

On the Bremsstrahlung in High-Density Plasmas

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SYNOPSIS

Emission and absorption coefficients of bremsstrahlung by high-temperature partially degenerate electrons are calculated for high-density plasmas where Coulomb coupling between ions is not weak. It is shown that the ion correlation substantially reduces these coefficients.

1. INTRODUCTION

Interactions between high-density plasmas and radiation have important effects on the behavior of high-temperature high-density matters. An example of these hot dense matters is the compressed laser target of the inertial confinement fusion.

The radiation interacts mainly with high-temperature electrons. Ions, however, do not necessarily have the same temperature as electrons due to large ion-electron mass ratio. We may therefore have the cases where the ion temperature is low enough and the Coulomb coupling between ions is not weak. We also note that electrons in dense matters may be partially degenerate even at high temperatures on account of large Fermi energies. In this paper, we calculate the bremsstrahlung by high-temperature partially degenerate electrons taking the effect of ion correlations into account.

We consider a plasma composed of classical ions and partially degenerate electrons. We denote the number density, the charge, and the temperature of ions by n_i , Ze , and T_i , respectively, and those of

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electrons by $n_e (=Zn_i)$, $-e$, and T_e . Nondimensional parameters of our system are

$$\Gamma = (4\pi n_i/3)^{1/3} (Ze)^2/k_B T_i = 2.32 \cdot 10^{-2} Z^2 (n_i'/10^{24})^{1/3} T_i' \quad (1)$$

related to ions,

$$r_s = (3/4\pi n_e)^{1/3}/a_B = 1.17 (10^{24}/n_e')^{1/3} \quad (2)$$

$$k_B T_e/E_F = 0.543 Z^{5/3} (r_s/\Gamma) (T_e/T_i) = 2.74 \cdot 10^{-2} T_e'/(n_e'/10^{24})^{2/3} \quad (3)$$

related to electrons, and

$$T_e/T_i \quad (4)$$

Here a_B is the Bohr radius, E_F is the Fermi energy of electrons, n_i' and n_e' are the values in cm^{-3} , and T_i' and T_e' , those in keV. We consider the case where

$$r_s < 1, \quad (5)$$

$$1 < \Gamma < 10, \quad (6)$$

and

$$T_e/T_i \gg 1. \quad (7)$$

An example of these parameters may be $n_i = n_e = 10^{25} \text{cm}^{-3}$ ($Z=1$), $T_e = 1 \text{keV}$, and $T_i = 30 \text{eV}$ with $\Gamma = 1.7$, $r_s = 0.54$ and $k_B T_e/E_F = 5.9$.

Since we are interested in the radiation emitted out of plasmas, we consider photons whose frequency ω is larger than the plasma frequency ω_p ,

$$\hbar\omega \gg \hbar\omega_p = \hbar(4\pi n_e e^2/m)^{1/2} = 3.71 \cdot 10^{-2} (n_e'/10^{24})^{1/2} \text{keV}, \quad (8)$$

where m is the electronic mass and \hbar is the planck's constant. For these frequencies, we may regard electrons as an ideal Fermi gas and the real part of the electronic dielectric function as unity. We also assume that Z is of order unity and the electron temperature satisfies the condition

$$mc^2 \gg k_B T_e \gg Z^2 (e^2/\hbar c)^2 mc^2 = 2.72 \cdot 10^{-2} Z^2 \text{keV}, \quad (9)$$

where $e^2/\hbar c = 1/137.036$ is the fine structure constant: The temperature of electrons is sufficiently high but still non-relativistic.

2. EMISSION AND ABSORPTION COEFFICIENTS

The cross section $d\sigma_{kp}'$ of the electric dipole emission of a

photon for an electron is given by⁽¹⁾

$$d\sigma_{kp} = \frac{(2\pi)^2 m}{\omega^3 \hbar p} |\vec{e}^* \cdot \vec{d}_{fi}|^2 \delta(E_i - E_f - \hbar\omega) \frac{d\vec{k}}{(2\pi)^3} \frac{d\vec{p}'}{(2\pi)^3}. \quad (10)$$

Here \vec{e} and \vec{k} are the polarization and the wave number of emitted photon, $\omega = ck$, $(\vec{d})_{fi} = (d^2\vec{d}/dt^2)_{fi}$ is the matrix element of the time derivative of the electric dipole moment \vec{d} between the initial state (energy $E_i = (\hbar p)^2/2m$ and asymptotic wave number \vec{p}) and the final state (energy $E_f = (\hbar p')^2/2m$ and asymptotic wave number \vec{p}') of the electron. Noting the condition (9), we here assume that

$$\xi = Ze^2 m / \hbar^2 p \ll 1, \quad (11)$$

$$\xi' = Ze^2 m / \hbar^2 p' \ll 1, \quad (12)$$

and calculate the matrix element in the Born approximation.⁽²⁾ For collisions with ions distributed at \vec{R}_j , $j=1,2,\dots$, the cross section is thus calculated as

$$\overline{d\sigma_{kp}} = \frac{Z^2 e^6}{3\pi^2 mc^3 \hbar p \omega q^2} n_i S(q) \delta(E_i - E_f - \hbar\omega) d\omega d\vec{p}'. \quad (13)$$

Here $\hbar\vec{q} = \hbar\vec{p} - \hbar\vec{p}'$ is the change of electronic momentum in the collision and $S(q)$ is the structure factor of ions defined by

$$S(q) = \langle |\sum_j \exp(i\vec{q} \cdot \vec{R}_j)|^2 \rangle / N_i, \quad (14)$$

where N_i is the number of ions and $\langle \rangle$ denotes the statistical average. In (13), the bar denotes that we have taken the average with respect to the polarizations and directions (solid angles) of photon.

The emission coefficient $E(\omega)d\omega$ (energy emitted per unit time, volume, solid angle, and polarization) is thus given by

$$E(\omega)d\omega = \int \hbar\omega \frac{\hbar p}{m} d\sigma_{kp} f(p) [1 - f(p')] \frac{2d\vec{p}'}{(2\pi)^3}. \quad (15)$$

Here $f(p)$ is the distribution function of electrons with momentum $\hbar p$

$$f(p) = \{\exp[(\hbar^2 p^2 / 2m - \mu) / k_B T_e] + 1\}^{-1}, \quad (16)$$

μ being the chemical potential. Integrating with respect to \vec{p} and \vec{p}' , we have

$$E(\omega)d\omega = \frac{2Z^2 e^6}{3\pi^3 c^3 \hbar^3} \frac{1}{\exp(\hbar\omega / k_B T_e) - 1} n_i k_B T_e d\omega \int_0^\infty dq S(q) F(q) / q, \quad (17)$$

where

$$F(q) = \ln \left\{ \frac{1 + \exp[\mu/k_B T_e - (\hbar^2/2mk_B T_e)(q/2 - m\omega/\hbar q)^2]}{1 + \exp[\mu/k_B T_e - (\hbar^2/2mk_B T_e)(q/2 + m\omega/\hbar q)^2]} \right\}. \quad (18)$$

When electrons are classical and ions are not correlated ($S(q)=1$), Eqs.(17) and (18) give the known result⁽³⁾

$$E(\omega)d\omega = \frac{2^{3/2} Z^2 e^6}{3\pi^{3/2} m^{3/2} c^3 (k_B T_e)^{1/2}} n_e n_i K_0(\hbar\omega/2k_B T_e) \exp(-\hbar\omega/2k_B T_e) d\omega. \quad (19)$$

Here $K_0(x)$ is the modified Bessel function of the 0-th order.

The rate of net absorption is the difference between the rates of absorption and stimulated emission. The energy $Q(\omega)d\omega$ absorbed per unit time, volume, solid angle, and polarization is thus given by

$$Q(\omega)d\omega = \int \hbar\omega \frac{\hbar p}{m} N_k \overline{d\sigma_{kp}} \{f(p') [1-f(p)] - f(p) [1-f(p')]\} \frac{2d\vec{p}}{(2\pi)^3} \quad (20)$$

Here N_k is the number of photons with a polarization and wave number k . The absorption coefficient $A(\omega)$ is related to the absorbed energy by

$$A(\omega) = \frac{Q(\omega)d\omega}{\hbar\omega^3 N_k d\omega / 8\pi^3 c^2}, \quad (21)$$

where the denominator is the photon energy spectrum (per solid angle and polarization) multiplied by c . Similarly performing the integrals with respect to \vec{p} and \vec{p}' , we have

$$A(\omega) = \frac{16Z^2 e^6}{3c\hbar^4 \omega^3} n_i k_B T_e \int_0^\infty dq S(q) / F(q) / q. \quad (22)$$

As is shown in Eqs.(17) and (22), the effect of ion correlation on these coefficients is expressed by the ratio R as

$$R = \frac{E(\omega)}{E[\omega, S(q)=1]} = \frac{A(\omega)}{A[\omega, S(q)=1]} = \frac{\int_0^\infty dq S(q) F(q) / q}{\int_0^\infty dq F(q) / q}. \quad (23)$$

In the above calculations, we have assumed that both the initial and final states of electron are described by the plane wave. When the final momentum of electron is too small to satisfy the condition (12), the emission probability is modified⁽²⁾ by a factor

$$g(\xi') = 2\pi\xi' / [1 - \exp(-2\pi\xi')], \quad (24)$$

By further multiplying a factor

$$1/g(\xi) = [1 - \exp(-2\pi\xi)] / 2\pi\xi, \quad (25)$$

we may extend⁽⁴⁾ the applicability of our calculation to the cases where the latter half of the condition (9) or the condition (11) does not strictly hold for the initial state of electron. The factor F included in Eqs. (17) and (22) is thus modified as

$$F'(q) = \frac{\hbar^2}{2mk_B T_e} \int_0^\infty dp_\perp^2 \{ [f(p) - f(\sqrt{p^2 + 2m\omega/\hbar})] g(\xi') / g(\xi) \}_{p_\parallel = m\omega/\hbar q - q/2}, \quad (26)$$

where $p^2 = p_\parallel^2 + p_\perp^2$.

3. EFFECT OF ION CORRELATION ON EMISSION AND ABSORPTION COEFFICIENTS

In Fig. 1, we show an example of the emission coefficient for $T_e = 1 \text{ keV}$ in the case of uncorrelated ions with $Z=1$ obtained from eq. (17) using F' . While calculations with F give the results dependent only on $\hbar\omega/k_B T_e$ and $k_B T_e/E_F$, those of calculations with F' depend also on $k_B T_e/mc^2$. Plotted are the values normalized by (19), the classical limit in the Born approximation calculated with F : Since $F' > F$, the emission coefficient in the classical limit calculated with F' is greater than (19). We see that the degeneracy of electrons decreases the emission coefficient through the factor $f(p)[1-f(p')]$ stemming from the Pauli principle.⁽⁵⁾

We now look at the contribution of various values of the momentum transfer $\hbar q$ to the emission and absorption coefficients. In Fig. 2, we show some typical examples of the values of F' . The behavior of F' is similar to that of F defined by (18) where q appears through the factor $\exp(-\hbar^2 q^2 / 8mk_B T_e - m\omega^2 / 2k_B T_e q^2)$. Thus the main contribution to the integral in (17) and (22) comes from the domain $(m/k_B T_e)^{1/2} \omega < q < (mk_B T_e)^{1/2} / \hbar$ when $\hbar\omega/k_B T_e < 1$, and from the domain centered at $q \sim (2m\omega/\hbar)^{1/2}$ when $\hbar\omega/k_B T_e > 1$. The boundary of the contributing domain of q becomes diffuse and extends to larger wave numbers as the electrons becomes classical.

The characteristic scale of length of the structure factor of classical ions with intermediate or strong coupling is the mean distance between ions $a = (3/4\pi n_i)^{1/3} = 1.919Z^{1/3} / k_F$, where $k_F = (3\pi^2 n_e)^{1/3}$

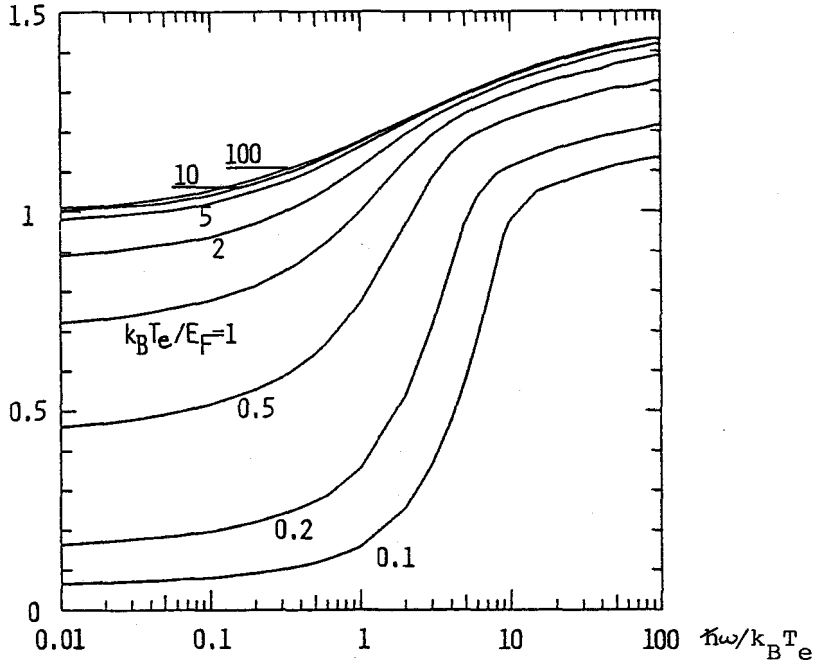


Fig. 1. Bremsstrahlung emission coefficient $E(\omega)$ of electrons with $T_e=1\text{keV}$ colliding against uncorrelated ions with $Z=1$. Values normalized by the Born approximation in the classical limit, (19), are plotted.

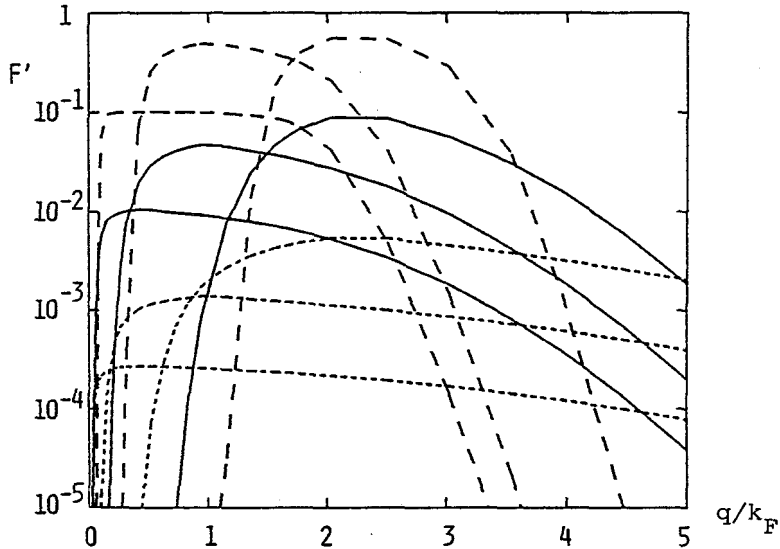


Fig. 2. Contribution of various values of electronic momentum change ξq to $E(\omega)$ and $A(\omega)$. Three broken (solid, or dotted) lines show, from left to right, the values of F' for $\xi\omega/E_F=0.2, 1, \text{ and } 5$, respectively, with $k_B T_e/E_F=0.2 (1, \text{ or } 5)$ and k_F is the Fermi wave number of electrons.

is the Fermi wave number of electrons. When we neglect the dielectric polarization of electrons, ions are regarded as the classical one-component plasma whose structure factor has been obtained by numerical experiments.⁽⁶⁾ Due to the long-range nature of the Coulomb interaction, $S(q)$ is proportional to q^2 for small values of q and approaches to unity for $qa \gg 1$. For $\Gamma \lesssim 10$, the oscillation of $S(q)$ around unity is small and $S(q)$ is substantially smaller than unity when $qa \lesssim 3$. If the integrals in (17) and (22) are effectively determined by the contribution from the domain where $S(q)$ is small, the ratio R is then reduced compared with the case of $S(q)=1$ or uncorrelated ions.

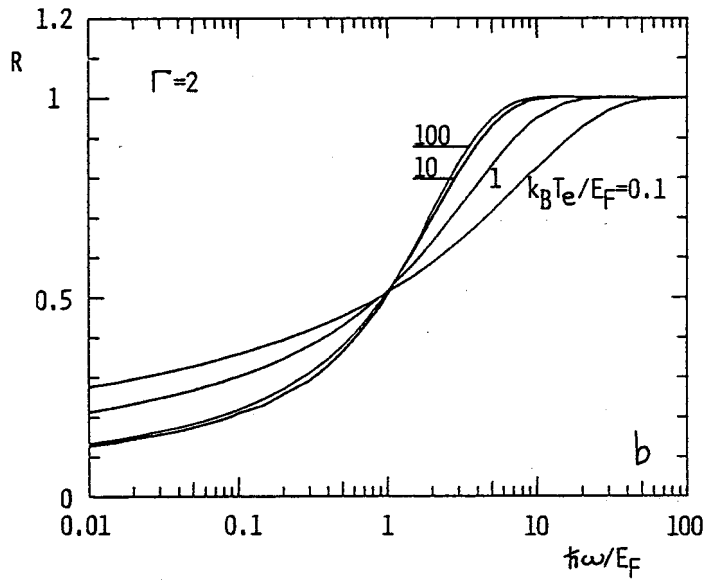
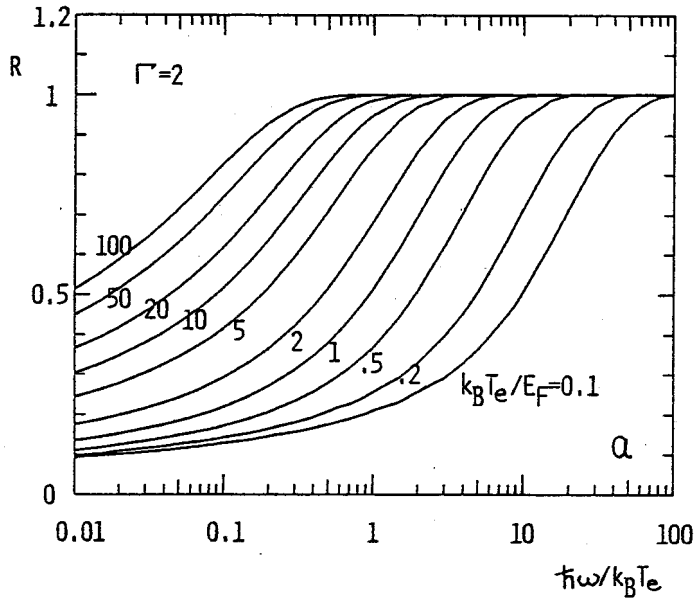
In Tables 1 and 2 and Figs. 3a, 3b, 4a, and 4b, we show the values of the ratio R for $\Gamma=2$ and 10 and several cases of electron degeneracy calculated with the structure factor of the one-component plasma.⁽⁶⁾ We see that the ion correlation has significant effect to reduce the emission and absorption coefficients in most cases. Small enhancement for the case of $\Gamma=10$ reflects the small overshooting of $S(q)$ beyond unity. These values of R are calculated with F' and we have put $T_e=1$ keV in the additional parameter $k_B T_e / mc^2$ of F' . The factors (24) and (25), however, have small effect on the ratio R and calculations with F give almost the same values: In fact, tabulated values are applicable with errors less than 2% in the domain $0.1 \text{ keV} < T_e < 10 \text{ keV}$.

Since the main contribution to the integral in (17) and (22) comes from the domain characterized by $q \sim (2m\omega/\hbar)^{1/2}$, the effect of ion correlation becomes important when $(2m\omega/\hbar)^{1/2} a \lesssim 1$ or $\hbar\omega/E_F \lesssim Z^{-2/3}$. This is clearly seen in Figs. 3b and 4b where the frequency is scaled by E_F .

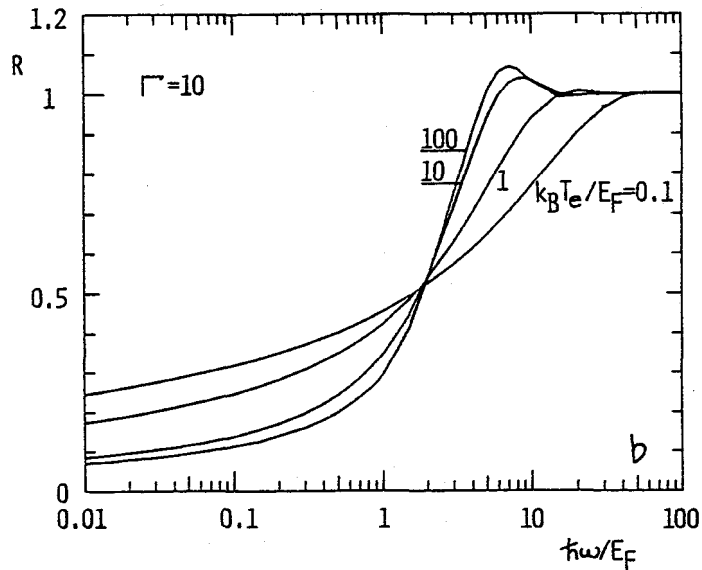
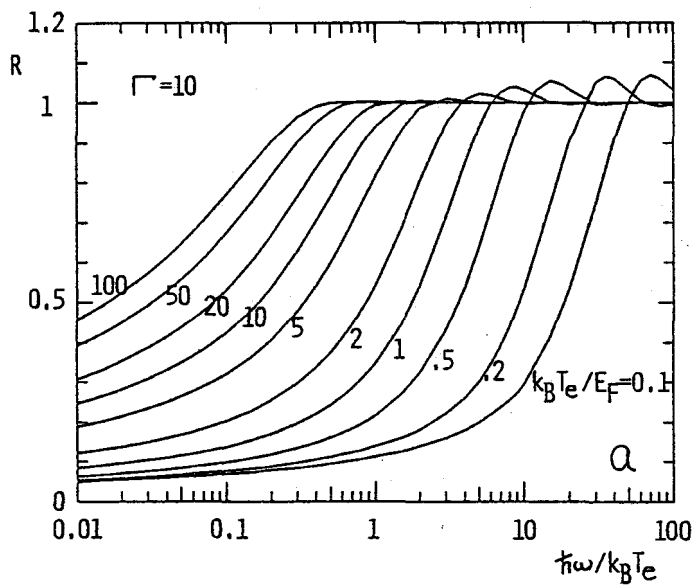
4. DISCUSSIONS AND CONCLUSION

When we are interested in the frequency such that $\hbar\omega/k_B T_e \sim 1$ and $\hbar\omega/E_F = (\hbar\omega/k_B T_e) (k_B T_e/E_F) \gg 1$ in the classical case, the integration with respect to q extends to very large values $\sim (mk_B T_e)^{1/2}/\hbar$ corresponding to the thermal de Broglie wavelength. The effect of the ion correlation in the domain of small wave numbers $\sim 1/a$ therefore has very small effect on the ratio R . For low frequencies such that $\hbar\omega/k_B T_e \ll 1$, however, R is affected by the ion correlation even in the classical case.

In reality, interactions between ions are screened by the polarization of electrons and the structure factor of ions differs from that of one-component plasmas: The Coulomb coupling between ions is effectively weakened and the structure factor becomes finite at the long-



Figs.3a and 3b. Effect of ion correlation on emission and absorption coefficients for $\Gamma=2$. The ratio of these coefficients to those without ion correlation is plotted.



Figs. 4a and 4b. The same as Fig. 3 for $\Gamma=10$.

wavelength limit.

The effect of screening on the structure factor of ions has been analysed⁽⁷⁾ by numerical experiments in the case where electrons and ions have the same temperature, and it has been shown that $S(0) \sim 0.3$ for $\Gamma=2$ and $r_s=1$ and $S(0) \sim 0.1$ for $\Gamma=10$ and $r_s=1$: An example is shown in Fig. 5. The polarizability of electrons depends on r_s and $k_B T_e / E_F$ and decreases with the decrease of r_s or the increase of $k_B T_e / E_F$. Since we are considering the case where $T_e \gg T_i$, the effect of electronic screening is expected to be much reduced compared with this result. We may thus use the structure factor of the one-component plasma as the first approximation.

We have calculated the bremsstrahlung emission and absorption from partially degenerate electrons in high-density plasmas. It has been shown that the ion correlation has substantial effect on the emission and absorption coefficients of bremsstrahlung.

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REFERENCES

- (1) For example, V.B.Berestetskii, E.M.Lifshitz and L.P.Pitaevskii, "Relativistic Quantum Theory", Part 1, Pergamon (1971), § 45.
- (2) Ref. 1, §90.
- (3) For example, G.Bekefi, "Radiation Processes in Plasmas", John Wiley and Sons (1966), Chap. 3.
- (4) W.Heitler, "The Quantum Theory of Radiation", Third Edition, Oxford at the Clarendon Press (1954), §25.
- (5) The effect of degeneracy of electrons on the bremsstrahlung has been discussed, for example, in S.Yamaguchi, S.Kawata, T.Abe, and K.Niu, Research Report of the Institute of Plasma Physics (Nagoya, Japan) IPPJ-553 (1982).
- (6) S.G.Galam and J.P.Hansen, Phys. Rev. A14 (1976), 816.
- (7) H.Totsuji and K.Tokami, to be published in Phys. Rev. A.

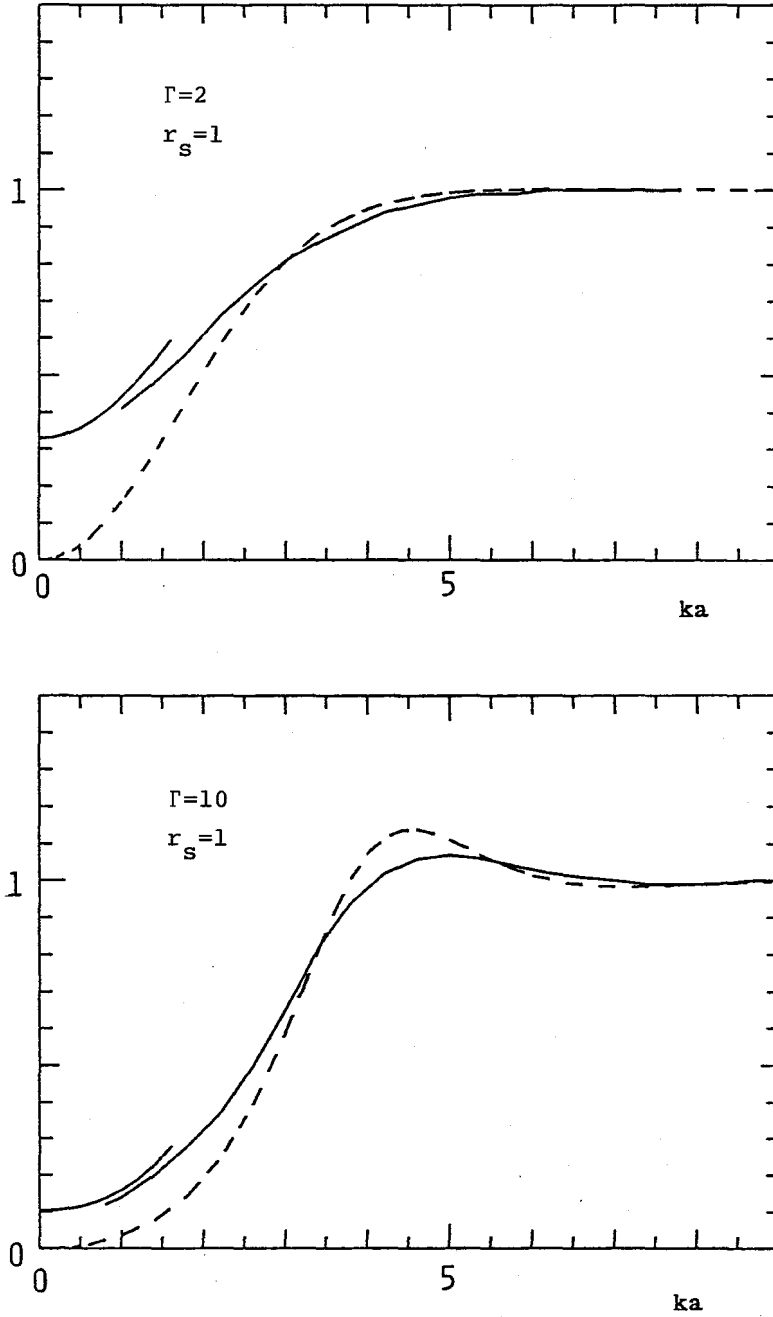


Fig. 5. The structure factor of ions in polarizing background of electrons with $r_s=1$ (solid lines), in comparison with the case of the one-component plasma (broken lines).