

## *Determination of Coefficients of Groundwater Flow in Multilayered Aquifers*

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### Synopsis

It is difficult to determine the coefficients of groundwater flow from the data which were obtained from the drawdown test in a multiaquifer system.

In this paper, new methods of analyzing drawdown tests are developed and illustrated with the example to determine aquifer coefficients. In a double-layered aquifer, the analytical solution of drawdown test, in which water is discharged from both layers, is derived. And also the theoretical solution to determine the coefficient of storage by using an index of elasticity of a confined aquifer is derived. From these solutions, methods of determining the coefficient of transmissibility in a double-layered aquifer and the coefficient of storage in a confined aquifer are got. The example analysis to determine aquifer coefficients is shown. As a result, the characteristics which were obtained by these methods are verified by the real drawdown test data.

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## 1. INTRODUCTION

In the drawdown tests, there are a few cases that the discharge well is prepared through more than one aquifer. If we pump up from multiaquifers at the same time, the problem, that is, how much value is discharged from each aquifer is occurred, so it is difficult to calculate aquifer coefficients.

In this paper, first, we will propose a method to determine aquifer coefficients by the drawdown tests used only one discharge well that is driven into two aquifers. Secondly, we will describe a method to determine the coefficient of storage from tidal fluctuations in a confined aquifer near the coast. The application of these methods will be shown for the actual drawdown test data.

## 2. DETERMINATION OF AQUIFER COEFFICIENTS IN A DOUBLE-LAYERED AQUIFER

Here we consider a double-layered aquifer system that is confined above and below by impervious layers, as illustrated in Fig.1. Each layer has its own seepage properties. The discharge well was driven in both layer and water was pumped at a constant rate  $Q$  from these layers. The drawdowns of pressure head are measured in each layer.

The coefficient of transmissibility ( $T_i$ ) can be expressed by Jacob's method as follows.

[ No.1 Aquifer ]

$$T_I = \frac{0.183Q_I}{S_I} \dots (1)$$

[ No.2 Aquifer ]

$$T_{II} = \frac{0.183Q_{II}}{S_{II}} \dots (2)$$

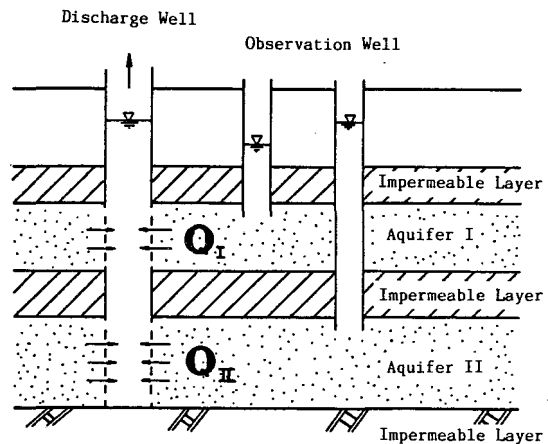


Fig.1. Schematic diagrams of double-layered aquifer

where  $Q_i$  is discharge rate,  $S_i$  is drawdown in one log cycle by Jacob's method, subscript I is meaning the upper aquifer and subscript II is the lower one.

In Eq.(1) and (2), as discharge rate from each aquifer  $Q_I, Q_{II}$  is unknown quantity, it is impossible to determine the coefficient of transmissibility for each aquifer. To determine the coefficient of transmissibility, drawdown tests in which water was pumped at a different rate ( $Q^1, Q^2$ ) from both layer were carried out. From the result the following equations are obtained for each aquifer.

[ Drawdown test No.1 ]

[ Drawdown test No.2 ]

( Discharge is  $Q^1$  )

( Discharge is  $Q^2$  )

$$T_I^1 = \frac{0.183 Q_I^1}{s_I^1} \dots\dots (3)$$

$$T_I^2 = \frac{0.183 Q_I^2}{s_I^2} \dots\dots (4)$$

$$T_{II}^1 = \frac{0.183 Q_{II}^1}{s_{II}^1} \dots\dots (5)$$

$$T_{II}^2 = \frac{0.183 Q_{II}^2}{s_{II}^2} \dots\dots (6)$$

$$Q^1 = Q_I^1 + Q_{II}^1 \dots\dots (7)$$

$$Q^2 = Q_I^2 + Q_{II}^2 \dots\dots (8)$$

In above equations, transmissibility for each layer is unique value, so the following equations are obtained.

[ for Aquifer I ]

[ for Aquifer II ]

$$T_I^1 = T_I^2 \dots\dots (9)$$

$$T_{II}^1 = T_{II}^2 \dots\dots (10)$$

substituting Eq.(9) and (10) into Eq.(3) through (6).

$$\frac{Q_I^1}{s_I^1} = \frac{Q_I^2}{s_I^2} \dots\dots (11)$$

$$\frac{Q_{II}^1}{s_{II}^1} = \frac{Q_{II}^2}{s_{II}^2} \dots\dots (12)$$

In order to simplify the equation, we let

$$\frac{Q_I^1}{Q_I^2} = \frac{s_I^1}{s_I^2} = \alpha \dots\dots (13)$$

$$\frac{Q_{II}^1}{Q_{II}^2} = \frac{s_{II}^1}{s_{II}^2} = \beta \dots\dots (14)$$

The discharge from each aquifer are obtained from following equations on referring to Eq.(7),(8),(11) through (14).

[ Drawdown test No.1 ]

[ Drawdown test No.2 ]

( Discharge is  $Q^1$  )

( Discharge is  $Q^2$  )

$$Q_I^1 = \frac{\alpha (\beta \cdot Q^2 - Q^1)}{\beta - \alpha} \dots\dots (15)$$

$$Q_I^2 = \frac{\beta \cdot Q^2 - Q^1}{\beta - \alpha} \dots\dots (16)$$

$$Q_{II}^1 = \frac{\beta (Q^1 - \alpha \cdot Q^2)}{\beta - \alpha} \dots\dots (17)$$

$$Q_{II}^2 = \frac{Q^1 - \alpha \cdot Q^2}{\beta - \alpha} \dots\dots (18)$$

As a result, the coefficient of transmissibility and storage in each aquifer is calculated by using Jacob's or Theis' method.

Summarizing the above method, to determine the multilayered aquifer coefficients, the drawdown tests should be performed same times as the number of aquifers with different discharge rates. But, if  $\alpha = \beta$  in Eq.(15) through (18), that is, the drawdown ratio is equal in each aquifer, it is impossible to apply this method.

### 3. DETERMINATION OF THE COEFFICIENT OF STORAGE IN A CONFINED AQUIFER

It is well known that water levels in a confined aquifer fluctuate due to the influence of barometric or tidal. This fluctuation expresses that the confined aquifer is elastic. And this phenomenon appears due to the fluctuation of the load on the confined aquifer. In an ideal confined aquifer, a theoretical method of determination of a coefficient of storage was introduced by Jacob as follows [1]. Firstly, the ratio of fluctuation of water levels in a confined aquifer due to barometric changes or tide is defined the barometric efficiency or the tidal efficiency respectively,

$$B = S_w / S_b \quad \dots (19)$$

$$C = S_w / S_t \quad \dots (20)$$

where  $B$  is the barometric efficiency,  $C$  is the tidal efficiency,  $S_b$  is barometric fluctuations,  $S_t$  is tidal fluctuations, and  $S_w$  is fluctuations of water levels in a confined aquifer.

The barometric efficiency and the tidal efficiency are led to Eq.(21),(22) by the equilibrium condition of stresses and water pressure.

$$B = \frac{n \cdot E_s}{n \cdot E_s + E_w} \quad \dots (21)$$

$$C = \frac{E_w}{n \cdot E_s + E_w} \quad \dots (22)$$

where  $E_s$  is the modulus of compression of the solid skeleton of the aquifer,  $E_w$  is the bulk modulus of elasticity of water (approximately  $E_w = 2.041 \times 10^7$  gf/cm<sup>2</sup> at 15°C), and  $n$  is porosity.

The confined aquifer has the nature of reacting to the barometric changes and the tidal fluctuations by the same mechanism, so there is following relation between  $B$  and  $C$ .

$$B + C = 1 \quad \dots (23)$$

By the theory of elasticity, the coefficient of storage is a function of elasticity and it can be expressed as follows [2]:

$$S = \gamma_w \cdot b \left( \frac{1}{E_s} + \frac{n}{E_w} \right) \quad \dots (24)$$

where  $\gamma_w$  is a unit weight of water,  $b$  is a thickness of a confined aquifer.

Eliminating the unknown parameter  $E_s$  by Eq.(21) and (24), coefficient of storage can be expressed as follows:

$$S = \frac{\gamma_w \cdot b \cdot n}{E_w \cdot B} \dots\dots (25)$$

$$S = \frac{\gamma_w \cdot b \cdot n}{E_w (1 - C)} \dots\dots (26)$$

According to this method, when the barometric or the tidal efficiency in a confined aquifer is measured, the coefficient of storage in that aquifer becomes determined by Eq. (25) and (26) theoretically.

#### 4. ANALYSIS OF DRAWDOWN TEST DATA

The following discussions give example calculations of the above methods. The drawdown test data were taken from a real multiaquifer that is located near the coast, Kobe City in Japan. The plane view of this region is shown in Fig.2. This region is bounded by the sea on the west.

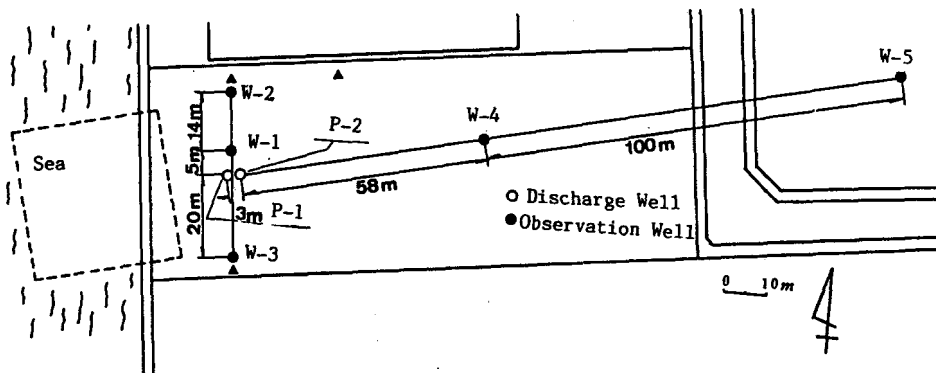


Fig.2. Plane view of the drawdown test site

The geological condition and the construction of wells are shown in Fig.3. Four sand-gravel layers ( Dug.1 - Dug.4 ) revealed as a confined aquifers exist in this region. The discharge well was sunk through Dug.2 and Dug.3 . Pumping up was performed from the two layers at the same time . Fractuations of water pressure in each aquifer was measured by the electric pore water pressure gauge which set at the each aquifer in the same observation well. The granular bentonite was used for material of seal.

4.1 Determination of The Coefficients of Permeability

Two drawdown tests were performed with different pumping rate,  $Q^1 = 300$  l/min ( test No.1 ) and  $Q^2 = 150$  l/min ( test No.2 ). The drawdown curves were obtained, as shown in Fig.4. The discharge rates from each aquifer were calculated by using Eq.(15) through (18). Aquifer coefficients, which were obtained from this method, are shown in Table-1. Coefficients of permeability were reported from in-situ permeability tests ( by Tube Method ) of pre-investigation in these aquifers as shown in Table-2. Comparing these results, they are nearly equal to that of draw-down test.

Table-2. Coefficients of permeability by Tube Method

Dug.2 Layer :  $k = 8.41 \times 10^{-3}$  cm/sec  
 Dug.3 Layer :  $k = 6.19 \times 10^{-3}$  cm/sec

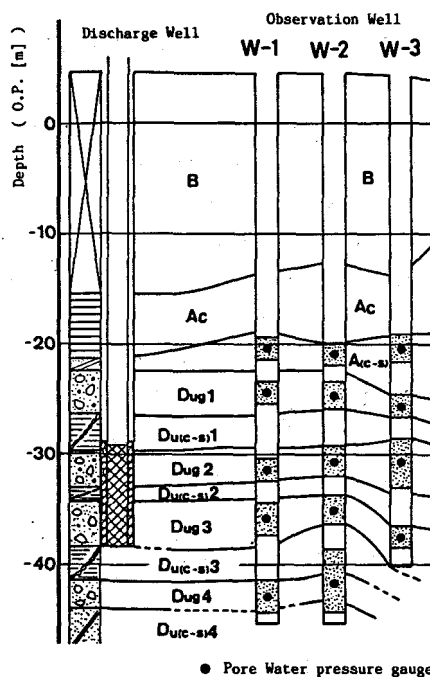


Fig.3. Geological condition and construction of wells

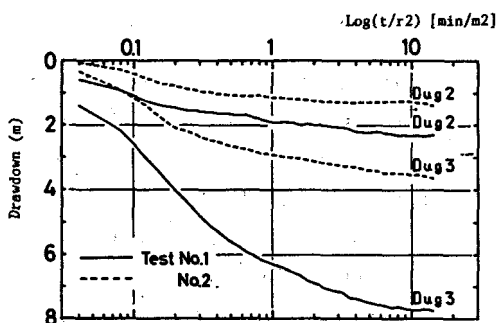


Fig.4. Time drawdown curve by Jacob's Method

Table-1. Coefficient of permeability from drawdown test

	TEST No.	Dug.2	Dug.3
S	①No.1	0.508	1.350
	②No.2	0.300	0.475
	①/②	1.693	2.842
Q	No.1	186.2	113.8
	No.2	110.0	40.0
K	(cm/sec)	$2.484 \times 10^{-2}$	$7.562 \times 10^{-3}$

S : Average Drawdown at log 1 cycle by Jacob's method  
 Q : Discharge  
 K : Coefficient of Permeability

### 4.2 Determination of The Coefficient of Storage From Tidal Efficiency

The relationship between tidal fluctuations (St) and water pressure changes (Sw) in each aquifer that were measured in the natural state are shown in Fig.5. Response time lag is very small and a strong correlation is observed between tide(St) and pore pressure (Sw). Then, relationship between Sw and St can be easily approximated by the method of least squares as one dimensional function.

$$Sw = a \cdot St + b \quad \dots\dots (27)$$

where a, b are constants.

In Eq.(27), tidal efficiency is equal to the gradient "a". Coefficient of storage is calculated by Eq.(26). Where  $E_w$  is  $2.041 \times 10^7$  gf/cm<sup>2</sup> and n is 0.328 according to soil tests. The above result are shown in Table-3. Comparison with coefficients of storage that calculated by Theis' and Jacob's method, is shown in Fig.6. In this figure, coefficients of storage from tidal efficiency is almost the average value of Theis' and Jacob's method.

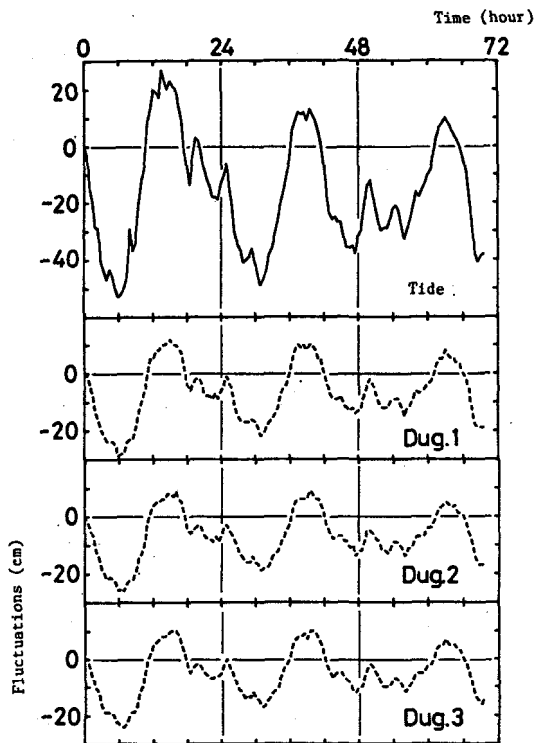


Fig.5. Relation between tidal fluctuations and water levels

Table-3. Coefficient of storage from tidal efficiency

Layer (Thickness)	Well No.	Tidal efficiency	Coefficient of Storage
Dug.1 *(330)	W-1	0.488	$1.036 \times 10^{-5}$
	W-2	0.464	$9.897 \times 10^{-6}$
	W-3	0.356	$8.237 \times 10^{-6}$
Dug.2 *(450)	W-1	0.401	$1.207 \times 10^{-5}$
	W-2	0.429	$1.267 \times 10^{-5}$
	W-3	0.353	$1.118 \times 10^{-5}$
Dug.3 *(340)	W-1	0.403	$9.153 \times 10^{-6}$
	W-2	0.435	$9.673 \times 10^{-6}$
	W-3	0.296	$7.762 \times 10^{-6}$

(\* Unit:cm )



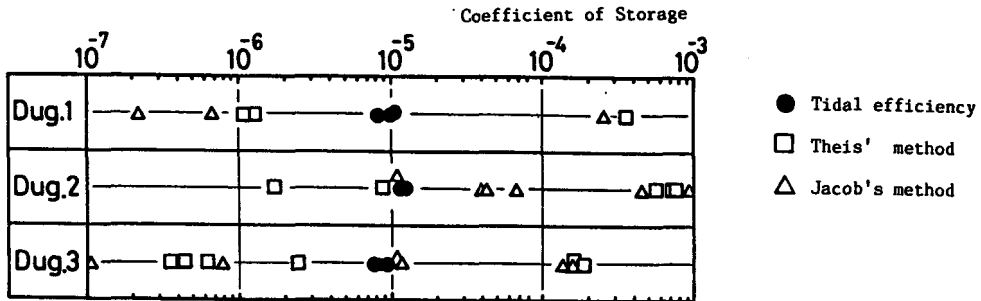


Fig.6. Comparisons of coefficient of storage

## 5. CONCLUSION

In this paper, the formulas and methods to determine aquifer coefficients in a double-layered aquifer from drawdown tests and to calculate the coefficient of storage in a confined aquifer from tidal fluctuations theoretically have been suggested.

The conclusions obtained in this paper are as follows :

- (1) An analysis of drawdown test with pumping from a double-layered aquifer is developed.
- (2) The theoretical solution of the coefficient of storage that is used an index of elasticity of a confined aquifer is derived.
- (3) By using these solutions, the methods of calculating the coefficient of transmissibility in a double-layered aquifer and the coefficient of storage in a confined aquifer are given.
- (4) The example analysis to determine aquifer coefficients are shown.
- (5) From these results, it becomes evident that proposed method are useful in analyzing the real drawdown test data.

## REFERENCES

- [1] Jacob, C.E. ; On the flow of water in a elastic artesian aquifer :Trans. Amer. Geophys. Union, vol.21,1940.
- [2] Jacob, C.E. ; Flow of ground water ; in Engineering hydraulics (H.Rouse, ed.), John Wiley and Sons, New York,1950.