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Internal Structure and Viscoelasticity Estimation by a Mechanical Impedance — In case of a vibrating disk —

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SYNOPSIS

In a stiffness estimation of living body, an internal structure under the skin influences the measured results. Because a different stiffness of body caused by bones and muscles is obtained. In this paper, by using a measurement system of mechanical impedance, the relations between a viscoelasticity and a distance from the surface of silicone-gel model is calculated. This relation is applied to a silicone-gel tumor model and a shape and a viscoelasticity of semi-sphere silicone-gel tumors are estimated. The obtained results are expressed as a reconstructed 3-D image of shape / viscoelasticity. The revised curve-fitting of mechanical impedance and the cancellation of peripheral vibration influence are proposed in order to increase an estimation accuracy.

1. INTRODUCTION

In a stiffness estimation of living body from a skin surface, an internal structure under the skin like bones and muscles influences the measurement results. This means conversely that it is possible to estimate a structure and viscoelasticity of the inside from the skin surface (1).

In a clinical diagnosis, X-ray CT, MRI, SPECT and Ultrasound Imaging have been used, although these methods have been chiefly applied to a structural evaluation. The recently developed CTs give an information of dynamic movements and functions of internal organs.

In this study, by using a random vibration (30-1000 Hz), a mechanical impedance at a vibrating point of model surface is measured. The experimental silicone-gel model includes two hemisphere gels whose stiffness is different from that of peripheral silicone-gel. By applying the mechanical impedance equation with a vibrating disk in a soft medium, an internal structure and viscoelasticity of silicone-gel model are estimated. Then a reconstructed 3-D images of estimated structure and viscoelasticity of the model are obtained.

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2. FORCED VIBRATION OF A RIGID DISK

2.1 Mechanical Impedance with an Vibrating Disk

The general theory of wave propagation in a homogeneous, isotropic, elastic, viscous, compressible medium was proposed by Oestreicher[2]. From its solution, the radiation impedance of a vibrating sphere is derived. When this equation is applied to a mechanical impedance measured with a vibrating disk in this study, it may give a rough approximation to the measurement result. In this study, the theoretical equation of mechanical impedance is obtained by expanding the analytical result in case of a vibrating disk in an elastic half-space.

The mechanical impedance is defined by force $f(t)$ and velocity $v(t)$ at a vibrating point. In the analysis of wave propagation in living tissues, a viscosity and elasticity of medium should be considered.

The displacement vector of wave equation is expressed in cylindrical coordinates[3] as follows :

$$(\lambda + 2\mu)\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\boldsymbol{u})\right] + \mu\frac{\partial^2 \boldsymbol{u}}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 \boldsymbol{w}}{\partial r \partial r} + \rho w^2 \boldsymbol{u} = 0 \quad \text{...(1)}$$

$$(\lambda + 2\mu)\frac{\partial^2 \boldsymbol{w}}{\partial z^2} + \frac{\boldsymbol{u}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \boldsymbol{w}}{\partial r}\right) + \frac{\lambda + \mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \boldsymbol{u}}{\partial r}\right) + \rho w^2 \boldsymbol{w} = 0 \quad \text{...(2)}$$

where λ and μ are the Lame constants and ρ is its density of the medium, $\mu = \mu_1 + j\omega\mu_2$, $\lambda = \lambda_1 + j\omega\lambda_2$, μ_1[N/m²]: coefficient of shear elasticity, μ_2[Ns/m²]: coefficient of shear viscosity, λ_1[N/m²]: coefficient of volume compressibility and λ_2[Ns/m²]: coefficient of volume viscosity. $\boldsymbol{u}$ and $\boldsymbol{w}$ are defined as follows:

$$\boldsymbol{u}(r, z) = \int_0^\infty y\bar{u}(y, z)\, J_1(yr)\, dy \quad \text{...(3)}$$

$$\boldsymbol{w}(r, z) = \int_0^\infty y\bar{w}(y, z)\, J_0(yr)\, dy \quad \text{...(4)}$$

The vertical stress at any point is obtained from Eqns. (3) and (4),

$$\sigma = -\mu\int_0^\infty y\left[\alpha^{-1}A\,(2y^2 - k^2)\, e^{-\alpha z} + 2\beta C\, e^{-\beta z}\right] J_0(yr)dy\, e^{-j\omega t} \quad \text{...(5)}$$

where $k^2 = \rho\omega^2/\mu$. Using the boundary condition in this study, the normal stress under the disk is obtained by putting $z = 0$ in Eqn. (5).

$$F = \frac{2\,c\,\mu\, e^{-j\omega t}}{\pi(1-\eta)\, r}\frac{d}{dr}\int_r^1 \frac{\tau\,\theta(\tau)\, dr}{\sqrt{\tau^2 - r^2}} \quad \text{...(6)}$$

The total force applied to the vibrating disk is given by the following equation:
where a [m]: radius of the disk and η: Poisson's ratio, $\eta = 1/2(\lambda + \mu)$.

$$F(j\omega) = \frac{4\,a\,\mu}{1-\eta}\, c\, e^{-j\omega t}\int_0^1 \theta(\tau)\, d\tau \quad \text{...(7)}$$

2.2 Viscoelasticity in Low Frequency

From a definition of mechanical impedance, Eqn. (7) is transformed,

$$Z(j\omega) = \frac{4a\mu}{j\omega(1-\eta)}\int_0^1 \theta(t)\,dt \qquad \ldots(8)$$

where $\theta(t)$ is a solution of Fredholm integral equation, $\theta(t)$ is written as follows:

$$\theta(t) = 1 + \sum_{n=1}^{\infty} k^n \theta_n(t) \qquad \ldots(9)$$

where $k^2 = \rho\omega^2/\mu$. $\theta_n(t)$ is easily obtained in case of n =1, $\theta_1(t)$ is

$$\theta_1(t) = -i\frac{I_1}{\pi} \qquad \ldots(10)$$

Eqn. (10) is substituted for Eqn. (9),

$$Z(j\omega) = \frac{4a\mu I_1}{\pi(1-\eta)}\left(\sqrt{\frac{\rho}{\mu}} + j\omega\right) \qquad \ldots(11)$$

I_1 is obtained from Poisson's ratio and I_1 = 2.621 when the ratio is 0.5 because $\lambda_1 = \infty$ and $\lambda_2 = 0$ in an incompressible medium like a living body. These conditions are substituted for Eqn. (11) and the mechanical impedance is defined as follows:

$$Z(j\omega) = 20.968\,a^2\frac{\sqrt{\rho\left(\sqrt{\mu_1^2+\omega^2\mu_2^2}+\mu_1\right)}}{2\pi} + 8\,a\mu_2 + j\omega 0.424\pi\rho a^3$$
$$+ j\,20.968\,a^2\frac{\sqrt{\rho\left(\sqrt{\mu_1^2+\omega^2\mu_2^2}-\mu_1\right)}}{2\pi} + \frac{8\,a\mu_1}{j\omega} \qquad \ldots(12)$$

The mechanical impedance spectrum has three characteristic patterns: a soft, intermediate, stiff pattern. Eqn. (12) can only express the soft spectrum pattern and its resistance spectrum of impedance can be fairly curve-fitted. The measured mechanical impedance spectrum is curve-fitted by Eqn. (12) and then μ_1, μ_2, a and ρ (constant) are calculated.

3. MEASUREMENT SYSTEM AND EXPERIMENTAL MODEL

3.1 Measurement System of Mechanical Impedance

Figure 1 shows a block diagram of measurement system of mechanical impedance [4].

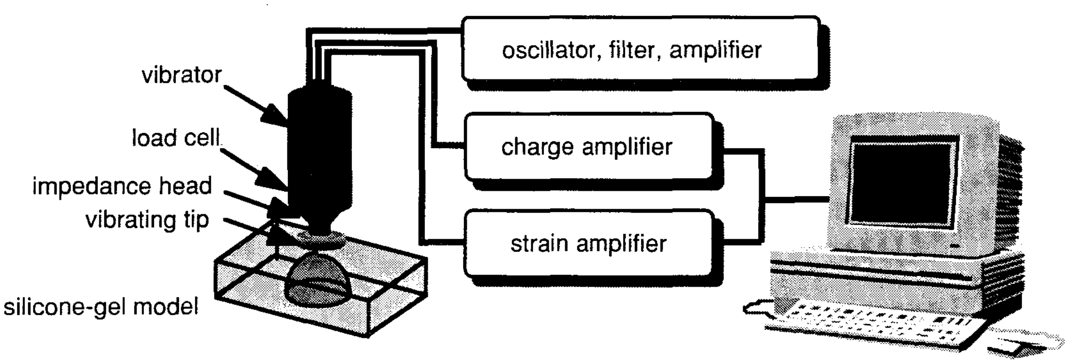

Fig. 1 Measurement system of a mechanical impedance.

The measuring probe is composed of a vibrator and an impedance-head and the surface is randomly vibrated at 30-1000 Hz through the vibrating disk of the probe. The force $f(t)$ and acceleration $a(t)$ at a driving point of the surface are detected by using the impedance-head. Both signals are transformed with FFT and the mechanical impedance spectrum is obtained.

3.2 Silicone-gel Double Layer Model

To obtain the relation between a depth and a viscoelasticity of silicone-gel model, the silicone-gel double layer model is made as shown in Fig. 2. In the silicone-gel AB or AC model, a gel A is laid on a gel B or C respectively. An upper and lower layer of this model influence each other. The thinner becomes the upper gel layer A, the stronger influences the viscoelasticity of lower gel layer B/C. The silicone-gel B is stiffer and the gel C is softer than the gel A. The penetration of the silicone-gels are as follows: A; 70, B; 50, C; 90.

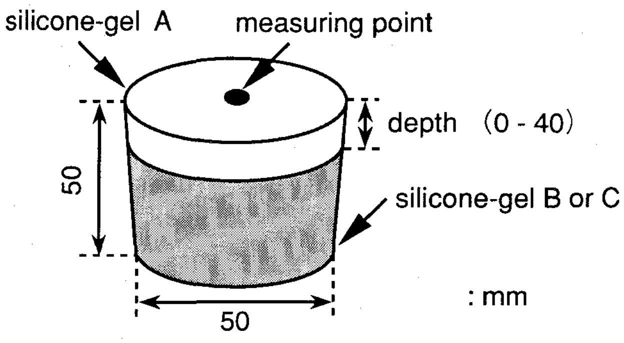

Fig. 2 Silicone-gel double layer model.

The vibrating tip is 10 mm in diameter and a contact preload onto the gel surface is 10 gf. The mechanical impedance of these models is measured with a space of 5 mm.

3.3 Silicone-gel Tumor Model

Figure 3 shows a silicone-gel tumor model . This model includes two different hemispheric tumors made of silicone-gel B and C inside the silicone-gel A. The diameter of hemisphere is 60 mm and the height of each tumor is 43 mm. The mechanical impedance is measured with a space of 5 mm and the measuring points on the tumor model are 861 (21×41). The vibrating tip is 10 mm in diameter and the contact preload is 10 gf.

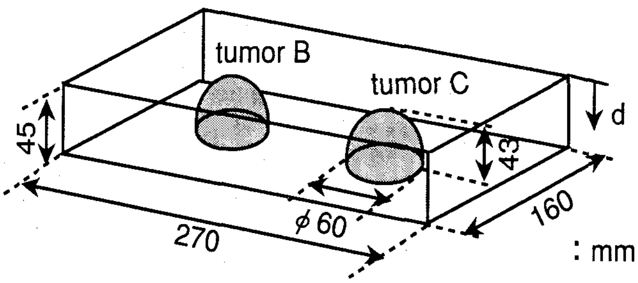

Fig. 3 Silicone-gel tumor model.

4. STRUCTURE AND VISCOELASTICITY ESTIMATION OF TUMOR MODEL

4.1 Structure Estimation

In order to estimate an internal structure of tumor model, the mechanical impedance of silicone-gel double layer model is measured. The mechanical impedance spectrum is curve-fitted by using Eqn. (12) and the effective vibrating radius a is obtained. The effective vibrating radius is reflecting a dependence upon an internal structure of silicone-gel tumor model[5]. Figure 4 shows the relation between the depth d (theoretical distance from the surface) and the effective vibrating radius a.

When the depth is deeper than 20 mm, the vibrating radius of silicone-gel AB and AC model does not change so much and the silicone-gel of lower layer does not give an influence to the measured results. It is possible to estimate the internal structure, when the internal object is located within the range of 20 mm.

In the silicone-gel AB and AC model, the relation between the depth and the effective vibrating radius is curve-fitted by using the following equations,

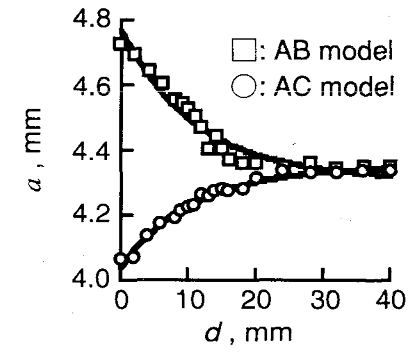

Fig. 4 Relation between the depth and the effective vibrating radius.

$$a = c_1 \pm c_2 e^{-c_3 d} \quad \text{...(13)}$$

where c_1, c_2, c_3 is a positive real number respectively.

Though the obtained effective vibrating radius includes an influence of peripheral vibration around the measuring point, that is not considered in Eqn. (13) . This is a reason why the estimated structure image does not appear clearly[6]. In an ultrasound imaging, a reconstruction algorithm by the ray tracing method is developed. In this study, we consider the influence of peripheral vibration.

Figure 5 shows the influence of peripheral vibration and Eqn. (13) is transformed as follows:

$$a = c_1 \pm c_2 \, exp\left[-c_3 d_1 - c_3 \sum_{i=2}^{69} \left\{ \frac{d_i - d_1}{|d_i - d_1|} \frac{d_0 - d_i}{d_0} \frac{d_i}{d_0} \right\} \right] \quad \text{...(14)}$$

where c_1, c_2, c_3 is a positive real number respectively and d_0 is a significant vibrating range in a semi-infinite medium. $(d_i - d_1) / |d_i - d_1|$ means a direction of peripheral vibration influence and $(d_0 - d_i)/d_0$ means an extent of influenced vibration. The influenced range of peripheral vibration is 68 measuring points, since the equivalent radius of influenced vibration is regarded as 20 mm as in Fig.4. d_i and d_0 are substituted for Eqn. (14) and then the distance to the tumor surface under the measuring point d_1 is obtained. Figure 6 shows a reconstructed image of the tumor model structure considering the influence of peripheral

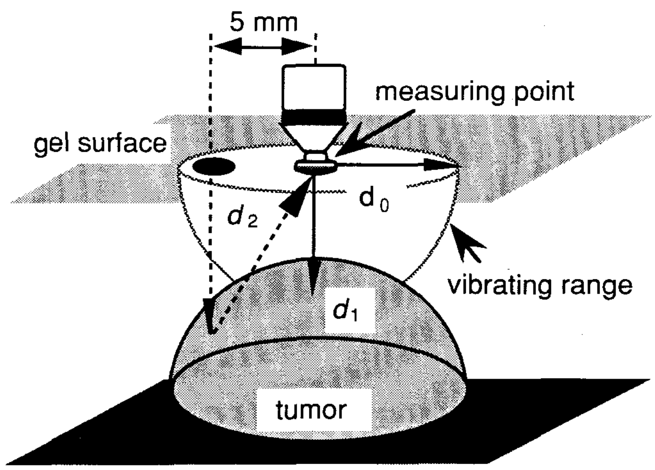

Fig. 5 Influence of peripheral vibration.

vibration, which is smoothed by using Median Filter. The value of d show a cross-section in the middle of model.

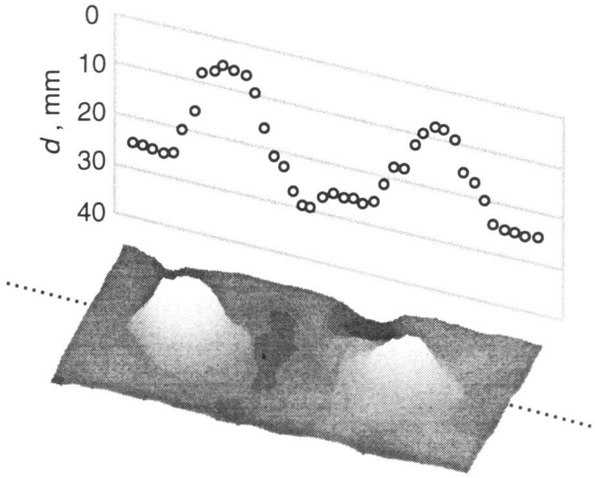

Fig. 6 Reconstructed structure image of tumors.

4.2 Viscoelasticity Estimation

Similarly to the structure estimation, the mechanical impedance spectrum of silicone-gel AB and AC model is curve-fitted by Eqn. (12) and the viscosity and elasticity are calculated in advance. Figure 7 shows the relation between the viscoelasticity and distance from the surface to lower gel layer.

The relation between μ (μ_1 and μ_2) and the depth obtained from the silicone-gel double layer model is as follows:

$$\mu_1 = c_4 + c_5 e^{-c_6 d} \quad \text{...(15)}$$

$$\mu_2 = c_4 + c_5 e^{-c_6 d} \quad \text{...(16)}$$

where c_4 , c_5, c_6 is a positive real number respectively.

Considering the influence of peripheral vibration, Eqns. (15) and (16) are transformed as follows:

$$\mu_1' = c_4 \pm c_5\, exp\left[- c_6 d_1 - c_6 \sum_{i=2}^{69} \left\{ \frac{d_i - d_1}{|d_i - d_1|} \; \frac{d_0 - d_i}{d_0} \; \frac{d_i}{d_0} \right\} \right] \quad \text{...(17)}$$

$$\mu_2' = c_4 \pm c_5\, exp\left[- c_6 d_1 - c_6 \sum_{i=2}^{69} \left\{ \frac{d_i - d_1}{|d_i - d_1|} \; \frac{d_0 - d_i}{d_0} \; \frac{d_i}{d_0} \right\} \right] \quad \text{...(18)}$$

As μ_1' and μ_2' are measured values at each measuring point, Eqn. (15) and (16) are substituted for Eqn. (17) and (18). Then μ_1 and μ_2 under the measuring point are obtained. The relation between the depth and μ_1 / μ_2, considering the peripheral vibration influence, is curve-fitted by an equivalent parallel model of gel A and B/C. Figure 8 shows the 3-D image of estimated vis-

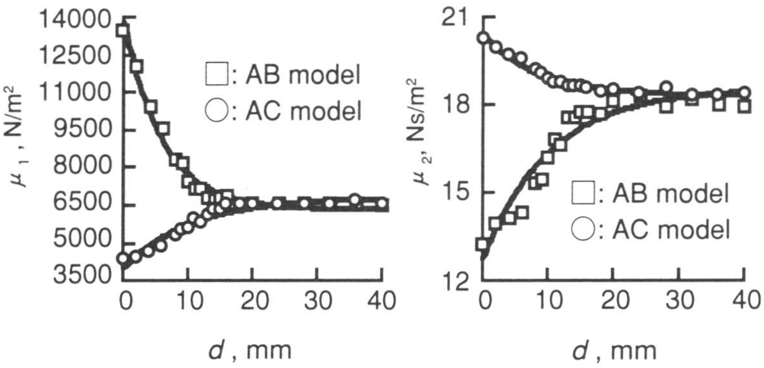

Fig. 7 Relation between the depth and the viscoelasticity.

coelasticity of internal hemisphere gel, which is smoothed by using V Filter. The values of viscoelasticity show a cross-section in the middle of model.

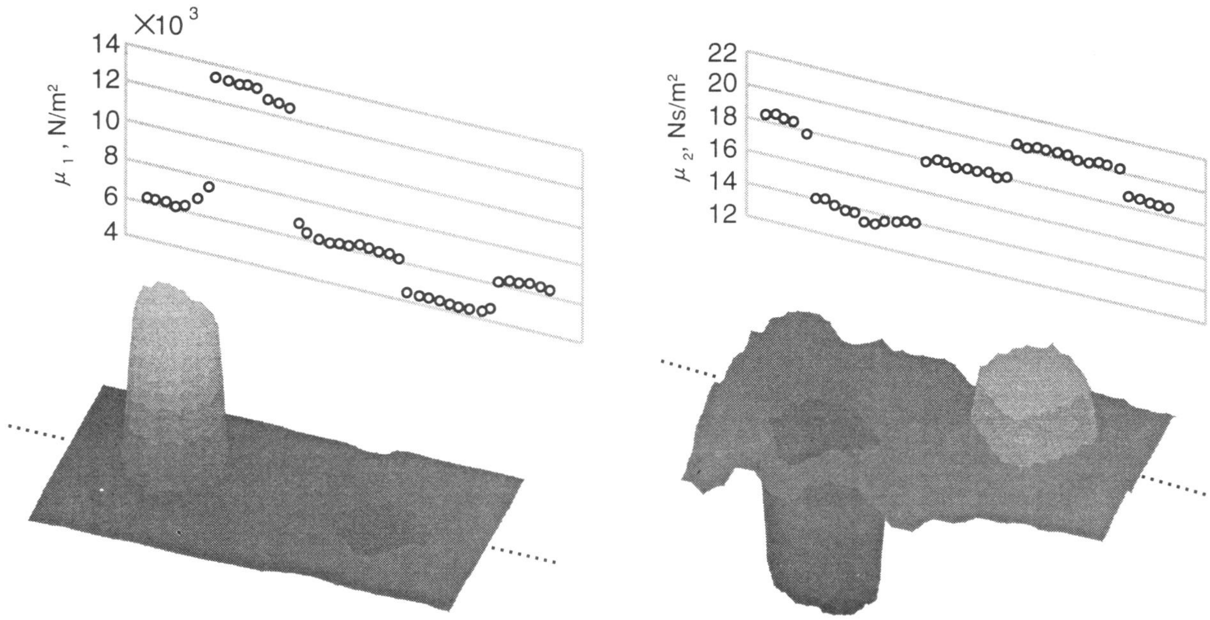

Fig. 8 Estimated elasticity and viscosity of internal hemisphere gels.

4.3 Discussions

As the effective vibration radius of silicone-gel double layer model does not change when d becomes deeper than 20 mm as in Fig. 4, the estimated depth limit is in 20 mm range. In Fig. 8, a structural and viscoelastic difference between the internal hemisphere gels and the peripheral gel A can be sharply observed.

Table 1 shows the measured / estimated diameter and viscoelasticity with a standard deviation of tumor B and C respectively. Considering the influence of peripheral vibration, the estimated values approach the measured values as it is obvious from the table.

Table 1 Estimated diameter and viscoelasticity of two hemisphere tumors.

	tumor B			tumor C		
	measured	estimated: not considering	estimated: considering	measured	estimated: not considering	estimated: considering
diameter (mm)	60.00	57.49 ±2.57	58.41 ±2.33	60.00	57.73 ±0.89	60.08 ±1.57
μ_1 (N/m²)	13826	11414 ± 64	13220 ± 279	3994	4515 ± 38	4277 ± 50
μ_2 (Ns/m²)	12.74	14.49 ± 0.31	13.70 ± 0.19	20.43	19.35 ± 0.09	20.29 ± 0.14

5. CONCLUSIONS

In this study, the relation between the depth from gel surface to lower layer gel and the viscoelasticity of the silicone-gel double layer model is obtained, by using the mechanical impedance equation with the vibrating disk. Applying this relation to the silicone-gel tumor model with two hemisphere tumors, the shape and viscoelasticity of internal silicone-gel tumors are estimated.

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