Price Determination Method Based on Price Elasticity

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In this paper, we propose the price determination method using the parameter of the price elasticity that shows the relation between price and demand. Firstly, the state of the price elasticity is examined under the condition that the relation between price and demand are assumed by the inverse proportional function, the linear function and the quadratic function. Secondly, the profit is estimated for each product by the break even point analysis. And the price is determined under the condition that the relation between the demand and price is shown by one of three demand−price functions above mentioned .

1. INTRODUCTION

The price determination plays an important role in the management strategy to yield a larger profit and to increase the market share. Many authors have be forecasted on the demand.[1][2][3][4]. In these researches, the amount of quantity was mainly improved to maximize the profit under the condition that the price was fixed and the due data was given. Recently the price of product changes depending on the consumers demand, and drastic price cut is required to compete in the market. However, there is no method for determining the price under these circumstances.

In this paper, we propose the price determination method based on the price elasticity function [5] and the break even point analysis under the condition that the demand−price function is given. [6] The price elasticity function is the index of the relation between price and demand. Firstly, we formulate the price elasticity function in three cases, that is, inverse proportional function, linear function and quadratic function and evaluate their characteristics precisely. Next, we estimate the profit of each product comparing with break even point and the price is determined to bring the profits.

2 PROPOSED METHOD

2.1 Symbols

We use the following symbols for determining the price based on the relation between price and demand.

S_i : The product's series, i=1,2, $\cdots$,k

DC_i : Limited price showing price zone of series S_i

C_{ij} : Price of products in series S_i. j=1,2, $\cdots$,n_i

D_i : Fixed cost for the products in series S_i

V_i : Variable cost for the products in series S_i

Q_i : Demand of products in series S_i : $Q_i = f(C_i)$

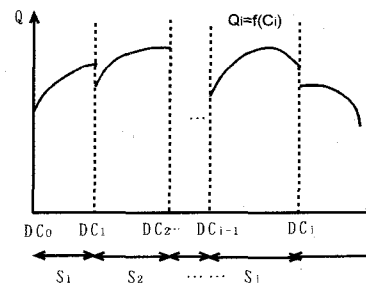η_i : Price elasticity function of the product

Fig.1 Demand−price function in each series

2.2 Restrictions

Until now, the relation between demand and price varies fairly in wide area, but now it is changing as follows. Firstly, the price of product is classified into some series (S_i). Secondly, the price in series (S_i) varies in the price zone [DC_{i-1}, DC_i] showing in Fig. 1. In each series (S_i), the order of price is $C_{i1} \leqq C_{i2} \cdots \leqq C_{in_i}$, and we assumed that the variable cost (V_i) and the fixed cost (D_i) of the product are constant.

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Further we assumed that the demand–price function (Q=f(C)) is given. The price is determined based on Price Elasticity Function (PEF) and the break even point analysis.

2.3   The Demand–price Function

In this study, three types of demand–price function are assumed. The inverse linear model is the most famous one, and the demand varies inversely for price (Fig.2). The linear model is a simple relation, and the demand varies linearly (Fig.3). For the quadratic function, the demand shows the maximum or minimum in the price zone (Fig.4).

2.4   Price Elasticity Function (PEF)

In the latter, the suffix i of variable is omitted to show simply the equation. The price elasticity function (PEF) can be shown as equation (1).

$$\eta =(dQ/Q)/(dC/C) \qquad (1)$$

PEF is the parameter of the sensitivity of change of the demand per unit price (Q/C) at (Q/C). As the demand is increased (decreased) for the price, PEF(η) is positive (negative). The price elasticity function is calculated to three functions.

(1) Case 1: Inverse proportional function (Q=a/C,a>0)

As the price is high/low, the demand becomes low/high. This is a natural phenomenon in the production. In this case,   PEF (η) in the series (S $_i$) becomes constant as $-$ 1 by the equation (2).

$$\eta =(dQ/Q)/(dC/C)=-1 \qquad (2)$$

PEF is independent of the price change and remains the constant. (Fig.5)

(2) Case 2:   Linear function   (Q=aC + b)

As the price is equal to zero, the demand becomes infinity by the inverse proportional function. From the view point of customer satisfaction, this phenomenon is not natural. So, the relation between price and demand is considered under the condition that the demand is constant, as price is equal to zero. As the relation between C and Q is given by the linear function, PEF(η) in the series (S $_i$)is given by equation (3).

$$\eta =1-b/(aDC+b) \qquad (3)$$

(a)   a>0 and aDC $_{i-1}$ +b>0

Fig. 6–(i) shows the relation between the price and η . The value of η is always positive, as the price is increased. In the " Bubble period in Japan" from 1985–1988 [7], as the price of land was increased, the demand was also increased. This phenomenon is explained by this equation. η is
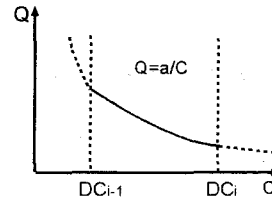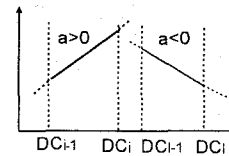

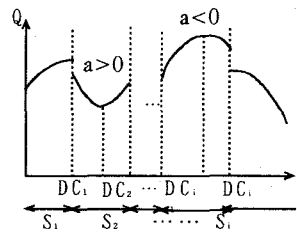Fig.2 Inverse propotional function

Fig.3 Linear function
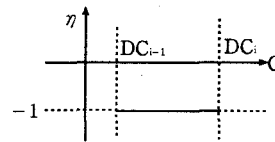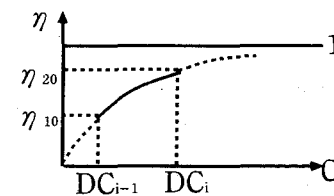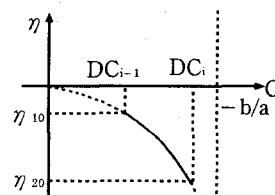

Fig.4 Quadratic function

Fig.5 Relation between price and η of Q=a/C

(i)   a>0 and aDC $_{i-1}$ >0

(ii)   a<0, aDC $_i$ +b>0

Fig.6 Relation between price and η of Q=aC + b

always positive, and as the price C is increased infinity, the limited value of η approaches to 1. In the price zone [DC $_{i-1}$, DC $_i$], η increased monotonously from η_{10} to η_{20}. Where η_{10} and η_{20} are calculated by the equation (4).

$$\eta_{10} = 1 - b/(aDC_{i-1} + b)$$

$$\eta_{20} = 1 - b/(aDC_i + b) \qquad (4)$$

(b) a<0 and aDC $_i$ +b>0

As the price is lowered, the demand is increased conversely. This relation is shown in Fig. 6—(ii). This price is belonged in the range of $0 \leq C < b/a$. As the demand—price function is decreased, η is negative and belongs in the area of [η_{10}. η_{20}].

(3) Case3: Quadratic function Q=a(C − b)^2 + d (DC $_{i-1} \leq$ b $\leq$ DC $_i$)

As the price is increased, the demand is changed the state, that is, decreasing from increasing or vice versa at the price b. This situation is modelled by the quadratic equation of Q=a(C−b)^2 +d. PEF (η) is given by the equation(5).

$$\eta = \{2a(C-b)C\}/\{a(C-b)^2 + d\} \qquad (5)$$

And the range of η is depended on the parameters of **a** and **d**. For the price values DC $_{i-1}$ and DC $_i$, η_{10} and η_{20} are calculated by the equation (6).

$$\eta_{10} = \{2a(DC_{i-1}-b)DC_{i-1}\}/\{a(DC_{i-1}-b)^2 + d\}$$

$$\eta_{20} = \{2a(DC_i-b)DC_i\}/\{a(DC_i-b)^2 + d\} \qquad (6)$$

(a) a>0 and d>0

① (b/2) $\leq$ DC $_{i-1}$:

η is increased monotonously from η_{10} to η_{20} in the area of price zone [DC $_{i-1}$, DC $_i$] in Fig.7—(i). And η is negative, as C is less than b and η is positive, as b $\leq$ C $\leq$ DC $_i$.

② DC $_{i-1}$ <(b/2)<DC $_i$:

The numerator of the equation (5) is attained the minimum value at C=b/2 and η_{00} is put as the value of η at C=b/2 by the equation (7).(Fig.7—(ii)).

$$\eta_{00} = (-ab)/(ab^2/4 + d) \qquad (7)$$

η is decreased from η_{10} to η_{00}, as C is increased from DC $_{i-1}$ to b/2, and η is increased monotonously η_{00} to η_{20}, as C is greater than b/2.

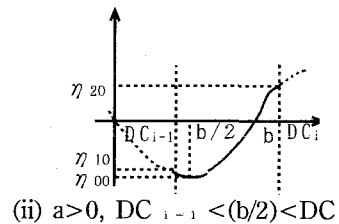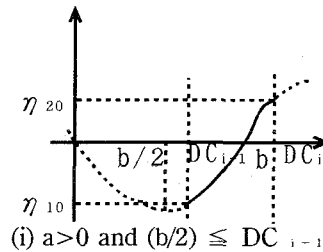(b) a<0, a(DC $_{i-1}$ −b)^2 +d>0, a(DC $_i$ −b)^2 +d>0, and DC $_{i-1} \leq$ b $\leq$ DC $_i$

① (b/2)<DC $_{i-1}$:

η is positive in the area of [DC $_{i-1}$,b]. And η is decreased from η_{10} (η_{10} >0) to 0. Further η is negative and decreased from 0 to η_{20} (η_{20} <0), as C is increased from b to DC $_i$.

② DC $_{i-1} \leq$ (b/2) $\leq$ DC $_i$:

η is positive in the area of [DC $_{i-1}$, b]. But η is increased from η_{10} to η_{00}, as C is increased from DC $_{i-1}$ to (b/2). And η is decreased monotonously from η_{00} to η_{20} (<0), as C is greater than (b/2).

(i) a>0 and (b/2) $\leq$ DC $_{i-1}$

(ii) a>0, DC $_{i-1}$ <(b/2)<DC $_i$

Fig.7 Relation of price and η of Q=a(C − b)^2 +d

(i) a<0 and (b/2)<DC $_{i-1}$

(ii) a<0 and (b/2) $\geq$ DC $_{i-1}$
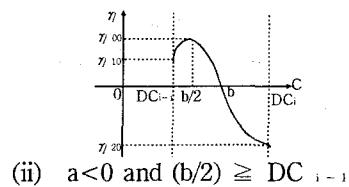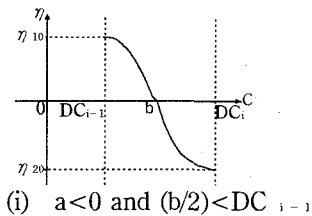
Fig.8 Relation between cost and η of Q=a(C−b)^2 +d

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Table1 shows the range of η in each demand–price function.

Table1, Characteristics of the price elasticity of demand

Function	PEF η	Range of η			
Q=a/C		a>0			−1
Q=aC+b	$\eta = 1 - b(aDC+b)$	a>0, a(DC$_{i-1}$+b)>0		$0 < \eta_{10} \leqq \eta \leqq \eta_{20} \leqq 1$	
		a<0, a(DC$_{i-1}$+b)>0		$0 > \eta_{10} \geqq \eta \geqq \eta_{20}$	
Q=a(C−b)^2 +d	$\eta = \dfrac{2aC(C-b)}{a(C-b)^2+d}$	a>0 d>0	(b/2)$\leqq$DC$_{i-1}$	$\eta_{10} \leqq \eta \leqq \eta_{20}$	
			DC$_{i-1}$<(b/2)<DC$_i$	$\eta_{10} \geqq \eta \geqq \eta_\infty, \ \eta_\infty \leqq \eta \leqq \eta_{20}$	
		a<0 a(DC$_{i-1}$−b)^2+d>0 a(DC$_i$−b)^2+d>0	(b/2)<DC$_{i-1}$	$\eta_{10} \geqq \eta \geqq \eta_{20}$	
			DC$_i \geqq$(b/2)$\geqq$DC$_{i-1}$	$\eta_{10} \leqq \eta \leqq \eta_\infty, \eta_\infty \geqq \eta \geqq \eta_{20}$	

2.5 Cost Determination Method

We can estimate the profit of each product by break even point analysis. The amount of break even point (TP) is given by the equation (8).

$$TP=D/(C-V) \qquad (8)$$

If Q is greater than TP, the product brings the profits and if Q is lesser than TP, we can't yield a profit. So we should change the price or fixed/variable cost.

(1) Case 1 : Q=a/C, a>0

If it is satisfied that {a/C−(D/(C−V)}>0, the profit can be brought, and we can get the limit price C $_0$ by the equation (9).

$$C_0=aV/(a-D) \qquad (9)$$

① V ≦ DC$_{i-1}$

(a) C$_0$ <DC$_{i-1}$

As price C$_0$ is less than DC$_{i-1}$, we can determine the price at any value in the price zone [DC$_{i-1}$, DC$_i$].

(b) DC$_{i-1}$≦ C$_0$≦ DC$_i$

The price area is restricted to [C$_0$, DC$_i$]. So the price of a product should be determined in this area.

② DC$_{i-1}$≦ V ≦ DC$_i$

The price area is determined as [C$_0$, DC$_i$] if C$_0$ is less than DC$_i$. As the price area [C$_0$, DC$_i$] becomes narrow, so we make effort to decrease the variable or fixed cost. In the other case, we can not determine the price area for the series.

(2) Case 2: Q=aC+b

(a) a>0 and aDC$_{i-1}$ +b>0

There are two prices (C$_{10}$ and C$_{20}$) satisfying D/(C−V)=aC+b and they are given by equation (10).

$$C_{10}=((aV-b)+\sqrt{DD})/2a$$
$$C_{20}=((aV-b)-\sqrt{DD})/2a \qquad (10)$$
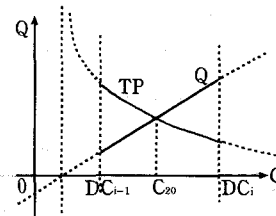Where, DD=(aV+b)^2 +4aD

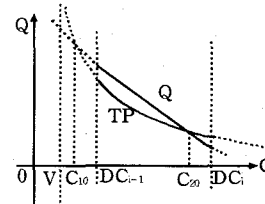And these prices satisfy the following relation.

C$_{10}$ <V<C$_{20}$

① V ≦ DC$_{i-1}$

(i) C$_{20}$ <DC$_{i-1}$: The price zone [DC$_{i-1}$, DC$_i$] is used to determine the price of a product.

(ii) DC$_{i-1}$≦ C$_{20}$≦ DC$_i$: As the price C is greater than C$_{20}$, we can yield the profit. But in this case, the price area [C$_{20}$, DC$_i$] becomes narrow, and the price should be determined highly.

(i) a>0 and aDC$_{i-1}$ +b>0

(ii) a<0 and aDC$_{i-1}$ +b>0

Fig.8 Price area of Q=aC+b

② DC $_{i-1}$ <V<DC $_i$

If C $_{20}$ is less than DC $_i$, the price can be determined in the narrow area of [C $_{20}$, DC $_i$].

(b) a<0 and aDC $_i$ +b>0:

In this case, we have two prices (C $_{10}$, C $_{20}$) under the condition that DD in equation (10) is positive. And V ≤ C $_{10}$ ≤ C $_{20}$ is always satisfied . The price area can be determined in the following cases.

① V<DC $_{i-1}$

(i) If C $_{10}$ <DC $_{i-1}$ and DC $_i$ <C $_{20}$, then the price is determined in the price zone [DC $_{i-1}$, DC $_i$].

(ii) If DC $_{i-1}$ <C $_{10}$ <DC $_i$ <C $_{20}$, then the price area is determined as [C $_{10}$, DC $_i$].

(iii) If DC $_{i-1}$ <C $_{10}$ <C $_{20}$ <DC $_i$, the price area is put as [C $_{10}$, C $_{20}$].

② DC $_{i-1}$ <V<DC $_i$:

As the relation of V ≤ C $_{10}$ ≤ C $_{20}$ is satisfied, then the price area becomes [C $_{10}$, DC $_i$] or [C $_{10}$, C $_{20}$].

(3) Case 3: Q=a(C−b) ^{2} +d, DC $_{i-1}$ ≤ b ≤ DC $_i$

(a) a>0 and d>0 :

The equation a(C−b) ^{2} +d=D/(C−V) has one or two or three roots. These roots are greater than V.

① Case of one root (C $_{10}$): V and C $_{10}$ are compared with DC $_{i-1}$ and DC $_i$. The price area is one of [DC $_{i-1}$, DC $_i$] or [C $_{10}$, DC $_i$].

② Case of two roots (C $_{10}$, C $_{20}$): Q=a(C−b) ^{2} +d and TP=D/(C−V) contacted at C $_{20}$ .The price area is determined as one of [DC $_{i-1}$,DC $_i$] or [C $_{10}$,DC $_i$].

③ Case of three roots (C $_{10}$, C $_{20}$, C $_{30}$):

We compare C $_{10}$ and C $_{20}$, C $_{30}$ with V, DC $_{i-1}$ and DC $_i$, and determine the price area which satisfies the relation of Q>TP.

(b) a<0, a(DC $_{i-1}$ −b) ^{2} +d>0, and a(DC $_i$ − b) ^{2} +d>0

There is always one root C $_{10}$ (<V) in any case. But this root can not use the price determination. So the price area is determined, in the case that the equation Q=TP has three roots (C $_{10}$, C $_{20}$, C $_{30}$).

(i) a>0 and d>0

(ii) a<0, a(DC $_{i-1}$ −b) ^{2} +d>0 and
a(DC $_i$ −b) ^{2} +d>0

Fig.9 Price area of Q=a(C−b) ^{2} +d, b>0

These roots satisfy the relation C $_{10}$ <V<C $_{20}$ <C $_{30}$. As we compare two roots C $_{20}$ and C $_{30}$ with V, DC $_{i-1}$ and DC $_i$, the price area that satisfies the relation of Q>TP is determined as one of [DC $_{i-1}$, DC $_i$], [C $_{20}$, DC $_i$] or [C $_{20}$, C $_{30}$].

3 . EXAMPLE

We apply the proposed method to determine the price of automobiles. There are four series (S $_i$) of automobiles. The price and demand in each series is given by Fig.10. [8]

(1) Demand−price function in each series: Demand−price function in each series are given as follows

S $_1$: Q=0.3C−20 × 10 ^{4} :Price zone: (DC $_0$ =100 × 10 ^{4} $, DC $_1$ =150 × 10 ^{4} $)

S $_2$: Q=(− 0.0024(C/10 ^{4} −220) ^{2} +30) × 10 ^{4}

                :Price zone: (DC $_1$ =150 × 10 ^{4} $, DC $_2$ =250 × 10 ^{4} $)

S $_3$: Q=0.07C−0.5 × 10 ^{4} :Price zone: (DC $_2$ =250 × 10 ^{4} $, DC $_3$ =350 × 10 ^{4} $)

S $_4$: Q=−0.07C+44.5 × 10 ^{4} :Price zone: (DC $_3$ =350 × 10 ^{4} $, DC $_4$ =400 × 10 ^{4} $)

(2) The price area in each series can be determined as follows:

① Series 1(S $_1$): The variable cost is equal to V $_1$ =90 × 10 ^{4} $ and the fixed cost D $_1$ is 260 × 10 ^{8} $.

As the demand−price function is linear and parameter a is positive, we calculate only one root C $_{20}$ by the equation (10) as follows. The profit will be yielded as the price is belonged in the price area of [110 × 10 ^{4} , 150 × 10 ^{4} $].
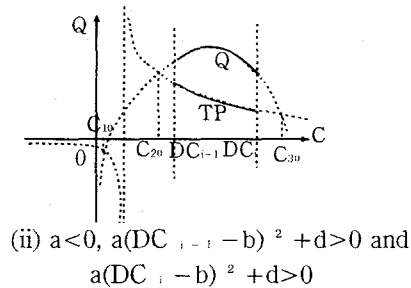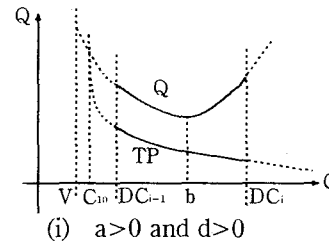
$$\sqrt{DD} = \sqrt{(b+aV)^2+4aD} \quad =\sqrt{((-20+0.3\times90)^2+4\times0.3\times260)\times10^8} =19\times10^4$$

$$C_{20}=248\times10^4 \ \$$$

② Series 2(S_2):In this series, the variable cost is equal to $V_2 =120 \times 10^4 \$$, and the fixed cost D_2 is $290 \times 10^8 \$$.

As $a=-0.0024$ is negative, one root C_{10} ($<150 \times 10^4 \$$) is always existed. TP=(290×10^8) $/(C-120 \times 10^4)$ is less than 9.67×10^4 in the price zone of [$150 \times 10^4 \$$, $250 \times 10^4 \$$]. $Q=(-0.0024(C-220)^2 +30) \times 10^4$ is greater than 18.4×10^4 in price zone of [$150 \times 10^4 \$$, 250×10^4]. Then Q is greater than TP in this price zone. So the two roots of Q=TP are satisfied the following conditions:

$V=120 \times 10^4 \$ <C_{20} <DC_1 =150 \times 10^4 \$$, and $DC_2 =250 \times 10^4 \$ \leq C_{30}$.

From these results, the price area is determined as [$150 \times 10^4 \$$, $250 \times 10^4 \$$].

③ Series 3 (S_3): In this series, the variable cost is equal to $V_3 =230 \times 10^4 \$$, and the fixed cost D_3 is $310 \times 10^8 \$$. C_{20} is less than $250 \times 10_4 \$$, than we can yield the profit, as the price is belonged in the price zone of [250×10^4 , 350×10^4 $ \$$].

$$\sqrt{DD} =18.17\times10^4 \qquad C_{20}=248\times10^4 \ \$$$

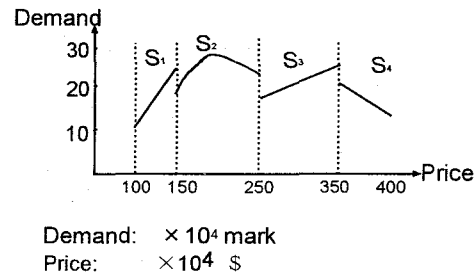Demand: $\times 10^4$ mark
Price: $\times 10^4$ $

Fig.10 Product series and price zone

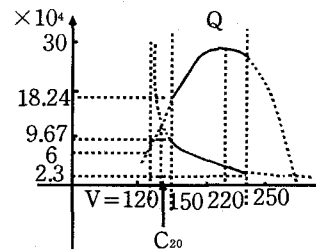Fig.11 Relation between demand (Q) and TP in S_2

④ Series 4 (S_4): In this series, the variable cost is equal to $V_4 =280 \times 10^4 \$$, and the fixed cost D_4 is $370 \times 10^8 \$$ .The profit will be brought, as the price is belonged in the area of [$350 \times 10^4 \$$, $372 \times 10^4 \$$].

$$\sqrt{DD} =22.72\times10^4 \qquad C_{10}=296\times10^4 \$ \qquad C_{20}=620\times10^4 \ \$$$

4. CONCLUSION

In this paper, we proposed the price determination method based on the price elasticity function and the break even point analysis under the condition that the demand—price function is given. The price elasticity function is the index of the relation between the price and demand. Firstly, we formulated the price elasticity function in three cases of inverse proportional function, linear function and quadratic function and evaluated their characteristics precisely. Next, we estimated the profit of each product comparing with break even point and the price is determined to bring the profit.

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