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Generalized Derivations with Commutativity and Anti-commutativity Conditions

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Generalized Derivations with Commutativity and Anti-commutativity Conditions

Howard E. Bell and Nadeem-ur Rehman

Abstract

Let R be a prime ring with 1, with char(R) $\neq 2$; and let F : R $\rightarrow$ R be a generalized derivation. We determine when one of the following holds for all x,y $\in$ R: (i) [F(x); F(y)] = 0; (ii) F(x)OF(y) = 0; (iii) F(x) O F(y) = x O y .

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GENERALIZED DERIVATIONS WITH COMMUTATIVITY AND ANTI-COMMUTATIVITY CONDITIONS

Howard E. BELL and Nadeem-Ur REHMAN

Abstract. Let R be a prime ring with 1, with $\text{char}(R) \neq 2$; and let $F : R \longrightarrow R$ be a generalized derivation. We determine when one of the following holds for all $x, y \in R$: (i) $[F(x), F(y)] = 0$; (ii) $F(x) \circ F(y) = 0$; (iii) $F(x) \circ F(y) = x \circ y$.

1. Introduction

Let R be an associative ring with center $Z = Z(R)$. For each $x, y \in R$ denote the commutator $xy - yx$ by $[x, y]$ and the anti-commutator $xy + yx$ by $x \circ y$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = \{0\}$ implies that $a = 0$ or $b = 0$. An additive mapping $d : R \longrightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. In particular, for fixed $a \in R$, the mapping $I_a : R \longrightarrow R$ given by $I_a(x) = [x, a]$ is a derivation called an inner derivation.

An additive function $F : R \longrightarrow R$ is called a generalized inner derivation if $F(x) = ax + xb$ for fixed $a, b \in R$. For such a mapping F, it is easy to see that

$$F(xy) = F(x)y + x[y, b] = F(x)y + xI_b(y) \text{ for all } x, y \in R.$$

This observation leads to the following definition, given in [6]: an additive mapping $F : R \longrightarrow R$ is called a generalized derivation with associated derivation d if

$$F(xy) = F(x)y + xd(y) \text{ for all } x, y \in R.$$

Familiar examples of generalized derivations are derivations and generalized inner derivations, and the later include left multipliers and right multipliers. Since the sum of two generalized derivations is a generalized derivation, every map of the form $F(x) = cx + d(x)$, where c is a fixed element of R and d is a derivation, is a generalized derivation; and if R has 1, all generalized derivations have this form.

Our primary purpose is to determine when a generalized derivation F satisfies $[F(x), F(y)] = 0$ for all $x, y \in R$, where R is a prime ring with 1 for

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which $\operatorname{char}(R) \neq 2$; and we also study the conditions $F(x) \circ F(y) = 0$ and $F(x) \circ F(y) = x \circ y$. Our results extend known results for derivations.

2. Preliminary results

We shall use without explicit mention the following basic identities:

$$[xy, z] = x[y, z] + [x, z]y$$

$$[x, yz] = y[x, z] + [x, y]z$$

$$x \circ (yz) = (x \circ y)z - y[x, z] = y(x \circ z) + [x, y]z$$

$$(xy) \circ z = x(y \circ z) - [x, z]y = (x \circ z)y + x[y, z]$$

We shall also use the elementary fact that if R is prime and d is a nonzero derivation, then $xd(R) = \{0\}$ or $d(R)x = \{0\}$ implies $x = 0$.

We shall require several lemmas, all but two of which are known.

Lemma 2.1. Let R be a prime ring and d a nonzero derivation of R.

(a) ([4, Theorem 2]). If $\operatorname{char}(R) \neq 2$ and $[d(x), d(y)] = 0$ for all $x, y \in R$, then R is commutative.

(b) ([5, Theorem 2]). If $\operatorname{char}(R) \neq 2$ and $[a, d(R)] = \{0\}$, then $a \in Z$.

Lemma 2.2 ([8, Corollary 3.2]). Let R be a prime ring. If R admits a nonzero generalized derivation F with associated derivation $d \neq 0$, such that $[F(x), x] = 0$ for all $x \in R$, then R is commutative.

Lemma 2.3 ([1, Theorem 4.3]). Let R be a prime ring with $\operatorname{char}(R) \neq 2$, and let I be a nonzero ideal of R. If R admits a nonzero derivation d such that $d(x) \circ d(y) = 0$ for all $x, y \in I$, then R is commutative.

Lemma 2.4. Let R be a prime ring with 1. Let F be a generalized derivation with associated derivation $d \neq 0$, such that $d(F(x)) = 0$ for all $x \in R$; and let $c = F(1)$. Then $cd(x)+d(x)c = 0$ for all $x \in R$. Moreover, if $\operatorname{char}(R) \neq 2$, $c^2 \in Z$; and if $c \in Z$, then $c = 0$ and $F = d$.

Proof. We have

$$d(F(x)) = 0 \text{ for all } x \in R. \tag{2.1}$$

Replacing x by xy in (2.1) and using (2.1), we get

$$F(x)d(y) + d(x)d(y) + xd^2(y) = 0 \quad \text{for all } x, y \in R. \tag{2.2}$$

Applying d again on (2.2) and using (2.1), we have

$$\begin{aligned} F(x)d^2(y) + d^2(x)d(y) + d(x)d^2(y) + d(x)d^2(y) \\ +xd^3(y) = 0 \qquad \text{for all } x, y \in R. \end{aligned} \tag{2.3}$$

But replacing y by $d(y)$ in (2.2), we get

$$F(x)d^2(y) + d(x)d^2(y) + xd^3(y) = 0; \tag{2.4}$$

and combining (2.3) and (2.4), we find that

$$d(x)d^2(y) + d^2(x)d(y) = 0 \text{ for all } x, y \in R. \tag{2.5}$$

Since R has 1,

$$F(x) = F(1x) = F(1)x + 1d(x) = cx + d(x) \text{ for all } x \in R. \tag{2.6}$$

Using the hypothesis that $d(F(R)) = \{0\}$, we get $d(c) = 0$; and by applying d to (2.6), we obtain

$$cd(x) + d^2(x) = 0 \text{ for all } x \in R. \tag{2.7}$$

Using this fact to substitute for $d^2(x)$ and $d^2(y)$ in (2.5), we get

$$d(x)(-cd(y)) + (-cd(x))d(y) = 0;$$

hence

$$(d(x)c + cd(x))d(y) = 0 \text{ for all } x, y \in R.$$

Thus,

$$cd(x) + d(x)c = 0 \text{ for all } x \in R. \tag{2.8}$$

Suppose now that $\text{char}(R) \neq 2$. It follows from (2.8) that $[c^2, d(R)] = \{0\}$, so that $c^2 \in Z$ by Lemma 2.1(b). If $c \in Z$, then (2.8) yields $2cd(R) = \{0\}$; hence $2c = 0 = c$ and $F = d$. □

Henceforth, except in our final section, R will always be a prime ring with extended centroid C and central closure $S = RC$. (For definitions and basic properties of C and S, see [7, Section 2] or [3, Chapter 1, Section 3]). Note that if R has 1, then C is the center $Z(S)$ of S.

Lemma 2.5 [2, Theorem 2.1]. Let R be a prime ring and let d, g, h be derivations on R for which there exist $a, b \in R \backslash Z$ such that $d(x) = ag(x) + h(x)b$ for all $x \in R$. Then there exists $\lambda \in C$ such that $d(x) = [\lambda ab, x]$, $g(x) = [\lambda b, x]$ and $h(x) = [\lambda a, x]$ for all $x \in R$.

Lemma 2.6. Let R be prime ring with 1. let F be a generalized derivation with associated derivation $d \neq 0$, such that $d(F(x)) = 0$ for all $x \in R$; and suppose $c = F(1) \notin Z$. Then

- (i) there exists $\lambda \in C$ such that $d(x) = [\lambda c, x]$ for all $x \in R$;
- (ii) F can be extended to a generalized derivation $\hat{F}$ on S;
- (iii) if $[F(x), F(y)] = 0$ for all $x, y \in R$, then $[\hat{F}(x), \hat{F}(y)] = 0$ for all $x, y \in S$.

Proof. (i) By Lemma 2.4, $cd(x) + d(x)c = 0$ for all $x \in R$; hence by Lemma 2.5, there exists $\lambda \in C$ such that $d(x) = [\lambda c, x]$ for all $x \in R$.

(ii) Define $\hat{F}(x) = cx + [\lambda c, x]$ for all $x \in S$.

(iii) Let $x \in S$, and write $x = \sum_{i=1}^{n} r_i u_i$, where $r_i \in R$, $u_i \in C$. Then using the fact that $u_i \in Z(S)$, we get $\hat{F}(x) = \sum cr_i u_i + \sum[\lambda c, r_i u_i] = \sum u_i(cr_i + [\lambda c, r_i]) = \sum u_i F(r_i)$; and (iii) follows at once. □

Lemma 2.7 [7, Theorem 3]. If R is prime and S satisfies a generalized polynomial identity over C, then S is primitive.

3. The condition $[F(x), F(y)] = 0$

In view of Lemma 2.1(a), it is natural to conjecture that if a prime ring R of characteristic different from 2 admits a nonzero generalized derivation F such that $[F(x), F(y)] = 0$ for all $x, y \in R$, then R is commutative. However, this is not the case.

Example 3.1. Let R be either the ring H of real quaternions or the subring K of H consisting of all elements $a + bi + cj + dk$ where a, b, c, d are integers. Define $F(x) = ix + xi$ for all $x \in R$. Then R is a noncommutative prime ring, and F is a generalized derivation such that $[F(x), F(y)] = 0$ for all $x, y \in R$.

Example 3.2. Let K be any field, and let R be the ring $M_2(K)$ of 2×2 matrices over K. Define $F(x) = cx + xc$, where c is either $e_{11} - e_{22}$ or e_{12}. It is easy to verify that, in either case, $[F(x), F(y)] = 0$ for all $x, y \in R$.

Example 3.3. Let R be the noncommutative prime ring $M_2(\mathbb{Z})$; and for arbitrary $x = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in R$, define $d(x) = \begin{bmatrix} 0 & -b \\ c & 0 \end{bmatrix}$. Define $F : R \longrightarrow R$ by $F(x) = (e_{11} - e_{22})x + d(x)$. It is easily verified that d is a derivation on R, so that F is a generalized derivation; and since $F\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & -d \end{bmatrix}$, $[F(x), F(y)] = 0$ for all $x, y \in R$. Note that F is the restriction to R of the map $\hat{F} : M_2(Q) \longrightarrow M_2(Q)$ given by $\hat{F}(x) = cx + xc$, where $c = \frac{1}{2}e_{11} - \frac{1}{2}e_{22}$.

In fact, for 2-torsion free prime rings with 1, these examples illustrate all possibilities, as our next (and principal) theorem shows.

Theorem 3.4. Let R be a prime ring with 1 such that $\mathrm{char}(R) \neq 2$. If R admits a nonzero generalized derivation F such that $[F(x), F(y)] = 0$ for all $x, y \in R$, then one of the following holds:

(i) R is commutative;

(ii) R is a noncommutative subring of a division ring Δ, and there exists $\delta \in \Delta$ such that $F(x) = \delta x + x\delta$ for all $x \in R$;

(iii) R is a noncommutative subring of a 2×2 total matrix ring M over a field, and there exists $m \in M$ such that $F(x) = mx + xm$ for all $x \in R$.

The following lemma is the first step in the proof.

Lemma 3.5. Let R be a noncommutative prime ring with 1 and with $\mathrm{char}(R) \neq 2$. Let F be a nonzero generalized derivation with associated derivation d, such that $[F(x), F(y)] = 0$ for all $x, y \in R$. Then

(i) $d(F(x)) = 0$ for all $x \in R$;

(ii) $c = F(1) \notin Z$ and $c^2 \in Z$;

(iii) S is primitive and there exists $s \in S$ such that $s^2 \in Z(S)$ and $\hat{F}(x) = sx + xs$ for all $x \in S$.

Proof. (i) If $d = 0$, then $c \neq 0$ and $[cx, cy] = 0$ for all $x, y \in R$; thus cR is a nonzero commutative right ideal. But a noncommutative prime ring cannot have such a right ideal, hence $d \neq 0$.

We have

$$[F(x), F(y)] = 0 \text{ for all } x, y \in R. \tag{3.1}$$

Replacing y by yz in (3.1) and using (3.1), we get

$$F(y)[F(x), z] + y[F(x), d(z)] + [F(x), y]d(z) = 0 \text{ for all } x, y, z \in R. \tag{3.2}$$

Now replacing y by ry in (3.2) gives

$$\begin{aligned} F(r)y[F(x), z] + rd(y)[F(x), z] + ry[F(x), d(z)] + r[F(x), y]d(z) \\ +[F(x), r]yd(z) = 0 \qquad \text{for all } x, y, z, r \in R; \end{aligned}$$

and hence application of (3.2) yields

$$F(r)y[F(x), z] + rd(y)[F(x), z] + [F(x), r]yd(z) - rF(y)[F(x), z] = 0.$$

Letting $z = F(x)$, we obtain $[F(x), r]yd(F(x)) = 0$ for all $x, y, r \in R$ -i.e.

$$[F(x), r]Rd(F(x)) = \{0\} \text{ for all } x, r \in R.$$

Since R is prime, for each $x \in R$, either $F(x) \in Z$ or $d(F(x)) = 0$. The sets of $x \in R$ for which these alternatives hold are additive subgroups whose union is R; therefore, either $F(R) \subseteq Z$ or $d(F(x)) = 0$ for all $x \in R$. But by Lemma 2.2, $F(R) \subseteq Z$ would force R to be a commutative; hence

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$d(F(R)) = \{0\}$.

(ii) Since R is not commutative, it follows from Lemmas 2.4 and 2.1 (a) that $c \notin Z$ and $c^2 \in Z$.

(iii) By Lemma 2.4 we now have $cd(x) + d(x)c = 0$ for all $x \in R$; and since $c \notin Z$, it follows by Lemma 2.6 that there exists $\lambda \in C$ such that $d(x) = [\lambda c, x] = \lambda[c, x]$ for all $x \in R$. Therefore $\hat{F}(x) = cx + \lambda[c, x]$ for all $x \in S$. Since $\hat{F}(1) = c$ and $[\hat{F}(x), \hat{F}(y)] = 0$ for all $x, y \in S$, we have

$$c(cx + \lambda[c, x]) = (cx + \lambda[c, x])c \text{ for all } x \in S. \tag{3.3}$$

Now $c^2 \in Z(S)$ by Lemma 2.4, so (3.3) can be written as

$$(2\lambda + 1)(c^2x - cxc) = 0 \text{ for all } x \in S,$$

from which it follows that

$$2\lambda + 1 = 0 \text{ or } c^2x - cxc = 0 \text{ for all } x \in S. \tag{3.4}$$

Since $c^2 \in Z(S)$, either c is regular or $c^2 = 0$. In the first case we see from (3.4) that $2\lambda + 1 \neq 0$ contradicts the fact that $c \notin Z$; in the second case $2\lambda + 1 \neq 0$ yields $c = 0$, contrary to our observation that $c \notin Z$. Therefore $\lambda = -\frac{1}{2}$ and for each $x \in S$, $\hat{F}(x) = cx - \frac{1}{2}(cx - xc) = sx + xs$, where $s = \frac{c}{2}$. Recalling that $[\hat{F}(x), \hat{F}(y)] = 0$ for all $x, y \in S$, we see that S satisfies the generalized polynomial identity $\frac{1}{4}(cx + xc)(cy + yc) = \frac{1}{4}(cy + yc)(cx + xc)$ over C; hence S is primitive by Lemma 2.7. □

Proof of Theorem 3.4. In view of Lemma 3.5 and Jacobson's density theorem, we may assume that R is a noncommutative dense ring of linear transformations on a vector space V over a division ring Δ, and that there exists $k \in R \setminus \{0\}$ such that $k^2 \in Z$ and $F(x) = kx + xk$ for all $x \in R$. We need only show that $\dim(V) \leq 2$ and that in the case $\dim(V) = 2$, Δ is a field. For any subset $W \subseteq V$, we denote by $< W >$ the subspace generated by W.

By a standard argument it follows that if $\dim(V) > 1$ and $k(u) \in < u >$ for each $u \in V$, then there exists $\beta \in \Delta \setminus \{0\}$ such that $k(u) = \beta u$ for all $u \in V$. But in this case we have $(kx + xk)(ky + yk)(u) = 4\beta^2 xy(u)$ and $(ky+yk)(kx+xk)(u) = 4\beta^2 yx(u)$ for all $u \in V$, contradicting our hypothesis that R is not commutative.

Assume that $\dim(V) \geq 3$, and choose $u \in V$ such that $k(u) = v \notin < u >$. Since $k^2 \in Z(R)$, there exists $\alpha \in Z(\Delta)$ such that $k^2(w) = \alpha w$ for all $w \in V$; therefore $k(v) = \alpha u$.

Suppose that $k(V) \not\subseteq < u, v >$, in which case there exists $z \in V \setminus < u, v >$ and $w \in V$ such that $k(w) = z$. Then $\{u, v, w\}$ is a linearly independent subset of V and there exist $a, b \in R$ such that $a(u) = v, a(v) = w, a(w) = u, b(u) = u, b(v) = 0$ and $b(w) = 0$. It is readily verified that the condition $(ka+ak)(kb+bk)(u) = (kb+bk)(ka+ak)(u)$ implies that $b(z) = z$. It follows that if $z \in < u, v, w >$, then $z = b(z) \in < u >$, contrary to the fact that $z \notin < u, v >$; therefore $\{u, v, w, z\}$ is a linearly independent subset of V and there exist $a', b' \in R$ such that $(a'(u), a'(v), a'(w), a'(z)) = (v, w, u, 0)$ and $(b'(u), b'(v), b'(w), b'(z)) = (u, 0, 0, 0)$. But the argument given for a and b shows that this is incompatible with the requirement that $[F(a'), F(b')] = 0$; therefore we must have $k(V) \subseteq < u, v >$.

Since $\dim(V) \geq 3$, $\ker(k) \neq \{0\}$ and there exists $t \in V \setminus \{0\}$ such that $k(t) = 0$. Therefore $k^2(t) = \alpha t = 0$, so that $\alpha = 0$, $k(v) = 0$ and $k^2 = 0$. Thus, if $q \in V$ and $k(q) = \gamma u + \delta(v)$, then $0 = k^2(q) = \gamma v$ so $\gamma = 0$. Hence $k(V) \subseteq < v >$ and $\ker(k)$ has dimension at least 2; and since $< u, v > \neq \ker(k)$, there exists $y \in \ker(k) \setminus < u, v >$. Choosing $a, b \in R$ such that $(a(u), a(v), a(y)) = (v, y, u)$ and $(b(u), b(v), b(y)) = (u, u, y)$, we get $(kb + bk)(ka + ak)(u) = 0$ and $(ka + ak)(kb + bk)(u) = y -$ a contradiction. Therefore $\dim(V) < 3$ as required.

Finally, assume $\dim(V) = 2$. As before, we have linearly independent u, v such that $k(u) = v$. Let $\beta, \gamma \in \Delta$ and consider $a, b \in R$ such that $(a(u), a(v)) = (0, \beta u)$ and $(b(u), b(v)) = (0, \gamma u)$. Then $(ka + ak)(u) = \beta u$ and $(kb+bk)(u) = \gamma u$, and the condition $[F(a), F(b)] = 0$ gives $\beta\gamma u = \gamma\beta u$, so that $\beta\gamma = \gamma\beta$. Thus Δ is a field. □

4. Anti-commutativity conditions

In our final section we present some more elementary results, which involve anti-commutativity hypotheses.

Theorem 4.1. Let R be a prime ring with 1 and $\mathrm{char}(R) \neq 2$. If F is a generalized derivation on R such that $F(x) \circ F(y) = 0$ for all $x, y \in R$, then $F = 0$.

Proof. Note that if R is commutative, it is a domain; and the condition $F(x) \circ F(y) = 0$ is just $2F(x)F(y) = 0$. Taking $y = x$ then shows that $F(x) = 0$ for all $x \in R$.

Assume that $F \neq 0$. Then R is not commutative; and since $F(1) \circ F(1) = 0$, we have $c^2 = 0$. Note that we cannot have $d = 0$, for in that case $F(1) \circ F(x) = 0$ becomes $cxc = 0$ for all $x \in R$, which implies that $c = 0$ and hence $F = 0$.

We now have $d \neq 0$ and

$$F(x) \circ F(y) = 0 \text{ for all } x, y \in R. \tag{4.1}$$

Replacing y by yz in (4.1) and using (4.1), we get

$$(F(x) \circ y)d(z) - F(y)[F(x), z] - y[F(x), d(z)] = 0 \text{ for all } x, y, z \in R. \tag{4.2}$$

Now replacing z by $zF(x)$ in (4.2) and using (4.2), we obtain

$$(F(x) \circ y)zd(F(x)) - yz[F(x), d(F(x))] - y[F(x), z]d(F(x)) = 0. \tag{4.3}$$

Finally, replacing y by ry in (4.3) and using (4.3), we conclude that

$$[F(x), r]yRd(F(x)) = \{0\} \text{ for all } x, y, r \in R.$$

Again, invoking the primeness of R, we learn that $F(R) \subseteq Z$ or $d(F(x)) = 0$ for all $x \in R$. But Lemma 2.2 implies that if $F(R) \subseteq Z$, then R is commutative, contrary to our assumption that $F \neq 0$; therefore $d(F(x)) = 0$ for all $x \in R$, and by Lemma 2.4 $cd(x) + d(x)c = 0$ for all $x \in R$. The condition that $F(1) \circ F(x) = 0 = c(cx + d(x)) + (cx + d(x))c$ reduces to $cxc = 0$; hence $c = 0$ and R is commutative by Lemma 2.3, so we have again contradicted our assumption that $F \neq 0$. Therefore, $F = 0$. □

Theorem 4.2. Let R be a 2-torsion free ring with 1. If F is a generalized derivation such that $F(x) \circ F(y) = x \circ y$ for all $x, y \in R$, then there exists c in Z such that $c^2 = 1$ and $F(x) = cx$ for all x in R. Thus, if R is prime, F is the identity map or its negative.

Proof. Since $F(1) \circ F(1) = 1 \circ 1$, we have $c^2 = 1$. Thus, the condition $F(x) \circ F(1) = x \circ 1$ reduces to

$$cxc + d(x)c + cd(x) = x. \tag{4.4}$$

Postmultiplying and premultiplying this equation by c and comparing the results yields $d(x) + cd(x)c = 0$; and premultiplying by c gives

$$cd(x) + d(x)c = 0. \tag{4.5}$$

It now follows from (4.4) that $cx = xc$ for all x in R, so that c is in Z; and since c is invertible, (4.5) shows that $d = 0$ and hence $F(x) = cx$ for all x in R. □

A similar proof yields our final theorem.

Theorem 4.3. Let R be a 2-torsion free ring with 1. If F is a generalized derivation such that $F(x) \circ F(y) + x \circ y = 0$ for all $x, y \in R$, then there exists c in Z such that $c^2 = -1$ and $F(x) = cx$ for all x in R.

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