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A Method of Direct Analog Simulation Using Transistor Switches and its Applications

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In the analog circuits for the systms governed by the differential equations with the coefficients which are a function of one or more of the dependent variables, the magnitudes of their circuit elements must be variable in accordance with the nature of the function. Such circuit elements can be realized by means of inserting or removing the additional elements with high speed switches in the analog circuits. Particularly, in the case of varying stepwise this method is effective. But as an analog circuit is regarded as a short-time or repetitive type analyzer, the above switches must be instantaneous operation. In this paper, first, it is made sure by the experiments that some of the transistor switches are met this condition, and moreover are very low closed resistance, very high open resistance and neglegible small voltage offsets. Next, the basic technique for the direct analog simulation usiug transistor switches is described about the simple example, i. e. the oscillatory system with varying cross-sectional surge tank. Finally, as its applications, the transient problems of these oscillatory systems are solved by means of these analog circuits.

§ 1. Introduction

Electrical equivalent circuits for the physical systems have the twofold significance. First, one can visualize the behavior of the original systems on the basis of a prior knowledge of the behavior of familiar electrical circuits. Second, one can set up experimental models or electrical analog circuits. Such analog circuits are known as "network analyzers".

Described in this paper is a method of the direct analog simulation for the oscillatory systems with the surge tank whose cross-sectional area in the vertical direction varies stepwise at the given points.

In the examples which are of interest in practice, the cross-sectional area A may be expressed mathematically by the following form.

$A = A_0$ at water level $y < y_1$

$A = A_0 + A_1$ at water level $y \geqq y_1$

where y_1 is the level corresponding to one of the above given points.

Here there is a one-to-one correspondence between each quantity in the two systems. The cross-sectional area of the surge tank may be corresponded to the capacitance of the capacitor, and the capacitance of the capacitor may be varied by inserting or removing the additional capacitors with high speed switches. These switches operate by the on-off signals sent from the control circuits.

In general, an analog circuit is regarded as a short-time or repetitive type anaglyzer. In this type analyzer the magnitude of the inductor or the capacitor may be much smaller than in the case of long-time type one. But it must be considered that the extremely high speed switches are demanded. In the electrical or electronic swiches a transistor switch will be suitable from the standpoints of both switching speed and operation control.

The basic technique for the design of the analog circuits for this purpose and the applications of such analog circuits are described in detail below.

§ 2. Considerations of the high speed swiches

The capacitance of the capacitor can be varied by inserting or removing the additional capacitors with high speed switches.

For these switches the following conditions must be met.

(1) The switches and their control circuits

must be instantaneous operation.

(2) The switches must be zero closed resistance, infinite open resistance, and no voltage offsets.

These conditions may be met by some of the transistor switches designed for this purpose.

Fig. 1 shows the circuit in which inserts or removes the additional capacitors. When the voltage across the capacitor of magnitude C_0 reaches the prescribed value v_1 at $t=t_1$, the switch S_{12} is closed and the switch S_{11} is opened simultaneously. The setting voltage across the capacitor of magnitude C_1 has been charged to the prescribed value v_1 by the battery. Now, equivalently the third capacitor of magnitude (C_0+C_1) can be obtained at $t=t_1$. If the voltage across each capacitor at $t=t_1-0$ is given, the voltage across the third capacitor at $t=t_1+0$ may be written as

$$v_0'=(C_0v_0+C_1v_1)/(C_0+C_1) \qquad (1)$$

where

v_0' = voltage across the third capacitor of magnitude (C_0+C_1) at $t=t_1+0$

v_0 = voltage across the capacitor of magnitude C_0 at $t=t_1-0$

v_1 = voltage across the capacitor of magnitude C_1 at $t=t_1-0$

From Eq. (1), then $v_0' \neq v_1$ when $v_0 \neq v_1$ and then $v_0'=v_1$ when $v_0=v_1$. In the above case v_0' must be equal to v_1. If the voltage across the capacitor of magnitude C_0 is v_1 at $t=t_1-\delta$ and v_0 at $t=t_1-0$, transient voltage varying with time v_0 will not be equal to v_1. Therefore the time δ must be neglegible small. For this reason the switching-time (the operatin time of the control circuit is contained within) must be neglegible small in comparison with the time of the transient variation of the voltage across the capacitor of magnitude C_0.

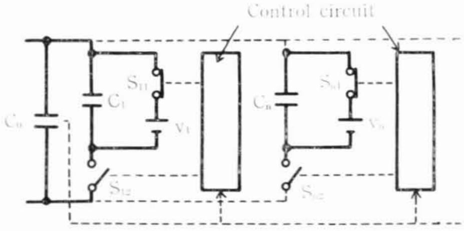

Fig. 1 Circuit which insert or remove additional capacitors.

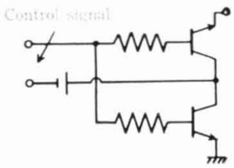

Fig. 2 Transistor switch for direct analog simulaton.

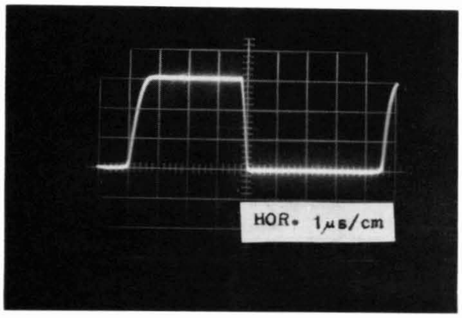

Fig. 3 Time-response of transistor switch.

When a rectangular disturbance is applied to the transistor switch shown in Fig. 2, the response is obtained in Fig. 3. According to this experimental data the switching-time is the order of μ sec. It is obvious that the first requirement is met by such a transistor switch.

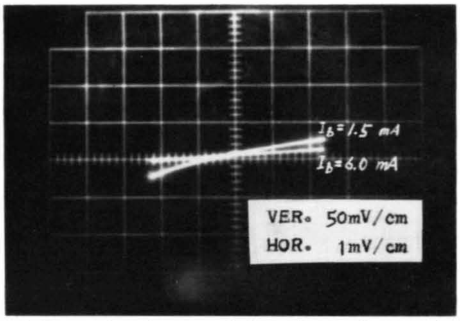

Fig. 4 Dynamic characteristics of transistor switch.

Fig. 4 shows the dynamic characteristics of the above transistor switch. These curves show that the second requirement is also met by the same one.

§ 3. Direct analog simulation for the oscillatory system with varying cross-sectional surge tank

Consider as an example the oscillatory system with the varying cross-sectional surge tank shown in Fig. 5 whose governing differential equations are

$$\frac{l}{ga}\frac{dq}{dz} \pm Kq^2 - y = 0 \qquad (2)$$

$$q + A(y)\frac{dy}{dz} = Q \qquad (3)$$

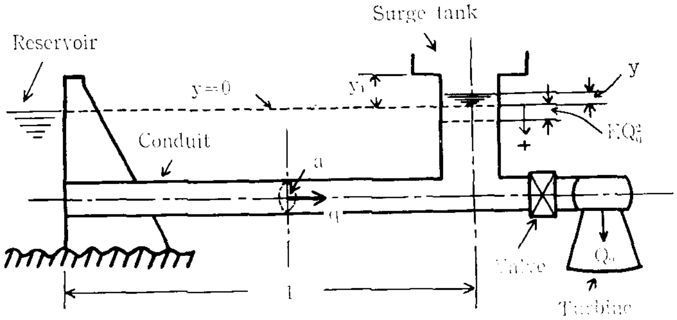

Fig. 5 Oscillatory system with varying cross-sectional surge tank.

The minus sign must be taken when water flows for the reservoir in the conduit ($q<0$). Where Q is the discharge through the valve, it is written in the following form.

$$Q=\begin{cases} Q_0 & \text{at } t<0 \\ 0 & \text{at } t>0 \end{cases} \text{ (load-rejection) (4)}$$

$$Q=\begin{cases} 0 & \text{at } t<0 \\ Q_0 & \text{at } t>0 \end{cases} \text{ (load-demand) (5)}$$

where Q_0 is the steady discharge for the given load.

The notation is as follows:

l = length of the conduit, (m)
a = area of the conduit, (m^2)
g = acceleration of gravity (m/sec^2)
K = a coefficient such that Kq^2 = total loses of head in the conduit, (sec^2/m^5)
$A(y)$ = area of the surge tank, (m^2)
y = departure of the water level in the surge tank from the reservoir level at any instant, (m)
q = discharge in the conduit at any instant, (m^3/sec)
τ = time in general, (sec)

A correspondence two systems is shown in Table 1. Making use of this correspondence and the following transformation equations, the analog circuit shown in Fig. 6 can be set up.

$$L=\frac{1}{N_t}\cdot\frac{N_i}{N_v}\cdot\frac{l}{ga} \quad (H)$$
$$C(v)=\frac{1}{N_t}\cdot\frac{N_v}{N_i}\cdot A(y) \; (F) \qquad (6)$$
$$R=\frac{N_i^2}{N_v}\cdot K \quad (\Omega/A)$$

where

N_t = time scale factor
N_v = voltage amplitude scale factor
N_i = current amplitude scale factor

Since the coefficient of nonlinear resistance is represented by means of the ordinary diode current limiter.

If the current source is suddenly removed or inserted in the analog circuit, the required transient phenomena may be produced. Thus the performance of load-rejection or load-demand can be analogically obtained.

Table I. Correspondence between surge tank system and electrical circuit.

Surge tank system		Electrical system	
l/ga	(sec^2/m^2)	L	(H)
$A(y)$	(m^2)	$C(y)$	(F)
K	(ses^2/m^5)	R	(Ω/A)
A_n	(m^2)	C_n	(F)
y	(m)	v	(V)
q	(m^3/sec)	i	(A)
Q	(m^3/sec)	S	(A)
$\|y_m\|$	(m)	$\|v_m\|$	(V)
y_n	(m)	v_n	(V)
Q_0	(m^3/sec)	S_0	(A)
τ	(sec)	t	(sec)

§ 4. Applications

The following numerical values are given for the system shown in Fig. 5

l = 5,000 (m)
g = 9.8 (m/sec^2)
a = 12.6 (m^2)
K = 0.00614 (sec^2/m^5)

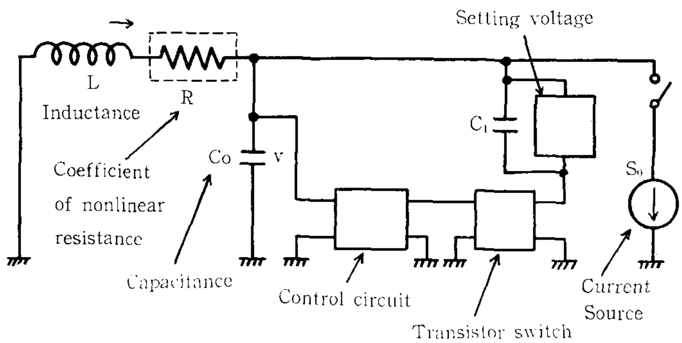

Fig. 6 Analog circuit for oscillatory system with varying cross-sectional surge tank.

$Q_0 = 15 \qquad (m^3/sec)$

Fig. 7 and Fig. 8 show all sorts of varying cross-sectional surge tanks. Fig. 9 shows a typical analog representation for the above type surge tanks.

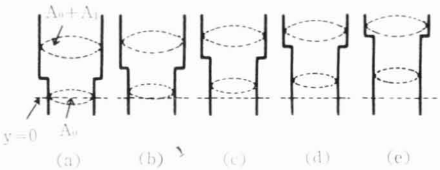

Fig. 7 Varying cross-sectional surge tanks.

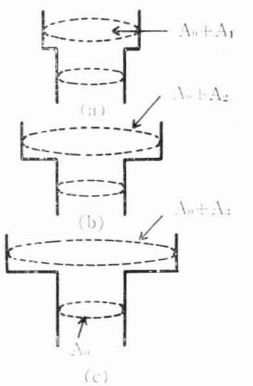

Fig. 8 Varying cross-sectional surge tanks.

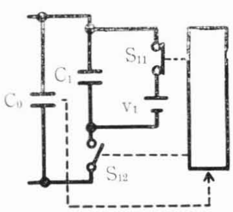

Fig. 9 Analog circuit for varying cross-sectional surge tank.

(Problem 1) Effect of the level arised the variation of the cross-sectional area upon performance curves (Fig. 7).

If the scale factor are $N_t = 10^6$, $N_v = 10$ and $N_i = 1.5 \times 10^4$, the values of the inductance and the capacitance shown in Fig. 6 then become

$L = 60.7 (mH)$, $C_0 = 0.03 (\mu F)$, $C_0 + C_1 = 0.06 (\mu F)$

Initial conditions at $t=0$ are

$i = S_0 = 1 \ (mA)$, $v = RS^2 = 0.138 \ (V)$

When the current source is removed from the circuit at $t=0$, the oscillograms shown in Fig. 10 are obtained. These are analogous to the original phenomena.

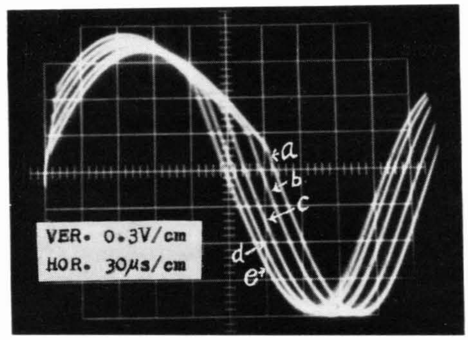

Fig. 10 Oscillograms of voltage corresponded to variation of water level in surge tank.

(Problem 2) Effect of the variation in size of the upper expanded area of the surge tank upon performance curves (Fig. 8).

Here $C_0 + G_1 = 0.06 \ (\mu F)$, $C_0 + C_2 = 0.12 \ (\mu F)$, $C_0 + C_3 = 0.18 \ (\mu F)$

Similarly, the oscillograms are shown in Fig. 11. In this case,

up-surging $|y_m|$ is given by

$|y_{m1}| = 9.7 \qquad (m)$

$|y_{m2}| = 7.9 \qquad (m)$

$|y_{m3}| = 7.0 \qquad (m)$

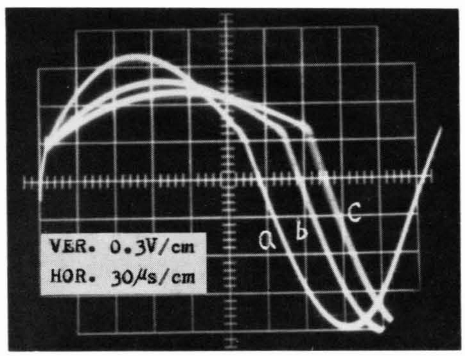

Fig. 11 Oscillograms of voltage corresponded to variation of water level in surge tank.

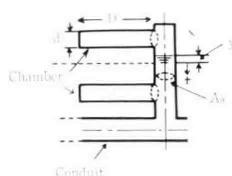

Fig. 12 Chamber surge tank system.

(Problem 3) Transient analysis of the chamber surge tank systems (Fig. 12, Fig. 13).

The numerical values for this system are given in Table II. Here each section of the chamber shown in Fig. 13 is simulated by a capacitor. Similarly, the oscillogram is shown in Fig. 14.

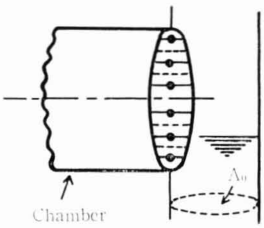

Fig. 13 Chamber divided into several sections at given levels.

Table Ⅱ. Numerical values for chamber surge tank system.

Notation	Value	
l	3,000	(m)
g	9.8	(m/sec^2)
a	12.6	(m^2)
K	0.0037	(sec^2/m^5)
Q_0	20	(m^3/sec)
A_0	7.065	(m^2)
d	6	(m)
D	18	(m)

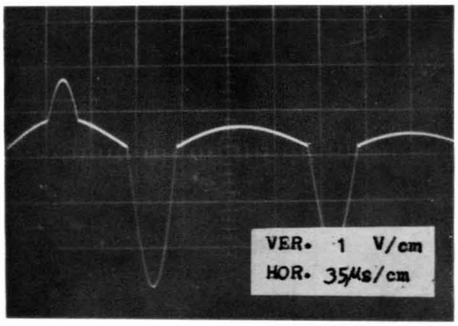

Fig. 14 Oscillograms of voltage corresponded to variation of water level in chamber surge tank.

(Accuracy check for the part (a) of the problem 2) The results of the numerical analysis obtained by use of the digital computer are shown in Table Ⅲ. The results by means of both methods for the comparison are shown in Fig. 15. Taking a view of the results, such a direct analog simulation for this problems has the accuracies of the order of a few per cent. these accuracies are allowable in practice.

§ 5. Conclusion

The conclusions are summarized as follows.

(1) The analog circuits for the systems governed by the differential equations with stepwise varying coefficients can be realized by using the transistor switches designed for this purpose.

Table Ⅲ. Calculated results by digital computer. (Problem 2-a)

(sec)	y (m)
0	+ 1.38
10	− 1.92
20	− 4.12
30	− 5.54
40	− 6.79
50	− 7.83
60	− 8.64
70	− 9.21
80	− 9.53
90	− 9.58
100	− 9.37
110	− 8.91
120	− 8.21
130	− 7.29
140	− 6.18
150	− 4.93
160	− 3.56
170	− 0.96
180	+ 2.19
190	+ 4.62
200	+ 7.04
210	+ 9.04
220	+10.53
230	+11.43
240	+11.70
250	+11.33
260	+10.35
270	+ 8.81
280	+ 6.81
290	+ 4.47
300	+ 1.92

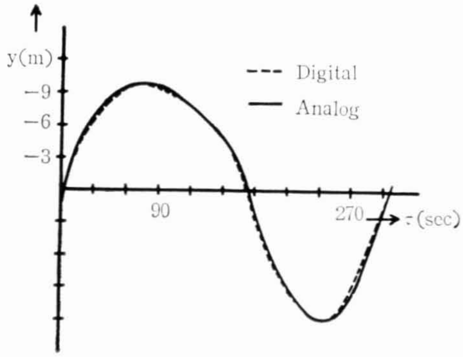

Fig. 15 Results by means of both direct analog simulation and digital computation. (Problem 2-a)

(2) Although only the problems of the surge tank are described, many other similar problems also can be solved.

(3) This simulation has an accuracy good enough for practical use in the above problems. But from now on, this points must be studied for the extension of the applications.

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