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Rectification of Digitized Aerial Photographic Image

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Synopsis

A practical example of digital rectification of tilted photographs using a drum scanning micro densitometer and general purpose computers is depicted. The present research covers rectification of projective distorsions, occuring when the camera axis is not truly vertical, and affine distorsions due to curvature of a drum of a scanner. For this purpose, fundamental mathematical expressions were derived. And some pixel interpolation methods necessary for image reconstruction were compared experimentally. The examples revealed, however, that the film was deformed so complexly that they could not be corrected sufficiently only by affine transformation. Accuracy of rectification was checked by use of stereo aerial photographs in terms of residual y-parallaxes. The result showed residual y-parallaxes of $\pm$ 1 pixel ($\pm$ 50 μm) and sometimes $\pm$ 2 pixels were observed. They seem to be caused mainly by film deformations which have not been eliminated, and their amount seems to exceed the photogrammetric tolerance.

1. Introduction

The word "Rectification" is used to mean, in a restricted sense,

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correction of image positions of tilted photographs and to make a equivalent vertical ones. Rectification has been performed using a specially designed instrument, "Rectifier", which processes photographs optically [1]. However, in the optically rectified photographs still remain image displacements due to not only ground reliefs, but also lens defects or film distorsions.

Recent prevailence of computers and cost down of computation make it practical to process digital images instead of optical ones [2,3]. Rectification by digital image processing has much advantage. For example, provided the camera and film calibration data are given, corrections of lens or film distorsions as well as the rectification of photographs taken in any inclined camera directions and heights can be easily accomploshed. This means that photographs taken with a nonmetric (amateur) camera are available for producing photo maps.

However, for the production of exact photo maps, corrections of image displacements due to ground reliefs must also be required. This process is called again rectification in a more comprehensive sense than defined above, or especially called differential rectification [4,5]. But we do not refer to it in this paper.

This paper describes off-line digital rectification techniques and a practical example using densitometric data quantized by a drum scanning micro densitometer and general purpose computers. This paper has five sections including this introduction. In the second section some pixel interpolation methods are explained. In the case of image reconstruction by off-line processing, each density at a grid point of newly defined lattice must be interpolated from adjacent density values. And, in addition, a result of comparison tests of those methods was presented from a view points of practical usefullness. It will be shown that a simple linear interpolation meets the demand sufficiently. In the forth section, a practical example of rectification is presented. Firstly, the outline of a sequence of procedures is explained. And secondly, a pair of stereo aerial photographs was rectified and the accuracy of the rectification was checked in terms of residual y-parallaxes. And, in the final section, some remarks as conclusion are presented.

2. Mathematical Expressions for Rectification

A camera axis of a aerial photograph is unavoidably a little

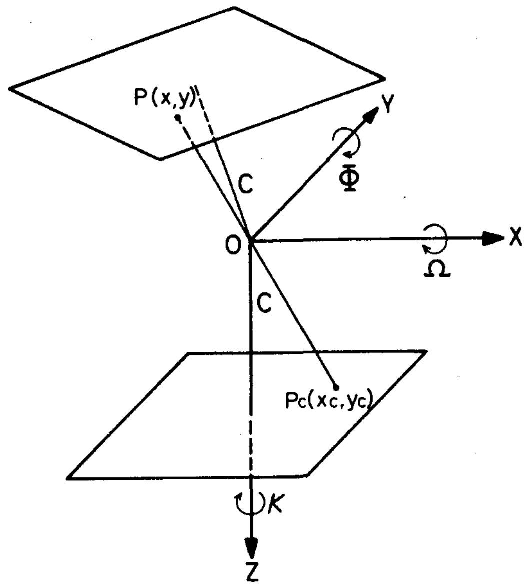


Fig.1 Tilted and Rectified Photographs

deviated from truly vertical axis, no matter what a stabilizing apparatus of high performance, for example, a level vial is equipped. For producing the equivalent vertical photograph, 6 independent exterior orientation parameters of a tilted photograph are needed. They consist of exposure station X_0, Y_0, Z_0 and angular orientation Φ, Ω, k in the ground coordinate system X_G Y_G Z_G. Fig.1 shows the relation of these two photographs. As the ground coordinate system, the left-hand one is used for computational convenience. That is, the direction of Z_G-axis is oriented vertical below. Still more, the exposure station is assumed to coincide with the origin of the ground coordinate system for simplicity, then $X_0=0$, $Y_0=0$, and $Z_0=0$. Angular orientation parameters Φ,Ω,K represent the amount and direction of camera axis, and are the amount of rotation about Y_G-, X_G-, Z_G-axes respectively. On the other hand, the photo coordinate system x y is defined usually such that its origin is at a principal point of a photograph, and the x- and y-axes pass through fiducial marks. Likewise, the photographic coordinate system x_c y_c of the rectified image is defined such that the origin is coincident with the principal point too, and x_c- and y_c-axes are parallel to X_G- and Y_G-axes respectively. This system is refered as the rectified photographic coordinate system.

Reffering to Fig.1, the ground coordinates X_p, Y_p, Z_p of any point p(x,y) on the tilted photograph are expressed as follows in a vector form;

$$\begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = D_\Phi D_\Omega D_K \begin{bmatrix} x \\ y \\ c \end{bmatrix}, \text{ or } \quad \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = D_{\Phi\Omega K} \begin{bmatrix} x \\ y \\ c \end{bmatrix}, \tag{1}$$

where c is a focal length of the camera, and D_Φ, D_Ω, D_K are rotation matrices associated with Φ, Ω, K respectively, and $D_{\Phi\Omega K}$ is a multiplication of these matrices. Therefore their elements can be written as thus;

$$\left.\begin{aligned} D_{\Phi} &= \begin{bmatrix} \cos\Phi & 0 & \sin\Phi \\ 0 & 1 & 0 \\ -\sin\Phi & 0 & \cos\Phi \end{bmatrix} \\ D_{\Omega} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega & -\sin\Omega \\ 0 & \sin\Omega & \cos\Omega \end{bmatrix} \\ D_{K} &= \begin{bmatrix} \cos K & -\sin K & 0 \\ \sin K & \cos K & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}\right\} \quad (2)$$

Of course, the scale of rectified image can be arbitrarily given. In this deduction the scale factor is set c, as the same as that of the tilted photograph. But the equations, if necessary, might be modified for the case of any scale factor.

The equation of the straight line $\overline{p0}$ is expressed as;

$$\frac{X}{X_p} = \frac{Y}{Y_p} = \frac{Z}{Z_p}. \quad (3)$$

The point $p_c(x_c, y_c)$ on the rectified image, corresponding to the point $p(x,y)$, can be obtained as the intersection of the straight line $\overline{p0}$ and the plane, the equation of which is

$$Z = c. \quad (4)$$

x_c and y_c are expressed in a vector form from Eq.(3) and (4) as thus;

$$\begin{bmatrix} x_c \\ y_c \\ c \end{bmatrix} = \frac{c}{Z_p} \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = \frac{c}{Z_p} D_{\Phi\Omega K} \begin{bmatrix} x \\ y \\ c \end{bmatrix}. \quad (5)$$

If the elements of the ratation matrix $D_{\Phi\Omega K}$ are written as follows;

$$D_{\Phi\Omega K} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}. \tag{6}$$

from Eq.(5) we obtain the projective relations;

$$\left.\begin{aligned} x_c &= \frac{d_{11}x + d_{12}y + d_{13}c}{d_{31}x + d_{32}y + d_{33}c}\,c \\ y_c &= \frac{d_{21}x + d_{22}y + d_{23}c}{d_{31}x + d_{32}y + d_{33}c}\,c. \end{aligned}\right\} \tag{7}$$

Eqs.(7) are fundamental expressions relating the tilted image with the equivalent vertical one. But when we rectify the digital image by off-line process, it is more convenient to invert the above relations and to express x, y as functions of x_c, y_c. The rotation matrix is an orthogonal matrix, i.e., the inverse is equal to the transpose, or

$$D_{\Phi\Omega K}^{-1} = D_{\Phi\Omega K}^{T}. \tag{8}$$

Using this property and Eq.(5), we obtain following relations likewise above;

$$\left.\begin{aligned} x &= \frac{d_{11}x_c + d_{21}y_c + d_{31}c}{d_{13}x_c + d_{23}y_c + d_{33}c}\,c \\ y &= \frac{d_{12}x_c + d_{22}y_c + d_{32}c}{d_{13}x_c + d_{23}y_c + d_{33}c}\,c. \end{aligned}\right\} \tag{9}$$

3. Pixel Interpolation

There exist a lot of interpolation methods proposed up to date. Representative methods among them are polinomial interpolation, least squares interpolation, interpolation by finite elements and spline functions and so forth [6]. However, because any image is usually quantized with equidistant spacing, the problem becomes easier than in the general cases of interpolation.

In this case, the interpolation method using "interpolation function" or "base function" is considered as the best by reason of its good compatibility to computer processing. This method is able to be considered as a special case of finite elements method. The interpolation function, denoted by $F_n(x, y)$ in the x y-coordinate system, is composed of functions of finite support. The support of a function is the region in which the function is not zero identically. And outside of the finite region the interpolation function is zero identically. In general a interpolation function is expressed as a piecewise polinomial.

Let a continuous, infinite dimensional image on the x y-plane be sampled with grid spacing Δx, and Δy along x- and y-axes respectively, and densities denoted by d_{ij} be obtained, where suffixes i, j indicate the position of grid points, the coordinates of which in the x y-plane are $x = i\,\Delta x$ and $y = j\,\Delta y$ ($i = 0, \pm 1, \pm 2, \ldots$; $j = 0, \pm 1, \pm 2, \ldots$). The continuous reconstructed function $f(x, y)$ is calculated by a linear combination of the density values d_{ij}'s and $F_n(x, y)$ thus;

$$f(x, y) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} d_{ij}\, F_n(x-i\Delta x,\, y-j\Delta y). \tag{10}$$

If $F_n(x, y)$ is zero-order, one-order, or three-order polinomials for each variable, x and y, the interpolation methods are called nearest neighbour, bi-linear, or cubic convolution method respectively. Our program has these three kinds of subroutines for interpolating image densities. They are herein briefly explained, because they are already very popular in the field of digital image processing [7].

(1) Nearest neighbour method

This is the simplest way of interpolation. The interpolation function has a box-car shape (Fig.2). The reconstructed signal by this function in the one-dimensional case is shown in Fig.3.

It is not continuous at nodal points. In the computational algorithm, the density value of newly defined pixel position can be obtained by substituting the density value of the nearest adjacent grid point for it.

(2) Bi-linear method

This method uses a linear function as interpolation function. Its shape is a triangle in the one-dimensional case. Fig.4 shows a triangle function, and a example of one dimensional linear interpolation is shown in Fig.5. Two-dimensional linear interpolation is performed similarly as the one dimensional case. As shown in Fig.6, if a grid spacing is assumed 1, any density value F is interpolated linearly from the density values of the surrounding grid points, F_{11}, F_{21}, F_{12} and F_{22} as follows;

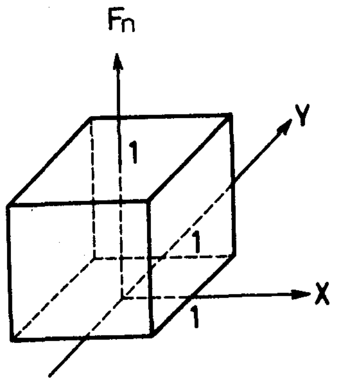


Fig.2 Box-car Function

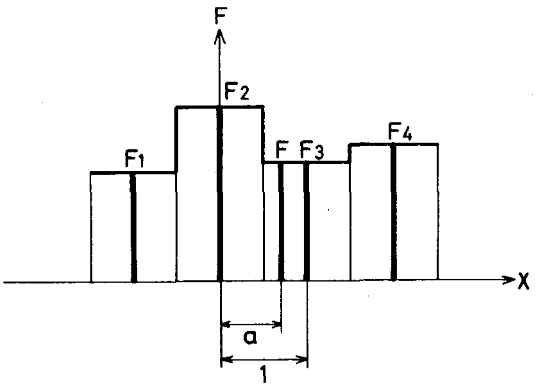


Fig.3 Nearest Neighbour Interpolation in One-dimensional Case

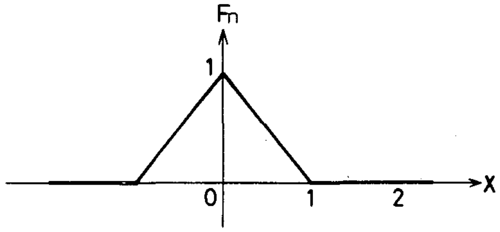


Fig.4 Triangle Function

$$\left.\begin{aligned} F &= g_1 + (g_2 - g_1)\,\beta \\ g_1 &= F_{11} + (F_{21} - F_{11})\,\alpha \\ g_2 &= F12 + (F_{22} - F_{12})\,\alpha . \end{aligned}\right\} \quad (11)$$

where α and β are distances of F from F_{11} in the x and y directions respectively.

(3) Cubic convolution method

Cubic convolution interpolation uses, as a interpolation function, an approximation of Sinc function, composed of cubic piecewise polinomials. According to the Fourier analysis, if the density values of a continuous image are sampled at the Nyquist rate, Sinc function interpolation is able to reconstruct the exact image from the sampled values. Fig.7 shows the one-dimensional cubic approximation function, which has continuity of itself and first derivative at the nodal points. Fig.8 shows the cubic convolution interpolation in one-dimensional case. In the two-dimension, interpolation is performed by 2 steps. Firstly, as shown in Fig.9, each F_j (j= 1,2,3,4) is caluculated from the densities in one row in the x direction, F_{1j}, F_{2j}, F_{3j} and F_{4j} by the following equation;

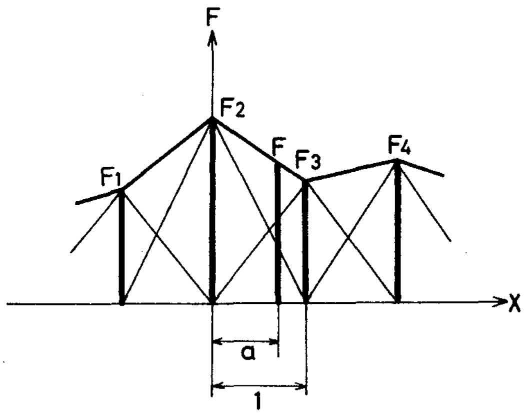


Fig.5 Bi-linear Interpolation in One-dimensional Case

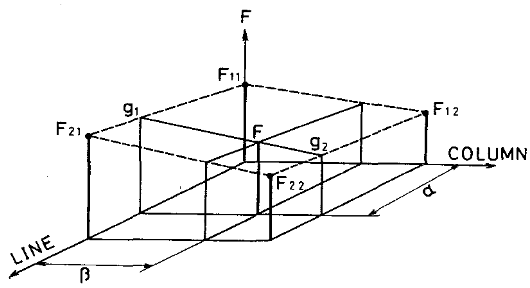


Fig.6 Bi-linear Interpolation in Two-dimensional Case

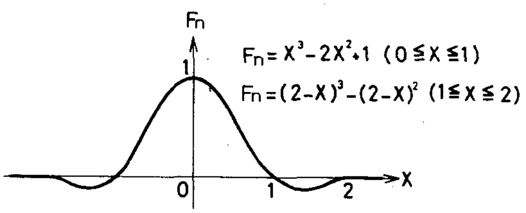


Fig.7 Cubic Approximation Function of Sinc Function

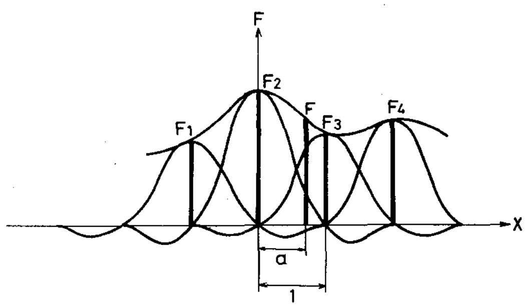


Fig.8 Cubic convolution interpolation in One-dimensional case

$$F_j = (F_{4j} - F_{3j} + F_{2j} - F_{1j}) + (F_{3j} - F_{4j} - 2\ F_{2j} + 2\ F_{1j}) + (F_{3j} - F_{1j}) + F_{2j} \quad (j = 1,2,3,4), \tag{12}$$

Secondly, F is calculated from these F_1, F_2, F_3 and F_4 in a likewise fashion;

$$F = (F_4 - F_3 + F_2 - F_1) + (F_3 - F_4 - 2\ F_2 + 2\ F_1) + (F_3 - F_1) + F_2. \tag{13}$$

Which is the best method for processing photographic density depends, of course, not only on the distribution property of spatial wave-numbers of density values of the original image and the grid spacing of the sampled discrete image, but also on consumed computation time and cost. Then these 3 methods were compared in practice in terms of computation time and accuracy.

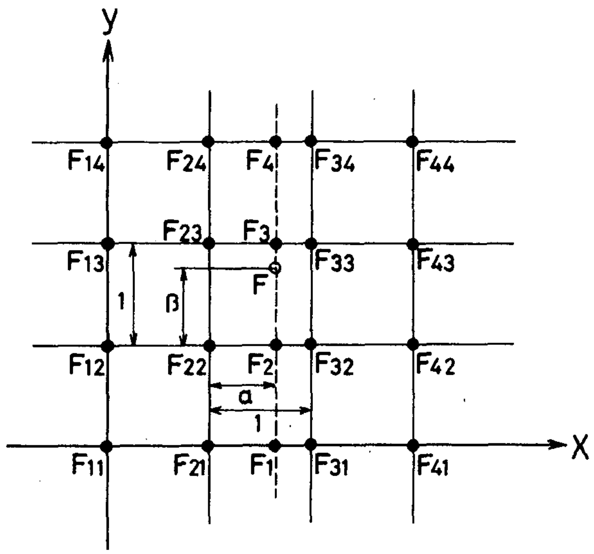
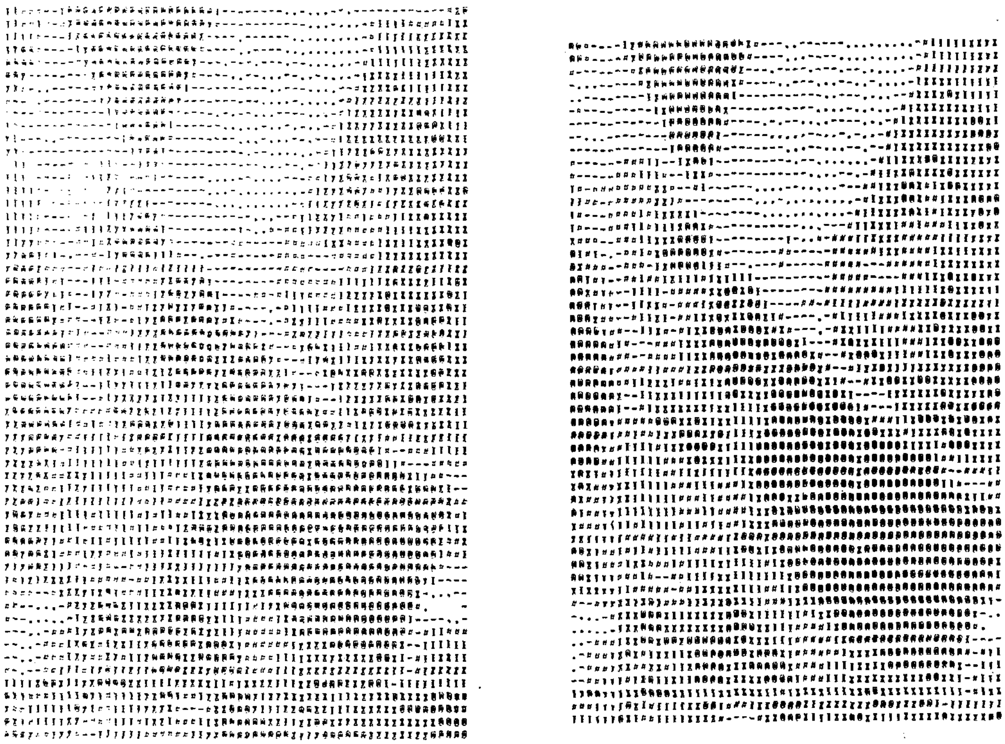


Fig.9 Cubic Convolution Interpolation in Two-dimensional Case

A small area of a tilted image, which will be explained in detail in the subsequent section, was rectified by each method. The test area is a square of 6 mm by 6 mm on the film, consisting of 120 by 120 pixels. A light spot of a scanner or a pixel of the sampled image is square-shaped and 50 μm by 50 μm of size. The consumed CPU times in interpolating 14,400 points are shown in Table 1. It is notable that the CPU time required for the interpolation by the bi-linear method is only ten per cent more than by the nearest neighbour method, but the cubic convolution method needs time twice as much as the bi-linear method.

Table 1 CPU Times Taken for Interpolation (14,400 points)

Method	Nearest Neighbour	Bi-linear	Cubic Convolution
CPU Time	12.25 sec.	14.40 sec.	24.15 sec.

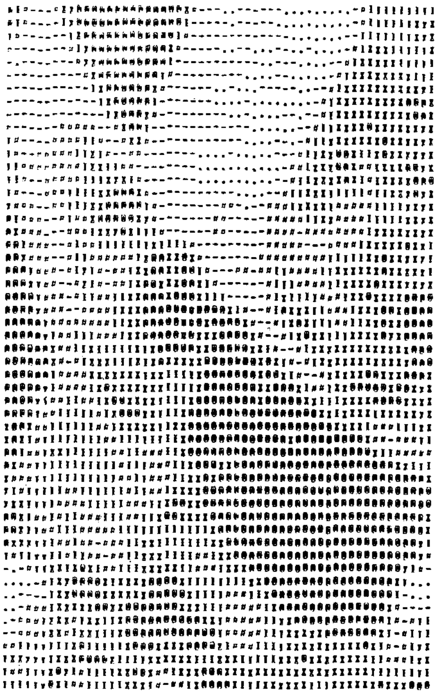


(a) Density Map of Original Digital Image

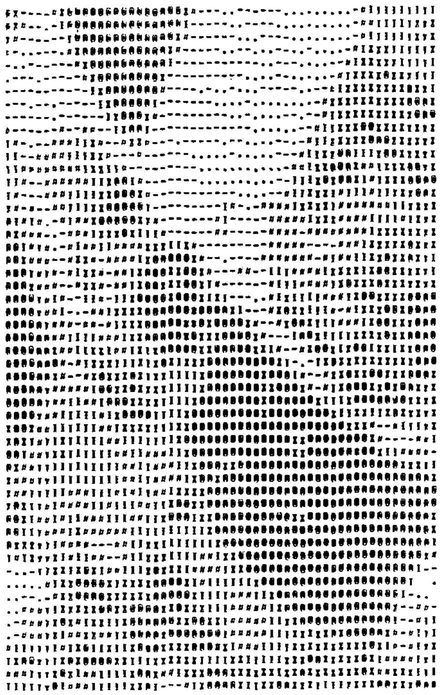
(b) Density Map of Reconstructed Image by Nearest Neighbour Method

Fig.10 Examples of Image Reconstruction

The interpolated images are shown in part together with the original digital image in Fig.10. which are printed out with a line printer. There appear no remarkable differences among them. If the Nyquist condition were fulfilled, cubic convolution interpolation could reconstruct the almost exact image. But it is impossible to digitize a photograph so dense enough to satisfy this condition. Then time consuming cubic convolution interpolation seems not to have a charm to use. On the other hand, when a tracking line of



(c) Density Map of Reconstructed Image by Bi-linear Method



(d) Density Map of Reconstructed Image by Cubic Convolution Method

Fig.10 - continued

a densitometer happens to run across a high contrast edge at a very acute angle, nearest neighbour interpolation tends to injure slightly continuity of the reconstructed edge line, and to make a false break line.

Compared to these interpolation, bi-linear interpolation seems to have a intermediate property between them, and the consumed time by this method is only a bit more than by nearest neighbour method. Therefore in the case of the sampling rate of every 50 μm spacing or so, this interpolation method might suffice for our practical necessity. However, according to Michail [8], et al., by some mensuration tests, the reproduced aerial film from the digital data scanned at the sampling rate of 20 μm spacing allowed skillfull observers to make pricking with a sufficient accuracy from the photogrammetric standpoint. Considering their result, a pixel size of 50 μm looks rather large. When the sampling rate is 20 μm spacing, 10 μm of maximal position error might occur in reconstructed image by nearest neighbour method, if no other error sources exist. Therefore bi-linear interpolation method is still preferable in this occasion from photogrammetric accuracy.

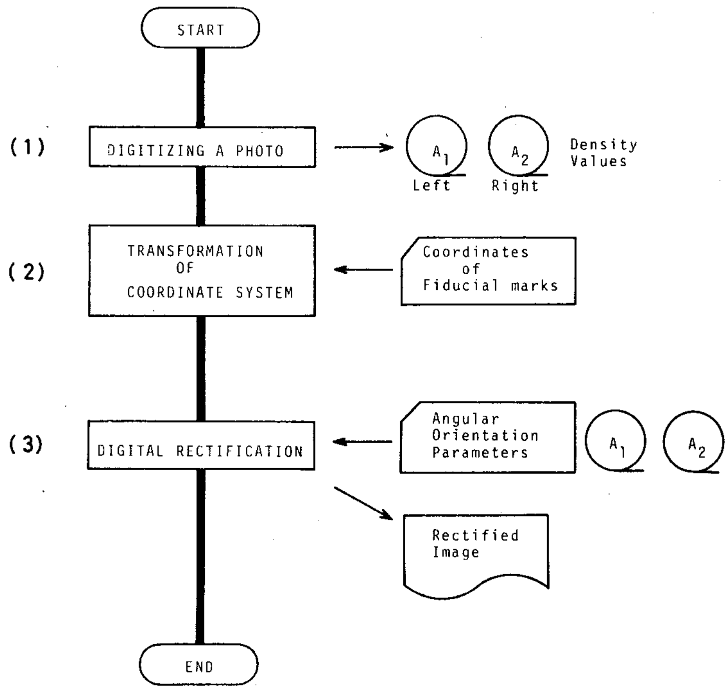


Fig.11 Flow Chart for Digital Rectification

4. A Practical Example

This section treats a practical example of digital rectification according to the flow chart of the procedure shown in Fig.11. Before detailing the procedure, we explain the items in Fig.11 brefly.

(1) First of all a photographic image is digitized with a drum scanning micro densitometer.

(2) Position of any pixel in a digitized image is indicated by a row and column number. These are hereafter reffered as the image coordinate system. The image coordinate system should be transformed to the photo coordinate system, because in digitizing the image the film deforms due to drum curvature.

(3) Digital rectification is performed using 6 angular orientation parameters, which are input as auxiliary parameters. At last the rectified image is output with a line printer.

In this experiment we utilize the property of stereo images for checking the rectification accuracy. For the accuracy check the image points and the corresponding ground points are required as references. They are called check points. But in this experiment the image has no available check points, except for 4 fiducial marks. But, if a pair of stereo photographs is rectified to a plane parallel to the air base, we can check the accuracy by means of marking y-parallaxes of arbitrary homologeous image points on the rectified images.

It is convenient for the consideration to introduce the model coordinate system X Y Z instead of the ground coordinate system X_g Y_g Z_g. That is, as shown in Fig.12, the origin is coincident with the point O_1, and X-axis passes through two exposure stations

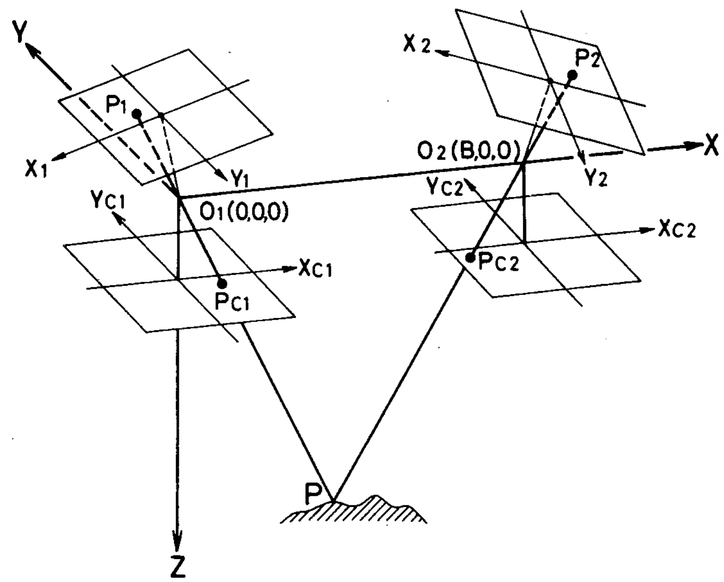


Fig.12 Rectification of Streo Photographs

O_1 and O_2. The Y- and Z-axes are set such that they are perpendicular to X-axis and the rotation angle around the Y-axis of the left image, ω_1, is null. The rectified image coordinate systems x_{c1} y_{c1} and x_{c2} y_{c2} are defined for the respective image such that their origins coincide with the principal points, and x_{c1}- and x_{c2}-axes are set in the direction of X. It is clear that on the rectified stereo pair, the homologeous points have the same y_c coordinates. For this reason the rectification accuracy is able to be checked by use of a stereo pair as mentioned above. It shoud be noticed, however, resultant errors due to both the left and the right rectification are included in observed y-parallaxes.

Angular orientation parameters ϕ_1, κ_1, ϕ_2, ω_2 and κ_2 (suffixes 1 and 2 represent the left and the right image respectively) necessary for the rectification can be calculated by the independent model method of relative orientation [9]. For this purpose, precise photographic coordinates of more than 5 pairs of homologeous image points are required. Usually for the mensuration a stereo comparator is used.

In the case of off-line process, coordinate systems used become fairly complex. Following 4 coordinate systems for each photograph should be distinguished (Fig.13):

(1) the comparator coordinate system x_k y_k,
(2) the image coordinate system ξ η,
(3) the photographic coordinate system x y,
(4) the rectified photographic coordinate system x_c y_c.

The photographic coordinate system is defined in this experiment such that its origin is set at the principal point of the photograph and x- and y-axes are set parallel to the comparator axes x_k and y_k respectively.

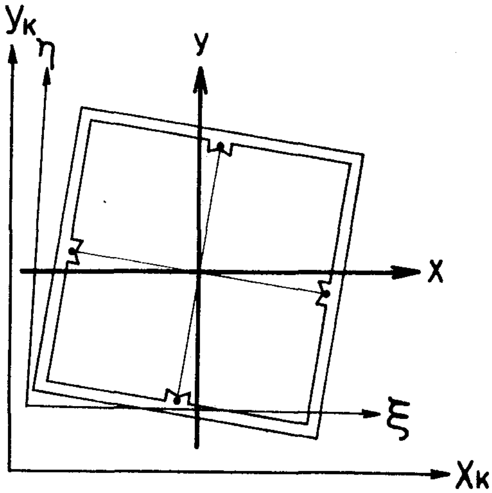


Fig.13 Relations of Coordinate systems Used for Measuring Image Points

The details of experiments are explained as follows:

1) Digitizing a photograph

The left photograph of the used pair is shown in Fig.14, and some data about them are shown in Table 2. Digitization was performed all over the photographs through a Joyce Leoble drum scanning micro densitometer, Scandig 3, technical data of which are listed in Table 3. In digitizing a film image, a grid spacing of 50 μm was selected, which corresponds to 1.3 m in width on the ground. It seems rather large from the photogrammetric point of view. As mentioned in section 3, though 20 μm grid spacing is more suitable for the photogrammetric needs, it had been nonrealistic to digitize

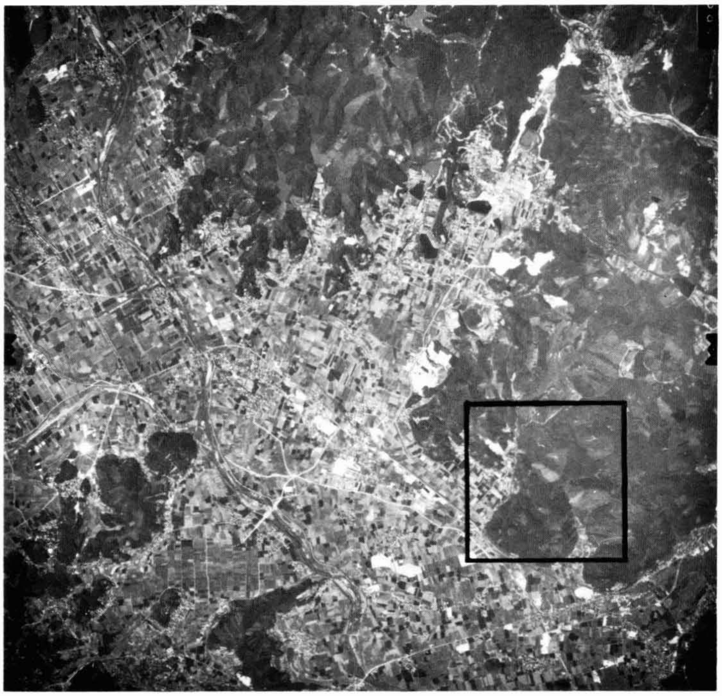


Fig.14 Left Photographs Used in the Experiment

Table 2 Data about the Photographs

Location	Kibitsu, Okayama Prefecture
Data	May 18th, 1970
Scale	1 : 26,000
Height	3,800
Camera	RMK 15/23
Focal Length	152.09 mm

Table 3 Technical Data about Scandig 3

Slit Width (μm)	12.5, 25, 50, 100, 200
Measuring Range	0-3 D (8 bit binary)
Densitometric Distorsion	Linearity $\leqq \pm$ 1/256
	Reconstructivity $\leqq \pm$ 2/256
Control Computer	NOVA 3/C (64KB)
Soft Ware	FSPP

at 20 μm grid spacing in this off-line experiment because of the great amount of output files. Measured densities are output to magnetic tapes in a format of 8-bit binary.

2) Correction of image coordinate system

The drum scanning micro densitometer can digitize a image speedily, but because of winding a film around the drum mannually, the film unavoidably deformed. As a result of a preliminary tests, along the curve of the drum the film shrank about 0.5 %. This distorsion, however, is thought of correctable with the affine transformation, unless the photograph is locally uplifted off the drum. That is, the following relation is assumed to exist for any point whose image coordinagtes are (ξ, η) and the photographic coordinates are (x, y);

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \qquad (14)$$

where a_{ij}, b_j $(i = 1,2;\ j = 1,2)$ are constant coefficients. They were determined by the least squares method from 4 fiducial marks as references whose photographic coordinates were measured with a stereo comparator. Table 4 shows the result of computations. The root mean squares of residuals were 0.64 pixels on the left image and 0.86 pixels on the right image respectively. They seem to be rather large and beyond photogrammetric tolerance. This means nonlinear complex deformations have occured in digitization.

3) Digital rectification [10]

The rectified region is shown in Fig. 14, enclosed with a rectangle which contains 1,024 by 1,024 pixels. The rectification was performed based on Eq.(9). The 6 orientation parameters were

Table 4 Coefficients of Affine Transformation

	a_{11}	a_{12}	a_{21}	a_{22}	b_1	b_2	Root mean square of Residuals
Left photo	20.0020 ±0.0078	0.4437 ±0.0078	-0.4295 ±0.0078	19.3896 ±0.0078	2272.50 ±0.6265	2290.00 ±0.6265	0.626 pixels
Right photo	20.0145 ±0.0103	-0.1649 ±0.0103	0.1572 ±0.0103	19.3996 ±0.0103	2276.37 ±0.8214	2229.12 ±0.8214	0.821 pixels

Unit of a_{ij} = pixel/mm
Unit of b_{ij} = pixel $(i = 1,2;\ j = 1,2)$

Table 5 Relative Orientation Parameters

κ_1 (rad) ϕ_1 (rad) κ_2 (rad) ω_2 (rad) ϕ_2 (rad)	-0.020007 ± 0.000046 -0.012601 ± 0.000087 -0.007417 ± 0.000048 -0.009581 ± 0.000044 -0.004799 ± 0.000095
Root Mean Square of y-parallaxes	0.0053 mm (0.1 pixels)

calculated by the independent model method. Table 5 shows the obtained angular parameters. The root mean squares of residual y-parallaxes are very small and the orientation error is considered to have no effect on the rectification. The rectified images were printed out with a line printer. The residual y-parallaxes between 2 images were checked. In consequence, the maximal y-parallaxes of ± 2 pixels were observed, but almost everywhere observed y-parallaxes were less than or equal to ± 1 pixel. These y-parallaxes are considered to have come mainly from the affine transformation errors.

5. Conclusion

The mathematical projection formulae and a practical example for digital rectification of tilted photographs were reported in this paper. The used method is based on the off-line processing using a drum scanning micro densitometer and general purpose computers. Firstly, the density interpolation methods of pixels necessary for off-line process were considered. And secondly, the practical example of rectification and related problems were explained.

Remarks obtained from the investigation are:

1) Bi-linear interpolation suffices for the reconstruction of digital images of ordinary aerial photographs with a grid spacing of 50 μm or so.

2) Image deformations in digitizing were beyond photogrammetric tolerance. For eliminating this defect, a film fitting mechanism to a drum should be improved. Or preferably a TV scanner equipped

with a flat film bed and a surbo-motor system for movement over the film should be developed.

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