Optimization of Inventories for Multiple Companies by Fuzzy Control Method

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In this research, Fuzzy control theory is applied to the inventory control of the supply chain between multiple companies. The proposed control method deals with the amount of inventories expressing supply chain between multiple companies. Referring past demand and tardiness, inventory amounts of raw materials are determined by Fuzzy inference. The method that an appropriate inventory control becomes possible optimizing fuzzy control gain by using SA method for Fuzzy control. The variation of uncertain demand is given to the proposal method. Furthermore, the method of forecasting demand and the effectiveness of fuzzy control method are confirmed by numerical experiments. As the results, the proposal method suppressed increase in an unnecessary cost when demand varies.

1 INTRODUCTION

In recent years, to perform decumulation of inventories and improve customer satisfaction, the importance of the supply chain management (SCM) that optimizes the entire supply chain along with the market diversification is pointed out[1]. The supply chain covers from procurement of raw material to flow until selling through production. The amount of demand varies according to time. Moreover, by the bullwhip effects, a few variation of the customer demand go back to the upper stream side of supply chain and it occurs a problem in SCM. To prove the problem adjustment of materials flow was proposed [2]. The amount of the raw procurement and the amount of demand are used for operation. However, inventory of the raw material and the finished product are not treated. In SCM, the conservation of the inventory is a factor of the substantial cost. Therefore, the inventory of the raw material and the product are vital problems in SCM.

In this research, the inventory control under the en-

vironment of an uncertain demand variation through supply chain is adopted as a problem. Relations between various factors in SCM are modelled in block diagram to ease the usage of control theory. And the inventory control method that uses fuzzy control theory are proposed. The more optimal inventory control is pursued to optimize the adjustment of fuzzy control gain by SA method. The effectiveness of the proposed method is examined through simulation.

2 EXPRESSION OF SUPPLY CHAIN BY BLOCK DIAGRAM

The control block diagram of the supply chain that treats by this research is shown in Fig.1.

The input is order amount $D(t)$ from the customer, and the output is order amount $O_i(t)$ from each company. G shows the transfer characteristic (production planning), and G_1 , G_2 , G_3 shows each company of the supply chain. $R_i(t)$ is an amount of procurement of each companies. It is shown as Z^{-l_i} because there

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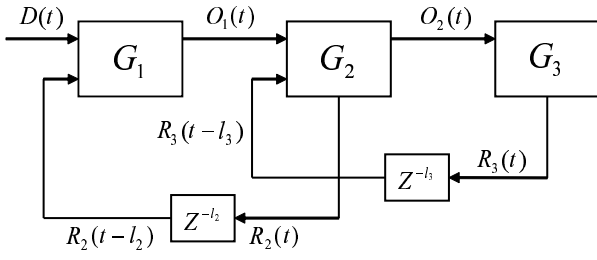*kawase@cntr.elec.okayama-u.ac.jp

Fig. 1: Block diagram for supply chain system

is a delay in distribution by the leadtime. In this research, G represents a production planning to attain at the total profit maximization in each company. As shown in Fig.1, information necessary to each production planning are $O_i(t)$ and $R_i(t)$. There exist demand transmission delay of change from customer to supplier.

3   PLANNING PROBLEM OF SUPPLY CHAIN

3.1   Supply Chain between Multiple Companies

The supply chain between multiple companies that target it by this research is shown in Fig.2.
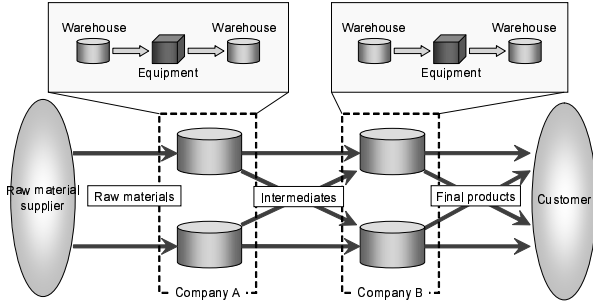

Fig. 2: Structure of supply chain of multiple companies

It is explained as follows about the supply chain planning model of each company. Each company is composed by the equipment for production and the warehouse for the inventory conservation. Company A is received in raw materials, and produced to intermediate products and ship it in company B. Company B is received in intermediate products, and produced to final products and ship it in customer. In each company, the raw material and the finished product are kept in the warehouse if necessary. Moreover, leadtime (l_1) between company A and the raw material supplier is set

as one period. And, leadtime (l_2) between company B and company A is set as one period. Shipping time between the warehouse and the machine is negligible. Processing time for jobs is given a priori. Once the production is started, it cannot be interrupted until the processing at each machine is completed.

3.2   Formulation of Production Planning Problem

In this paragraph, the formulation of the production plan is described. Each company is planning production plan individually. Whenever the amount of demand of time period t from the customer is given, company B plans the production plan at the time period K. Similarly, whenever the amount of demand of time period t from company B is given, company A plans the production plan at time period K. And, the production planning problem to the demand fluctuation is done by repeatedly solving the production planning problem whenever the period advances. Formulation of supply chain model is shown as follows. Moreover, the explanation of the variable is described to the appendix.

Object function :

$$\max : J_t$$
$$J_t = \sum_{s \in S} \sum_{i,t} (\mu_{i,s,t} S_{i,s,t} - \nu_{i,s,t} P_{i,s,t} - \zeta_{i,s,t} L_{i,s,t}$$
$$-h_{i,s,t} I_{i,s,t} - q_{i,s,t} R_{i,s,t} - \gamma_{i,s,t} Z_{i,s,t}) \quad (1)$$

subject to :

$$D_{i,s,t'} = \theta_{i,s,t'} \quad (\forall i \in I^{FP}; \forall s \in S^{FP}; t' = t) \quad (2)$$

$$D_{i,s,t'} = R_{i,s',t'+1}$$
$$(\forall i \in I^{IP}; \forall s \in S^{IP}; \forall s' \in S^{FP}; t' = t) \quad (3)$$

$$D_{i,s,t'} = \bar{\theta}_{i,s,t+1}$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; t' = t+1, \cdots, t+K-1) \quad (4)$$

$$P_{i,s,t} = n_{i,s,t} \cdot V_{i,s,t}$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; \forall t \in T) \quad (5)$$

$$\sum_{i \in I \setminus I^{RM}} n_{i,s,t} \cdot p_{i,s,t} \leq H_{i,s,t} \quad (\forall s \in S; \forall t \in T) \quad (6)$$

$$C_{i,s,t} = \sum_{\forall i \in f_{i,i',s}} f_{i,i',s} \cdot P_{i,s,t}$$
$$(\forall i \in I \setminus I^{RM}; \forall i' \in I \setminus I^{FP}; \forall s \in S; \forall t \in T) \quad (7)$$

$$I_{i,s,t} = I_{i,s,(t-1)} + \sum_{s \in S} \cdot R_{i,s,t} - C_{i,s,t}$$
$$(\forall i \in I \setminus I^{FP}; \forall s \in S; \forall t \in T) \quad (8)$$

$$I_{i,s,t} = I_{i,s,(t-1)} + P_{i,s,t} - S_{i,s,t}$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; \forall t \in T) \quad (9)$$

$$S_{i,s,t} + L_{i,s,t} = D_{i,s,t} + L_{i,s,(t-1)}$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; \forall t \in T) \quad (10)$$

$$Z_{i,s,t} \geq \mid I_{i,s,t} - I_{i,s,t}^{T} \mid$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; \forall t \in T) \quad (11)$$

$$0 \leq I_{i,s,t} \leq I_{i,s,t}^{MAX}$$
$$(\forall i \in I; \forall s \in S; \forall t \in T) \quad (12)$$

$$I_{i,s,t}, C_{i,s,t}, L_{i,s,t}, n_{i,s,t}, P_{i,s,t}, R_{i,s,t}, S_{i,s,t}, Z_{i,s,t} \leq 0$$
$$(\forall i \in I; \forall s \in S; \forall t \in T) \quad (13)$$

Eq. (1) represents a object function in each company. The object function is that attracted each cost from the total profit at time period t. Eq. (2) specify the demand to company B from a customer. Here, the demand decided than a random number generated by normal distribution on average $\bar{\theta}$ and variation σ. Eq. (3) specify the demand of the intermediate product to company A from a company B. Eq. (4) denotes that the amount of demand of period in the future is for values calculated by a demand prediction. Eq. (5) is concerned with the relationship between production amount and the number of jobs. Eq. (6) restricts production capacity. Eq. (7) expresses amount of necessary raw materials for the production of products. Eq. (8) is balance of payments constraints of amount of supply and the consumption of raw materials. Eq. (9) is balance of payments constraints of produce amount of product and amount of shipment. Eq. (10) is related to shortage of delivery to final product supplier company indicating that delivery shortage at previous time period is added to demand from final product supplier company at current time period. Eq. (11) expresses a penalty for the gap with the targeted safety stock level. Eq. (12) denotes the maximum inventory constraints. Eq. (13) denotes the nonnegative constraints on each variable.

The inventory management method is explained below. The decision method of procurement amount of raw materials and the targeted safety inventory level of product is described below. At first the amount of temporary demand $\theta_{i,s,(t+1)}^{*}$ to exceed amount of demand of time period t in probability $\lambda_{i,s}$ is calculated [3].

$$Pr[\theta_{i,s,(t'+1)}^{*} \leq \theta_{i,s,t'}] = \lambda_{i,s}$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; t' = t) \quad (14)$$

Eq. (15) [4] is provided when Eq. (14) is expanded reciprocal.

$$\theta_{i,s,t'}^{*} = \bar{\theta}_{i,s,t'} + \sigma_{i,s,t'} \cdot \Phi^{-1}(\lambda_{i,s})$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; t' = t) \quad (15)$$

Here, the temporary amount of production that can satisfy temporary amount of demand, a shortage amount of delivery at present, the targeted safety stock level is introduced.

$$I_{i,s,(t-1)} + P_{i,s,t'}^{*} \leq \theta_{i,s,t'}^{*} + L_{i,s,(t'-1)} + I_{i,s,(t'-1)}^{T}$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; t' = t + 1) \quad (16)$$

$$C_{i,s,t'} = \sum_{\forall i \in f_{i,i',s}} f_{i,i',s} \cdot P_{i,s,t'}^{*}$$
$$(\forall i \in I \setminus I^{RM}; \forall i' \in I \setminus I^{FP}; \forall s \in S; t' = t + 1) \quad (17)$$

The amount of targeted safety stock level are calculated than Eq. (18).

$$I_{i,s,t'}^{T} = \sigma_{i,s,t'} \cdot \Phi^{-1}(\lambda_{i,s})$$
$$(\forall i \in I \setminus I^{RM}; \forall s \in S; t' = t) \quad (18)$$

These expressions are considered to be a constrained condition of the production planning. Therefore, there can treat it as the production planning for uncertain demand [5, 6, 7].

4   INVENTORY CONTROL METHOD

4.1   Forecast Method of the Demand

It explains the forecast method as follows about the amount of the demand forecast used by Eq. (4).

4.1.1   Forecasting demand by the moving average method

It is explained below the moving average method [4] as one of the prediction methods. This method is a method for forecasting the demand amount of next time period $(t + 1)$ based on the history of past demand for each companies. When it is used demand data for the past p period, forecasting amount $\bar{\theta}_{i,s,t+1}$ of the demand of time period t becomes Eq. (19).

$$\bar{\theta}_{i,s,t+1} = \frac{\Sigma_{j=1}^{p} D_{t-j}}{p} \quad (19)$$

Here, D_t is amount of true demand.

4.1.2 Forecasting demand by the exponential smoothing method

It is explained below the exponential smoothing method [4] as one of the prediction methods. This method is forecasting amount of demand of the next time period $(t + 1)$ used by parameter $alpha$ for smoothing. The amount of forecast of the demand is calculated by Eq. (20).

$$\bar{\theta}_{i,s,t+1} = \alpha D_{t-1} + (1 - \alpha)\bar{\theta}_{i,s,t} \qquad (20)$$

4.2 Comparison of Forecasting Demand Method

In this paragraph, the moving average method is compared with the exponential smoothing method.

Therefore, the amount of the demand forecast of each company when temporary demand from the customer is given is compared with actual demand. Fig.3 and Fig.4 show a demand forecast and actual demand in company B. Fig.3 uses moving average method, and Fig.4 uses exponential smoothing method. Then a past history in the moving average method was assumed to be $p = 4$ period, and a smooth parameter in the exponential smoothing method was assumed to be $\alpha = 0.8$.
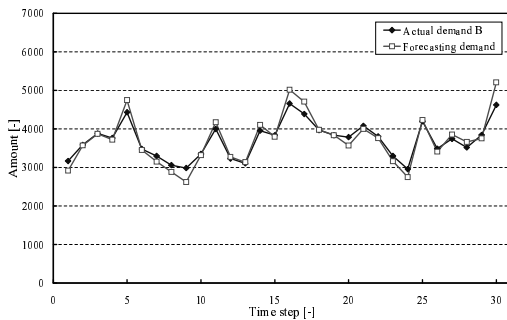

Fig. 3: The moving average method (p=4)

When the moving average method compares with exponential smoothing method from Fig.3 and Fig.4, a difference of the moving average method with actual demand is small . Therefore, it is judged that the moving average method is desirable as the demand forecast method.
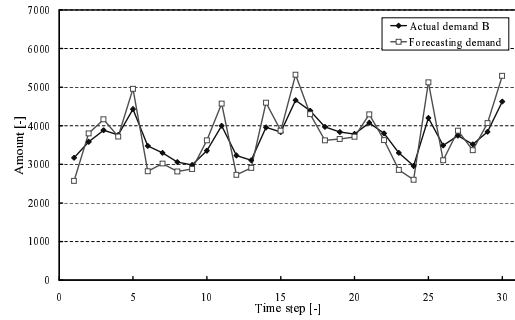

Fig. 4: the exponential smoothing method (α =0.8)

4.3 Example of Production Planning Problem

Next, the move of demand from the customer greatly increasing by the 10 time step, and decreasing greatly by the 20 time step was given, and the production planning at 30 periods was done.

The forecasting demand method assumed it moving average method. The period of production planning K in each period is 3 periods and the period of prediction p of the moving average method is 4 periods. Fig.5 show order amount of raw materials and procurement amount of raw materials in company A. Fig.6 show inventory amount tardiness and delivery shortage of intermediates product in company A.
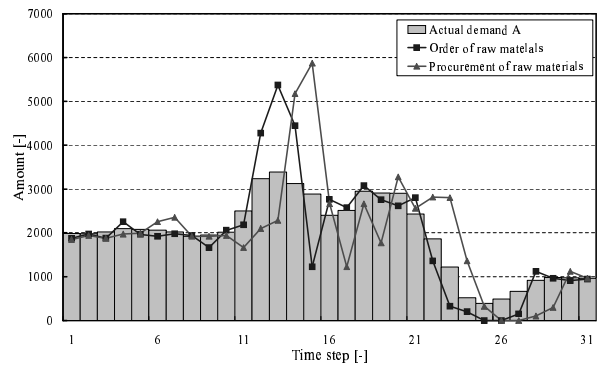

Fig. 5: Order and procurement of intermediates

When the amount of demand increases by the 10 time step, it becomes delivery shortage due to raw material shortage. To supplement this, the amount of the order grows. The leadtime between company A and B is one period. However, the amount of procurement of the 11 time step has been becoming small compared with the amount of the order. This is because company A cannot also ship it to a sudden order from company
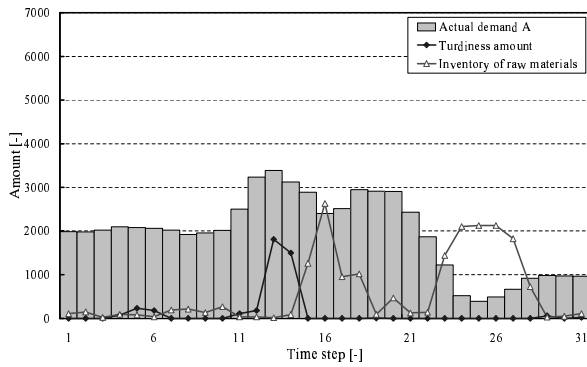
Fig. 6: Inventory of intermediates and delivery shortage

B because it is raw material shortage. The amount of procurement increases greatly at 12 time step after two periods when one period of the leadtime between company A and the raw material supplier was added. An unnecessary inventory by an excessive order increases by the 13 time step not to consider this and to order based on a present amount of delivery shortage. Fuzzy control method that operates the amount of the order when demand move to solve such a problem is described as follows.

5   OPTIMIZATION OF INVENTORY AMOUNT BY FUZZY CONTROL

5.1   Control Model of the Amount of Inventoriy

It proposes the control method of the amount of inventory in which the amount of the order is operated by using the algorithm of fuzzy control from the production planning result to solve the problem of showing by paragraph 4.2. Fuzzy control is a control algorithm that shows the function of the judgment of human by the IF-THEN routine and executes it [8, 9]. The block diagram of the supply chain between multiple companies that use fuzzy control for Fig.7 is shown.

Here, $D(t)$ is amount of the order from a customer to company B. $O_B(t)$ is amount of the order from company B to company A. $O_A(t)$ is amount of the order from company A to the raw materials supplier. $R_B(t)$ is amount of delivery from company A to company B. $R_A(t)$ is amount of delivery from raw materials supplier to company A. Company B (G_B) holds the production plan based on the amount of the order of final products

from the customer. Amount of the intermediate product $O_B(t)$ decided by the production plan is ordered to company A (G_A). Company A also similarly holds the production plan. Amount of the raw material $O_B(t)$ is ordered to raw material supplier (G_R). Because the leadtime is set as one period, the delay of the arrival of raw materials between each companies is shown as Z^{-1}.
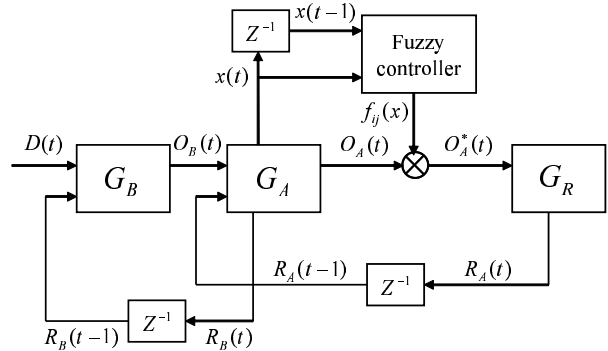

Fig. 7: Inventory control using fuzzy algorism

The difference between the amount of delivery shortage and the amount of inventory is assumed $x(t)$ as an tardiness loss. In the fuzzy controller, compensation gain $f_{ij}(x)$ is decided by using IF-THEN rule and the gain table from data on tardiness amount $x(t)$ and tardiness amount $x(t-1)$ according to fuzzy membership function (NB, NS, M, PS, PB). The compensation amount of the order $O_A^*(t)$ is decided by multiplying the compensation gain decided by the fuzzy controller and the amount of order of intermediate product $O_A(t)$ decided by production plan of company A.

5.2   The Adjustment of Fuzzy Gains

It was described to compensate the amount of the order by the compensation gain in the foregoing paragraph. However, when the production plan begins, fuzzy membership function that decides the compensation gain is arbitrarily given. Therefore, it is not necessarily the optimal fuzzy gain. Then, the compensation fuzzy gain value by fuzzy control is optimized every time the production planning is performed. In doing like this, the improvement of the solution is hoped for. Actually, SA method [10] that is one of the optimization method as the method for optimizing the compensation fuzzy gain is used. The algorithm is shown as follows.

5.3   Algorithm of SA Method

The algorithm is shown below.

STEP1 Setting of initial value and initial solution
An initial value of fuzzy gain and the threshold amount is set. At this time, each variable used by SA method is initialized.

STEP2 Neighborhood operation
Operated fuzzy gain table or threshold amount is selected, and adjusted at random.

STEP3 Production planning
The production planning is performed in each company. In that case, the profit is adopted as a provisional solution.

STEP4 Evaluation of solution
The solution is evaluated by SA method. The provisional solution is compared with the last solution, and fuzzy gain at the provisional solution is preserved if solution is improvement solution.

STEP5 End judgment
The iteration count in the same temperature is filled in SA method. Moreover, whether it arrived at the frozen temperature is judged. It returns to Step2 if it doesn't satisfy condition. It advances to Step6 if it meets each condition.

STEP6 End
The production planning of the following period is prepared.

6   NUMERICAL EXPERIMENTS

Fuzzy control was applied for the raw material order. Here, the upper stream company is adopted as one and the lower stream company is adopted as one. This time, fuzzy control was applied only to the amount of the raw material order of the upper stream company that the bullwip effect greatly influenced. Therefore, only the upper stream company is shown about the result.

Fuzzy gain table was composed of the difference between a tardiness amount and the inventory amount of raw materials $x(t)$ and $x(t-1)$ for fuzzy control (Table 1).

The production planning at 30 periods was performed by using fuzzy gain table in Table 1. The result when

Table 1: Gain of Fuzzy control

		$x(t)$				
		PB	PS	M	NS	NB
	PB	0.6	1.0	1.0	1.0	1.0
	PS	0.7	1.0	1.0	1.0	1.0
$x(t-1)$	M	0.6	1.0	1.0	1.0	0.95
	NS	1.0	1.0	1.0	0.9	0.7
	NB	1.0	1.0	1.0	0.9	0.7

fuzzy control is not applied to Fig.8 and the result and when fuzzy control is applied to Fig.9 are shown respectively.
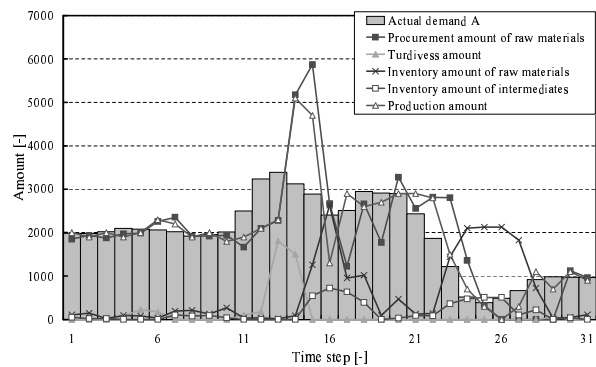

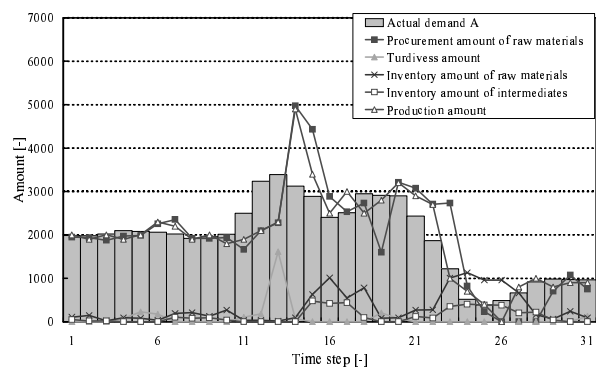Fig. 8: Transition of each amount in Company A

Fig. 9: Transition of each amount in Company A

Because the amount of the order of the raw material is adjusted by tardiness amount $x(t)$ in Fig.9 that applies fuzzy control compared with Fig.8, it has been reduced the possession of an excessive amount of inventory. An excessive amount of procurement is generated, and the amount of the raw material inventory increases in Fig.8. In Fig2, fuzzy gain table is set that

the raw material order is suppressed when tardiness amount of the previous period $x(t-1)$ is large. As a result, the amount of the raw material order has been decreased.

The final evaluation values obtained by optimizing the objective function are compared. It was 1416.5 [-] when fuzzy control was not applied, and it became 1503.7 [-] when applying. As a result, the solution has been improved.

7   CONCLUSION

In this research, the supply chain problem between multiple companies was expressed as the block diagram model, and is proposed. It proposed control method of the inventory amount by using the idea of the fuzzy control theory. Moreover, the demand forecast method coping to the variation of uncertain demand was examined. And, the improvement of the solution was examined by optimizing Fuzzy control gains by SA method, which adjusts the compensation gain in the fuzzy control. The proposed method showed that the increase in an unnecessary cost was suppressed for demand variations. Moreover, it was shown that it caused a decrease in an unnecessary inventory cost of other companies of the supply chain by improving the inventory control of one company in the supply chain, and the profit of the entire companies that belonged to the supply chain improved.

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Appendix

A. Description of Each Variable Number

The set, the variable, the cost coefficient, and the constant used for the formulation are shown.

Sets :

Set of products $I \equiv i$

Set of companies of supply chain $S \equiv s$

Set of time periods $T \equiv t$

Here, the set $I = I^{RM} \cup I^{IP} \cup I^{FP}$ consists of three sets. It's raw materials set (I^{RM}), intermediate product set (I^{IP}) and final product set (I^{FP}). The set $S = SI^{IP} \cup S^{FP}$ consists of two sets. It's intermediate product production company (S^{IP}) and final product production company (S^{FP}).

Variable number :

$I_{i,s,t}$: amount of inventories for raw materials and intermediate products $i \in I \setminus I^{FP}$ at company $s \in S^{IP}$ at the end of time period t, or amount of inventories for intermediate products and final products $i \in I \setminus I^{RM}$ at company $s \in S^{FP}$ at the end of time period t

$C_{i,s,t}$: consumption of raw materials $i \in I^{RM}$ at company $s \in S^{IP}$ in time period t, or consumption of intermediate products $i \in I^{IP}$ at company $s \in S^{FP}$ in time period t,

$L_{i,s,t}$: shortage of delivery amount of products $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or shortage of delivery amount of products $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t,

$n_{i,s,t}$: number of jobs for product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or number of jobs for product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t,

$P_{i,s,t}$: production amount of product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or production amount of product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t,

$R_{i,s,t}$: amount of procurement for products $i \in I \backslash I_{FP}$ at company $s \in S$ in time period t,

$S_{i,s,t}$: delivery amount of final product $i \in I^{FP}$ from the company $s \in S^{FP}$ to customer in time period t,

$Z_{i,s,t}$: penalty for deviation from targeted safety stock level for product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or penalty for deviation from targeted safety stock level for product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t.

Cost coefficients :

$h_{i,s,t}$: unit inventory holding cost for product $i \in I \backslash I_{FP}$ at company $s \in S^{IP}$ at the end of time period t, or unit inventory holding cost for product $i \in I \backslash I_{RM}$ at company $s \in S^{FP}$ at the end of time period t,

$q_{i,s,t}$: unit procurement cost for product $i \in I^{RM}$ at company $s \in S^{IP}$ in time period t, or unit procurement cost for product $i \in I^{IP}$ at company $s \in S^{FP}$ in time period t,

$\mu_{i,s,t}$: unit revenue of product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or unit revenue of product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t

$\gamma_{i,s,t}$: penalty cost for deviation from targeted safety stock for product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or penalty cost for deviation from targeted safety stock for product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t,

$\nu_{i,s,t}$: unit production cost for product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or unit production cost for product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t,

$\zeta_{i,s,t}$: penalty for shortage of delivery for product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, penalty for shortage of delivery for product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t.

General data :

$D_{i,s,t}$: demand for product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or demand for product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t

$f_{i,i',s}$: raw material $i \in I^{RM}$ to produce unit amount of intermediate product $i' \in I^{IP}$ at company $s \in S^{IP}$, intermediate product $i \in I^{IP}$ to produce unit amount of final product $i' \in I^{FP}$ at company $s \in S^{FP}$,

H_t : lengh of hours in time period t,

$I_{i,s}^{\max}$: capacity for warehouse of product $i \in I \backslash I_{FP}$ at company $s \in S^{IP}$, or capacity for warehouse of product $i \in I \backslash I_{RM}$ at company $s \in S^{FP}$

$I_{i,s,t}^{T}$: targeted safety stock level for product $i \in I^{IP}$ at company $s \in S^{IP}$ in time period t, or targeted safety stock level for product $i \in I^{FP}$ at company $s \in S^{FP}$ in time period t,

$p_{i,s,t}$: processing time of product $i \in I^{IP}$ per one job at company $s \in S^{IP}$ in time period t, or processing time of product $i \in I^{FP}$ per one job at company $s \in S^{FP}$ in time period t,

$V_{i,s,t}$: production amount of product $i \in I^{IP}$ per one job at company $s \in S^{IP}$ in time period t, or production amount of product $i \in I^{IP}$ per one job at company $s \in S^{IP}$ in time period t.