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On the Phase Adjusting of a Magic T-Coupled Oscillators System

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Synopsis

A magic T-coupled two oscillators system having arbitrary amount of phase adjusting errors is studied both theoretically and experimentally. Denoting two independent phase parameters of the system measured from their optimum values as $\Delta\phi$ and $\Delta\psi$, it is derived analytically that the optimum operation is possible in principle in a definite region of $\Delta\phi$ - $\Delta\psi$ plane, so we have a considerable amount of margin for phase adjusting error. Experimental result also confirms the existence of some phase error margin, though the measured magnitude of margin is smaller to some extent than the theoretical prediction.

1. Introduction

As an application of locking phenomena in oscillators, several attempts to compose two or more oscillators output have recently been carried out along with the demand to obtain much power at microwave frequencies[1 - 4]. The most elementary and simplest system will be the one in which two oscillators are coupled together through a hybrid element such as a magic T. It is known that a magic T-coupled oscillators system has another merit of avoiding the instantaneous interruption of a communication system[5].

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Though detail analysis on the system is given in [4] and the method for optimum phase adjustment is derived, permissible amount of phase adjusting error is left unknown. This paper studies the behavior of the system having arbitrary adjusting errors in two independent phase parameters, and theoretical result on the permissible phase adjusting error is compared with experiment.

2. Differential Equations of Coupled System

Coupled oscillators system treated in this paper is such that two microwave oscillators are mounted at each side of a magic T and coupled together through the E-arm as shown in Fig.1. In the figure, S_1 and S_2 are the reference planes of the oscillators, and S'_1 - S'_4 are those of the magic T. Coupling strength, which we denote k , can be varied using an attenuator inserted in the E-arm. Distances ℓ_1 , ℓ_2 and ℓ_4 should be adjusted so that the system may operate at an optimum condition. The circuitry which lies between S_1 and S_2 can be regarded as a two-port circuit, so its characteristic may be represented by a Y-matrix. Then an equivalent circuit for the system is obtained as shown in Fig.2,

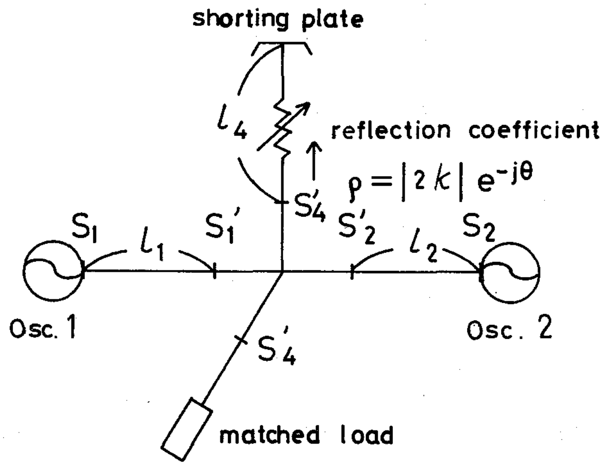

Fig.1 A magic T coupled oscillators system.

where C, L and G_R are capacitance, inductance and loss conductance respectively when the tuning circuit of the oscillator is represented by a parallel resonant circuit, Y_0 is the characteristic admittance of the output line, and $-gY_0$ is negative conductance.

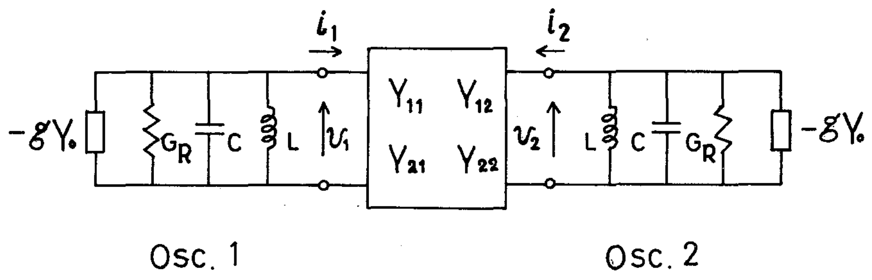

Fig.2 Equivalent circuit of the system

From Fig.2 the differential equation for one oscillator is written as

$$\frac{d^2v_n}{dt^2} + \omega_0{}^2 v_n + \frac{\omega_0}{Q_0}\frac{dv_n}{dt} - \frac{\omega_0}{Q_{ex}}\frac{d(gv_n)}{dt} + \frac{\omega_0}{Y_0 Q_{ex}}\frac{di_n}{dt} = 0, \qquad (1)$$

$(n = 1, 2)$

where

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad Q_0 = \frac{\omega_0 C}{G_R}, \quad Q_{ex} = \frac{\omega_0 C}{Y_0}, \qquad (2)$$

and i_n is related to v_1 and v_2 by

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} . \qquad (3)$$

Now we introduce a new time variable $\tau = \Omega t$, Ω being the angular oscillation requency when mutual locking is attained, and consider the circuit parameters y_{mn} as operators

$$y_{mn} \equiv \frac{Y_{mn}}{Y_0} = g_{mn} + b'_{mn}\frac{d}{d\tau} - b''_{mn}\int d\tau \qquad (4)$$

Thus, assuming that time variation of v_n is nearly harmonic, we obtain as the differential equation which describes the coupled oscillators system

$$\frac{d^2v_n}{d\tau^2} + v_n = (h_0 + \frac{b_{nn}}{Q_{ex}})\, v_n - \frac{1}{Q_{L,n}}\frac{dv_n}{d\tau} + \frac{1}{Q_{ex}}\frac{d(gv_n)}{d\tau}$$

$$- \frac{g_{nm}}{Q_{ex}}\frac{dv_n}{d\tau} + \frac{b_{nm}}{Q_{ex}}\, v_m , \qquad (5)$$

$(n,m = 1,2, \ n \neq m)$

where

$$b_{nm} \equiv b'_{nm} + b''_{nm} ,$$
$$h_0 \equiv 2(\Omega - \omega_0) / \omega_0 , \tag{6}$$
$$1/Q_{L,n} \equiv 1/Q_0 + g_{nn}/Q_{ex} .$$

Supposing that the right side of Eq.(5) is small as compared with v_n, we assume

$$v_n = V_n \cos(\tau + \theta_n) ,$$
$$\frac{dv_n}{d\tau} = -V_n \sin(\tau + \theta_n) . \tag{7}$$

By use of the averaging method, we obtain the reduced equation for Eq.(5)

$$2\frac{dV_n}{d\tau} = -\left\{ g(V_n) - \frac{1}{Q_{L,n}} \right\} V_n - \frac{V_m}{Q_{ex}} \left\{ g_{nm}\cos(\theta_n - \theta_m) + b_{nm}\sin(\theta_n - \theta_m) \right\} , \tag{8}$$

$$2\frac{d\theta_n}{d\tau} = -\left(h_0 + \frac{b_{nn}}{Q_{ex}}\right) + \frac{1}{Q_{ex}} \cdot \frac{V_m}{V_n} \left\{ g_{nm}\sin(\theta_n - \theta_m) - b_{nm}\cos(\theta_n - \theta_m) \right\} . \tag{9}$$

These are the basic equations for discussions which follow.

3. Theoretical Characteristic of the Oscillator System

The operation of the magic T-coupled system of Fig.1 is optimum under the condition that $\beta(\ell_n + \ell_4)$, β being phase constant, is equal to some integral multiple of π. We consider the case when the phase adjustment error occurs, i.e. $\beta(\ell_n + \ell_4) \doteq N_n\pi + \Delta\psi_n$ (mod.π). The Y matrix components in Fig.2 are then

$$g_{nn} = \frac{1}{A} \left\{ 1 - 2k\cos 2(\Delta\phi \mp \Delta\psi) \right\} \tag{10a}$$

$$b_{nn} = \pm \frac{2k}{A} \left\{ k\sin 4\Delta\phi \mp \sin 2(\Delta\psi \pm \Delta\phi) \right\}$$

(the upper and the lower sign corresponds to $n = 1$ and $n = 2$, respectively)

$$g_{12} = g_{21} = (-1)^{N_1+N_2+1} \cdot \frac{2k}{A} \cdot (\cos 2\Delta\psi - 2k\cos 2\Delta\phi)$$

$$b_{12} = b_{21} = (-1)^{N_1+N_2} \cdot \frac{2k}{A} \cdot \sin 2\Delta\psi \,, \tag{10b}$$

where

$$\Delta\psi \equiv \frac{\Delta\psi_1 + \Delta\psi_2}{2} \quad , \quad \Delta\phi \equiv \frac{\Delta\psi_1 - \Delta\psi_2}{2}$$

and

$$A \equiv 1 - 4k\cos 2\Delta\phi . \cos 2\Delta\psi + 4k^2\cos^2 2\Delta\phi .$$

Under the approximation $V_1 \doteqdot V_2$, in the steady state ($dV_n/d\tau = d\theta_n/d\tau = 0$), we obtain from Eq.(9)

$$-(h_o + \frac{b_{11}}{Q_{ex}}) + \frac{1}{Q_{ex}} (g_{12}\sin\theta - b_{12}\cos\theta) = 0 \tag{11}$$

$$(h_o + \frac{b_{22}}{Q_{ex}}) + \frac{1}{Q_{ex}} (g_{21}\sin\theta + b_{21}\cos\theta) = 0 \,, \tag{12}$$

where $\theta \equiv \theta_1 - \theta_2$.
Using Eq.(10), these lead to

$$\sin\theta = \frac{b_{11} - b_{22}}{2g_{12}} = (-1)^{N_1+N_2} . \sin 2\Delta\phi \tag{13}$$

$$\cos\theta = -\frac{Q_{ex}}{b_{12}} h_o - \frac{b_{11} + b_{22}}{2b_{12}} = \frac{(-1)^{N_1+N_2+1} A.Q_{ex}}{2k\sin 2\Delta\psi} . h_o + (-1)^{N_1+N_2}\cos 2\Delta\phi . \tag{14}$$

From Eq.(13) and Eq.(14) we obtain h_o , accordingly the mutually locked oscillator frequency:

$$h_o \equiv \frac{2(\Omega - \omega_o)}{\omega_o} = \begin{cases} 0 & \text{....... Case A} \\ \dfrac{4k.\sin 2\Delta\psi . \cos 2\Delta\phi}{A.Q_{ex}} & \text{....... Case B} \end{cases} \tag{15}$$

The oscillation phase difference $\theta \equiv \theta_1 - \theta_2$ is obtained from Eq.(14) and Eq.(15) as

$$\theta = \begin{cases} 2\Delta\phi + (N_1 + N_2)\pi & \text{for Case A,} \\ -2\Delta\phi + (N_1 + N_2 + 1)\pi & \text{for Case B.} \end{cases} \quad (16)$$

$(N_1, N_2$: integer)

The stability condition for the obtained periodic solution is given from Eq.(9) by using Routh-Hurwitz criterion

$$g_{12}\cos\theta < 0 . \quad (17)$$

From Eq.(16) and Eq.(17), Case A in Eq.(15) is stable for

$$F \equiv (\cos 2\Delta\psi - 2k\cos 2\Delta\phi)\cos 2\Delta\phi > 0$$

and Case B for

$$F < 0 .$$

The stable regions for the periodic solutions in the $\Delta\psi - \Delta\phi$ plane are shown in Fig.3 and Fig.4. Combined power P_{out} is proportional to

$$\frac{1 + \cos\Phi}{1 - 2k^2(1 - \cos\Phi)} ,$$

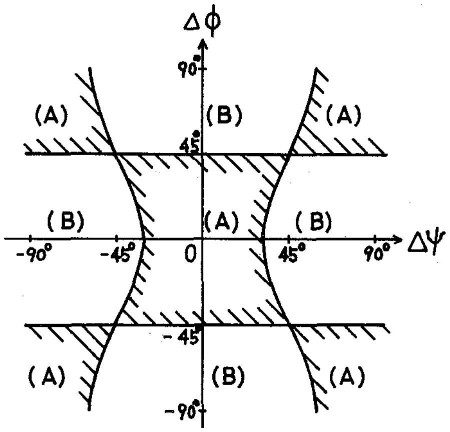

Fig.3 Stable regions in $\Delta\psi - \Delta\phi$ plane. ($k = 1/4$)

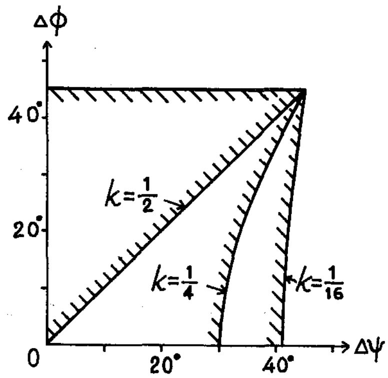

Fig. 4 Stable region (A) vs k

where

$$\Phi \equiv \theta - 2\Delta\phi - (N_1 - N_2)\pi$$

is the phase difference between two waves of equal amplitude coming into the magic T from both sides. Accordingly,

$$P_{out} = 2P_1 \qquad \text{for Case A},$$
$$= \frac{1 - \cos 4\Delta\phi}{1 - 2k^2 \; 1 + \cos 4\Delta\phi} \cdot P_1 \qquad \text{for Case B}, \qquad (18)$$

P_1 being one oscillator power at single operation. The change of output power with $\Delta\phi$ is shown in Fig.5 for Case B. The Case A corresponds to in-phase combination practically useful.

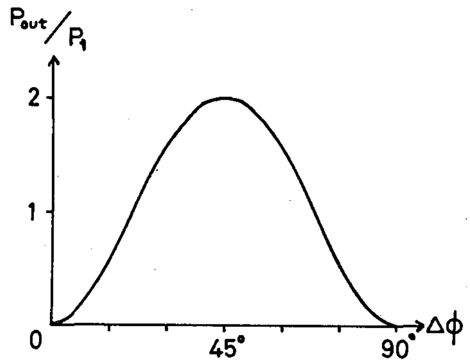

Fig.5
P_{out} vs $\Delta\phi$ for Case B ($k = 1/4$).

4. Experiment

Using two Gunn oscillators the experiment was carried out. Circuit arrangement for experiment is shown in Fig.6. Both Gunn oscillators are adjusted to equal oscillation frequency, 9164MHz , and output power, 30mW , which is the value when matched load is connected to Osc.1 or Osc.2. The measured region of in-phase combination for $k = \frac{1}{10}$, $\frac{1}{4}$ and $\frac{1}{2}$ are given in Fig.7. Though they are generally a little smaller than predicted theoretically , qualitative coincidence can be seen. The measured oscillator frequency and combined power in the region of in-phase combination are shown in Tab.1. Theoretically they are constant, but experimentally they vary a little. It can be thought that the difference between theoretical and experimental value was caused

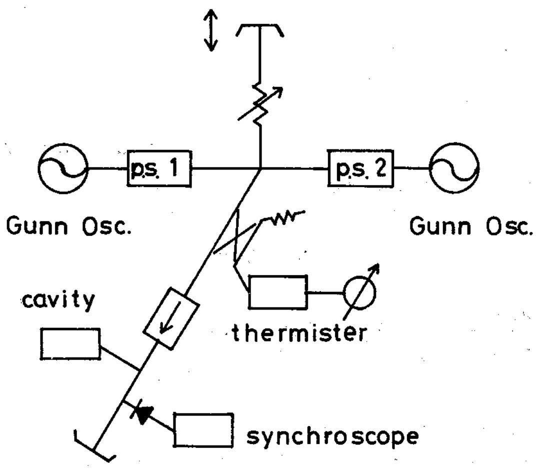

Fig.6 Experimental circuit.

partly by the fact that the V.S.W.R. of E-arm of the used magic T was about 1.3 and partly because the used Gunn oscillators was considerably sensitive.

Obvious discrepancy between theory and experiment in case of $k= 1/2$ will be due to two reasons. Firstly, the analysis developed here is not valid for $k \simeq 1/2$, because then the right side of Eq.(5) is no longer small as compared with V_n . Secondly, the characteristic of used

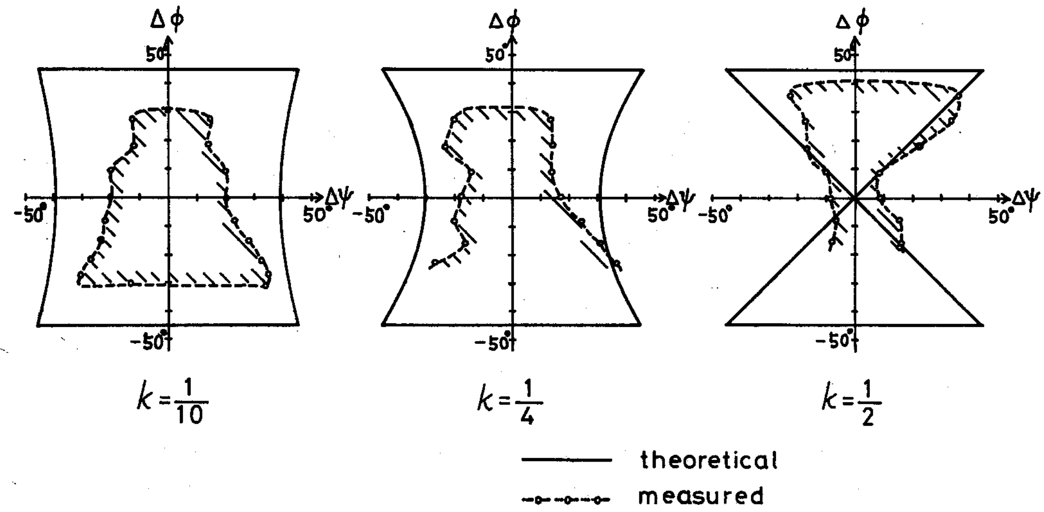

Fig.7 Measured regions of in-phase combination.

magic T deviates more or less from the ideal one.

5. Conclusions

In a magic T-coupled oscillators system, it was confirmed both

Table 1 Combined power and oscillation frequency in the region of in-phase combination

k	combined power(mW)	frequency(GHz)
1/10	*4.9 ∿ 5.4*	*9.162 ∿ 9.165*
1/4	*4.5 ∿ 5.6*	*9.163 ∿ 9.167*
1/2	*5.6 ∿ 6.2*	*9.163 ∿ 9.165*

theoretically and experimentally that region of in-phase combination in the $\Delta\phi$ - $\Delta\psi$ plane is considerably large as far as the coupling strength does not approach to *1/2*. It was also shown that we have a constant oscillation frequency and a constant level of combined power in the region of in-phase combination. Thus, for small value of k , we have a considerable amount of margin for phase adjusting error. On the other hand, small value of k is not preferable for obtaining sufficiently large locking range. Therefore an adequate value of k must be chosen at the compromise between two facts above described.

References

(1) H.Fukui : Proc.IEEE,54(1966),1475
(2) S.Mizushina : Proc.IEEE,55(1967),2166
(3) J.R.Nevarez and G.J.Herskowitz : IEEE Trans. Microwave Theory and Tech.,MTT-17(1966),2
(4) J.Ikenoue and K.Fukui : J.IECE, Japan, 49(1966), 2339
(5) K.Fukui, J.Ikenoue, K.Yamazaki, N.Hirano and K.Imamura : I.C.M.C.I. (1964),M14-2