v_4x_3^8 \otimes x_3^{12} + v_4x_3^8 \otimes x_3^2x_{15}^2 + x_5^4 \otimes x_5^2 + v_4x_5^4 \otimes x_5^2x_{15}^2 \\
&+ x_3^4 \otimes x_9^2 + v_4x_3^4 \otimes x_9^2x_{15}^2 + a_{134}v_4x_3^8x_5^6 \otimes x_3^2 + a_{135}v_4x_3^{12}x_9^2 \otimes x_3^2 \\
&+ a_{136}v_4x_3^2x_5^6x_9^2 \otimes x_3^2 + a_{137}v_4x_3^8x_{15}^2 \otimes x_3^2 + a_{138}v_4x_3^2x_9^2x_{15}^2 \otimes x_3^2 \\
&+ a_{139}v_4x_3^{10}x_5^4 \otimes x_5^2 + a_{140}v_4x_3^4x_5^4x_9^2 \otimes x_5^2 + a_{141}v_4x_5^4x_{15}^2 \otimes x_5^2 \\
&+ a_{142}v_4x_3^8x_5^4 \otimes x_3^2x_5^2 + a_{143}v_4x_3^2x_5^4x_9^2 \otimes x_3^2x_5^2 + a_{144}v_4x_3^{14} \otimes x_9^2 \\
&+ a_{145}v_4x_3^4x_5^6 \otimes x_9^2 + a_{146}v_4x_3^8x_9^2 \otimes x_9^2 + a_{147}v_4x_3^4x_{15}^2 \otimes x_9^2 \\
&+ a_{148}v_4x_3^{12} \otimes x_3^8 + a_{148}v_4x_3^{12} \otimes x_3^2x_9^2 + a_{149}v_4x_3^2x_5^6 \otimes x_3^8 \\
&+ a_{149}v_4x_3^2x_5^6 \otimes x_3^2x_9^2 + a_{150}v_4x_3^6x_9^2 \otimes x_3^8 + a_{150}v_4x_3^6x_9^2 \otimes x_3^2x_9^2 \\
&+ a_{151}v_4x_3^2x_{15}^2 \otimes x_3^8 + a_{151}v_4x_3^2x_{15}^2 \otimes x_3^2x_9^2 + a_{152}v_4x_3^4x_5^4 \otimes x_5^2x_9^2 \\
&+ a_{153}v_4x_3^{10} \otimes x_{15}^2 + a_{154}v_4x_5^6 \otimes x_{15}^2 + a_{155}v_4x_3^4x_9^2 \otimes x_{15}^2 \\
&+ a_{156}v_4x_{15}^2 \otimes x_{15}^2.
\end{aligned}
$$

Then we obtain $b_{27} = b_{28} = b_{31} = a_{134} = a_{135} = a_{136} = a_{138} = a_{139} = a_{140} = a_{143} = a_{144} = a_{145} = a_{146} = a_{149} = a_{150} = a_{151} = a_{153} = a_{154} = a_{155} = 0$ and $b_{26} = b_{29} = b_{30} = a_{137} = a_{141} = a_{142} = a_{147} = a_{148} = a_{152} = a_{156} = 1$. We compare the coefficient of $Q_1\bar{\psi}(x_{29})$ and $\bar{\psi}(Q_1x_{29})$:

$$
\begin{aligned}
\bar{\psi}(Q_1x_{29}) \equiv\ & x_3^4 \otimes x_5^4 + x_5^4 \otimes x_3^4 + b_{59}v_4x_3^{14} \otimes x_5^4 + b_{59}v_4x_5^4 \otimes x_3^{14} \\
&+ b_{59}v_4x_3^2x_5^4 \otimes x_3^{12} + b_{59}v_4x_3^2 \otimes x_3^{12}x_5^4 + b_{59}v_4x_3^4x_5^4 \otimes x_3^{10} \\
&+ b_{59}v_4x_3^4 \otimes x_3^{10}x_5^4 + b_{59}v_4x_3^6x_5^4 \otimes x_3^8 + b_{59}v_4x_3^6 \otimes x_3^8x_5^4 \\
&+ b_{59}v_4x_3^8x_5^4 \otimes x_3^6 + b_{59}v_4x_3^8 \otimes x_3^6x_5^4 + b_{59}v_4x_3^{10}x_5^4 \otimes x_3^4 \\
&+ b_{59}v_4x_3^{10} \otimes x_3^4x_5^4 + b_{59}v_4x_3^{12}x_5^4 \otimes x_3^2 + b_{59}v_4x_3^{12} \otimes x_3^2x_5^4 \\
&+ b_{60}v_4x_3^8x_5^4 \otimes x_9^2 + b_{60}v_4x_3^8x_9^2 \otimes x_5^4 + b_{60}v_4x_3^8 \otimes x_5^4x_9^2 \\
&+ b_{60}v_4x_5^4x_9^2 \otimes x_3^8 + b_{60}v_4x_5^4 \otimes x_3^8x_9^2 + b_{60}v_4x_9^2 \otimes x_3^8x_5^4 \\
&+ b_{61}v_4x_3^4x_5^4 \otimes x_{15}^2 + b_{61}v_4x_3^4x_{15}^2 \otimes x_5^4 + b_{61}v_4x_3^4 \otimes x_5^4x_{15}^2
\end{aligned}
$$

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$$
\begin{aligned}
&+ b_{61}v_4x_5^4x_{15}^2 \otimes x_3^4 + b_{61}v_4x_5^4 \otimes x_3^4x_{15}^2 + b_{61}v_4x_{15}^2 \otimes x_3^4x_5^4 \\
&+ b_{61}v_4x_3^{12}x_5^4 \otimes x_3^2 + b_{61}v_4x_3^{12} \otimes x_3^2x_5^4 + b_{61}v_4x_3^8x_5^4 \otimes x_3^6 \\
&+ b_{61}v_4x_3^8 \otimes x_3^6x_5^4 + b_{61}v_4x_3^4x_5^4 \otimes x_5^6 + b_{61}v_4x_5^4 \otimes x_3^4x_5^6 \\
&+ b_{61}v_4x_3^8x_5^4 \otimes x_9^2 + b_{61}v_4x_3^8 \otimes x_5^4x_9^2 + b_{61}v_4x_3^4x_5^4 \otimes x_3^4x_9^2 \\
&+ b_{61}v_4x_3^4 \otimes x_3^4x_5^4x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_1\bar{\psi}(x_{29}) \equiv\ & v_4x_3^8 \otimes x_3^6x_5^4 + v_4x_3^8 \otimes x_5^4x_9^2 + x_5^4 \otimes x_3^4 + v_4x_5^4 \otimes x_3^4x_5^6 \\
&+ v_4x_5^4 \otimes x_3^4x_{15}^2 + x_3^4 \otimes x_5^4 + v_4x_3^4 \otimes x_3^4x_5^4x_9^2 + v_4x_3^4 \otimes x_5^4x_{15}^2 \\
&+ a_{131}v_4x_3^{12}x_5^4 \otimes x_3^2 + a_{132}v_4x_3^6x_5^4x_9^2 \otimes x_3^2 + a_{133}v_4x_3^2x_5^4x_{15}^2 \otimes x_3^2 \\
&+ v_4x_5^4x_{15}^2 \otimes x_3^4 + v_4x_3^8x_5^4 \otimes x_9^2 + v_4x_3^8x_5^4 \otimes x_3^6 + v_4x_3^4x_{15}^2 \otimes x_5^4 \\
&+ v_4x_3^{12} \otimes x_3^2x_5^4 + v_4x_3^4x_5^4 \otimes x_{15}^2 + v_4x_3^4x_5^4 \otimes x_3^4x_9^2 + v_4x_3^4x_5^4 \otimes x_5^6 \\
&+ v_4x_{15}^2 \otimes x_3^4x_5^4.
\end{aligned}
$$

Then we obtain $b_{59} = b_{60} = a_{132} = a_{133} = 0$ and $b_{61} = a_{131} = 1$. We compare the coefficient of $Q_2\bar{\psi}(x_{29})$ and $\bar{\psi}(Q_2x_{29})$:

$$
\begin{aligned}
\bar{\psi}(Q_2x_{29}) \equiv\ & x_3^4 \otimes x_3^8 + x_3^8 \otimes x_3^4 + b_{89}v_4x_3^{12} \otimes x_5^6 + b_{89}v_4x_5^6 \otimes x_3^{12} \\
&+ b_{89}v_4x_3^4x_5^6 \otimes x_3^8 + b_{89}v_4x_3^4 \otimes x_3^8x_5^6 + b_{89}v_4x_3^8x_5^6 \otimes x_3^4 \\
&+ b_{89}v_4x_3^8 \otimes x_3^4x_5^6 + b_{89}v_4x_3^{12}x_5^2 \otimes x_5^4 + b_{89}v_4x_5^2 \otimes x_3^{12}x_5^4 \\
&+ b_{89}v_4x_3^{12}x_5^4 \otimes x_5^2 + b_{89}v_4x_5^4 \otimes x_3^{12}x_5^2 + b_{89}v_4x_3^4x_5^2 \otimes x_3^8x_5^4 \\
&+ b_{89}v_4x_3^4x_5^4 \otimes x_3^8x_5^2 + b_{89}v_4x_3^8x_5^2 \otimes x_3^4x_5^4 + b_{89}v_4x_3^8x_5^4 \otimes x_3^4x_5^2 \\
&+ b_{90}v_4x_3^6x_5^6 \otimes x_9^2 + b_{90}v_4x_3^6x_9^2 \otimes x_5^6 + b_{90}v_4x_3^6 \otimes x_5^6x_9^2 \\
&+ b_{90}v_4x_5^6x_9^2 \otimes x_3^6 + b_{90}v_4x_5^6 \otimes x_3^6x_9^2 + b_{90}v_4x_9^2 \otimes x_3^6x_5^6 \\
&+ b_{90}v_4x_3^2x_5^6x_9^2 \otimes x_3^4 + b_{90}v_4x_3^2x_5^6 \otimes x_3^4x_9^2 + b_{90}v_4x_3^2x_9^2 \otimes x_3^4x_5^6 \\
&+ b_{90}v_4x_3^2 \otimes x_3^4x_5^6x_9^2 + b_{90}v_4x_3^4x_5^6x_9^2 \otimes x_3^2 + b_{90}v_4x_3^4x_5^6 \otimes x_3^2x_9^2 \\
&+ b_{90}v_4x_3^4x_9^2 \otimes x_3^2x_5^6 + b_{90}v_4x_3^4 \otimes x_3^2x_5^6x_9^2 + b_{90}v_4x_3^6x_5^2x_9^2 \otimes x_5^4 \\
&+ b_{90}v_4x_3^6x_5^2 \otimes x_5^4x_9^2 + b_{90}v_4x_5^2x_9^2 \otimes x_3^6x_5^4 + b_{90}v_4x_5^2 \otimes x_3^6x_5^4x_9^2 \\
&+ b_{90}v_4x_3^6x_5^4x_9^2 \otimes x_5^2 + b_{90}v_4x_3^6x_5^4 \otimes x_5^2x_9^2 + b_{90}v_4x_5^4x_9^2 \otimes x_3^6x_5^2 \\
&+ b_{90}v_4x_5^4 \otimes x_3^6x_5^2x_9^2 + b_{90}v_4x_3^2x_5^2x_9^2 \otimes x_3^4x_5^4 \\
&+ b_{90}v_4x_3^2x_5^2 \otimes x_3^4x_5^4x_9^2 + b_{90}v_4x_3^2x_5^4x_9^2 \otimes x_3^4x_5^2 \\
&+ b_{90}v_4x_3^2x_5^4 \otimes x_3^4x_5^2x_9^2 + b_{90}v_4x_3^4x_5^2x_9^2 \otimes x_3^2x_5^4 \\
&+ b_{90}v_4x_3^4x_5^2 \otimes x_3^2x_5^4x_9^2 + b_{90}v_4x_3^4x_5^4x_9^2 \otimes x_3^2x_5^2 \\
&+ b_{90}v_4x_3^4x_5^4 \otimes x_3^2x_5^2x_9^2 + b_{91}v_4x_3^{12} \otimes x_{15}^2 + b_{91}v_4x_{15}^2 \otimes x_3^{12}
\end{aligned}
$$

$$
\begin{aligned}
&+ b_{91}v_4x_3^4x_{15}^2 \otimes x_3^8 + b_{91}v_4x_3^4 \otimes x_3^8x_{15}^2 + b_{91}v_4x_3^8x_{15}^2 \otimes x_3^4\\
&+ b_{91}v_4x_3^8 \otimes x_3^4x_{15}^2 + b_{91}v_4x_3^8 \otimes x_3^{14} + b_{91}v_4x_3^{12} \otimes x_3^{10}\\
&+ b_{91}v_4x_5^4 \otimes x_3^{12}x_5^2 + b_{91}v_4x_3^4x_5^4 \otimes x_3^8x_5^2 + b_{91}v_4x_3^8x_5^4 \otimes x_3^4x_5^2\\
&+ b_{91}v_4x_3^{12}x_5^4 \otimes x_5^2 + b_{91}v_4x_3^4 \otimes x_3^{12}x_9^2 + b_{91}v_4x_3^8 \otimes x_3^8x_9^2\\
&+ b_{91}v_4x_3^{12} \otimes x_3^4x_9^2 + b_{92}v_4x_3^2x_5^6 \otimes x_{15}^2 + b_{92}v_4x_3^2x_{15}^2 \otimes x_5^6\\
&+ b_{92}v_4x_3^2 \otimes x_5^6x_{15}^2 + b_{92}v_4x_5^6x_{15}^2 \otimes x_3^2 + b_{92}v_4x_5^6 \otimes x_3^2x_{15}^2\\
&+ b_{92}v_4x_{15}^2 \otimes x_3^2x_5^6 + b_{92}v_4x_3^2x_5^2x_{15}^2 \otimes x_5^4 + b_{92}v_4x_3^2x_5^2 \otimes x_5^4x_{15}^2\\
&+ b_{92}v_4x_5^2x_{15}^2 \otimes x_3^2x_5^4 + b_{92}v_4x_5^2 \otimes x_3^2x_5^4x_{15}^2 + b_{92}v_4x_3^2x_5^4x_{15}^2 \otimes x_5^2\\
&+ b_{92}v_4x_3^2x_5^4 \otimes x_5^2x_{15}^2 + b_{92}v_4x_5^4x_{15}^2 \otimes x_3^2x_5^2 + b_{92}v_4x_5^4 \otimes x_3^2x_5^2x_{15}^2\\
&+ b_{92}v_4x_3^8 \otimes x_3^4x_5^6 + b_{92}v_4x_3^8x_5^2 \otimes x_3^4x_5^4 + b_{92}v_4x_3^8x_5^4 \otimes x_3^4x_5^2\\
&+ b_{92}v_4x_3^8x_5^6 \otimes x_3^4 + b_{92}v_4x_3^{10} \otimes x_3^2x_5^6 + b_{92}v_4x_3^{10}x_5^2 \otimes x_3^2x_5^4\\
&+ b_{92}v_4x_3^{10}x_5^4 \otimes x_3^2x_5^2 + b_{92}v_4x_3^{10}x_5^6 \otimes x_3^2 + b_{92}v_4x_3^2x_5^6 \otimes x_5^6\\
&+ b_{92}v_4x_5^6 \otimes x_3^2x_5^6 + b_{92}v_4x_3^4 \otimes x_3^2x_5^6x_9^2 + b_{92}v_4x_3^4x_5^2 \otimes x_3^2x_5^4x_9^2\\
&+ b_{92}v_4x_3^4x_5^4 \otimes x_3^2x_5^2x_9^2 + b_{92}v_4x_3^4x_5^6 \otimes x_3^2x_9^2 + b_{92}v_4x_3^6 \otimes x_5^6x_9^2\\
&+ b_{92}v_4x_3^6x_5^2 \otimes x_5^4x_9^2 + b_{92}v_4x_3^6x_5^4 \otimes x_5^2x_9^2 + b_{92}v_4x_3^6x_5^6 \otimes x_9^2\\
&+ b_{93}v_4x_3^6x_9^2 \otimes x_{15}^2 + b_{93}v_4x_3^6x_{15}^2 \otimes x_9^2 + b_{93}v_4x_3^6 \otimes x_9^2x_{15}^2\\
&+ b_{93}v_4x_9^2x_{15}^2 \otimes x_3^6 + b_{93}v_4x_9^2 \otimes x_3^6x_{15}^2 + b_{93}v_4x_{15}^2 \otimes x_3^6x_9^2\\
&+ b_{93}v_4x_3^2x_9^2x_{15}^2 \otimes x_3^4 + b_{93}v_4x_3^2x_9^2 \otimes x_3^4x_{15}^2 + b_{93}v_4x_3^2x_{15}^2 \otimes x_3^4x_9^2\\
&+ b_{93}v_4x_3^2 \otimes x_3^4x_9^2x_{15}^2 + b_{93}v_4x_3^4x_9^2x_{15}^2 \otimes x_3^2 + b_{93}v_4x_3^4x_9^2 \otimes x_3^2x_{15}^2\\
&+ b_{93}v_4x_3^4x_{15}^2 \otimes x_3^2x_9^2 + b_{93}v_4x_3^4 \otimes x_3^2x_9^2x_{15}^2 + b_{93}v_4x_3^8x_9^2 \otimes x_3^8\\
&+ b_{93}v_4x_3^8 \otimes x_3^8x_9^2 + b_{93}v_4x_3^{10}x_9^2 \otimes x_3^6 + b_{93}v_4x_3^{10} \otimes x_3^6x_9^2\\
&+ b_{93}v_4x_3^{12}x_9^2 \otimes x_3^4 + b_{93}v_4x_3^{12} \otimes x_3^4x_9^2 + b_{93}v_4x_3^{14}x_9^2 \otimes x_3^2\\
&+ b_{93}v_4x_3^{14} \otimes x_3^2x_9^2 + b_{93}v_4x_5^4x_9^2 \otimes x_3^6x_5^2 + b_{93}v_4x_5^4 \otimes x_3^6x_5^2x_9^2\\
&+ b_{93}v_4x_3^2x_5^4x_9^2 \otimes x_3^4x_5^2 + b_{93}v_4x_3^2x_5^4 \otimes x_3^4x_5^2x_9^2\\
&+ b_{93}v_4x_3^4x_5^4x_9^2 \otimes x_3^2x_5^2 + b_{93}v_4x_3^4x_5^4 \otimes x_3^2x_5^2x_9^2\\
&+ b_{93}v_4x_3^6x_5^4x_9^2 \otimes x_5^2 + b_{93}v_4x_3^6x_5^4 \otimes x_5^2x_9^2 + b_{93}v_4x_3^4x_9^2 \otimes x_3^6x_9^2\\
&+ b_{93}v_4x_3^6x_9^2 \otimes x_3^4x_9^2 + b_{93}v_4x_3^8x_9^2 \otimes x_3^2x_9^2 + b_{93}v_4x_3^{10}x_9^2 \otimes x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_2\bar{\psi}(x_{29}) \equiv\ & x_3^8 \otimes x_3^4 + v_4x_3^8 \otimes x_3^{14} + v_4x_3^8 \otimes x_3^8x_9^2 + v_4x_3^8 \otimes x_3^4x_{15}^2\\
&+ v_4x_5^4 \otimes x_3^{12}x_5^2
\end{aligned}
$$

$$
\begin{aligned}
&+ x_3^4 \otimes x_3^8 + v_4 x_3^4 \otimes x_3^{12} x_9^2 + v_4 x_3^4 \otimes x_3^8 x_{15}^2 + v_4 x_3^{12} x_5^4 \otimes x_5^2 \\
&+ v_4 x_3^8 x_{15}^2 \otimes x_3^4 + v_4 x_3^8 x_5^4 \otimes x_3^4 x_5^2 + v_4 x_3^4 x_{15}^2 \otimes x_3^8 + v_4 x_3^{12} \otimes x_{15}^2 \\
&+ v_4 x_3^{12} \otimes x_3^{10} + v_4 x_3^{12} \otimes x_3^4 x_9^2 + v_4 x_3^4 x_5^4 \otimes x_3^8 x_5^2 + v_4 x_{15}^2 \otimes x_3^{12}.
\end{aligned}
$$

Then we obtain $b_{89} = b_{90} = b_{92} = b_{93} = 0$ and $b_{91} = 1$. We compare the coefficient of $Q_3\bar{\psi}(x_{29})$ and $\bar{\psi}(Q_3 x_{29})$:

$$
\begin{aligned}
\bar{\psi}(Q_3 x_{29}) \equiv\ & x_3^8 \otimes x_5^4 + x_5^4 \otimes x_3^8 + b_{124} v_4 x_3^{12} x_5^4 \otimes x_9^2 + b_{124} v_4 x_3^{12} x_9^2 \otimes x_5^4 \\
&+ b_{124} v_4 x_3^{12} \otimes x_5^4 x_9^2 + b_{124} v_4 x_5^4 x_9^2 \otimes x_3^{12} + b_{124} v_4 x_5^4 \otimes x_3^{12} x_9^2 \\
&+ b_{124} v_4 x_9^2 \otimes x_3^{12} x_5^4 + b_{124} v_4 x_3^8 x_5^4 x_9^2 \otimes x_3^4 + b_{124} v_4 x_3^8 x_5^4 \otimes x_3^4 x_9^2 \\
&+ b_{124} v_4 x_3^8 x_9^2 \otimes x_3^4 x_5^4 + b_{124} v_4 x_3^8 \otimes x_3^4 x_5^4 x_9^2 + b_{124} v_4 x_3^4 x_5^4 x_9^2 \otimes x_3^8 \\
&+ b_{124} v_4 x_3^4 x_5^4 \otimes x_3^8 x_9^2 + b_{124} v_4 x_3^4 x_9^2 \otimes x_3^8 x_5^4 + b_{124} v_4 x_3^4 \otimes x_3^8 x_5^4 x_9^2 \\
&+ b_{125} v_4 x_3^8 x_5^4 \otimes x_{15}^2 + b_{125} v_4 x_3^8 x_{15}^2 \otimes x_5^4 + b_{125} v_4 x_3^8 \otimes x_5^4 x_{15}^2 \\
&+ b_{125} v_4 x_5^4 x_{15}^2 \otimes x_3^8 + b_{125} v_4 x_5^4 \otimes x_3^8 x_{15}^2 + b_{125} v_4 x_{15}^2 \otimes x_3^8 x_5^4 \\
&+ b_{125} v_4 x_3^8 x_5^4 \otimes x_3^{10} + b_{125} v_4 x_3^8 \otimes x_3^{10} x_5^4 + b_{125} v_4 x_3^8 x_5^4 \otimes x_5^6 \\
&+ b_{125} v_4 x_5^4 \otimes x_3^8 x_5^6 + b_{125} v_4 x_3^{12} x_5^4 \otimes x_9^2 + b_{125} v_4 x_3^{12} \otimes x_5^4 x_9^2 \\
&+ b_{125} v_4 x_3^4 x_5^4 \otimes x_3^8 x_9^2 + b_{125} v_4 x_3^4 \otimes x_3^8 x_5^4 x_9^2 + b_{126} v_4 x_3^2 x_5^4 x_9^2 \otimes x_{15}^2 \\
&+ b_{126} v_4 x_3^2 x_5^4 x_{15}^2 \otimes x_9^2 + b_{126} v_4 x_3^2 x_5^4 \otimes x_9^2 x_{15}^2 \\
&+ b_{126} v_4 x_3^2 x_9^2 x_{15}^2 \otimes x_5^4 + b_{126} v_4 x_3^2 x_9^2 \otimes x_5^4 x_{15}^2 \\
&+ b_{126} v_4 x_3^2 x_{15}^2 \otimes x_5^4 x_9^2 + b_{126} v_4 x_3^2 \otimes x_5^4 x_9^2 x_{15}^2 \\
&+ b_{126} v_4 x_5^4 x_9^2 x_{15}^2 \otimes x_3^2 + b_{126} v_4 x_5^4 x_9^2 \otimes x_3^2 x_{15}^2 \\
&+ b_{126} v_4 x_5^4 x_{15}^2 \otimes x_3^2 x_9^2 + b_{126} v_4 x_5^4 \otimes x_3^2 x_9^2 x_{15}^2 \\
&+ b_{126} v_4 x_9^2 x_{15}^2 \otimes x_3^2 x_5^4 + b_{126} v_4 x_9^2 \otimes x_3^2 x_5^4 x_{15}^2 \\
&+ b_{126} v_4 x_{15}^2 \otimes x_3^2 x_5^4 x_9^2 + b_{126} v_4 x_3^{10} x_5^4 x_9^2 \otimes x_3^2 \\
&+ b_{126} v_4 x_3^{10} x_5^4 \otimes x_3^2 x_9^2 + b_{126} v_4 x_3^{10} x_9^2 \otimes x_3^2 x_5^4 \\
&+ b_{126} v_4 x_3^{10} \otimes x_3^2 x_5^4 x_9^2 + b_{126} v_4 x_3^8 x_5^4 x_9^2 \otimes x_3^4 + b_{126} v_4 x_3^8 x_5^4 \otimes x_3^4 x_9^2 \\
&+ b_{126} v_4 x_3^8 x_9^2 \otimes x_3^4 x_5^4 + b_{126} v_4 x_3^8 \otimes x_3^4 x_5^4 x_9^2 + b_{126} v_4 x_3^2 x_5^4 x_9^2 \otimes x_5^6 \\
&+ b_{126} v_4 x_3^2 x_5^4 \otimes x_5^6 x_9^2 + b_{126} v_4 x_5^4 x_9^2 \otimes x_3^2 x_5^6 + b_{126} v_4 x_5^4 \otimes x_3^2 x_5^6 x_9^2 \\
&+ b_{126} v_4 x_3^6 x_5^4 x_9^2 \otimes x_9^2 + b_{126} v_4 x_3^6 x_9^2 \otimes x_5^4 x_9^2 \\
&+ b_{126} v_4 x_3^4 x_5^4 x_9^2 \otimes x_3^2 x_9^2 + b_{126} v_4 x_3^4 x_9^2 \otimes x_3^2 x_5^4 x_9^2,
\end{aligned}
$$

$$
\begin{aligned}
Q_3\bar{\psi}(x_{29}) \equiv\ & x_3^8 \otimes x_5^4 + v_4 x_3^8 \otimes x_3^{10} x_5^4 + v_4 x_3^8 \otimes x_5^4 x_{15}^2 + x_5^4 \otimes x_3^8 \\
&+ v_4 x_5^4 \otimes x_3^8 x_5^6 + v_4 x_5^4 \otimes x_3^8 x_{15}^2 + v_4 x_3^4 \otimes x_3^8 x_5^4 x_9^2 + v_4 x_3^{12} x_5^4 \otimes x_9^2
\end{aligned}
$$

$$+ v_4x_3^8x_{15}^2 \otimes x_5^4 + v_4x_5^4x_{15}^2 \otimes x_3^8 + v_4x_3^8x_5^4 \otimes x_{15}^2 + v_4x_3^8x_5^4 \otimes x_3^{10}$$
$$+ v_4x_3^8x_5^4 \otimes x_5^6 + v_4x_3^{12} \otimes x_5^4x_9^2 + v_4x_3^4x_5^4 \otimes x_3^8x_9^2 + v_4x_{15}^2 \otimes x_3^8x_5^4.$$

Then we obtain $b_{124} = b_{126} = 0$ and $b_{125} = 1$.

Lemma 3.2. *We have*

$a_i = 0$ *for* $i = 5, 9, 16, 17, 26, 27, 28, 30, 31, 33, 36, 38, 46, 50, 54, 63, 64, 65, 67, 68, 70, 71, 72, 76, 78, 83, 85, 86, 90, 92, 93, 97, 105, 107, 108, 110, 111, 112, 114, 116, 117, 119, 120, 124, 125, 129, 132, 133, 134, 135, 136, 138, 139, 140, 143, 144, 145, 146, 149, 150, 151, 153, 154, 155$;

$a_i = 1$ *for* $i = 1, 2, 3, 4, 6, 7, 8, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24, 25, 29, 32, 34, 35, 37, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 66, 69, 73, 74, 75, 77, 79, 80, 81, 82, 84, 87, 88, 89, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 106, 109, 113, 115, 118, 121, 122, 123, 126, 127, 128, 130, 131, 137, 141, 142, 147, 148, 152, 156$;

$b_i = 0$ *for* $i = 4, 5, 7, 8, 11, 13, 14, 15, 16, 18, 20, 23, 24, 27, 28, 31, 32, 38, 40, 43, 47, 50, 54, 57, 59, 60, 66, 70, 71, 75, 77, 79, 82, 83, 88, 89, 90, 92, 93, 94, 97, 100, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 117, 119, 121, 122, 124, 126$;

$b_i = 1$ *for* $i = 1, 2, 3, 6, 9, 10, 12, 17, 19, 21, 22, 25, 26, 29, 30, 33, 34, 35, 36, 37, 39, 41, 42, 44, 45, 46, 48, 49, 51, 52, 53, 55, 56, 58, 61, 62, 63, 64, 65, 67, 68, 69, 72, 73, 74, 76, 78, 80, 81, 84, 85, 86, 87, 91, 95, 96, 98, 99, 101, 102, 105, 114, 116, 118, 120, 123, 125.$

Notation 3.3. For simplicity we write

$$\bar\psi(x_3) \equiv v_4f_{(3,1)}, \quad \bar\psi(x_5) \equiv v_4f_{(5,1)}, \quad \bar\psi(x_9) \equiv v_4f_{(9,1)}, \quad \bar\psi(x_{17}) \equiv v_4f_{(17,1)},$$
$$\bar\psi(x_{15}) \equiv x_3^4 \otimes x_3 + x_5^2 \otimes x_5 + x_3^2 \otimes x_9 + v_4f_{(15,1)},$$
$$\bar\psi(x_{23}) \equiv x_5^4 \otimes x_3 + x_9^2 \otimes x_5 + x_3^2 \otimes x_{17} + v_4f_{(23,1)},$$
$$\bar\psi(x_{27}) \equiv x_3^8 \otimes x_3 + x_9^2 \otimes x_9 + x_5^2 \otimes x_{17} + v_4f_{(27,1)},$$
$$\bar\psi(x_{29}) \equiv x_3^8 \otimes x_5 + x_5^4 \otimes x_9 + x_3^4 \otimes x_{17} + v_4f_{(29,1)};$$

$$Q_0x_3 \equiv v_4g_{(0,3,1)}, \qquad Q_0x_5 \equiv x_3^2 + v_4g_{(0,5,1)},$$
$$Q_0x_9 \equiv x_5^2 + v_4g_{(0,9,1)}, \qquad Q_0x_{15} \equiv x_3^2x_5^2 + v_4g_{(0,15,1)},$$

$Q_0x_{17} \equiv x_9^2 + v_4g_{(0,17,1)}$, $Q_0x_{23} \equiv x_3^8 + x_3^2x_9^2 + v_4g_{(0,23,1)}$,
$Q_0x_{27} \equiv x_5^2x_9^2 + v_4g_{(0,27,1)}$, $Q_0x_{29} \equiv x_{15}^2 + v_4g_{(0,29,1)}$,
$Q_1x_3 \equiv x_3^2 + v_4g_{(1,3,1)}$, $Q_1x_5 \equiv v_4g_{(1,5,1)}$,
$Q_1x_9 \equiv x_3^4 + v_4g_{(1,9,1)}$, $Q_1x_{15} \equiv x_9^2 + x_3^6 + v_4g_{(1,15,1)}$,
$Q_1x_{17} \equiv x_5^4 + v_4g_{(1,17,1)}$, $Q_1x_{23} \equiv x_3^2x_5^4 + v_4g_{(1,23,1)}$,
$Q_1x_{27} \equiv x_{15}^2 + x_3^4x_9^2 + x_5^6 + v_4g_{(1,27,1)}$,
$Q_1x_{29} \equiv x_3^4x_5^4 + v_4g_{(1,29,1)}$,
$Q_2x_3 \equiv x_5^2 + v_4g_{(2,3,1)}$, $Q_2x_5 \equiv x_3^4 + v_4g_{(2,5,1)}$,
$Q_2x_9 \equiv v_4g_{(2,9,1)}$, $Q_2x_{15} \equiv x_3^4x_5^2 + v_4g_{(2,15,1)}$,
$Q_2x_{17} \equiv x_3^8 + v_4g_{(2,17,1)}$,
$Q_2x_{23} \equiv x_{15}^2 + x_3^{10} + x_3^4x_9^2 + v_4g_{(2,23,1)}$,
$Q_2x_{27} \equiv x_3^8x_5^2 + v_4g_{(2,27,1)}$, $Q_2x_{29} \equiv x_3^{12} + v_4g_{(2,29,1)}$,
$Q_3x_3 \equiv x_9^2 + v_4g_{(3,3,1)}$, $Q_3x_5 \equiv x_5^4 + v_4g_{(3,5,1)}$,
$Q_3x_9 \equiv x_3^8 + v_4g_{(3,9,1)}$, $Q_3x_{15} \equiv x_{15}^2 + x_3^{10} + x_5^6$,
$Q_3x_{17} \equiv v_4g_{(3,17,1)}$, $Q_3x_{23} \equiv x_5^4x_9^2 + v_4g_{(3,23,1)}$,
$Q_3x_{27} \equiv x_3^8x_9^2 + v_4g_{(3,27,1)}$, $Q_3x_{29} \equiv x_3^8x_5^4 + v_4g_{(3,29,1)}$,

where $\equiv$ is mod $I(1,0,0,\cdots)$.

The elements x_3^{16}, x_5^8, x_9^4, x_{15}^4, x_{17}^2, x_{23}^2, x_{27}^2, x_{29}^2 are primitive mod $I(1,0,0,\cdots)$, but there is no primitive element mod $I(1,0,0,\cdots)$ in their degree. Thus all c_i are zero for $1 \le i \le 26$.

Next we consider mod $I(2,0,0,\cdots)$ in the same way. We can put

$$\begin{aligned}\bar\psi(x_3) \equiv\ & v_4f_{(3,1)} + a_{157}v_4^2x_3^{10}x_5^6 \otimes x_3 + a_{158}v_4^2x_3^{14}x_9^2 \otimes x_3 \\ & + a_{159}v_4^2x_3^4x_5^6x_9^2 \otimes x_3 + a_{160}v_4^2x_3^{10}x_{15}^2 \otimes x_3 + a_{161}v_4^2x_5^6x_{15}^2 \otimes x_3 \\ & + a_{162}v_4^2x_3^4x_9^2x_{15}^2 \otimes x_3 + a_{163}v_4^2x_3^{10}x_5^2x_9^2 \otimes x_5 + a_{164}v_4^2x_3^6x_5^2x_{15}^2 \otimes x_5 \\ & + a_{165}v_4^2x_5^2x_9^2x_{15}^2 \otimes x_5 + a_{166}v_4^2x_3^8x_5^6 \otimes x_9 + a_{167}v_4^2x_3^{12}x_9^2 \otimes x_9 \\ & + a_{168}v_4^2x_3^2x_5^6x_9^2 \otimes x_9 + a_{169}v_4^2x_3^8x_{15}^2 \otimes x_9 + a_{170}v_4^2x_3^2x_9^2x_{15}^2 \otimes x_9 \\ & + a_{171}v_4^2x_3^6x_5^6 \otimes x_{15} + a_{172}v_4^2x_3^{10}x_9^2 \otimes x_{15} + a_{173}v_4^2x_5^6x_9^2 \otimes x_{15} \\ & + a_{174}v_4^2x_3^6x_{15}^2 \otimes x_{15} + a_{175}v_4^2x_9^2x_{15}^2 \otimes x_{15} + a_{176}v_4^2x_3^{12}x_5^2 \otimes x_{17} \\ & + a_{177}v_4^2x_3^6x_5^2x_9^2 \otimes x_{17} + a_{178}v_4^2x_3^2x_5^2x_{15}^2 \otimes x_{17} + a_{179}v_4^2x_3^{10}x_5^2 \otimes x_{23} \\ & + a_{180}v_4^2x_3^4x_5^2x_9^2 \otimes x_{23} + a_{181}v_4^2x_5^2x_{15}^2 \otimes x_{23} + a_{182}v_4^2x_3^{12} \otimes x_{27}\end{aligned}$$

$$
\begin{aligned}
&+ a_{183}v_4^2x_3^2x_5^6 \otimes x_{27} + a_{184}v_4^2x_3^6x_9^2 \otimes x_{27} + a_{185}v_4^2x_3^2x_{15}^2 \otimes x_{27} \\
&+ a_{186}v_4^2x_3^8x_5^2 \otimes x_{29} + a_{187}v_4^2x_3^2x_5^2x_9^2 \otimes x_{29}, \\
\bar\psi(x_5) \equiv{}& v_4f_{(5,1)} + a_{188}v_4^2x_3^{14}x_5^4 \otimes x_3 + a_{189}v_4^2x_3^8x_5^4x_9^2 \otimes x_3 \\
&+ a_{190}v_4^2x_3^4x_5^4x_{15}^2 \otimes x_3 + a_{191}v_4^2x_3^{10}x_5^6 \otimes x_5 + a_{192}v_4^2x_3^{14}x_9^2 \otimes x_5 \\
&+ a_{193}v_4^2x_3^4x_5^6x_9^2 \otimes x_5 + a_{194}v_4^2x_3^{10}x_{15}^2 \otimes x_5 + a_{195}v_4^2x_5^6x_{15}^2 \otimes x_5 \\
&+ a_{196}v_4^2x_3^4x_9^2x_{15}^2 \otimes x_5 + a_{197}v_4^2x_3^{12}x_5^4 \otimes x_9 + a_{198}v_4^2x_3^6x_5^4x_9^2 \otimes x_9 \\
&+ a_{199}v_4^2x_3^2x_5^4x_{15}^2 \otimes x_9 + a_{200}v_4^2x_3^{10}x_5^4 \otimes x_{15} + a_{201}v_4^2x_3^4x_5^4x_9^2 \otimes x_{15} \\
&+ a_{202}v_4^2x_5^4x_{15}^2 \otimes x_{15} + a_{203}v_4^2x_3^6x_5^6 \otimes x_{17} + a_{204}v_4^2x_3^{10}x_9^2 \otimes x_{17} \\
&+ a_{205}v_4^2x_5^6x_9^2 \otimes x_{17} + a_{206}v_4^2x_3^6x_{15}^2 \otimes x_{17} + a_{207}v_4^2x_9^2x_{15}^2 \otimes x_{17} \\
&+ a_{208}v_4^2x_3^{14} \otimes x_{23} + a_{209}v_4^2x_3^4x_5^6 \otimes x_{23} + a_{210}v_4^2x_3^8x_9^2 \otimes x_{23} \\
&+ a_{211}v_4^2x_3^4x_{15}^2 \otimes x_{23} + a_{212}v_4^2x_3^6x_5^4 \otimes x_{27} + a_{213}v_4^2x_5^4x_9^2 \otimes x_{27} \\
&+ a_{214}v_4^2x_3^{12} \otimes x_{29} + a_{215}v_4^2x_3^2x_5^6 \otimes x_{29} + a_{216}v_4^2x_3^6x_9^2 \otimes x_{29} \\
&+ a_{217}v_4^2x_3^2x_{15}^2 \otimes x_{29}, \\
\bar\psi(x_9) \equiv{}& v_4f_{(9,1)} + a_{218}v_4^2x_3^{12}x_5^6 \otimes x_3 + a_{219}v_4^2x_3^6x_5^6x_9^2 \otimes x_3 \\
&+ a_{220}v_4^2x_3^{12}x_{15}^2 \otimes x_3 + a_{221}v_4^2x_3^2x_5^6x_{15}^2 \otimes x_3 + a_{222}v_4^2x_3^6x_9^2x_{15}^2 \otimes x_3 \\
&+ a_{223}v_4^2x_3^{12}x_5^2x_9^2 \otimes x_5 + a_{224}v_4^2x_3^8x_5^2x_{15}^2 \otimes x_5 \\
&+ a_{225}v_4^2x_3^2x_5^2x_9^2x_{15}^2 \otimes x_5 + a_{226}v_4^2x_3^{10}x_5^6 \otimes x_9 + a_{227}v_4^2x_3^{14}x_9^2 \otimes x_9 \\
&+ a_{228}v_4^2x_3^4x_5^6x_9^2 \otimes x_9 + a_{229}v_4^2x_3^{10}x_{15}^2 \otimes x_9 + a_{230}v_4^2x_5^6x_{15}^2 \otimes x_9 \\
&+ a_{231}v_4^2x_3^4x_9^2x_{15}^2 \otimes x_9 + a_{232}v_4^2x_3^8x_5^6 \otimes x_{15} + a_{233}v_4^2x_3^{12}x_9^2 \otimes x_{15} \\
&+ a_{234}v_4^2x_3^2x_5^6x_9^2 \otimes x_{15} + a_{235}v_4^2x_3^8x_{15}^2 \otimes x_{15} + a_{236}v_4^2x_3^2x_9^2x_{15}^2 \otimes x_{15} \\
&+ a_{237}v_4^2x_3^{14}x_5^2 \otimes x_{17} + a_{238}v_4^2x_3^8x_5^2x_9^2 \otimes x_{17} + a_{239}v_4^2x_3^4x_5^2x_{15}^2 \otimes x_{17} \\
&+ a_{240}v_4^2x_3^{12}x_5^2 \otimes x_{23} + a_{241}v_4^2x_3^6x_5^2x_9^2 \otimes x_{23} + a_{242}v_4^2x_3^2x_5^2x_{15}^2 \otimes x_{23} \\
&+ a_{243}v_4^2x_3^{14} \otimes x_{27} + a_{244}v_4^2x_3^4x_5^6 \otimes x_{27} + a_{245}v_4^2x_3^8x_9^2 \otimes x_{27} \\
&+ a_{246}v_4^2x_3^4x_{15}^2 \otimes x_{27} + a_{247}v_4^2x_3^{10}x_5^2 \otimes x_{29} + a_{248}v_4^2x_3^4x_5^2x_9^2 \otimes x_{29} \\
&+ a_{249}v_4^2x_5^2x_{15}^2 \otimes x_{29}, \\
\bar\psi(x_{15}) \equiv{}& x_3^4 \otimes x_3 + x_5^2 \otimes x_5 + x_3^2 \otimes x_9 + v_4f_{(15,1)} \\
&+ a_{250}v_4^2x_3^{14}x_5^6 \otimes x_3 + a_{251}v_4^2x_3^8x_5^6x_9^2 \otimes x_3 + a_{252}v_4^2x_3^{14}x_{15}^2 \otimes x_3 \\
&+ a_{253}v_4^2x_3^4x_5^6x_{15}^2 \otimes x_3 + a_{254}v_4^2x_3^8x_9^2x_{15}^2 \otimes x_3 + a_{255}v_4^2x_3^{14}x_5^2x_9^2 \otimes x_5 \\
&+ a_{256}v_4^2x_3^{10}x_5^2x_{15}^2 \otimes x_5 + a_{257}v_4^2x_3^4x_5^2x_9^2x_{15}^2 \otimes x_5 + a_{258}v_4^2x_3^{12}x_5^6 \otimes x_9
\end{aligned}
$$

$$
\begin{aligned}
&+ a_{259}v_4^2x_3^6x_5^6x_9^2 \otimes x_9 + a_{260}v_4^2x_3^{12}x_{15}^2 \otimes x_9 + a_{261}v_4^2x_3^2x_5^6x_{15}^2 \otimes x_9\\
&+ a_{262}v_4^2x_3^6x_9^2x_{15}^2 \otimes x_9 + a_{263}v_4^2x_3^{10}x_5^6 \otimes x_{15} + a_{264}v_4^2x_3^{14}x_9^2 \otimes x_{15}\\
&+ a_{265}v_4^2x_3^4x_5^6x_9^2 \otimes x_{15} + a_{266}v_4^2x_3^{10}x_{15}^2 \otimes x_{15} + a_{267}v_4^2x_5^6x_{15}^2 \otimes x_{15}\\
&+ a_{268}v_4^2x_3^4x_9^2x_{15}^2 \otimes x_{15} + a_{269}v_4^2x_3^{10}x_5^2x_9^2 \otimes x_{17}\\
&+ a_{270}v_4^2x_3^6x_5^2x_{15}^2 \otimes x_{17} + a_{271}v_4^2x_5^2x_9^2x_{15}^2 \otimes x_{17} + a_{272}v_4^2x_3^{14}x_5^2 \otimes x_{23}\\
&+ a_{273}v_4^2x_3^8x_5^2x_9^2 \otimes x_{23} + a_{274}v_4^2x_3^4x_5^2x_{15}^2 \otimes x_{23} + a_{275}v_4^2x_3^6x_5^6 \otimes x_{27}\\
&+ a_{276}v_4^2x_3^{10}x_9^2 \otimes x_{27} + a_{277}v_4^2x_5^6x_9^2 \otimes x_{27} + a_{278}v_4^2x_3^6x_{15}^2 \otimes x_{27}\\
&+ a_{279}v_4^2x_9^2x_{15}^2 \otimes x_{27} + a_{280}v_4^2x_3^{12}x_5^2 \otimes x_{29} + a_{281}v_4^2x_3^6x_5^2x_9^2 \otimes x_{29}\\
&+ a_{282}v_4^2x_3^2x_5^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar\psi(x_{17}) \equiv{}& v_4f_{(17,1)} + a_{283}v_4^2x_3^{12}x_5^4x_9^2 \otimes x_3 + a_{284}v_4^2x_3^8x_5^4x_{15}^2 \otimes x_3\\
&+ a_{285}v_4^2x_3^2x_5^4x_9^2x_{15}^2 \otimes x_3 + a_{286}v_4^2x_3^{14}x_5^6 \otimes x_5 + a_{287}v_4^2x_3^8x_5^6x_9^2 \otimes x_5\\
&+ a_{288}v_4^2x_3^{14}x_{15}^2 \otimes x_5 + a_{289}v_4^2x_3^4x_5^6x_{15}^2 \otimes x_5 + a_{290}v_4^2x_3^8x_9^2x_{15}^2 \otimes x_5\\
&+ a_{291}v_4^2x_3^{10}x_5^4x_9^2 \otimes x_9 + a_{292}v_4^2x_3^6x_5^4x_{15}^2 \otimes x_9 + a_{293}v_4^2x_5^4x_9^2x_{15}^2 \otimes x_9\\
&+ a_{294}v_4^2x_3^{14}x_5^4 \otimes x_{15} + a_{295}v_4^2x_3^8x_5^4x_9^2 \otimes x_{15} + a_{296}v_4^2x_3^4x_5^4x_{15}^2 \otimes x_{15}\\
&+ a_{297}v_4^2x_3^{10}x_5^6 \otimes x_{17} + a_{298}v_4^2x_3^{14}x_9^2 \otimes x_{17} + a_{299}v_4^2x_3^4x_5^6x_9^2 \otimes x_{17}\\
&+ a_{300}v_4^2x_3^{10}x_{15}^2 \otimes x_{17} + a_{301}v_4^2x_5^6x_{15}^2 \otimes x_{17} + a_{302}v_4^2x_3^4x_9^2x_{15}^2 \otimes x_{17}\\
&+ a_{303}v_4^2x_3^8x_5^6 \otimes x_{23} + a_{304}v_4^2x_3^{12}x_9^2 \otimes x_{23} + a_{305}v_4^2x_3^2x_5^6x_9^2 \otimes x_{23}\\
&+ a_{306}v_4^2x_3^8x_{15}^2 \otimes x_{23} + a_{307}v_4^2x_3^2x_9^2x_{15}^2 \otimes x_{23} + a_{308}v_4^2x_3^{10}x_5^4 \otimes x_{27}\\
&+ a_{309}v_4^2x_3^4x_5^4x_9^2 \otimes x_{27} + a_{310}v_4^2x_5^4x_{15}^2 \otimes x_{27} + a_{311}v_4^2x_3^6x_5^6 \otimes x_{29}\\
&+ a_{312}v_4^2x_3^{10}x_9^2 \otimes x_{29} + a_{313}v_4^2x_5^6x_9^2 \otimes x_{29} + a_{314}v_4^2x_3^6x_{15}^2 \otimes x_{29}\\
&+ a_{315}v_4^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar\psi(x_{23}) \equiv{}& x_5^4 \otimes x_3 + x_9^2 \otimes x_5 + x_3^2 \otimes x_{17} + v_4f_{(23,1)}\\
&+ a_{316}v_4^2x_3^{14}x_5^4x_9^2 \otimes x_3 + a_{317}v_4^2x_3^{10}x_5^4x_{15}^2 \otimes x_3\\
&+ a_{318}v_4^2x_3^4x_5^4x_9^2x_{15}^2 \otimes x_3 + a_{319}v_4^2x_3^{10}x_5^6x_9^2 \otimes x_5\\
&+ a_{320}v_4^2x_3^6x_5^6x_{15}^2 \otimes x_5 + a_{321}v_4^2x_3^{10}x_9^2x_{15}^2 \otimes x_5 + a_{322}v_4^2x_5^6x_9^2x_{15}^2 \otimes x_5\\
&+ a_{323}v_4^2x_3^{12}x_5^4x_9^2 \otimes x_9 + a_{324}v_4^2x_3^8x_5^4x_{15}^2 \otimes x_9\\
&+ a_{325}v_4^2x_3^2x_5^4x_9^2x_{15}^2 \otimes x_9 + a_{326}v_4^2x_3^{10}x_5^4x_9^2 \otimes x_{15}\\
&+ a_{327}v_4^2x_3^6x_5^4x_{15}^2 \otimes x_{15} + a_{328}v_4^2x_5^4x_9^2x_{15}^2 \otimes x_{15} + a_{329}v_4^2x_3^{12}x_5^6 \otimes x_{17}\\
&+ a_{330}v_4^2x_3^6x_5^6x_9^2 \otimes x_{17} + a_{331}v_4^2x_3^{12}x_{15}^2 \otimes x_{17} + a_{332}v_4^2x_3^2x_5^6x_{15}^2 \otimes x_{17}
\end{aligned}
$$

$$
\begin{aligned}
&+ a_{333}v_4^2x_3^6x_9^2x_{15}^2 \otimes x_{17} + a_{334}v_4^2x_3^{10}x_5^6 \otimes x_{23} + a_{335}v_4^2x_3^{14}x_9^2 \otimes x_{23}\\
&+ a_{336}v_4^2x_3^4x_5^6x_9^2 \otimes x_{23} + a_{337}v_4^2x_3^{10}x_{15}^2 \otimes x_{23} + a_{338}v_4^2x_5^6x_{15}^2 \otimes x_{23}\\
&+ a_{339}v_4^2x_3^4x_9^2x_{15}^2 \otimes x_{23} + a_{340}v_4^2x_3^{12}x_5^4 \otimes x_{27} + a_{341}v_4^2x_3^6x_5^4x_9^2 \otimes x_{27}\\
&+ a_{342}v_4^2x_3^2x_5^4x_{15}^2 \otimes x_{27} + a_{343}v_4^2x_3^8x_5^6 \otimes x_{29} + a_{344}v_4^2x_3^{12}x_9^2 \otimes x_{29}\\
&+ a_{345}v_4^2x_3^2x_5^6x_9^2 \otimes x_{29} + a_{346}v_4^2x_3^8x_{15}^2 \otimes x_{29} + a_{347}v_4^2x_3^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar{\psi}(x_{27}) \equiv\ & x_3^8 \otimes x_3 + x_9^2 \otimes x_9 + x_5^2 \otimes x_{17} + v_4 f_{(27,1)}\\
&+ a_{348}v_4^2x_3^{12}x_5^6x_9^2 \otimes x_3 + a_{349}v_4^2x_3^8x_5^6x_{15}^2 \otimes x_3 + a_{350}v_4^2x_3^{12}x_9^2x_{15}^2 \otimes x_3\\
&+ a_{351}v_4^2x_3^2x_5^6x_9^2x_{15}^2 \otimes x_3 + a_{352}v_4^2x_3^{14}x_5^2x_{15}^2 \otimes x_5\\
&+ a_{353}v_4^2x_3^8x_5^2x_9^2x_{15}^2 \otimes x_5 + a_{354}v_4^2x_3^{10}x_5^6x_9^2 \otimes x_9\\
&+ a_{355}v_4^2x_3^6x_5^6x_{15}^2 \otimes x_9 + a_{356}v_4^2x_3^{10}x_9^2x_{15}^2 \otimes x_9 + a_{357}v_4^2x_5^6x_9^2x_{15}^2 \otimes x_9\\
&+ a_{358}v_4^2x_3^{14}x_5^6 \otimes x_{15} + a_{359}v_4^2x_3^8x_5^6x_9^2 \otimes x_{15} + a_{360}v_4^2x_3^{14}x_{15}^2 \otimes x_{15}\\
&+ a_{361}v_4^2x_3^4x_5^6x_{15}^2 \otimes x_{15} + a_{362}v_4^2x_3^8x_9^2x_{15}^2 \otimes x_{15}\\
&+ a_{363}v_4^2x_3^{14}x_5^2x_9^2 \otimes x_{17} + a_{364}v_4^2x_3^{10}x_5^2x_{15}^2 \otimes x_{17}\\
&+ a_{365}v_4^2x_3^4x_5^2x_9^2x_{15}^2 \otimes x_{17} + a_{366}v_4^2x_3^{12}x_5^2x_9^2 \otimes x_{23}\\
&+ a_{367}v_4^2x_3^8x_5^2x_{15}^2 \otimes x_{23} + a_{368}v_4^2x_3^2x_5^2x_9^2x_{15}^2 \otimes x_{23}\\
&+ a_{369}v_4^2x_3^{10}x_5^6 \otimes x_{27} + a_{370}v_4^2x_3^{14}x_9^2 \otimes x_{27} + a_{371}v_4^2x_3^4x_5^6x_9^2 \otimes x_{27}\\
&+ a_{372}v_4^2x_3^{10}x_{15}^2 \otimes x_{27} + a_{373}v_4^2x_5^6x_{15}^2 \otimes x_{27} + a_{374}v_4^2x_3^4x_9^2x_{15}^2 \otimes x_{27}\\
&+ a_{375}v_4^2x_3^{10}x_5^2x_9^2 \otimes x_{29} + a_{376}v_4^2x_3^6x_5^2x_{15}^2 \otimes x_{29}\\
&+ a_{377}v_4^2x_5^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar{\psi}(x_{29}) \equiv\ & x_3^8 \otimes x_5 + x_5^4 \otimes x_9 + x_3^4 \otimes x_{17} + v_4 f_{(29,1)}\\
&+ a_{378}v_4^2x_3^{12}x_5^4x_{15}^2 \otimes x_3 + a_{379}v_4^2x_3^6x_5^4x_9^2x_{15}^2 \otimes x_3\\
&+ a_{380}v_4^2x_3^{12}x_5^6x_9^2 \otimes x_5 + a_{381}v_4^2x_3^8x_5^6x_{15}^2 \otimes x_5 + a_{382}v_4^2x_3^{12}x_9^2x_{15}^2 \otimes x_5\\
&+ a_{383}v_4^2x_3^2x_5^6x_9^2x_{15}^2 \otimes x_5 + a_{384}v_4^2x_3^{14}x_5^4x_9^2 \otimes x_9\\
&+ a_{385}v_4^2x_3^{10}x_5^4x_{15}^2 \otimes x_9 + a_{386}v_4^2x_3^4x_5^4x_9^2x_{15}^2 \otimes x_9\\
&+ a_{387}v_4^2x_3^{12}x_5^4x_9^2 \otimes x_{15} + a_{388}v_4^2x_3^8x_5^4x_{15}^2 \otimes x_{15}\\
&+ a_{389}v_4^2x_3^2x_5^4x_9^2x_{15}^2 \otimes x_{15} + a_{390}v_4^2x_3^{14}x_5^6 \otimes x_{17}\\
&+ a_{391}v_4^2x_3^8x_5^6x_9^2 \otimes x_{17} + a_{392}v_4^2x_3^{14}x_{15}^2 \otimes x_{17} + a_{393}v_4^2x_3^4x_5^6x_{15}^2 \otimes x_{17}\\
&+ a_{394}v_4^2x_3^8x_9^2x_{15}^2 \otimes x_{17} + a_{395}v_4^2x_3^{12}x_5^6 \otimes x_{23} + a_{396}v_4^2x_3^6x_5^6x_9^2 \otimes x_{23}\\
&+ a_{397}v_4^2x_3^{12}x_{15}^2 \otimes x_{23} + a_{398}v_4^2x_3^2x_5^6x_{15}^2 \otimes x_{23}
\end{aligned}
$$

$$+ a_{399}v_4^2x_3^6x_9^2x_{15}^2 \otimes x_{23} + a_{400}v_4^2x_3^{14}x_5^4 \otimes x_{27} + a_{401}v_4^2x_3^8x_5^4x_9^2 \otimes x_{27}$$
$$+ a_{402}v_4^2x_3^4x_5^4x_{15}^2 \otimes x_{27} + a_{403}v_4^2x_3^{10}x_5^6 \otimes x_{29} + a_{404}v_4^2x_3^{14}x_9^2 \otimes x_{29}$$
$$+ a_{405}v_4^2x_3^4x_5^6x_9^2 \otimes x_{29} + a_{406}v_4^2x_3^{10}x_{15}^2 \otimes x_{29} + a_{407}v_4^2x_5^6x_{15}^2 \otimes x_{29}$$
$$+ a_{408}v_4^2x_3^4x_9^2x_{15}^2 \otimes x_{29};$$

$$Q_0x_3 \equiv v_4g_{(0,3,1)} + b_{127}v_4^2x_3^{12}x_5^2x_9^2 + b_{128}v_4^2x_3^8x_5^2x_{15}^2 + b_{129}v_4^2x_3^2x_5^2x_9^2x_{15}^2,$$
$$Q_0x_5 \equiv x_3^2 + v_4g_{(0,5,1)} + b_{130}v_4^2x_3^{12}x_5^6 + b_{131}v_4^2x_3^6x_5^6x_9^2 + b_{132}v_4^2x_3^{12}x_{15}^2$$
$$+ b_{133}v_4^2x_3^2x_5^6x_{15}^2 + b_{134}v_4^2x_3^6x_9^2x_{15}^2,$$
$$Q_0x_9 \equiv x_5^2 + v_4g_{(0,9,1)} + b_{135}v_4^2x_3^{14}x_5^2x_9^2 + b_{136}v_4^2x_3^{10}x_5^2x_{15}^2$$
$$+ b_{137}v_4^2x_3^4x_5^2x_9^2x_{15}^2,$$
$$Q_0x_{15} \equiv x_3^2x_5^2 + v_4g_{(0,15,1)} + b_{138}v_4^2x_3^{12}x_5^2x_{15}^2 + b_{139}v_4^2x_3^6x_5^2x_9^2x_{15}^2,$$
$$Q_0x_{17} \equiv x_9^2 + v_4g_{(0,17,1)} + b_{140}v_4^2x_3^{10}x_5^6x_9^2 + b_{141}v_4^2x_3^6x_5^6x_{15}^2 + b_{142}v_4^2x_3^{10}x_9^2x_{15}^2$$
$$+ b_{143}v_4^2x_5^6x_9^2x_{15}^2,$$
$$Q_0x_{23} \equiv x_3^8 + x_3^2x_9^2 + v_4g_{(0,23,1)} + b_{144}v_4^2x_3^{12}x_5^6x_9^2 + b_{145}v_4^2x_3^8x_5^6x_{15}^2$$
$$+ b_{146}v_4^2x_3^{12}x_9^2x_{15}^2 + b_{147}v_4^2x_3^2x_5^6x_9^2x_{15}^2,$$
$$Q_0x_{27} \equiv x_5^2x_9^2 + v_4g_{(0,27,1)} + b_{148}v_4^2x_3^{10}x_5^2x_9^2x_{15}^2,$$
$$Q_0x_{29} \equiv x_{15}^2 + v_4g_{(0,29,1)} + b_{149}v_4^2x_3^{14}x_5^6x_9^2 + b_{150}v_4^2x_3^{10}x_5^6x_{15}^2$$
$$+ b_{151}v_4^2x_3^{14}x_9^2x_{15}^2 + b_{152}v_4^2x_3^4x_5^6x_9^2x_{15}^2,$$
$$Q_1x_3 \equiv x_3^2 + v_4g_{(1,3,1)} + b_{153}v_4^2x_3^{12}x_5^6 + b_{154}v_4^2x_3^6x_5^6x_9^2 + b_{155}v_4^2x_3^{12}x_{15}^2$$
$$+ b_{156}v_4^2x_3^2x_5^6x_{15}^2 + b_{157}v_4^2x_3^6x_9^2x_{15}^2,$$
$$Q_1x_5 \equiv v_4g_{(1,5,1)} + b_{158}v_4^2x_3^{10}x_5^4x_9^2 + b_{159}v_4^2x_3^6x_5^4x_{15}^2 + b_{160}v_4^2x_5^4x_9^2x_{15}^2,$$
$$Q_1x_9 \equiv x_3^4 + v_4g_{(1,9,1)} + b_{161}v_4^2x_3^{14}x_5^6 + b_{162}v_4^2x_3^8x_5^6x_9^2 + b_{163}v_4^2x_3^{14}x_{15}^2$$
$$+ b_{164}v_4^2x_3^4x_5^6x_{15}^2 + b_{165}v_4^2x_3^8x_9^2x_{15}^2,$$
$$Q_1x_{15} \equiv x_9^2 + x_3^6 + v_4g_{(1,15,1)} + b_{166}v_4^2x_3^{10}x_5^6x_9^2 + b_{167}v_4^2x_3^6x_5^6x_{15}^2$$
$$+ b_{168}v_4^2x_3^{10}x_9^2x_{15}^2 + b_{169}v_4^2x_5^6x_9^2x_{15}^2,$$
$$Q_1x_{17} \equiv x_5^4 + v_4g_{(1,17,1)} + b_{170}v_4^2x_3^{14}x_5^4x_9^2 + b_{171}v_4^2x_3^{10}x_5^4x_{15}^2$$
$$+ b_{172}v_4^2x_3^4x_5^4x_9^2x_{15}^2,$$
$$Q_1x_{23} \equiv x_3^2x_5^4 + v_4g_{(1,23,1)} + b_{173}v_4^2x_3^{12}x_5^4x_{15}^2 + b_{174}v_4^2x_3^6x_5^4x_9^2x_{15}^2,$$
$$Q_1x_{27} \equiv x_{15}^2 + x_3^4x_9^2 + x_5^6 + v_4g_{(1,27,1)} + b_{175}v_4^2x_3^{14}x_5^6x_9^2 + b_{176}v_4^2x_3^{10}x_5^6x_{15}^2$$
$$+ b_{177}v_4^2x_3^{14}x_9^2x_{15}^2 + b_{178}v_4^2x_3^4x_5^6x_9^2x_{15}^2,$$

$$
\begin{aligned}
Q_1x_{29} &\equiv x_3^4x_5^4 + v_4g_{(1,29,1)} + b_{179}v_4^2x_3^{14}x_5^4x_{15}^2 + b_{180}v_4^2x_3^8x_5^4x_9^2x_{15}^2,\\
Q_2x_3 &\equiv x_5^2 + v_4g_{(2,3,1)} + b_{181}v_4^2x_3^{14}x_5^2x_9^2 + b_{182}v_4^2x_3^{10}x_5^2x_{15}^2\\
&\quad + b_{183}v_4^2x_3^4x_5^2x_9^2x_{15}^2,\\
Q_2x_5 &\equiv x_3^4 + v_4g_{(2,5,1)} + b_{184}v_4^2x_3^{14}x_5^6 + b_{185}v_4^2x_3^8x_5^6x_9^2 + b_{186}v_4^2x_3^{14}x_{15}^2\\
&\quad + b_{187}v_4^2x_3^4x_5^6x_{15}^2 + b_{188}v_4^2x_3^8x_9^2x_{15}^2,\\
Q_2x_9 &\equiv v_4g_{(2,9,1)} + b_{189}v_4^2x_3^{12}x_5^2x_{15}^2 + b_{190}v_4^2x_3^6x_5^2x_9^2x_{15}^2,\\
Q_2x_{15} &\equiv x_3^4x_5^2 + v_4g_{(2,15,1)} + b_{191}v_4^2x_3^{14}x_5^2x_{15}^2 + b_{192}v_4^2x_3^8x_5^2x_9^2x_{15}^2,\\
Q_2x_{17} &\equiv x_3^8 + v_4g_{(2,17,1)} + b_{193}v_4^2x_3^{12}x_5^6x_9^2 + b_{194}v_4^2x_3^8x_5^6x_{15}^2 + b_{195}v_4^2x_3^{12}x_9^2x_{15}^2\\
&\quad + b_{196}v_4^2x_3^2x_5^6x_9^2x_{15}^2,\\
Q_2x_{23} &\equiv x_{15}^2 + x_3^{10} + x_3^4x_9^2 + v_4g_{(2,23,1)} + b_{197}v_4^2x_3^{14}x_5^6x_9^2 + b_{198}v_4^2x_3^{10}x_5^6x_{15}^2\\
&\quad + b_{199}v_4^2x_3^{14}x_9^2x_{15}^2 + b_{200}v_4^2x_3^4x_5^6x_9^2x_{15}^2,\\
Q_2x_{27} &\equiv x_3^8x_5^2 + v_4g_{(2,27,1)} + b_{201}v_4^2x_3^{12}x_5^2x_9^2x_{15}^2,\\
Q_2x_{29} &\equiv x_3^{12} + v_4g_{(2,29,1)} + b_{202}v_4^2x_3^{12}x_5^6x_{15}^2 + b_{203}v_4^2x_3^6x_5^6x_9^2x_{15}^2,\\
Q_3x_3 &\equiv x_9^2 + v_4g_{(3,3,1)} + b_{204}v_4^2x_3^{10}x_5^6x_9^2 + b_{205}v_4^2x_3^6x_5^6x_{15}^2 + b_{206}v_4^2x_3^{10}x_9^2x_{15}^2\\
&\quad + b_{207}v_4^2x_5^6x_9^2x_{15}^2,\\
Q_3x_5 &\equiv x_5^4 + v_4g_{(3,5,1)} + b_{208}v_4^2x_3^{14}x_5^4x_9^2 + b_{209}v_4^2x_3^{10}x_5^4x_{15}^2\\
&\quad + b_{210}v_4^2x_3^4x_5^4x_9^2x_{15}^2,\\
Q_3x_9 &\equiv x_3^8 + v_4g_{(3,9,1)} + b_{211}v_4^2x_3^{12}x_5^6x_9^2 + b_{212}v_4^2x_3^8x_5^6x_{15}^2 + b_{213}v_4^2x_3^{12}x_9^2x_{15}^2\\
&\quad + b_{214}v_4^2x_3^2x_5^6x_9^2x_{15}^2,\\
Q_3x_{15} &\equiv x_{15}^2 + x_3^{10} + x_5^6 + b_{215}v_4^2x_3^{14}x_5^6x_9^2 + b_{216}v_4^2x_3^{10}x_5^6x_{15}^2 + b_{217}v_4^2x_3^{14}x_9^2x_{15}^2\\
&\quad + b_{218}v_4^2x_3^4x_5^6x_9^2x_{15}^2,\\
Q_3x_{17} &\equiv v_4g_{(3,17,1)} + b_{219}v_4^2x_3^{14}x_5^4x_{15}^2 + b_{220}v_4^2x_3^8x_5^4x_9^2x_{15}^2,\\
Q_3x_{23} &\equiv x_5^4x_9^2 + v_4g_{(3,23,1)} + b_{221}v_4^2x_3^{10}x_5^4x_9^2x_{15}^2,\\
Q_3x_{27} &\equiv x_3^8x_9^2 + v_4g_{(3,27,1)} + b_{222}v_4^2x_3^{14}x_5^6x_{15}^2 + b_{223}v_4^2x_3^8x_5^6x_9^2x_{15}^2,\\
Q_3x_{29} &\equiv x_3^8x_5^4 + v_4g_{(3,29,1)} + b_{224}v_4^2x_3^{12}x_5^4x_9^2x_{15}^2;
\end{aligned}
$$

$$
\begin{aligned}
x_3^{16} &\equiv c_{27}v_4^2x_3^{10}x_5^6x_9^2x_{15}^2, & x_5^8 &\equiv c_{28}v_4^2x_3^{14}x_5^2x_9^2x_{15}^2,\\
x_9^4 &\equiv c_{29}v_4^2x_3^{12}x_5^6x_{15}^2 + c_{30}v_4^2x_3^6x_5^6x_9^2x_{15}^2, & x_{15}^4 &\equiv c_{31}v_4^2x_3^{14}x_5^6x_9^2x_{15}^2,\\
x_{17}^2 &\equiv c_{32}v_4^2x_3^{12}x_5^2x_9^2x_{15}^2, & x_{23}^2 &\equiv 0,\\
x_{27}^2 &\equiv c_{33}v_4^2x_3^{12}x_5^6x_9^2x_{15}^2, & x_{29}^2 &\equiv 0,
\end{aligned}
$$

where $\equiv$ is mod $I(2,0,0,\cdots)$ and $a_i, b_i, c_i = 0, 1$. It is easy to calculate that

$$\bar\psi(x_3^2) \equiv v_4x_9^2\otimes x_9^2 + v_4^2x_3^8x_9^2\otimes x_3^8 + v_4^2x_3^8\otimes x_3^8x_9^2, \qquad \bar\psi(x_3^4)\equiv 0,$$
$$\bar\psi(x_5^2)\equiv v_4x_5^4\otimes x_5^4, \qquad \bar\psi(x_5^4)\equiv 0, \qquad \bar\psi(x_9^2)\equiv v_4x_3^8\otimes x_3^8,$$
$$\begin{aligned}
\bar\psi(x_{15}^2) \equiv{}& x_3^8\otimes x_3^2 + x_5^4\otimes x_5^2 + x_3^4\otimes x_9^2\\
&+ v_4x_3^2\otimes x_3^8x_5^6 + v_4x_3^2\otimes x_3^8x_{15}^2 + v_4x_3^2x_5^2\otimes x_3^8x_5^4 + v_4x_3^2x_5^6\otimes x_3^8\\
&+ v_4x_3^2x_{15}^2\otimes x_3^8 + v_4x_3^4\otimes x_3^{10}x_9^2 + v_4x_3^4\otimes x_5^6x_9^2 + v_4x_3^4\otimes x_9^2x_{15}^2\\
&+ v_4x_3^4x_5^2\otimes x_5^4x_9^2 + v_4x_3^4x_5^6\otimes x_9^2 + v_4x_3^4x_{15}^2\otimes x_9^2 + v_4x_3^6\otimes x_3^8x_9^2\\
&+ v_4x_3^{10}\otimes x_3^{10} + v_4x_3^{10}\otimes x_5^6 + v_4x_3^{10}\otimes x_{15}^2 + v_4x_3^{10}x_5^2\otimes x_5^4\\
&+ v_4x_3^{12}\otimes x_3^8 + v_4x_3^{14}\otimes x_9^2 + v_4x_5^2\otimes x_3^{10}x_5^4 + v_4x_5^2\otimes x_5^4x_{15}^2\\
&+ v_4x_5^2x_{15}^2\otimes x_5^4 + v_4x_5^6\otimes x_3^{10} + v_4x_5^6\otimes x_5^6 + v_4x_5^6\otimes x_{15}^2\\
&+ v_4x_{15}^2\otimes x_3^{10} + v_4x_{15}^2\otimes x_5^6 + v_4x_{15}^2\otimes x_{15}^2\\
&+ v_4^2x_3^2x_5^2\otimes x_3^8x_5^4x_{15}^2 + v_4^2x_3^2x_5^2x_{15}^2\otimes x_3^8x_5^4 + v_4^2x_3^2x_5^6\otimes x_3^8x_5^6\\
&+ v_4^2x_3^2x_5^6\otimes x_3^8x_{15}^2 + v_4^2x_3^2x_{15}^2\otimes x_3^8x_5^6 + v_4^2x_3^2x_{15}^2\otimes x_3^8x_{15}^2\\
&+ v_4^2x_3^4x_5^2\otimes x_3^{10}x_5^4x_9^2 + v_4^2x_3^4x_5^2\otimes x_5^4x_9^2x_{15}^2 + v_4^2x_3^4x_5^2x_{15}^2\otimes x_5^4x_9^2\\
&+ v_4^2x_3^4x_5^6\otimes x_3^{10}x_9^2 + v_4^2x_3^4x_5^6\otimes x_5^6x_9^2 + v_4^2x_3^4x_5^6\otimes x_9^2x_{15}^2\\
&+ v_4^2x_3^4x_{15}^2\otimes x_3^{10}x_9^2 + v_4^2x_3^4x_{15}^2\otimes x_5^6x_9^2 + v_4^2x_3^4x_{15}^2\otimes x_9^2x_{15}^2\\
&+ v_4^2x_3^6\otimes x_3^8x_5^6x_9^2 + v_4^2x_3^6\otimes x_3^8x_9^2x_{15}^2 + v_4^2x_3^6x_5^2\otimes x_3^8x_5^4x_9^2\\
&+ v_4^2x_3^6x_5^6\otimes x_3^8x_9^2 + v_4^2x_3^6x_{15}^2\otimes x_3^8x_9^2 + v_4^2x_3^8x_5^2x_9^2\otimes x_5^4x_9^2\\
&+ v_4^2x_3^8x_5^6x_9^2\otimes x_9^2 + v_4^2x_3^8x_9^2\otimes x_3^{10}x_9^2 + v_4^2x_3^8x_9^2\otimes x_5^6x_9^2\\
&+ v_4^2x_3^8x_9^2\otimes x_9^2x_{15}^2 + v_4^2x_3^8x_9^2x_{15}^2\otimes x_9^2 + v_4^2x_3^{10}x_5^2\otimes x_3^{10}x_5^4\\
&+ v_4^2x_3^{10}x_5^2\otimes x_5^4x_{15}^2 + v_4^2x_3^{12}\otimes x_3^8x_5^6 + v_4^2x_3^{12}\otimes x_3^8x_{15}^2\\
&+ v_4^2x_3^{12}x_5^2\otimes x_3^8x_5^4 + v_4^2x_3^{14}\otimes x_3^{10}x_9^2 + v_4^2x_3^{14}\otimes x_5^6x_9^2\\
&+ v_4^2x_3^{14}\otimes x_9^2x_{15}^2 + v_4^2x_3^{14}x_5^2\otimes x_5^4x_9^2 + v_4^2x_5^2x_9^2\otimes x_3^8x_5^4x_9^2\\
&+ v_4^2x_5^2x_{15}^2\otimes x_3^{10}x_5^4 + v_4^2x_5^2x_{15}^2\otimes x_5^4x_{15}^2 + v_4^2x_5^6x_9^2\otimes x_3^8x_9^2\\
&+ v_4^2x_9^2\otimes x_3^8x_5^6x_9^2 + v_4^2x_9^2\otimes x_3^8x_9^2x_{15}^2 + v_4^2x_9^2x_{15}^2\otimes x_3^8x_9^2,
\end{aligned}$$

where $\equiv$ is mod $I(2,0,0,\cdots)$. In a similar way, comparing the coefficient of $Q_i\bar\psi(x_j)$ and $\bar\psi(Q_ix_j)$, we can determine all the coefficients a_i and b_i.

Lemma 3.4. *We have*

$$a_i = 0 \text{ for } i = 157, 160, 161, 164, 171, 174, 176, 182, 192, 193, 194, 196, 198, 201, 203, 204, 205, 209, 215, 216, 219, 221, 222, 223, 225, 227, 228, 230, 231, 233, 234, 236, 237, 238, 241, 242, 243, 245, 247, 248, 250, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 267, 268, 269, 276, 281, 285, 286, 287, 288, 289, 291, 292, 294, 296, 298, 299, 303, 305, 307, 308, 311, 312, 313, 314, 317, 320, 321, 327, 329, 330, 336, 337, 340, 343, 345, 349, 351, 352, 355, 357, 358, 360, 361, 363, 368, 370, 373, 375, 376, 379, 380, 382, 383, 384, 386, 387, 289, 391, 394, 395, 396, 398, 399, 400, 401, 404, 405, 408;$$

$$a_i = 1 \text{ for } i = 158, 159, 162, 163, 165, 166, 167, 168, 169, 170, 172, 173, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 195, 197, 199, 200, 202, 206, 207, 208, 210, 211, 212, 213, 214, 217, 218, 220, 224, 226, 229, 232, 235, 239, 240, 244, 246, 249, 251, 254, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 282, 283, 284, 290, 293, 295, 297, 300, 301, 302, 304, 306, 309, 310, 315, 316, 318, 319, 322, 323, 324, 325, 326, 328, 331, 332, 333, 334, 335, 338, 339, 341, 342, 344, 346, 347, 348, 350, 353, 354, 356, 359, 362, 364, 365, 366, 367, 369, 371, 372, 374, 377, 378, 381, 385, 388, 390, 392, 393, 397, 402, 403, 406, 407;$$

$$b_i = 0 \text{ for } i = 131, 132, 134, 135, 137, 138, 139, 141, 146, 149, 151, 152, 156, 158, 162, 164, 165, 167, 169, 170, 176, 178, 180, 182, 185, 186, 190, 191, 193, 196, 198, 199, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224;$$

$$b_i = 1 \text{ for } i = 127, 128, 129, 130, 133, 136, 140, 142, 143, 144, 145, 147, 148, 150, 153, 154, 155, 157, 159, 160, 161, 163, 166, 168, 171, 172, 173, 174, 175, 177, 179, 181, 183, 184, 187, 188, 189, 192, 194, 195, 197, 200, 201, 202, 220.$$

Notation 3.5. For simplicity we write

$$\bar{\psi}(x_3) \equiv v_4 f_{(3,1)} + v_4^2 f_{(3,2)}, \qquad \bar{\psi}(x_5) \equiv v_4 f_{(5,1)} + v_4^2 f_{(5,2)},$$
$$\bar{\psi}(x_9) \equiv v_4 f_{(9,1)} + v_4^2 f_{(9,2)}, \qquad \bar{\psi}(x_{17}) \equiv v_4 f_{(17,1)} + v_4^2 f_{(17,2)},$$

$$\bar{\psi}(x_{15}) \equiv x_3^4 \otimes x_3 + x_5^2 \otimes x_5 + x_3^2 \otimes x_9 + v_4 f_{(15,1)} + v_4^2 f_{(15,2)},$$
$$\bar{\psi}(x_{23}) \equiv x_5^4 \otimes x_3 + x_9^2 \otimes x_5 + x_3^2 \otimes x_{17} + v_4 f_{(23,1)} + v_4^2 f_{(23,2)},$$
$$\bar{\psi}(x_{27}) \equiv x_3^8 \otimes x_3 + x_9^2 \otimes x_9 + x_5^2 \otimes x_{17} + v_4 f_{(27,1)} + v_4^2 f_{(27,2)},$$
$$\bar{\psi}(x_{29}) \equiv x_3^8 \otimes x_5 + x_5^4 \otimes x_9 + x_3^4 \otimes x_{17} + v_4 f_{(29,1)} + v_4^2 f_{(29,2)};$$

$$Q_0 x_3 \equiv v_4 g_{(0,3,1)} + v_4^2 g_{(0,3,2)}, \qquad Q_0 x_5 \equiv x_3^2 + v_4 g_{(0,5,1)} + v_4^2 g_{(0,5,2)},$$
$$Q_0 x_9 \equiv x_5^2 + v_4 g_{(0,9,1)} + v_4^2 g_{(0,9,2)},$$
$$Q_0 x_{15} \equiv x_3^2 x_5^2 + v_4 g_{(0,15,1)} + v_4^2 g_{(0,15,2)},$$
$$Q_0 x_{17} \equiv x_9^2 + v_4 g_{(0,17,1)} + v_4^2 g_{(0,17,2)},$$
$$Q_0 x_{23} \equiv x_3^8 + x_3^2 x_9^2 + v_4 g_{(0,23,1)} + v_4^2 g_{(0,23,2)},$$
$$Q_0 x_{27} \equiv x_5^2 x_9^2 + v_4 g_{(0,27,1)} + v_4^2 g_{(0,27,2)},$$
$$Q_0 x_{29} \equiv x_{15}^2 + v_4 g_{(0,29,1)} + v_4^2 g_{(0,29,2)},$$
$$Q_1 x_3 \equiv x_3^2 + v_4 g_{(1,3,1)} + v_4^2 g_{(1,3,2)}, \qquad Q_1 x_5 \equiv v_4 g_{(1,5,1)} + v_4^2 g_{(1,5,2)},$$
$$Q_1 x_9 \equiv x_3^4 + v_4 g_{(1,9,1)} + v_4^2 g_{(1,9,2)},$$
$$Q_1 x_{15} \equiv x_9^2 + x_3^6 + v_4 g_{(1,15,1)} + v_4^2 g_{(1,15,2)},$$
$$Q_1 x_{17} \equiv x_5^4 + v_4 g_{(1,17,1)} + v_4^2 g_{(1,17,2)},$$
$$Q_1 x_{23} \equiv x_3^2 x_5^4 + v_4 g_{(1,23,1)} + v_4^2 g_{(1,23,2)},$$
$$Q_1 x_{27} \equiv x_{15}^2 + x_3^4 x_9^2 + x_5^6 + v_4 g_{(1,27,1)} + v_4^2 g_{(1,27,2)},$$
$$Q_1 x_{29} \equiv x_3^4 x_5^4 + v_4 g_{(1,29,1)} + v_4^2 g_{(1,29,2)},$$
$$Q_2 x_3 \equiv x_5^2 + v_4 g_{(2,3,1)} + v_4^2 g_{(1,3,2)}, \qquad Q_2 x_5 \equiv x_3^4 + v_4 g_{(2,5,1)} + v_4^2 g_{(2,5,2)},$$
$$Q_2 x_9 \equiv v_4 g_{(2,9,1)} + v_4^2 g_{(2,9,2)},$$
$$Q_2 x_{15} \equiv x_3^4 x_5^2 + v_4 g_{(2,15,1)} + v_4^2 g_{(2,15,2)},$$
$$Q_2 x_{17} \equiv x_3^8 + v_4 g_{(2,17,1)} + v_4^2 g_{(2,17,2)},$$
$$Q_2 x_{23} \equiv x_{15}^2 + x_3^{10} + x_3^4 x_9^2 + v_4 g_{(2,23,1)} + v_4^2 g_{(2,23,2)},$$
$$Q_2 x_{27} \equiv x_3^8 x_5^2 + v_4 g_{(2,27,1)} + v_4^2 g_{(2,27,2)},$$
$$Q_2 x_{29} \equiv x_3^{12} + v_4 g_{(2,29,1)} + v_4^2 g_{(2,29,2)},$$
$$Q_3 x_3 \equiv x_9^2 + v_4 g_{(3,3,1)}, \qquad Q_3 x_5 \equiv x_5^4 + v_4 g_{(3,5,1)},$$
$$Q_3 x_9 \equiv x_3^8 + v_4 g_{(3,9,1)}, \qquad Q_3 x_{15} \equiv x_{15}^2 + x_3^{10} + x_5^6,$$
$$Q_3 x_{17} \equiv v_4 g_{(3,17,1)} + v_4^2 g_{(3,17,2)}, \qquad Q_3 x_{23} \equiv x_5^4 x_9^2 + v_4 g_{(3,23,1)},$$
$$Q_3 x_{27} \equiv x_3^8 x_9^2 + v_4 g_{(3,27,1)}, \qquad Q_3 x_{29} \equiv x_3^8 x_5^4 + v_4 g_{(3,29,1)},$$

where $\equiv$ is mod $I(2,0,0,\cdots)$.

The elements x_3^{16}, x_5^8, x_9^4, x_{15}^4, x_{17}^2, x_{23}^2, x_{27}^2, x_{29}^2 are primitive mod $I(2,0,0,\cdots)$, but there is no primitive element mod $I(2,0,0,\cdots)$ in their degree. Thus all c_i are zero.

Next we consider mod $I(3,0,0,\cdots)$ in the same way. For degree reasons we can see that

$$x_3^{16} \equiv x_5^8 \equiv x_9^4 \equiv x_{15}^4 \equiv x_{17}^2 \equiv x_{23}^2 \equiv x_{27}^2 \equiv x_{29}^2 \equiv 0.$$

We can put

$$\begin{aligned}
\bar\psi(x_3) \equiv\ & v_4 f_{(3,1)} + v_4^2 f_{(3,2)} + a_{409} v_4^3 x_3^{14} x_5^6 x_9^2 \otimes x_3 + a_{410} v_4^3 x_3^{10} x_5^6 x_{15}^2 \otimes x_3 \\
& + a_{411} v_4^3 x_3^{14} x_9^2 x_{15}^2 \otimes x_3 + a_{412} v_4^3 x_3^4 x_5^6 x_9^2 x_{15}^2 \otimes x_3 \\
& + a_{413} v_4^3 x_3^{10} x_5^2 x_9^2 x_{15}^2 \otimes x_5 + a_{414} v_4^3 x_3^{12} x_5^6 x_9^2 \otimes x_9 \\
& + a_{415} v_4^3 x_3^8 x_5^6 x_{15}^2 \otimes x_9 + a_{416} v_4^3 x_3^{12} x_9^2 x_{15}^2 \otimes x_9 \\
& + a_{417} v_4^3 x_3^2 x_5^6 x_9^2 x_{15}^2 \otimes x_9 + a_{418} v_4^3 x_3^{10} x_5^6 x_9^2 \otimes x_{15} \\
& + a_{419} v_4^3 x_3^6 x_5^6 x_{15}^2 \otimes x_{15} + a_{420} v_4^3 x_3^{10} x_9^2 x_{15}^2 \otimes x_{15} \\
& + a_{421} v_4^3 x_5^6 x_9^2 x_{15}^2 \otimes x_{15} + a_{422} v_4^3 x_3^{12} x_5^2 x_{15}^2 \otimes x_{17} \\
& + a_{423} v_4^3 x_3^6 x_5^2 x_9^2 x_{15}^2 \otimes x_{17} + a_{424} v_4^3 x_3^{14} x_5^2 x_9^2 \otimes x_{23} \\
& + a_{425} v_4^3 x_3^{10} x_5^2 x_{15}^2 \otimes x_{23} + a_{426} v_4^3 x_3^4 x_5^2 x_9^2 x_{15}^2 \otimes x_{23} \\
& + a_{427} v_4^3 x_3^{12} x_5^6 \otimes x_{27} + a_{428} v_4^3 x_3^6 x_5^6 x_9^2 \otimes x_{27} \\
& + a_{429} v_4^3 x_3^{12} x_{15}^2 \otimes x_{27} + a_{430} v_4^3 x_3^2 x_5^6 x_{15}^2 \otimes x_{27} \\
& + a_{431} v_4^3 x_3^6 x_9^2 x_{15}^2 \otimes x_{27} + a_{432} v_4^3 x_3^{12} x_5^2 x_9^2 \otimes x_{29} \\
& + a_{433} v_4^3 x_3^8 x_5^2 x_{15}^2 \otimes x_{29} + a_{434} v_4^3 x_3^2 x_5^2 x_9^2 x_{15}^2 \otimes x_{29}, \\
\bar\psi(x_5) \equiv\ & v_4 f_{(5,1)} + v_4^2 f_{(5,2)} + a_{435} v_4^3 x_3^{14} x_5^4 x_{15}^2 \otimes x_3 + a_{436} v_4^3 x_3^8 x_5^4 x_9^2 x_{15}^2 \otimes x_3 \\
& + a_{437} v_4^3 x_3^{14} x_5^6 x_9^2 \otimes x_5 + a_{438} v_4^3 x_3^{10} x_5^6 x_{15}^2 \otimes x_5 \\
& + a_{439} v_4^3 x_3^{14} x_9^2 x_{15}^2 \otimes x_5 + a_{440} v_4^3 x_3^4 x_5^6 x_9^2 x_{15}^2 \otimes x_5 \\
& + a_{441} v_4^3 x_3^{12} x_5^4 x_{15}^2 \otimes x_9 + a_{442} v_4^3 x_3^6 x_5^4 x_9^2 x_{15}^2 \otimes x_9 \\
& + a_{443} v_4^3 x_3^{14} x_5^4 x_9^2 \otimes x_{15} + a_{444} v_4^3 x_3^{10} x_5^4 x_{15}^2 \otimes x_{15} \\
& + a_{445} v_4^3 x_3^4 x_5^4 x_9^2 x_{15}^2 \otimes x_{15} + a_{446} v_4^3 x_3^{10} x_5^6 x_9^2 \otimes x_{17} \\
& + a_{447} v_4^3 x_3^6 x_5^6 x_{15}^2 \otimes x_{17} + a_{448} v_4^3 x_3^{10} x_9^2 x_{15}^2 \otimes x_{17} \\
& + a_{449} v_4^3 x_5^6 x_9^2 x_{15}^2 \otimes x_{17} + a_{450} v_4^3 x_3^{14} x_5^6 \otimes x_{23} + a_{451} v_4^3 x_3^8 x_5^6 x_9^2 \otimes x_{23} \\
& + a_{452} v_4^3 x_3^{14} x_{15}^2 \otimes x_{23} + a_{453} v_4^3 x_3^4 x_5^6 x_{15}^2 \otimes x_{23} \\
& + a_{454} v_4^3 x_3^8 x_9^2 x_{15}^2 \otimes x_{23} + a_{455} v_4^3 x_3^{10} x_5^4 x_9^2 \otimes x_{27}
\end{aligned}$$

$$
\begin{aligned}
&+ a_{456}v_4^3x_3^6x_5^4x_{15}^2 \otimes x_{27} + a_{457}v_4^3x_5^4x_9^2x_{15}^2 \otimes x_{27} + a_{458}v_4^3x_3^{12}x_5^6 \otimes x_{29} \\
&+ a_{459}v_4^3x_3^6x_5^6x_9^2 \otimes x_{29} + a_{460}v_4^3x_3^{12}x_{15}^2 \otimes x_{29} + a_{461}v_4^3x_3^2x_5^6x_{15}^2 \otimes x_{29} \\
&+ a_{462}v_4^3x_3^6x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar\psi(x_9) \equiv\ & v_4f_{(9,1)} + v_4^2f_{(9,2)} + a_{463}v_4^3x_3^{12}x_5^6x_{15}^2 \otimes x_3 + a_{464}v_4^3x_3^6x_5^6x_9^2x_{15}^2 \otimes x_3 \\
&+ a_{465}v_4^3x_3^{12}x_5^2x_9^2x_{15}^2 \otimes x_5 + a_{466}v_4^3x_3^{14}x_5^6x_9^2 \otimes x_9 \\
&+ a_{467}v_4^3x_3^{10}x_5^6x_{15}^2 \otimes x_9 + a_{468}v_4^3x_3^{14}x_9^2x_{15}^2 \otimes x_9 \\
&+ a_{469}v_4^3x_3^4x_5^6x_9^2x_{15}^2 \otimes x_9 + a_{470}v_4^3x_3^{12}x_5^6x_9^2 \otimes x_{15} \\
&+ a_{471}v_4^3x_3^8x_5^6x_{15}^2 \otimes x_{15} + a_{472}v_4^3x_3^{12}x_9^2x_{15}^2 \otimes x_{15} \\
&+ a_{473}v_4^3x_3^2x_5^6x_9^2x_{15}^2 \otimes x_{15} + a_{474}v_4^3x_3^{14}x_5^2x_{15}^2 \otimes x_{17} \\
&+ a_{475}v_4^3x_3^8x_5^2x_9^2x_{15}^2 \otimes x_{17} + a_{476}v_4^3x_3^{12}x_5^2x_{15}^2 \otimes x_{23} \\
&+ a_{477}v_4^3x_3^6x_5^2x_9^2x_{15}^2 \otimes x_{23} + a_{478}v_4^3x_3^{14}x_5^6 \otimes x_{27} \\
&+ a_{479}v_4^3x_3^8x_5^6x_9^2 \otimes x_{27} + a_{480}v_4^3x_3^{14}x_{15}^2 \otimes x_{27} + a_{481}v_4^3x_3^4x_5^6x_{15}^2 \otimes x_{27} \\
&+ a_{482}v_4^3x_3^8x_9^2x_{15}^2 \otimes x_{27} + a_{483}v_4^3x_3^{14}x_5^2x_9^2 \otimes x_{29} \\
&+ a_{484}v_4^3x_3^{10}x_5^2x_{15}^2 \otimes x_{29} + a_{485}v_4^3x_3^4x_5^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar\psi(x_{15}) \equiv\ & x_3^4 \otimes x_3 + x_5^2 \otimes x_5 + x_3^2 \otimes x_9 + v_4f_{(15,1)} + v_4^2f_{(15,2)} \\
&+ a_{486}v_4^3x_3^{14}x_5^6x_{15}^2 \otimes x_3 + a_{487}v_4^3x_3^8x_5^6x_9^2x_{15}^2 \otimes x_3 \\
&+ a_{488}v_4^3x_3^{14}x_5^2x_9^2x_{15}^2 \otimes x_5 + a_{489}v_4^3x_3^{12}x_5^6x_{15}^2 \otimes x_9 \\
&+ a_{490}v_4^3x_3^6x_5^6x_9^2x_{15}^2 \otimes x_9 + a_{491}v_4^3x_3^{14}x_5^6x_9^2 \otimes x_{15} \\
&+ a_{492}v_4^3x_3^{10}x_5^6x_{15}^2 \otimes x_{15} + a_{493}v_4^3x_3^{14}x_9^2x_{15}^2 \otimes x_{15} \\
&+ a_{494}v_4^3x_3^4x_5^6x_9^2x_{15}^2 \otimes x_{15} + a_{495}v_4^3x_3^{10}x_5^2x_9^2x_{15}^2 \otimes x_{17} \\
&+ a_{496}v_4^3x_3^{14}x_5^2x_{15}^2 \otimes x_{23} + a_{497}v_4^3x_3^8x_5^2x_9^2x_{15}^2 \otimes x_{23} \\
&+ a_{498}v_4^3x_3^{10}x_5^6x_9^2 \otimes x_{27} + a_{499}v_4^3x_3^6x_5^6x_{15}^2 \otimes x_{27} \\
&+ a_{500}v_4^3x_3^{10}x_9^2x_{15}^2 \otimes x_{27} + a_{501}v_4^3x_5^6x_9^2x_{15}^2 \otimes x_{27} \\
&+ a_{502}v_4^3x_3^{12}x_5^2x_{15}^2 \otimes x_{29} + a_{503}v_4^3x_3^6x_5^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar\psi(x_{17}) \equiv\ & v_4f_{(17,1)} + v_4^2f_{(17,2)} + a_{504}v_4^3x_3^{12}x_5^4x_9^2x_{15}^2 \otimes x_3 \\
&+ a_{505}v_4^3x_3^{14}x_5^6x_{15}^2 \otimes x_5 + a_{506}v_4^3x_3^8x_5^6x_9^2x_{15}^2 \otimes x_5 \\
&+ a_{507}v_4^3x_3^{10}x_5^4x_9^2x_{15}^2 \otimes x_9 + a_{508}v_4^3x_3^{14}x_5^4x_{15}^2 \otimes x_{15} \\
&+ a_{509}v_4^3x_3^8x_5^4x_9^2x_{15}^2 \otimes x_{15} + a_{510}v_4^3x_3^{14}x_5^6x_9^2 \otimes x_{17} \\
&+ a_{511}v_4^3x_3^{10}x_5^6x_{15}^2 \otimes x_{17} + a_{512}v_4^3x_3^{14}x_9^2x_{15}^2 \otimes x_{17}
\end{aligned}
$$

$$
\begin{aligned}
&+ a_{513}v_4^3x_3^4x_5^6x_9^2x_{15}^2 \otimes x_{17} + a_{514}v_4^3x_3^{12}x_5^6x_9^2 \otimes x_{23} \\
&+ a_{515}v_4^3x_3^8x_5^6x_{15}^2 \otimes x_{23} + a_{516}v_4^3x_3^{12}x_9^2x_{15}^2 \otimes x_{23} \\
&+ a_{517}v_4^3x_3^2x_5^6x_9^2x_{15}^2 \otimes x_{23} + a_{518}v_4^3x_3^{14}x_5^4x_9^2 \otimes x_{27} \\
&+ a_{519}v_4^3x_3^{10}x_5^4x_{15}^2 \otimes x_{27} + a_{520}v_4^3x_3^4x_5^4x_9^2x_{15}^2 \otimes x_{27} \\
&+ a_{521}v_4^3x_3^{10}x_5^6x_9^2 \otimes x_{29} + a_{522}v_4^3x_3^6x_5^6x_{15}^2 \otimes x_{29} \\
&+ a_{523}v_4^3x_3^{10}x_9^2x_{15}^2 \otimes x_{29} + a_{524}v_4^3x_5^6x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar{\psi}(x_{23}) \equiv\ & x_5^4 \otimes x_3 + x_9^2 \otimes x_5 + x_3^2 \otimes x_{17} + v_4 f_{(23,1)} + v_4^2 f_{(23,2)} \\
&+ a_{525}v_4^3x_3^{14}x_5^4x_9^2x_{15}^2 \otimes x_3 + a_{526}v_4^3x_3^{10}x_5^6x_9^2x_{15}^2 \otimes x_5 \\
&+ a_{527}v_4^3x_3^{12}x_5^4x_9^2x_{15}^2 \otimes x_9 + a_{528}v_4^3x_3^{10}x_5^4x_9^2x_{15}^2 \otimes x_{15} \\
&+ a_{529}v_4^3x_3^{12}x_5^6x_{15}^2 \otimes x_{17} + a_{530}v_4^3x_3^6x_5^6x_9^2x_{15}^2 \otimes x_{17} \\
&+ a_{531}v_4^3x_3^{14}x_5^6x_9^2 \otimes x_{23} + a_{532}v_4^3x_3^{10}x_5^6x_{15}^2 \otimes x_{23} \\
&+ a_{533}v_4^3x_3^{14}x_9^2x_{15}^2 \otimes x_{23} + a_{534}v_4^3x_3^4x_5^6x_9^2x_{15}^2 \otimes x_{23} \\
&+ a_{535}v_4^3x_3^{12}x_5^4x_{15}^2 \otimes x_{27} + a_{536}v_4^3x_3^6x_5^4x_9^2x_{15}^2 \otimes x_{27} \\
&+ a_{537}v_4^3x_3^{12}x_5^6x_9^2 \otimes x_{29} + a_{538}v_4^3x_3^8x_5^6x_{15}^2 \otimes x_{29} \\
&+ a_{539}v_4^3x_3^{12}x_9^2x_{15}^2 \otimes x_{29} + a_{540}v_4^3x_3^2x_5^6x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar{\psi}(x_{27}) \equiv\ & x_3^8 \otimes x_3 + x_9^2 \otimes x_9 + x_5^2 \otimes x_{17} + v_4 f_{(27,1)} + v_4^2 f_{(27,2)} \\
&+ a_{541}v_4^3x_3^{12}x_5^6x_9^2x_{15}^2 \otimes x_3 + a_{542}v_4^3x_3^{10}x_5^6x_9^2x_{15}^2 \otimes x_9 \\
&+ a_{543}v_4^3x_3^{14}x_5^6x_{15}^2 \otimes x_{15} + a_{544}v_4^3x_3^8x_5^6x_9^2x_{15}^2 \otimes x_{15} \\
&+ a_{545}v_4^3x_3^{14}x_5^2x_9^2x_{15}^2 \otimes x_{17} + a_{546}v_4^3x_3^{12}x_5^2x_9^2x_{15}^2 \otimes x_{23} \\
&+ a_{547}v_4^3x_3^{14}x_5^6x_9^2 \otimes x_{27} + a_{548}v_4^3x_3^{10}x_5^6x_{15}^2 \otimes x_{27} \\
&+ a_{549}v_4^3x_3^{14}x_9^2x_{15}^2 \otimes x_{27} + a_{550}v_4^3x_3^4x_5^6x_9^2x_{15}^2 \otimes x_{27} \\
&+ a_{551}v_4^3x_3^{10}x_5^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}
$$

$$
\begin{aligned}
\bar{\psi}(x_{29}) \equiv\ & x_3^8 \otimes x_5 + x_5^4 \otimes x_9 + x_3^4 \otimes x_{17} + v_4 f_{(27,1)} + v_4^2 f_{(27,2)} \\
&+ a_{552}v_4^3x_3^{12}x_5^6x_9^2x_{15}^2 \otimes x_5 + a_{553}v_4^3x_3^{14}x_5^4x_9^2x_{15}^2 \otimes x_9 \\
&+ a_{554}v_4^3x_3^{12}x_5^4x_9^2x_{15}^2 \otimes x_{15} + a_{555}v_4^3x_3^{14}x_5^6x_{15}^2 \otimes x_{17} \\
&+ a_{556}v_4^3x_3^8x_5^6x_9^2x_{15}^2 \otimes x_{17} + a_{557}v_4^3x_3^{12}x_5^6x_{15}^2 \otimes x_{23} \\
&+ a_{558}v_4^3x_3^6x_5^6x_9^2x_{15}^2 \otimes x_{23} + a_{559}v_4^3x_3^{14}x_5^4x_{15}^2 \otimes x_{27} \\
&+ a_{560}v_4^3x_3^8x_5^4x_9^2x_{15}^2 \otimes x_{27} + a_{561}v_4^3x_3^{14}x_5^6x_9^2 \otimes x_{29} \\
&+ a_{562}v_4^3x_3^{10}x_5^6x_{15}^2 \otimes x_{29} + a_{563}v_4^3x_3^{14}x_9^2x_{15}^2 \otimes x_{29}
\end{aligned}
$$

$$+ a_{564}v_4^3x_3^4x_5^6x_9^2x_{15}^2 \otimes x_{29};$$

$$Q_0x_3 \equiv v_4g_{(0,3,1)} + v_4^2g_{(0,3,2)} + b_{225}v_4^3x_3^{12}x_5^2x_9^2x_{15}^2,$$
$$Q_0x_5 \equiv x_3^2 + v_4g_{(0,5,1)} + v_4^2g_{(0,5,2)} + b_{226}v_4^3x_3^{12}x_5^6x_{15}^2 + b_{227}v_4^3x_3^6x_5^6x_9^2x_{15}^2,$$
$$Q_0x_9 \equiv x_5^2 + v_4g_{(0,9,1)} + v_4^2g_{(0,9,2)} + b_{228}v_4^3x_3^{14}x_5^2x_9^2x_{15}^2,$$
$$Q_0x_{15} \equiv x_3^2x_5^2 + v_4g_{(0,15,1)},$$
$$Q_0x_{17} \equiv x_9^2 + v_4g_{(0,17,1)} + v_4^2g_{(0,17,2)} + b_{229}v_4^3x_3^{10}x_5^6x_9^2x_{15}^2,$$
$$Q_0x_{23} \equiv x_3^8 + x_3^2x_9^2 + v_4g_{(0,23,1)} + v_4^2g_{(0,23,2)} + b_{230}v_4^3x_3^{12}x_5^6x_9^2x_{15}^2,$$
$$Q_0x_{27} \equiv x_5^2x_9^2 + v_4g_{(0,27,1)} + v_4^2g_{(0,27,2)},$$
$$Q_0x_{29} \equiv x_{15}^2 + v_4g_{(0,29,1)} + v_4^2g_{(0,29,2)} + b_{231}v_4^3x_3^{14}x_5^6x_9^2x_{15}^2,$$
$$Q_1x_3 \equiv x_3^2 + v_4g_{(1,3,1)} + v_4^2g_{(1,3,2)} + b_{232}v_4^3x_3^{12}x_5^6x_{15}^2 + b_{233}v_4^3x_3^6x_5^6x_9^2x_{15}^2,$$
$$Q_1x_5 \equiv v_4g_{(1,5,1)} + v_4^2g_{(1,5,2)} + b_{234}v_4^3x_3^{10}x_5^4x_9^2x_{15}^2,$$
$$Q_1x_9 \equiv x_3^4 + v_4g_{(1,9,1)} + v_4^2g_{(1,9,2)} + b_{235}v_4^3x_3^{14}x_5^6x_{15}^2 + b_{236}v_4^3x_3^8x_5^6x_9^2x_{15}^2,$$
$$Q_1x_{15} \equiv x_9^2 + x_3^6 + v_4g_{(1,15,1)} + v_4^2g_{(1,15,2)} + b_{237}v_4^3x_3^{10}x_5^6x_9^2x_{15}^2,$$
$$Q_1x_{17} \equiv x_5^4 + v_4g_{(1,17,1)} + v_4^2g_{(1,17,2)} + b_{238}v_4^3x_3^{14}x_5^4x_9^2x_{15}^2,$$
$$Q_1x_{23} \equiv x_3^2x_5^4 + v_4g_{(1,23,1)} + v_4^2g_{(1,23,2)},$$
$$Q_1x_{27} \equiv x_{15}^2 + x_3^4x_9^2 + x_5^6 + v_4g_{(1,27,1)} + v_4^2g_{(1,27,2)} + b_{239}v_4^3x_3^{14}x_5^6x_9^2x_{15}^2,$$
$$Q_1x_{29} \equiv x_3^4x_5^4 + v_4g_{(1,29,1)} + v_4^2g_{(1,29,2)},$$
$$Q_2x_3 \equiv x_5^2 + v_4g_{(2,3,1)} + v_4^2g_{(2,3,2)} + b_{240}v_4^3x_3^{14}x_5^2x_9^2x_{15}^2,$$
$$Q_2x_5 \equiv x_3^4 + v_4g_{(2,5,1)} + v_4^2g_{(2,5,2)} + b_{241}v_4^3x_3^{14}x_5^6x_{15}^2 + b_{242}v_4^3x_3^8x_5^6x_9^2x_{15}^2,$$
$$Q_2x_9 \equiv v_4g_{(2,9,1)} + v_4^2g_{(2,9,2)},$$
$$Q_2x_{15} \equiv x_3^4x_5^2 + v_4g_{(2,15,1)} + v_4^2g_{(2,15,2)},$$
$$Q_2x_{17} \equiv x_3^8 + v_4g_{(2,17,1)} + v_4^2g_{(2,17,2)} + b_{243}v_4^3x_3^{12}x_5^6x_9^2x_{15}^2,$$
$$Q_2x_{23} \equiv x_{15}^2 + x_3^{10} + x_3^4x_9^2 + v_4g_{(2,23,1)} + v_4^2g_{(2,23,2)} + b_{244}v_4^3x_3^{14}x_5^6x_9^2x_{15}^2,$$
$$Q_2x_{27} \equiv x_3^8x_5^2 + v_4g_{(2,27,1)} + v_4^2g_{(2,27,2)},$$
$$Q_2x_{29} \equiv x_3^{12} + v_4g_{(2,29,1)} + v_4^2g_{(2,29,2)},$$
$$Q_3x_3 \equiv x_9^2 + v_4g_{(3,3,1)} + b_{245}v_4^3x_3^{10}x_5^6x_9^2x_{15}^2,$$
$$Q_3x_5 \equiv x_5^4 + v_4g_{(3,5,1)} + b_{246}v_4^3x_3^{14}x_5^4x_9^2x_{15}^2,$$
$$Q_3x_9 \equiv x_3^8 + v_4g_{(3,9,1)} + b_{247}v_4^3x_3^{12}x_5^6x_9^2x_{15}^2,$$
$$Q_3x_{15} \equiv x_{15}^2 + x_3^{10} + x_5^6 + b_{248}v_4^3x_3^{14}x_5^6x_9^2x_{15}^2,$$

$$Q_3x_{17} \equiv v_4g_{(3,17,1)} + v_4^2g_{(3,17,2)},$$
$$Q_3x_{23} \equiv x_5^4x_9^2 + v_4g_{(3,23,1)},$$
$$Q_3x_{27} \equiv x_3^8x_9^2 + v_4g_{(3,27,1)},$$
$$Q_3x_{29} \equiv x_3^8x_5^4 + v_4g_{(3,29,1)},$$

where $\equiv$ is mod $I(3,0,0,\cdots)$ and $a_i, b_i = 0, 1$. It is easy to calculate that

$$\bar\psi(x_3^2) \equiv v_4x_9^2 \otimes x_9^2 + v_4^2x_3^8x_9^2 \otimes x_3^8 + v_4^2x_3^8 \otimes x_3^8x_9^2, \qquad \bar\psi(x_3^4) \equiv 0,$$
$$\bar\psi(x_5^2) \equiv v_4x_5^4 \otimes x_5^4, \qquad \bar\psi(x_5^4) \equiv 0, \qquad \bar\psi(x_9^2) \equiv v_4x_3^8 \otimes x_3^8,$$
$$\begin{aligned}
\bar\psi(x_{15}^2) \equiv\ & x_3^8 \otimes x_3^2 + x_5^4 \otimes x_5^2 + x_3^4 \otimes x_9^2 \\
& + v_4x_3^2 \otimes x_3^8x_5^6 + v_4x_3^2 \otimes x_3^8x_{15}^2 + v_4x_3^2x_5^2 \otimes x_3^8x_5^4 + v_4x_3^2x_5^6 \otimes x_3^8 \\
& + v_4x_3^2x_{15}^2 \otimes x_3^8 + v_4x_3^4 \otimes x_3^{10}x_9^2 + v_4x_3^4 \otimes x_5^6x_9^2 + v_4x_3^4 \otimes x_9^2x_{15}^2 \\
& + v_4x_3^4x_5^2 \otimes x_5^4x_9^2 + v_4x_3^4x_5^6 \otimes x_9^2 + v_4x_3^4x_{15}^2 \otimes x_9^2 + v_4x_3^6 \otimes x_3^8x_9^2 \\
& + v_4x_3^{10} \otimes x_3^{10} + v_4x_3^{10} \otimes x_5^6 + v_4x_3^{10} \otimes x_{15}^2 + v_4x_3^{10}x_5^2 \otimes x_5^4 \\
& + v_4x_3^{12} \otimes x_3^8 + v_4x_3^{14} \otimes x_9^2 + v_4x_5^2 \otimes x_3^{10}x_5^4 + v_4x_5^2 \otimes x_5^4x_{15}^2 \\
& + v_4x_5^2x_{15}^2 \otimes x_5^4 + v_4x_5^6 \otimes x_3^{10} + v_4x_5^6 \otimes x_5^6 + v_4x_5^6 \otimes x_{15}^2 \\
& + v_4x_{15}^2 \otimes x_3^{10} + v_4x_{15}^2 \otimes x_5^6 + v_4x_{15}^2 \otimes x_{15}^2 \\
& + v_4^2x_3^2x_5^2 \otimes x_3^8x_5^4x_{15}^2 + v_4^2x_3^2x_5^2x_{15}^2 \otimes x_3^8x_5^4 + v_4^2x_3^2x_5^6 \otimes x_3^8x_5^6 \\
& + v_4^2x_3^2x_5^6 \otimes x_3^8x_{15}^2 + v_4^2x_3^2x_{15}^2 \otimes x_3^8x_5^6 + v_4^2x_3^2x_{15}^2 \otimes x_3^8x_{15}^2 \\
& + v_4^2x_3^4x_5^2 \otimes x_3^{10}x_5^4x_9^2 + v_4^2x_3^4x_5^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^2x_3^4x_5^2x_{15}^2 \otimes x_5^4x_9^2 \\
& + v_4^2x_3^4x_5^6 \otimes x_3^{10}x_9^2 + v_4^2x_3^4x_5^6 \otimes x_5^6x_9^2 + v_4^2x_3^4x_5^6 \otimes x_9^2x_{15}^2 \\
& + v_4^2x_3^4x_{15}^2 \otimes x_3^{10}x_9^2 + v_4^2x_3^4x_{15}^2 \otimes x_5^6x_9^2 + v_4^2x_3^4x_{15}^2 \otimes x_9^2x_{15}^2 \\
& + v_4^2x_3^6 \otimes x_3^8x_5^6x_9^2 + v_4^2x_3^6 \otimes x_3^8x_9^2x_{15}^2 + v_4^2x_3^6x_5^2 \otimes x_3^8x_5^4x_9^2 \\
& + v_4^2x_3^6x_5^6 \otimes x_3^8x_9^2 + v_4^2x_3^6x_{15}^2 \otimes x_3^8x_9^2 + v_4^2x_3^8x_5^2x_9^2 \otimes x_5^4x_9^2 \\
& + v_4^2x_3^8x_5^6x_9^2 \otimes x_9^2 + v_4^2x_3^8x_9^2 \otimes x_3^{10}x_9^2 + v_4^2x_3^8x_9^2 \otimes x_5^6x_9^2 \\
& + v_4^2x_3^8x_9^2 \otimes x_9^2x_{15}^2 + v_4^2x_3^8x_9^2x_{15}^2 \otimes x_9^2 + v_4^2x_3^{10}x_5^2 \otimes x_3^{10}x_5^4 \\
& + v_4^2x_3^{10}x_5^2 \otimes x_5^4x_{15}^2 + v_4^2x_3^{12} \otimes x_3^8x_5^6 + v_4^2x_3^{12} \otimes x_3^8x_{15}^2 \\
& + v_4^2x_3^{12}x_5^2 \otimes x_3^8x_5^4 + v_4^2x_3^{14} \otimes x_3^{10}x_9^2 + v_4^2x_3^{14} \otimes x_5^6x_9^2 \\
& + v_4^2x_3^{14} \otimes x_9^2x_{15}^2 + v_4^2x_3^{14}x_5^2 \otimes x_5^4x_9^2 + v_4^2x_5^2x_9^2 \otimes x_3^8x_5^4x_9^2 \\
& + v_4^2x_5^2x_{15}^2 \otimes x_3^{10}x_5^4 + v_4^2x_5^2x_{15}^2 \otimes x_5^4x_{15}^2 + v_4^2x_5^6x_9^2 \otimes x_3^8x_9^2 \\
& + v_4^2x_9^2 \otimes x_3^8x_5^6x_9^2 + v_4^2x_9^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^2x_9^2x_{15}^2 \otimes x_3^8x_9^2
\end{aligned}$$

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$$
\begin{aligned}
&+ v_4^3x_3^2x_5^2x_{15}^2 \otimes x_3^8x_5^4x_{15}^2 + v_4^3x_3^4x_5^2x_{15}^2 \otimes x_3^{10}x_5^4x_9^2 \\
&+ v_4^3x_3^4x_5^2x_{15}^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^3x_3^6x_5^2 \otimes x_3^8x_5^4x_9^2x_{15}^2 \\
&+ v_4^3x_3^6x_5^2x_{15}^2 \otimes x_3^8x_5^4x_9^2 + v_4^3x_3^6x_5^6 \otimes x_3^8x_5^6x_9^2 + v_4^3x_3^6x_5^6 \otimes x_3^8x_9^2x_{15}^2 \\
&+ v_4^3x_3^6x_{15}^2 \otimes x_3^8x_5^6x_9^2 + v_4^3x_3^6x_{15}^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^3x_3^8x_5^2x_9^2 \otimes x_3^{10}x_5^4x_9^2 \\
&+ v_4^3x_3^8x_5^2x_9^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^3x_3^8x_5^2x_9^2x_{15}^2 \otimes x_5^4x_9^2 + v_4^3x_3^8x_5^6x_9^2 \otimes x_3^{10}x_9^2 \\
&+ v_4^3x_3^8x_5^6x_9^2 \otimes x_5^6x_9^2 + v_4^3x_3^8x_5^6x_9^2 \otimes x_9^2x_{15}^2 + v_4^3x_3^8x_9^2x_{15}^2 \otimes x_3^{10}x_9^2 \\
&+ v_4^3x_3^8x_9^2x_{15}^2 \otimes x_5^6x_9^2 + v_4^3x_3^8x_9^2x_{15}^2 \otimes x_9^2x_{15}^2 + v_4^3x_3^{10}x_5^6x_9^2 \otimes x_3^8x_9^2 \\
&+ v_4^3x_3^{10}x_9^2x_{15}^2 \otimes x_3^8x_9^2 + v_4^3x_3^{12}x_5^2 \otimes x_3^8x_5^4x_{15}^2 + v_4^3x_3^{14}x_5^2 \otimes x_3^{10}x_5^4x_9^2 \\
&+ v_4^3x_3^{14}x_5^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^3x_5^2x_9^2 \otimes x_3^8x_5^4x_9^2x_{15}^2 + v_4^3x_5^2x_9^2x_{15}^2 \otimes x_3^8x_5^4x_9^2 \\
&+ v_4^3x_5^6x_9^2 \otimes x_3^8x_5^6x_9^2 + v_4^3x_5^6x_9^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^3x_9^2x_{15}^2 \otimes x_3^8x_5^6x_9^2 \\
&+ v_4^3x_9^2x_{15}^2 \otimes x_3^8x_9^2x_{15}^2,
\end{aligned}
$$

where $\equiv$ is mod $I(3,0,0,\cdots)$. Comparing the coefficient of $Q_i\bar{\psi}(x_j)$ and $\bar{\psi}(Q_ix_j)$, we can determine all the coefficients a_i and b_i.

Lemma 3.6. *We have*

$a_i = 0$ *for* $i =$ 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 425, 430, 435, 437, 438, 349, 440, 441, 442, 443, 444, 445, 451, 452, 455, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 477, 479, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 496, 498, 499, 501, 502, 503, 505, 508, 514, 517, 518, 522, 525, 526, 527, 528, 532, 533, 539, 541, 542, 543, 544, 548, 550, 552, 553, 554, 556, 558, 560, 561, 563, 564;

$a_i = 1$ *for* $i =$ 422, 423, 424, 426, 427, 428, 429, 431, 432, 433, 434, 436, 446, 447, 448, 449, 450, 453, 454, 456, 457, 458, 461, 474, 476, 478, 480, 484, 495, 497, 500, 504, 506, 507, 509, 510, 511, 512, 513, 515, 516, 519, 520, 521, 523, 524, 529, 530, 531, 534, 535, 536, 537, 538, 540, 545, 546, 547, 549, 551, 555, 557, 559, 562;

$b_i = 0$ *for* $i =$ 225, 226, 227, 228, 230, 231, 232, 233, 235, 236, 237, 239, 240, 241, 244, 245, 246, 247, 248;

$b_i = 1$ *for* $i =$ 229, 234, 238, 242, 243.

Notation 3.7. For simplicity we write

$$\bar{\psi}(x_3) \equiv v_4 f_{(3,1)} + v_4^2 f_{(3,2)} + v_4^3 f_{(3,3)},$$
$$\bar{\psi}(x_5) \equiv v_4 f_{(5,1)} + v_4^2 f_{(5,2)} + v_4^3 f_{(5,3)},$$
$$\bar{\psi}(x_9) \equiv v_4 f_{(9,1)} + v_4^2 f_{(9,2)} + v_4^3 f_{(9,3)},$$
$$\bar{\psi}(x_{17}) \equiv v_4 f_{(17,1)} + v_4^2 f_{(17,2)} + v_4^3 f_{(17,3)},$$
$$\bar{\psi}(x_{15}) \equiv x_3^4 \otimes x_3 + x_5^2 \otimes x_5 + x_3^2 \otimes x_9 + v_4 f_{(15,1)} + v_4^2 f_{(15,2)} + v_4^3 f_{(15,3)},$$
$$\bar{\psi}(x_{23}) \equiv x_5^4 \otimes x_3 + x_9^2 \otimes x_5 + x_3^2 \otimes x_{17} + v_4 f_{(23,1)} + v_4^2 f_{(23,2)} + v_4^3 f_{(23,3)},$$
$$\bar{\psi}(x_{27}) \equiv x_3^8 \otimes x_3 + x_9^2 \otimes x_9 + x_5^2 \otimes x_{17} + v_4 f_{(27,1)} + v_4^2 f_{(27,2)} + v_4^3 f_{(27,3)},$$
$$\bar{\psi}(x_{29}) \equiv x_3^8 \otimes x_5 + x_5^4 \otimes x_9 + x_3^4 \otimes x_{17} + v_4 f_{(29,1)} + v_4^2 f_{(29,2)} + v_4^3 f_{(29,3)};$$

$$Q_0 x_3 \equiv v_4 g_{(0,3,1)} + v_4^2 g_{(0,3,2)}, \qquad Q_0 x_5 \equiv x_3^2 + v_4 g_{(0,5,1)} + v_4^2 g_{(0,5,2)},$$
$$Q_0 x_9 \equiv x_5^2 + v_4 g_{(0,9,1)} + v_4^2 g_{(0,9,2)}, \qquad Q_0 x_{15} \equiv x_3^2 x_5^2 + v_4 g_{(0,15,1)},$$
$$Q_0 x_{17} \equiv x_9^2 + v_4 g_{(0,17,1)} + v_4^2 g_{(0,17,2)} + v_4^3 g_{(0,17,3)},$$
$$Q_0 x_{23} \equiv x_3^8 + x_3^2 x_9^2 + v_4 g_{(0,23,1)} + v_4^2 g_{(0,23,2)},$$
$$Q_0 x_{27} \equiv x_5^2 x_9^2 + v_4 g_{(0,27,1)} + v_4^2 g_{(0,27,2)},$$
$$Q_0 x_{29} \equiv x_{15}^2 + v_4 g_{(0,29,1)} + v_4^2 g_{(0,29,2)},$$
$$Q_1 x_3 \equiv x_3^2 + v_4 g_{(1,3,1)} + v_4^2 g_{(1,3,2)},$$
$$Q_1 x_5 \equiv v_4 g_{(1,5,1)} + v_4^2 g_{(1,5,2)} + v_4^3 g_{(1,5,3)},$$
$$Q_1 x_9 \equiv x_3^4 + v_4 g_{(1,9,1)} + v_4^2 g_{(1,9,2)},$$
$$Q_1 x_{15} \equiv x_9^2 + x_3^6 + v_4 g_{(1,15,1)} + v_4^2 g_{(1,15,2)},$$
$$Q_1 x_{17} \equiv x_5^4 + v_4 g_{(1,17,1)} + v_4^2 g_{(1,17,2)} + v_4^3 g_{(1,17,3)},$$
$$Q_1 x_{23} \equiv x_3^2 x_5^4 + v_4 g_{(1,23,1)} + v_4^2 g_{(1,23,2)},$$
$$Q_1 x_{27} \equiv x_{15}^2 + x_3^4 x_9^2 + x_5^6 + v_4 g_{(1,27,1)} + v_4^2 g_{(1,27,2)},$$
$$Q_1 x_{29} \equiv x_3^4 x_5^4 + v_4 g_{(1,29,1)} + v_4^2 g_{(1,29,2)},$$
$$Q_2 x_3 \equiv x_5^2 + v_4 g_{(2,3,1)} + v_4^2 g_{(2,3,2)},$$
$$Q_2 x_5 \equiv x_3^4 + v_4 g_{(2,5,1)} + v_4^2 g_{(2,5,2)} + v_4^3 g_{(2,5,3)},$$
$$Q_2 x_9 \equiv v_4 g_{(2,9,1)} + v_4^2 g_{(2,9,2)},$$
$$Q_2 x_{15} \equiv x_3^4 x_5^2 + v_4 g_{(2,15,1)} + v_4^2 g_{(2,15,2)},$$
$$Q_2 x_{17} \equiv x_3^8 + v_4 g_{(2,17,1)} + v_4^2 g_{(2,17,2)} + v_4^3 g_{(2,17,3)},$$
$$Q_2 x_{23} \equiv x_{15}^2 + x_3^{10} + x_3^4 x_9^2 + v_4 g_{(2,23,1)} + v_4^2 g_{(2,23,2)},$$

$Q_2x_{27} \equiv x_3^8x_5^2 + v_4g_{(2,27,1)} + v_4^2g_{(2,27,2)}$,
$Q_2x_{29} \equiv x_3^{12} + v_4g_{(2,29,1)} + v_4^2g_{(2,29,2)}$,
$Q_3x_3 \equiv x_9^2 + v_4g_{(3,3,1)}$, $Q_3x_5 \equiv x_5^4 + v_4g_{(3,5,1)}$,
$Q_3x_9 \equiv x_3^8 + v_4g_{(3,9,1)}$, $Q_3x_{15} \equiv x_{15}^2 + x_3^{10} + x_5^6$,
$Q_3x_{17} \equiv v_4g_{(3,17,1)} + v_4^2g_{(3,17,2)}$, $Q_3x_{23} \equiv x_5^4x_9^2 + v_4g_{(3,23,1)}$,
$Q_3x_{27} \equiv x_3^8x_9^2 + v_4g_{(3,27,1)}$, $Q_3x_{29} \equiv x_3^8x_5^4 + v_4g_{(3,29,1)}$,
where $\equiv$ is mod $I(3,0,0,\cdots)$.

Next we consider mod $I(4,0,0,\cdots)$ in the same way. For degree reasons we can see that

$$x_3^{16} \equiv x_5^8 \equiv x_9^4 \equiv x_{15}^4 \equiv x_{17}^2 \equiv x_{23}^2 \equiv x_{27}^2 \equiv x_{29}^2 \equiv 0,$$

and also that

$Q_0x_3 \equiv v_4g_{(0,3,1)} + v_4^2g_{(0,3,2)}$, $Q_0x_5 \equiv x_3^2 + v_4g_{(0,5,1)} + v_4^2g_{(0,5,2)}$,
$Q_0x_9 \equiv x_5^2 + v_4g_{(0,9,1)} + v_4^2g_{(0,9,2)}$, $Q_0x_{15} \equiv x_3^2x_5^2 + v_4g_{(0,15,1)}$,
$Q_0x_{17} \equiv x_9^2 + v_4g_{(0,17,1)} + v_4^2g_{(0,17,2)} + v_4^3g_{(0,17,3)}$,
$Q_0x_{23} \equiv x_3^8 + x_3^2x_9^2 + v_4g_{(0,23,1)} + v_4^2g_{(0,23,2)}$,
$Q_0x_{27} \equiv x_5^2x_9^2 + v_4g_{(0,27,1)} + v_4^2g_{(0,27,2)}$,
$Q_0x_{29} \equiv x_{15}^2 + v_4g_{(0,29,1)} + v_4^2g_{(0,29,2)}$,
$Q_1x_3 \equiv x_3^2 + v_4g_{(1,3,1)} + v_4^2g_{(1,3,2)}$,
$Q_1x_5 \equiv v_4g_{(1,5,1)} + v_4^2g_{(1,5,2)} + v_4^3g_{(1,5,3)}$,
$Q_1x_9 \equiv x_3^4 + v_4g_{(1,9,1)} + v_4^2g_{(1,9,2)}$,
$Q_1x_{15} \equiv x_9^2 + x_3^6 + v_4g_{(1,15,1)} + v_4^2g_{(1,15,2)}$,
$Q_1x_{17} \equiv x_5^4 + v_4g_{(1,17,1)} + v_4^2g_{(1,17,2)} + v_4^3g_{(1,17,3)}$,
$Q_1x_{23} \equiv x_3^2x_5^4 + v_4g_{(1,23,1)} + v_4^2g_{(1,23,2)}$,
$Q_1x_{27} \equiv x_{15}^2 + x_3^4x_9^2 + x_5^6 + v_4g_{(1,27,1)} + v_4^2g_{(1,27,2)}$,
$Q_1x_{29} \equiv x_3^4x_5^4 + v_4g_{(1,29,1)} + v_4^2g_{(1,29,2)}$,
$Q_2x_3 \equiv x_5^2 + v_4g_{(2,3,1)} + v_4^2g_{(2,3,2)}$,
$Q_2x_5 \equiv x_3^4 + v_4g_{(2,5,1)} + v_4^2g_{(2,5,2)} + v_4^3g_{(2,5,3)}$,
$Q_2x_9 \equiv v_4g_{(2,9,1)} + v_4^2g_{(2,9,2)}$,
$Q_2x_{15} \equiv x_3^4x_5^2 + v_4g_{(2,15,1)} + v_4^2g_{(2,15,2)}$,
$Q_2x_{17} \equiv x_3^8 + v_4g_{(2,17,1)} + v_4^2g_{(2,17,2)} + v_4^3g_{(2,17,3)}$,

$Q_2x_{23} \equiv x_{15}^2 + x_3^{10} + x_3^4x_9^2 + v_4g_{(2,23,1)} + v_4^2g_{(2,23,2)}$,
$Q_2x_{27} \equiv x_3^8x_5^2 + v_4g_{(2,27,1)} + v_4^2g_{(2,27,2)}$,
$Q_2x_{29} \equiv x_3^{12} + v_4g_{(2,29,1)} + v_4^2g_{(2,29,2)}$,
$Q_3x_3 \equiv x_9^2 + v_4g_{(3,3,1)}$, $Q_3x_5 \equiv x_5^4 + v_4g_{(3,5,1)}$,
$Q_3x_9 \equiv x_3^8 + v_4g_{(3,9,1)}$, $Q_3x_{15} \equiv x_{15}^2 + x_3^{10} + x_5^6$,
$Q_3x_{17} \equiv v_4g_{(3,17,1)} + v_4^2g_{(3,17,2)}$, $Q_3x_{23} \equiv x_5^4x_9^2 + v_4g_{(3,23,1)}$,
$Q_3x_{27} \equiv x_3^8x_9^2 + v_4g_{(3,27,1)}$, $Q_3x_{29} \equiv x_3^8x_5^4 + v_4g_{(3,29,1)}$,

where $\equiv$ is mod $I(4,0,0,\cdots)$. We can put

$$\begin{aligned}
\bar\psi(x_3) \equiv\ & v_4f_{(3,1)} + v_4^2f_{(3,2)} + v_4^3f_{(3,3)} + a_{565}v_4^4x_3^{14}x_5^6x_9^2x_{15}^2 \otimes x_3 \\
&+ a_{566}v_4^4x_3^{12}x_5^6x_9^2x_{15}^2 \otimes x_9 + a_{567}v_4^4x_3^{10}x_5^6x_9^2x_{15}^2 \otimes x_{15} \\
&+ a_{568}v_4^4x_3^{14}x_5^2x_9^2x_{15}^2 \otimes x_{23} + a_{569}v_4^4x_3^{12}x_5^6x_{15}^2 \otimes x_{27} \\
&+ a_{570}v_4^4x_3^6x_9^2x_5^6x_{15}^2 \otimes x_{27} + a_{571}v_4^4x_3^{12}x_5^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}$$

$$\begin{aligned}
\bar\psi(x_5) \equiv\ & v_4f_{(5,1)} + v_4^2f_{(5,2)} + v_4^3f_{(5,3)} + a_{572}v_4^4x_3^{14}x_5^6x_9^2x_{15}^2 \otimes x_5 \\
&+ a_{573}v_4^4x_3^{14}x_5^4x_9^2x_{15}^2 \otimes x_{15} + a_{574}v_4^4x_3^{10}x_5^6x_9^2x_{15}^2 \otimes x_{17} \\
&+ a_{575}v_4^4x_3^{14}x_5^6x_{15}^2 \otimes x_{23} + a_{576}v_4^4x_3^8x_5^6x_9^2x_{15}^2 \otimes x_{23} \\
&+ a_{577}v_4^4x_3^{10}x_5^4x_9^2x_{15}^2 \otimes x_{27} + a_{578}v_4^4x_3^{12}x_5^6x_{15}^2 \otimes x_{29} \\
&+ a_{579}v_4^4x_3^6x_5^6x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}$$

$$\begin{aligned}
\bar\psi(x_9) \equiv\ & v_4f_{(9,1)} + v_4^2f_{(9,2)} + v_4^3f_{(9,3)} + a_{580}v_4^4x_3^{14}x_5^6x_9^2x_{15}^2 \otimes x_9 \\
&+ a_{581}v_4^4x_3^{12}x_5^6x_9^2x_{15}^2 \otimes x_{15} + a_{582}v_4^4x_3^{14}x_5^6x_{15}^2 \otimes x_{27} \\
&+ a_{583}v_4^4x_3^8x_5^6x_9^2x_{15}^2 \otimes x_{27} + a_{584}v_4^4x_3^{14}x_5^2x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}$$

$$\begin{aligned}
\bar\psi(x_{15}) \equiv\ & x_3^4 \otimes x_3 + x_5^2 \otimes x_5 + x_3^2 \otimes x_9 + v_4f_{(15,1)} + v_4^2f_{(15,2)} + v_4^3f_{(15,3)} \\
&+ a_{585}v_4^4x_3^{14}x_5^6x_9^2x_{15}^2 \otimes x_{15} + a_{586}v_4^4x_3^{10}x_5^6x_9^2x_{15}^2 \otimes x_{27},
\end{aligned}$$

$$\begin{aligned}
\bar\psi(x_{17}) \equiv\ & v_4f_{(17,1)} + v_4^2f_{(17,2)} + v_4^3f_{(17,3)} + a_{587}v_4^4x_3^{14}x_5^6x_9^2x_{15}^2 \otimes x_{17} \\
&+ a_{588}v_4^4x_3^{12}x_5^6x_9^2x_{15}^2 \otimes x_{23} + a_{589}v_4^4x_3^{14}x_5^4x_9^2x_{15}^2 \otimes x_{27} \\
&+ a_{590}v_4^4x_3^{10}x_5^6x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}$$

$$\begin{aligned}
\bar\psi(x_{23}) \equiv\ & x_5^4 \otimes x_3 + x_9^2 \otimes x_5 + x_3^2 \otimes x_{17} + v_4f_{(23,1)} + v_4^2f_{(23,2)} + v_4^3f_{(23,3)} \\
&+ a_{591}v_4^4x_3^{14}x_5^6x_9^2x_{15}^2 \otimes x_{23} + a_{592}v_4^4x_3^{12}x_5^6x_9^2x_{15}^2 \otimes x_{29},
\end{aligned}$$

$$\begin{aligned}
\bar\psi(x_{27}) \equiv\ & x_3^8 \otimes x_3 + x_9^2 \otimes x_9 + x_5^2 \otimes x_{17} + v_4f_{(27,1)} + v_4^2f_{(27,2)} + v_4^3f_{(27,3)} \\
&+ a_{593}v_4^4x_3^{14}x_5^6x_9^2x_{15}^2 \otimes x_{27},
\end{aligned}$$

$$\begin{aligned}\bar\psi(x_{29}) \equiv\ & x_3^8\otimes x_5 + x_5^4\otimes x_9 + x_3^4\otimes x_{17} + v_4 f_{(29,1)} + v_4^2 f_{(29,2)} + v_4^3 f_{(29,3)}\\ &+ a_{594} v_4^4 x_3^{14}x_5^6x_9^2x_{15}^2\otimes x_{29},\end{aligned}$$

where $\equiv$ is mod $I(4,0,0,\cdots)$ and $a_i = 0,1$. It is easy to calculate that

$$\bar\psi(x_3^2)\equiv v_4x_9^2\otimes x_9^2 + v_4^2x_3^8x_9^2\otimes x_3^8 + v_4^2x_3^8\otimes x_3^8x_9^2, \qquad \bar\psi(x_3^4)\equiv 0,$$

$$\bar\psi(x_5^2)\equiv v_4x_5^4\otimes x_5^4, \qquad \bar\psi(x_5^4)\equiv 0, \qquad \bar\psi(x_9^2)\equiv v_4x_3^8\otimes x_3^8,$$

$$\begin{aligned}\bar\psi(x_{15}^2)\equiv\ & x_3^8\otimes x_3^2 + x_5^4\otimes x_5^2 + x_3^4\otimes x_9^2\\
&+ v_4x_3^2\otimes x_3^8x_5^6 + v_4x_3^2\otimes x_3^8x_{15}^2 + v_4x_3^2x_5^2\otimes x_3^8x_5^4 + v_4x_3^2x_5^6\otimes x_3^8\\
&+ v_4x_3^2x_{15}^2\otimes x_3^8 + v_4x_3^4\otimes x_3^{10}x_9^2 + v_4x_3^4\otimes x_5^6x_9^2 + v_4x_3^4\otimes x_9^2x_{15}^2\\
&+ v_4x_3^4x_5^2\otimes x_5^4x_9^2 + v_4x_3^4x_5^6\otimes x_9^2 + v_4x_3^4x_{15}^2\otimes x_9^2 + v_4x_3^6\otimes x_3^8x_9^2\\
&+ v_4x_3^{10}\otimes x_3^{10} + v_4x_3^{10}\otimes x_5^6 + v_4x_3^{10}\otimes x_{15}^2 + v_4x_3^{10}x_5^2\otimes x_5^4\\
&+ v_4x_3^{12}\otimes x_3^8 + v_4x_3^{14}\otimes x_9^2 + v_4x_5^2\otimes x_3^{10}x_5^4 + v_4x_5^2\otimes x_5^4x_{15}^2\\
&+ v_4x_5^2x_{15}^2\otimes x_5^4 + v_4x_5^6\otimes x_3^{10} + v_4x_5^6\otimes x_5^6 + v_4x_5^6\otimes x_{15}^2\\
&+ v_4x_{15}^2\otimes x_3^{10} + v_4x_{15}^2\otimes x_5^6 + v_4x_{15}^2\otimes x_{15}^2\\
&+ v_4^2x_3^2x_5^2\otimes x_3^8x_5^4x_{15}^2 + v_4^2x_3^2x_5^2x_{15}^2\otimes x_3^8x_5^4 + v_4^2x_3^2x_5^6\otimes x_3^8x_5^6\\
&+ v_4^2x_3^2x_5^6\otimes x_3^8x_{15}^2 + v_4^2x_3^2x_{15}^2\otimes x_3^8x_5^6 + v_4^2x_3^2x_{15}^2\otimes x_3^8x_{15}^2\\
&+ v_4^2x_3^4x_5^2\otimes x_3^{10}x_5^4x_9^2 + v_4^2x_3^4x_5^2\otimes x_5^4x_9^2x_{15}^2 + v_4^2x_3^4x_5^2x_{15}^2\otimes x_5^4x_9^2\\
&+ v_4^2x_3^4x_5^6\otimes x_3^{10}x_9^2 + v_4^2x_3^4x_5^6\otimes x_5^6x_9^2 + v_4^2x_3^4x_5^6\otimes x_9^2x_{15}^2\\
&+ v_4^2x_3^4x_{15}^2\otimes x_3^{10}x_9^2 + v_4^2x_3^4x_{15}^2\otimes x_5^6x_9^2 + v_4^2x_3^4x_{15}^2\otimes x_9^2x_{15}^2\\
&+ v_4^2x_3^6\otimes x_3^8x_5^6x_9^2 + v_4^2x_3^6\otimes x_3^8x_9^2x_{15}^2 + v_4^2x_3^6x_5^2\otimes x_3^8x_5^4x_9^2\\
&+ v_4^2x_3^6x_5^6\otimes x_3^8x_9^2 + v_4^2x_3^6x_{15}^2\otimes x_3^8x_9^2 + v_4^2x_3^8x_5^2x_9^2\otimes x_5^4x_9^2\\
&+ v_4^2x_3^8x_5^6x_9^2\otimes x_9^2 + v_4^2x_3^8x_9^2\otimes x_3^{10}x_9^2 + v_4^2x_3^8x_9^2\otimes x_5^6x_9^2\\
&+ v_4^2x_3^8x_9^2\otimes x_9^2x_{15}^2 + v_4^2x_3^8x_9^2x_{15}^2\otimes x_9^2 + v_4^2x_3^{10}x_5^2\otimes x_3^{10}x_5^4\\
&+ v_4^2x_3^{10}x_5^2\otimes x_5^4x_{15}^2 + v_4^2x_3^{12}\otimes x_3^8x_5^6 + v_4^2x_3^{12}\otimes x_3^8x_{15}^2\\
&+ v_4^2x_3^{12}x_5^2\otimes x_3^8x_5^4 + v_4^2x_3^{14}\otimes x_3^{10}x_9^2 + v_4^2x_3^{14}\otimes x_5^6x_9^2\\
&+ v_4^2x_3^{14}\otimes x_9^2x_{15}^2 + v_4^2x_3^{14}x_5^2\otimes x_5^4x_9^2 + v_4^2x_5^2x_9^2\otimes x_3^8x_5^4x_9^2\\
&+ v_4^2x_5^2x_{15}^2\otimes x_3^{10}x_5^4 + v_4^2x_5^2x_{15}^2\otimes x_5^4x_{15}^2 + v_4^2x_5^6x_9^2\otimes x_3^8x_9^2\\
&+ v_4^2x_9^2\otimes x_3^8x_5^6x_9^2 + v_4^2x_9^2\otimes x_3^8x_9^2x_{15}^2 + v_4^2x_9^2x_{15}^2\otimes x_3^8x_9^2\\
&+ v_4^3x_3^2x_5^2x_{15}^2\otimes x_3^8x_5^4x_{15}^2 + v_4^3x_3^4x_5^2x_{15}^2\otimes x_3^{10}x_5^4x_9^2\\
&+ v_4^3x_3^4x_5^2x_{15}^2\otimes x_5^4x_9^2x_{15}^2 + v_4^3x_3^6x_5^2\otimes x_3^8x_5^4x_9^2x_{15}^2\end{aligned}$$

$$
\begin{aligned}
&+ v_4^3x_3^6x_5^2x_{15}^2 \otimes x_3^8x_5^4x_9^2 + v_4^3x_3^6x_5^6 \otimes x_3^8x_5^6x_9^2 + v_4^3x_3^6x_5^6 \otimes x_3^8x_9^2x_{15}^2 \\
&+ v_4^3x_3^6x_{15}^2 \otimes x_3^8x_5^6x_9^2 + v_4^3x_3^6x_{15}^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^3x_3^8x_5^2x_9^2 \otimes x_3^{10}x_5^4x_9^2 \\
&+ v_4^3x_3^8x_5^2x_9^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^3x_3^8x_5^2x_9^2x_{15}^2 \otimes x_5^4x_9^2 + v_4^3x_3^8x_5^6x_9^2 \otimes x_3^{10}x_9^2 \\
&+ v_4^3x_3^8x_5^6x_9^2 \otimes x_5^6x_9^2 + v_4^3x_3^8x_5^6x_9^2 \otimes x_9^2x_{15}^2 + v_4^3x_3^8x_9^2x_{15}^2 \otimes x_3^{10}x_9^2 \\
&+ v_4^3x_3^8x_9^2x_{15}^2 \otimes x_5^6x_9^2 + v_4^3x_3^8x_9^2x_{15}^2 \otimes x_9^2x_{15}^2 + v_4^3x_3^{10}x_5^6x_9^2 \otimes x_3^8x_9^2 \\
&+ v_4^3x_3^{10}x_9^2x_{15}^2 \otimes x_3^8x_9^2 + v_4^3x_3^{12}x_5^2 \otimes x_3^8x_5^4x_{15}^2 + v_4^3x_3^{14}x_5^2 \otimes x_3^{10}x_5^4x_9^2 \\
&+ v_4^3x_3^{14}x_5^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^3x_5^2x_9^2 \otimes x_3^8x_5^4x_9^2x_{15}^2 + v_4^3x_5^2x_9^2x_{15}^2 \otimes x_3^8x_5^4x_9^2 \\
&+ v_4^3x_5^6x_9^2 \otimes x_3^8x_5^6x_9^2 + v_4^3x_5^6x_9^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^3x_9^2x_{15}^2 \otimes x_3^8x_5^6x_9^2 \\
&+ v_4^3x_9^2x_{15}^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^4x_3^6x_5^2x_{15}^2 \otimes x_3^8x_5^4x_9^2x_{15}^2 \\
&+ v_4^4x_3^8x_5^2x_9^2x_{15}^2 \otimes x_3^{10}x_5^4x_9^2 + v_4^4x_3^8x_5^2x_9^2x_{15}^2 \otimes x_5^4x_9^2x_{15}^2 \\
&+ v_4^4x_3^{10}x_5^2x_9^2x_{15}^2 \otimes x_3^8x_5^4x_9^2 + v_4^4x_3^{10}x_5^6x_9^2 \otimes x_3^8x_5^6x_9^2 \\
&+ v_4^4x_3^{10}x_5^6x_9^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^4x_3^{10}x_9^2x_{15}^2 \otimes x_3^8x_5^6x_9^2 \\
&+ v_4^4x_3^{10}x_9^2x_{15}^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^4x_5^2x_9^2x_{15}^2 \otimes x_3^8x_5^4x_9^2x_{15}^2,
\end{aligned}
$$

where $\equiv$ is mod $I(4,0,0,\cdots)$. Comparing the coefficient of $Q_i\bar{\psi}(x_j)$ and $\bar{\psi}(Q_ix_j)$, we can determine all the coefficients a_i and b_i.

Lemma 3.8. *We have*

$$
\begin{aligned}
a_i = 0 \text{ for } i = {}& 565, 566, 567, 568, 569, 570, 571, 572, 573, 575, 578, 579, 580, \\
& 581, 582, 583, 584, 585, 586, 591, 592, 593, 594; \\
a_i = 1 \text{ for } i = {}& 574, 576, 577, 587, 588, 589, 590.
\end{aligned}
$$

For degree reasons, no new term appears mod $I(k,0,0,\cdots)$ for $k \geq 5$. Thus we have obtained the bi-algebra structure of $k(4)^*(E_8)$.

The similar arguments mod $I(0,1,0,0,\cdots)$, $I(1,1,0,0,\cdots)$ and $I(2,1,0,0,\cdots)$ imply that the above formulas do not contain the terms with v_5, v_4v_5 and $v_4^2v_5$ respectively. Similarly for degree reasons, no new term appears mod $I(k,1,0,\cdots)$, that is, no new terms with $v_4^kv_5$ appear for $k \geq 3$. The similar argument mod $I(0,2,0,0,\cdots)$ implies that the above formulas do not contain terms with v_5^2. For degree reasons, no new term appears mod $I(k,2,0,\cdots)$ for $k \geq 1$. The similar argument mod $I(0,0,1,0,0,\cdots)$ implies that the above formulas do not contain the term with v_6. For degree reasons, no new term appears mod $I(L)$ for any other sequences L. Thus we have obtained Main Theorem.

To conclude the paper we give the following, which are obtained during the process of proving the main theorem.

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Remark 3.9. The coproducts of x_3^2, x_3^4, x_5^2, x_5^4, x_9^2 and x_{15}^2 are as follows:

$$\bar{\psi}(x_3^2) = v_4x_9^2 \otimes x_9^2 + v_4^2x_3^8x_9^2 \otimes x_3^8 + v_4^2x_3^8 \otimes x_3^8x_9^2, \qquad \bar{\psi}(x_3^4) = 0,$$

$$\bar{\psi}(x_5^2) = v_4x_5^4 \otimes x_5^4, \qquad \bar{\psi}(x_5^4) = 0, \qquad \bar{\psi}(x_9^2) = v_4x_3^8 \otimes x_3^8,$$

$$\begin{aligned}
\bar{\psi}(x_{15}^2) = {} & x_3^8 \otimes x_3^2 + x_5^4 \otimes x_5^2 + x_3^4 \otimes x_9^2 \\
& + v_4x_3^2 \otimes x_3^8x_5^6 + v_4x_3^2 \otimes x_3^8x_{15}^2 + v_4x_3^2x_5^2 \otimes x_3^8x_5^4 + v_4x_3^2x_5^6 \otimes x_3^8 \\
& + v_4x_3^2x_{15}^2 \otimes x_3^8 + v_4x_3^4 \otimes x_3^{10}x_9^2 + v_4x_3^4 \otimes x_5^6x_9^2 + v_4x_3^4 \otimes x_9^2x_{15}^2 \\
& + v_4x_3^4x_5^2 \otimes x_5^4x_9^2 + v_4x_3^4x_5^6 \otimes x_9^2 + v_4x_3^4x_{15}^2 \otimes x_9^2 + v_4x_3^6 \otimes x_3^8x_9^2 \\
& + v_4x_3^{10} \otimes x_3^{10} + v_4x_3^{10} \otimes x_5^6 + v_4x_3^{10} \otimes x_{15}^2 + v_4x_3^{10}x_5^2 \otimes x_5^4 \\
& + v_4x_3^{12} \otimes x_3^8 + v_4x_3^{14} \otimes x_9^2 + v_4x_5^2 \otimes x_3^{10}x_5^4 + v_4x_5^2 \otimes x_5^4x_{15}^2 \\
& + v_4x_5^2x_{15}^2 \otimes x_5^4 + v_4x_5^6 \otimes x_3^{10} + v_4x_5^6 \otimes x_5^6 + v_4x_5^6 \otimes x_{15}^2 \\
& + v_4x_{15}^2 \otimes x_3^{10} + v_4x_{15}^2 \otimes x_5^6 + v_4x_{15}^2 \otimes x_{15}^2 \\
& + v_4^2x_3^2x_5^2 \otimes x_3^8x_5^4x_{15}^2 + v_4^2x_3^2x_5^2x_{15}^2 \otimes x_3^8x_5^4 + v_4^2x_3^2x_5^6 \otimes x_3^8x_5^6 \\
& + v_4^2x_3^2x_5^6 \otimes x_3^8x_{15}^2 + v_4^2x_3^2x_{15}^2 \otimes x_3^8x_5^6 + v_4^2x_3^2x_{15}^2 \otimes x_3^8x_{15}^2 \\
& + v_4^2x_3^4x_5^2 \otimes x_3^{10}x_5^4x_9^2 + v_4^2x_3^4x_5^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^2x_3^4x_5^2x_{15}^2 \otimes x_5^4x_9^2 \\
& + v_4^2x_3^4x_5^6 \otimes x_3^{10}x_9^2 + v_4^2x_3^4x_5^6 \otimes x_5^6x_9^2 + v_4^2x_3^4x_5^6 \otimes x_9^2x_{15}^2 \\
& + v_4^2x_3^4x_{15}^2 \otimes x_3^{10}x_9^2 + v_4^2x_3^4x_{15}^2 \otimes x_5^6x_9^2 + v_4^2x_3^4x_{15}^2 \otimes x_9^2x_{15}^2 \\
& + v_4^2x_3^6 \otimes x_3^8x_5^6x_9^2 + v_4^2x_3^6 \otimes x_3^8x_9^2x_{15}^2 + v_4^2x_3^6x_5^2 \otimes x_3^8x_5^4x_9^2 \\
& + v_4^2x_3^6x_5^6 \otimes x_3^8x_9^2 + v_4^2x_3^6x_{15}^2 \otimes x_3^8x_9^2 + v_4^2x_3^8x_5^2x_9^2 \otimes x_5^4x_9^2 \\
& + v_4^2x_3^8x_5^6x_9^2 \otimes x_9^2 + v_4^2x_3^8x_9^2 \otimes x_3^{10}x_9^2 + v_4^2x_3^8x_9^2 \otimes x_5^6x_9^2 \\
& + v_4^2x_3^8x_9^2 \otimes x_9^2x_{15}^2 + v_4^2x_3^8x_9^2x_{15}^2 \otimes x_9^2 + v_4^2x_3^{10}x_5^2 \otimes x_3^{10}x_5^4 \\
& + v_4^2x_3^{10}x_5^2 \otimes x_5^4x_{15}^2 + v_4^2x_3^{12} \otimes x_3^8x_5^6 + v_4^2x_3^{12} \otimes x_3^8x_{15}^2 \\
& + v_4^2x_3^{12}x_5^2 \otimes x_3^8x_5^4 + v_4^2x_3^{14} \otimes x_3^{10}x_9^2 + v_4^2x_3^{14} \otimes x_5^6x_9^2 \\
& + v_4^2x_3^{14} \otimes x_9^2x_{15}^2 + v_4^2x_3^{14}x_5^2 \otimes x_5^4x_9^2 + v_4^2x_5^2x_9^2 \otimes x_3^8x_5^4x_9^2 \\
& + v_4^2x_5^2x_{15}^2 \otimes x_3^{10}x_5^4 + v_4^2x_5^2x_{15}^2 \otimes x_5^4x_{15}^2 + v_4^2x_5^6x_9^2 \otimes x_3^8x_9^2 \\
& + v_4^2x_9^2 \otimes x_3^8x_5^6x_9^2 + v_4^2x_9^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^2x_9^2x_{15}^2 \otimes x_3^8x_9^2 \\
& + v_4^3x_3^2x_5^2x_{15}^2 \otimes x_3^8x_5^4x_{15}^2 + v_4^3x_3^4x_5^2x_{15}^2 \otimes x_3^{10}x_5^4x_9^2 \\
& + v_4^3x_3^4x_5^2x_{15}^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^3x_3^6x_5^2 \otimes x_3^8x_5^4x_9^2x_{15}^2 \\
& + v_4^3x_3^6x_5^2x_{15}^2 \otimes x_3^8x_5^4x_9^2 + v_4^3x_3^6x_5^6 \otimes x_3^8x_5^6x_9^2 \\
& + v_4^3x_3^6x_5^6 \otimes x_3^8x_9^2x_{15}^2 + v_4^3x_3^6x_{15}^2 \otimes x_3^8x_5^6x_9^2 + v_4^3x_3^6x_{15}^2 \otimes x_3^8x_9^2x_{15}^2
\end{aligned}$$

$$+ v_4^3x_3^8x_5^2x_9^2 \otimes x_3^{10}x_5^4x_9^2 + v_4^3x_3^8x_5^2x_9^2 \otimes x_5^4x_9^2x_{15}^2$$
$$+ v_4^3x_3^8x_5^2x_9^2x_{15}^2 \otimes x_5^4x_9^2 + v_4^3x_3^8x_5^6x_9^2 \otimes x_3^{10}x_9^2 + v_4^3x_3^8x_5^6x_9^2 \otimes x_5^6x_9^2$$
$$+ v_4^3x_3^8x_5^6x_9^2 \otimes x_9^2x_{15}^2 + v_4^3x_3^8x_9^2x_{15}^2 \otimes x_3^{10}x_9^2 + v_4^3x_3^8x_9^2x_{15}^2 \otimes x_5^6x_9^2$$
$$+ v_4^3x_3^8x_9^2x_{15}^2 \otimes x_9^2x_{15}^2 + v_4^3x_3^{10}x_5^6x_9^2 \otimes x_3^8x_9^2 + v_4^3x_3^{10}x_9^2x_{15}^2 \otimes x_3^8x_9^2$$
$$+ v_4^3x_3^{12}x_5^2 \otimes x_3^8x_5^4x_{15}^2 + v_4^3x_3^{14}x_5^2 \otimes x_3^{10}x_5^4x_9^2 + v_4^3x_3^{14}x_5^2 \otimes x_5^4x_9^2x_{15}^2$$
$$+ v_4^3x_5^2x_9^2 \otimes x_3^8x_5^4x_9^2x_{15}^2 + v_4^3x_5^2x_9^2x_{15}^2 \otimes x_3^8x_5^4x_9^2 + v_4^3x_5^6x_9^2 \otimes x_3^8x_5^6x_9^2$$
$$+ v_4^3x_5^6x_9^2 \otimes x_3^8x_9^2x_{15}^2 + v_4^3x_9^2x_{15}^2 \otimes x_3^8x_5^6x_9^2 + v_4^3x_9^2x_{15}^2 \otimes x_3^8x_9^2x_{15}^2$$
$$+ v_4^4x_3^6x_5^2x_{15}^2 \otimes x_3^8x_5^4x_9^2x_{15}^2 + v_4^4x_3^8x_5^2x_9^2x_{15}^2 \otimes x_3^{10}x_5^4x_9^2$$
$$+ v_4^4x_3^8x_5^2x_9^2x_{15}^2 \otimes x_5^4x_9^2x_{15}^2 + v_4^4x_3^{10}x_5^2x_9^2x_{15}^2 \otimes x_3^8x_5^4x_9^2$$
$$+ v_4^4x_3^{10}x_5^6x_9^2 \otimes x_3^8x_5^6x_9^2 + v_4^4x_3^{10}x_5^6x_9^2 \otimes x_3^8x_9^2x_{15}^2$$
$$+ v_4^4x_3^{10}x_9^2x_{15}^2 \otimes x_3^8x_5^6x_9^2 + v_4^4x_3^{10}x_9^2x_{15}^2 \otimes x_3^8x_9^2x_{15}^2$$
$$+ v_4^4x_5^2x_9^2x_{15}^2 \otimes x_3^8x_5^4x_9^2x_{15}^2 + v_4^5x_3^{10}x_5^2x_9^2x_{15}^2 \otimes x_3^8x_5^4x_9^2x_{15}^2.$$

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Tetsu Nishimoto
Kinki Welfare University
Fukusaki-cho
Hyogo 679-2217, Japan
e-mail address: nishimoto@kinwu.ac.jp

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