MEMOIRS OF THE SCHOOL OF ENGINEERING, OKAYAMA UNIVERSITY, Vol. 2, No. 1, APRIL, 1967

On the Pendulation of a Water Column in an Open Vertical U-Tube

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(Received December 28, 1966)

The friction coefficient for unsteady flow of fluids in pipes has not been studied so well. Usually the friction coefficient for steady flow has been used in place of one for unsteady flow. For a checkup of shis institution, in this paper, the pedulation of water column in an open vertical U-tabe is studied analytically and experimentally. The results of the numerical calculations coincide with the experimental results fairly well.

. Introduction

ʼor the steady flow of a fluid in a pipe, the s head h in the pipe length l is customary ɔressed in the form

$$h = \lambda \frac{l}{d}\frac{v^2}{2g} \quad \text{or} \quad h = \zeta \frac{v^2}{2g}$$

.ere d is pipe diameter, v is mean flow ocity and g is gravitational acceleration.

Ɔimensionless coefficient λ and ζ are fre- ɘntly called a friction coefficient and a loss ːfficient respectively. They are a function of ynolds Number $R_e = \frac{vd}{\nu}$. (ν is kinematic cosity of fluid.)

Ր he values of these coefficients for unsteady w have not been studied sufficiently up to w. Usually it is assumed that the coefficient unsteady flow is same one for steady flow.

n this paper, as an example of unsteady v, the pedulation of a water column in an ɘn vertical U-tube is studied analytically and ɔerimentally, and then the propriety of this ,umptive institution is checked up.

. Analytical Studies

Ր he equation of motion on the pendulation of ɾater column in an open vertical U-tube is as lows.

$$l\frac{d^2x}{dt^2} \pm gh + 2gx = 0 \qquad (1)$$

,ere l is the length of a water column, x is the tantaneous deflection of free water surface m the line of static equilibrium, h is the loss ɜd in the length l and t is the time. (Fig. 1)

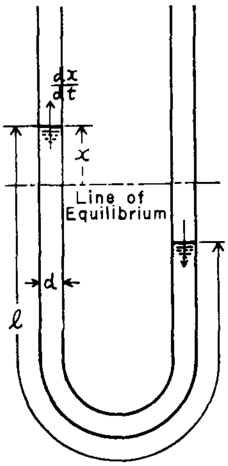

Fig. 1 The pendulation of water column in a U-tube.

Concerning the double sign, the positive sign is used for $\frac{dx}{dt} > 0$ and negative sign for $\frac{dx}{dt} < 0$.

As h is expressed in the form $h = \lambda \frac{l}{d}\frac{v^2}{2g}$, equation (1) becomes as follows.

$$l\frac{d^2x}{dt^2} + \lambda \frac{l}{2d}\left|\frac{dx}{dt}\right|\left(\frac{dx}{dt}\right) + 2gx = 0 \qquad (2)$$

Furthermore assuming the value for steady flow is accepted for λ in the equation (2), and λ is expressed in the form $\lambda = aR_e^n$, where a and n are the constants which are decided by the experiments, then equation (2) is as follows.

$$\frac{d^2x}{dt^2} + \frac{a}{2}\frac{d^{n-1}}{\nu^n}\left|\frac{dx}{dt}\right|^{n+1}\left(\frac{dx}{dt}\right) + \frac{2g}{l}x = 0 \qquad (3)$$

Equation (3) must be solved numerica ly for the given initial conditions. In this paper, the method of Runge-Kutta-Gill was adopted, and

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ʼresented at the Meeting, Okayama, November 18, ,6, of the Chugoku and Shikoku Branch of JSME.

the calculation was performed by using the digital computer (NEAC 2203) at School of Science, Okayama Univercity.

§ 3. Experimental Studies

The U-tube, which was used for the experiments, was made of acrylic plastic tube of inner diameter 30 mm, and the total length was about 3 m. The detailed construction of the U-tube is shown in Fig. 2. It was deviced to insert a orifice plate at the middle part of the bottom of the U-tube. The ratio of the used orifice area to the pipe area was 3/4, 2/3 and 1/2.

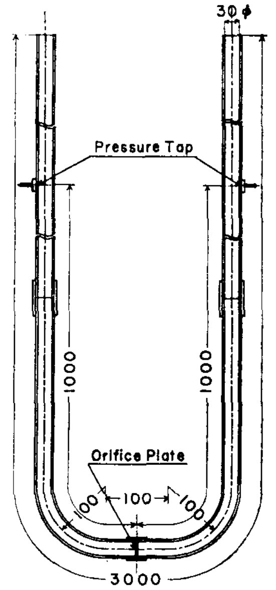

Fig. 2 Constrnction detail of the U-tube.

To record the pendulation of the water column of the length $l=2$ m, the 16-mm moving picture camera (Bell & Howell 70 DR) and the high sensitive film (ASA-200) were used. The water was coloured slightly by blue ink for easy distinguishment of the position of free water surface. In the field of the camera, the electric counter with decatorons which was operated by the audio-frequency oscillator and the scale were added. These were usefull to measure the time interval and the deflection of the free water surface respectively. The crank-speed was 24 frames for a second and then the time of exposure was 1/42 second for the standard state. For the improvement of the difiniteness of the image on the film of the moving free water surface, the rotary-shutter of the camera was worked up to cut down the open area of it to about half, and then the time of exposure was reduced to about 1/100 second.

The electrical signal of 1 KC/sec. sine wave was injected to the counter by the audio-freqnency oscillator which was calibrated by the standard signal wave of JJY during the measurement occasionally to retain the precision.

The loss head h for the steady flow was measured by the two pressure taps of 1 mm diameter in Fig. 2. In this measurement, the U-tube was set horizontally and water was dashed by the pump from the one end, and the volume rate of flow was measured by the weight-method at the other end.

§ 4. Results and Discussion

Loss Head for Steady Flow

Fig. 3 shows the friction coefficient λ of the U-tube versus Reynolds number R_e. Assuming these relations are represented by the function $\lambda = aR_e^n$, Table 1 represents the values of a and n. In Fig. 3 the solid lines show these functions and the lowest line shows the Blasius formula $\lambda=0.3164\ R_e^{-0.25}(a=0.3164,\ n=-0.25)$. Perhapse the difference of the Blasius formula line and the line of no orifice (area ratio 1/1) will be due to the bends of the pipe.

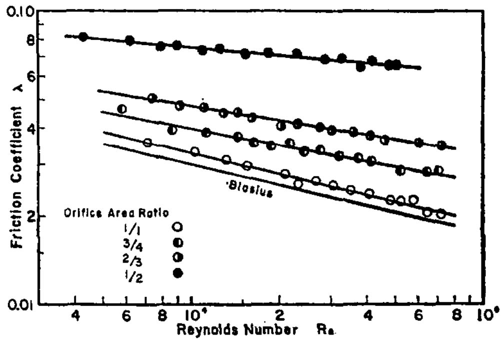

Fig. 3 Friction coeficients of the U-tube.

Table 1 Values a and n

orifice area ratio	a	n
1/1	0.296	−0.239
3/4	0.224	−0.188
2/3	0.208	−0.161
1/2	0.176	−0.0918

The Pendulation of the Water Column

Projecting the film on a screen, the change of the deflection of the free surface with time was measured. The solid lines of Figs. 4, 5, 6 and 7 show these results, for the case of orifice

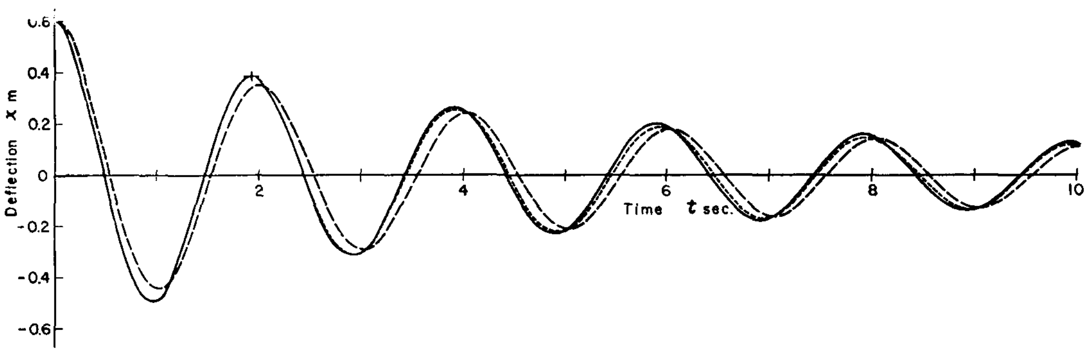

Fig. 4 Change of deflection of free water surface with time. (Orifice area ratio 1/1 i. e. no-orifice)

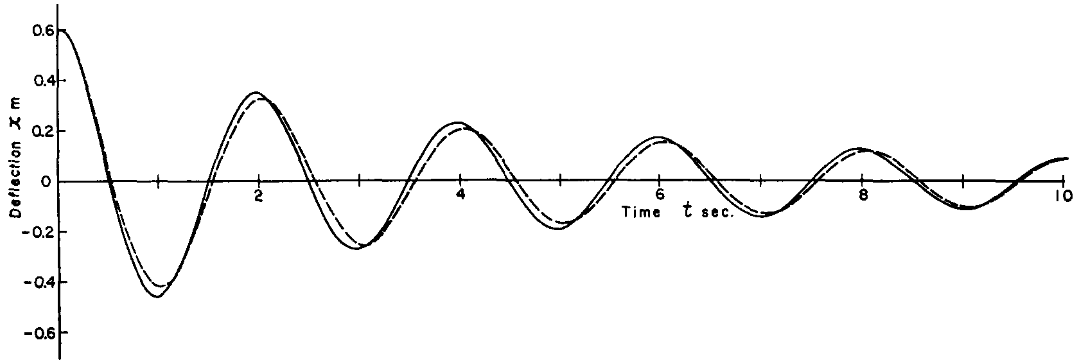

Fig. 5 Change of deflection of free water surface with time. (Orifice area ratio 3/4)

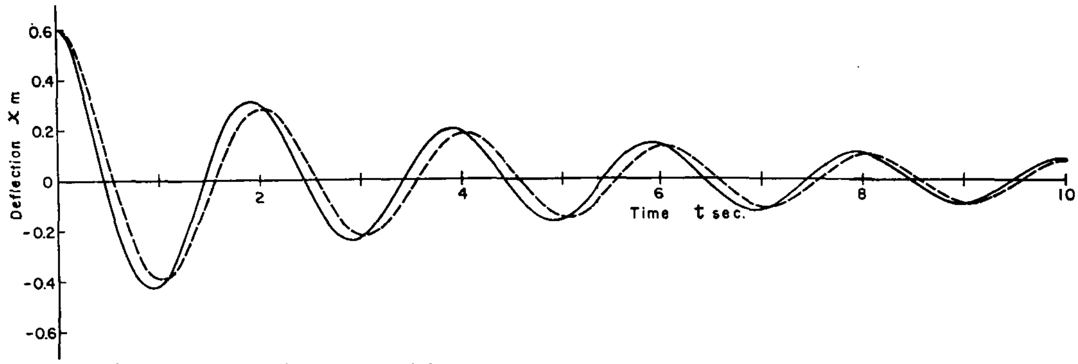

Fig. 6 Change of deflection of free water surface with time. (Orifice area ratio 2/3)

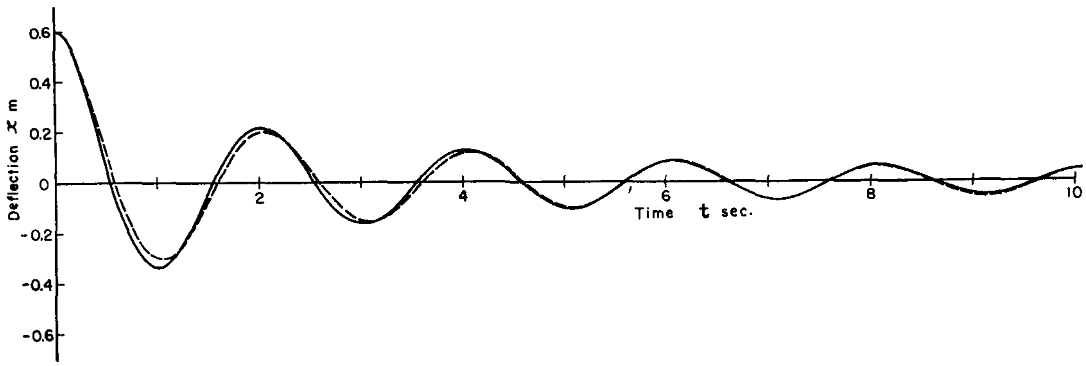

Fig. 7 Change of deflection of free water surface with time. (Orifice area ratio 1/2)

area ratio 1/1 (no-orifice), 3/4, 2/3 and 1/2 respectively. The initial condition of the experiments was $x_0 = 0.60\,\text{m}$, $\left(\frac{dx}{dt}\right)_0 = 0$ at $t = 0$ in every case.

On the other hand, equation (3) was solved numerically with the same initial condition and the time interval $dt = 0.1$ sec.. The obtained results are shown by the broken lines in the figures.

Although in Figs. 5 and 7 the corespondence is tolerable, in Figs. 4 and 6 it is not so. But in latter cases, the phase difference of the pedulation at the first period are carried over to the after periods. It is considered that this is caused by the unsatisfactory initial condition at the experiments. Then the equation (3) was solved again for the case of no-orifice with the new initial condition $x_0 = 0.385\,\text{m}$ and $\left(\frac{dx}{dt}\right)_0 = 0$ at $t = 0$, which is the maximum deflection after one period from starting in Fig. 4, and the result is shown by the dotted line in Fig. 4. The solid and dotted lines are brought in line tolerably this time.

For every case, the detailed observation gives that the damping of the pendulation is slightly remarkable in the calculation than in the experiment. Concerning this fact, next two points will be taken into consideration, but have not been studied yet.

(1) The influence of having the free surfaces on both sides of the water column i. e. the non-uniformity of the velocity distribution.

(2) The properiety of using the friction coefficient $\lambda = aR_e^n$, a and n given in Table 1, for all Reynolds number range. In these experiments the maximum Reynolds number was about 4×10^4.

Ignoring viscous stress, the period of the pedulation of the water column in a U-tube is

$$T = 2\pi\sqrt{\frac{l}{2g}} \tag{4}$$

Putting $l = 2\,\text{m}$, then $T = 2.01$ sec.. This value agrees with the experimental results, wherefore, so far as the period is disputed, it will be sufficient to calculate the period by equation (4).

§ 5. Summary

The pedulation of the water column in an open vertical U-tube has been solved numerically using the friction coefficient for the steady flow. The calculating results have been compared with the experimental results. The tolerable correspondence of both results has regarded the foregoing method as appropriate.

As the maximum Reynolds number of the velocity of the water column was about 4×10^4, for the pedulation with larger Reynolds number i. e. larger maximum deflection, nothing can be obtained.