Memoirs of the School of Engineering Okayama University Vol. 12, February 1978

Analysis of Pump Test Data for Partial Penetrating wells

Iichiro Kono* and Makoto Nishigaki*

(Received November 17, 1977)

Synopsis

The solutions of unsteady phreatic flow toward a partially penetrating well in an aquifer of finite thickness are described. Firstly the solution for a confined aquifer is shown. In this case,three methods of analyzing field data with partially penetrating well are given, that is,"Log-Log Method, Log-Log Distance Drawdown Method and Jacob's Method Ajusted for Partial Penetration". By using these methods the hydraulic conductivities and the specific storage of the aquifer may be determined.

Secondly the solution for an unconfined aquifer is shown. In this case,also two methods of analyzing field data with partially penetrating well are given. By using these methods, the anisotropic permeability and the storage coefficient(effective porosity) of the aquifer may be determined.

Moreover in each case, the effects of partial penetration are discussed and the limits of adapting the Theis' and Jacob's methods are setted. From these analytic results, some cosiderations are added to determine the anisotropy of permeability and to evaluate the storage coefficient.

*Depertment of Civil Engineering

1. Introduction

To obtain the formation constants from pumping test data, Theis' Method or Jacob's Method are commonly used. However, we are often confronted with the case in which we can not obtain the formation constants, using their methods. For this problem, it is necessary to consider again the assumptions on which Theis' and Jacob's Methods stand as follows:

(1) Flow within the porous medium obeys Darcy's Law.
(2) The layer is homogeneous and istropic with respect to permeability.
(3) Storage coefficient is time independent.
(4) The system is considered to be of infinite radial extent with the well at its center.
(5) Only single phase(or saturated) flow occurs in the aquifer.
(6) The well is assumed to have no surface of seepage.
(7) Head losses through the well screen are neglected.
(8) The pumping well is totaly penetrated in the aquifer.

It seems that some assumptions of those do not satisfy the conditions of field pumping test. In this conception, we consider that it is quite common in developing aquifer storage projects not to open up the entire aquifer thickness. In other words, the last assumption dose not satisfy the conditions of pumping test. Therefor, it is necessary to understand the effects of partial penetration and to consider the deviations from simple radial flow.

As illustrated in **Fig.1.1**, in this case, water moving toward the pumping well has to converge in some manner into the open well from all parts of the aquifer. This diversion of flow-lines from the holizontal leads to a more complicated pressure distribution pattern around the pumping well than is the case with complete penetration.

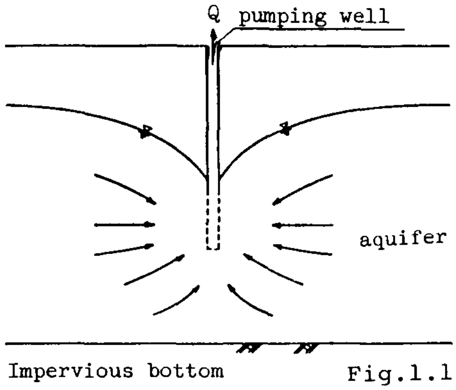

Fig.1.1
Aquifer with partially penetrating wells.

This produces pressure changes in the aquifer that may by substantially above or below those that would be predicted using the Theis' solution.

There may be situations where the total thickness of the aquifer is not known. As will be discussed below, the effects of partial penetration may be used to determine the thickness of the aquifer or, that part of the total thickness that is responding to the pumping test, and may be considered the anisotropy of permeability.

Therefore, the pumping test with pertially penetrating well would be more useful than that with completly penetrating well.

In this paper, methods of handling pertiall penetration problems are discussed.

2. Analytic Solution for Partially Penetrating Well in a Confined Aquifer

Wells, of which the water-entry section is less than the aquifer they penetrate, are called partially penetrating wells. Unlike the flow toward completely penetrating wells where the main flow takes place essentially in planes parallel to the bedding planes of the formation, the flow toward partially penetrating wells is three-dimensional. Consequently, the drawdown observed in partially penetrating wells will depend, among other variables, on the length and space position of the screened portion(water-entry section) of the observation wells, as well as on that of the pumping or flowing well.

In aquifer where the horizontal conductivity is several times greater than the vertical, the yield of partially penetrating wells may be appreciably smaller than that of equivalent wells in isotropic auifer.

In treating the problem of flow toward partially penetrating well (Fig.2.1), the following assumptions are made

(1) The aquifer is homogeneous, anisotropic and extend infinitelly with impermeable clay layers above and below.

(2) The aquifer shall also be considered to be horizontal and water saturated at all times.

(3) The conductivities of main aqufer in the horizontal and vertical directions have different, but constant values, k_r, k_z, respectively.

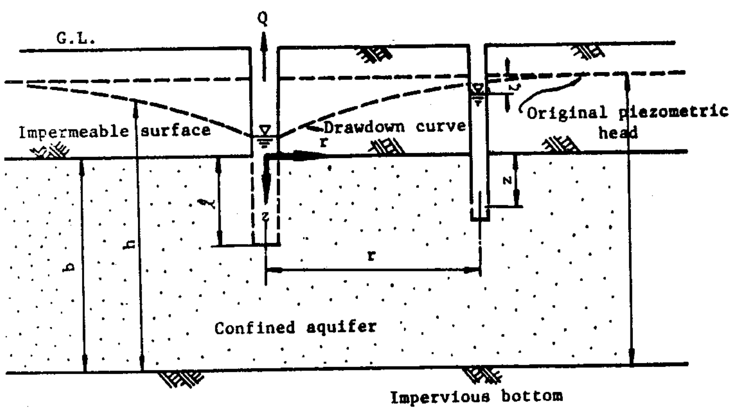

Fig.2.1
Partially penetrating wells in a confined aquifer.

(4) The well is of a vanishingly small radius and discharging at a constant rate.

2.1 Basic Equations and Solution

The differential equation that describes the fluid movement is given by

$$k_r\frac{\partial^2 h}{\partial r^2}+k_r\frac{1}{r}\frac{\partial h}{\partial r}+k_z\frac{\partial^2 h}{\partial z^2}=S_s\frac{\partial h}{\partial t} \tag{2.1}$$

where the hydraulic head, $h(=\psi+z)$, is now a function of r, z, and t. The initial and boundary conditions to be imposed on the solution are as follows:

$$h(r,z,0)=H \quad \text{(head initially constant)} \tag{2.2}$$

$$h(\infty,z,t)=H \quad \text{(head at infinity remains constat)} \tag{2.3}$$

$$\frac{\partial h}{\partial z}(r,0,t)=0 \quad \text{(no flow across upper boundary)} \tag{2.4}$$

$$\frac{\partial h}{\partial z}(r,b,t)=0 \quad \text{(no flow across lower boundary)} \tag{2.5}$$

$$\lim_{r\to 0} 2\pi k_r r\int_{b-\ell}^{\ell}\frac{\partial h}{\partial r}dz=-Q \quad \text{(flow rate into well of zero radius remains constat)} \tag{2.6}$$

Hantush has studied this problem for a more complicated situation where there is also clay leakage into the aquifer that is being pumped[1,2]. By imposing the condition of no clay leakage, i.e. Eqs.(2.4),(2.5), the Hantush's solution simplifies to

$$\left.\begin{aligned}&\zeta_r^*=W(u_r)+f(u_r,r^*/b,\ell/b,z/b)\\ &\text{where}\quad \zeta_r^*=\frac{4\pi k_r b}{Q}\zeta\ ,\quad \zeta=H-h,\quad u_r=\frac{(r^*)^2}{4(k_r/S_s)t}\end{aligned}\right\} \tag{2.7}$$

Noting that

$$W(u_r)=\int_{u_r}^{\infty}\frac{e^{-\omega}}{\omega}d\omega \qquad \text{(well function)} \tag{2.8}$$

and

$$f(u_r,r^*/b,\ell/b,z/b)=\frac{2b}{\pi\ell}\sum_{n=1}^{\infty}\frac{1}{n}\cos(\frac{n\pi z}{b})\sin(\frac{n\pi\ell}{b})\int_{u_r}^{\infty}\exp[-\omega-\frac{(n\pi r^*)^2}{4\omega b^2}]\frac{d\omega}{\omega} \tag{2.9}$$

where

$$r^*=\sqrt{k_z/k_r}\ r \tag{2.10}$$

describing another form of the Hantush solution

$$\zeta_r^{**}=\int_{u_r}^{\infty}\frac{e^{-\omega}}{\omega}\left[\sum_{n=1}^{\infty}\left\{\mathrm{erf}\frac{(2nb+\ell+z)\sqrt{\omega}}{r^*}-\mathrm{erf}\frac{(2nb-\ell-z)\sqrt{\omega}}{r^*}+\mathrm{erf}\frac{(2nb+\ell-z)\sqrt{\omega}}{r^*}-\mathrm{erf}\frac{(2nb-\ell+z)\sqrt{\omega}}{r^*}\right\}+\mathrm{erf}\frac{(\ell+z)\sqrt{\omega}}{r^*}-\mathrm{erf}\frac{(\ell-z)\sqrt{\omega}}{r^*}\right]d\omega \tag{2.11}$$

where $\zeta_r^{**}=\frac{8\pi k_r \ell}{Q}\zeta$

for infinite aquifer thickness($b\to\infty$)

$$\zeta_r^{**}=\int_{u_r}^{\infty}\frac{e^{-\omega}}{\omega}\left\{\mathrm{erf}\frac{(\ell-z)\sqrt{\omega}}{r^*}+\mathrm{erf}\frac{(\ell+z)\sqrt{\omega}}{r^*}\right\}d\omega \tag{2.12}$$

Substituting $k_r=k_z=k$ in Eq.(2.7), the solution for an isotropic aquifer is obtained as follows:

$$\zeta^*= W(u) +f(u,r/b,\ell/b,z/b) \tag{2.13}$$

where:

$$\zeta^*=\frac{4\pi kb}{Q}\zeta \tag{2.14}$$

$$u=\frac{r^2}{4(k/S_s)t} \tag{2.15}$$

Javandel shows in some detail how Eq.(2.13) can derived and has used a heat transfer model as an indepent means of verifying the solution.[3] By comparing Eq.(2.13) with the Theis' solution for the pumping well with complete penetration, it is evident that $f(u,r/b,\ell/b,z/b)$ is simply added to the exponetial integral to describe the effects of partial penetration. For full penetration, ℓ is equal to b in Eq.(2.13) and the result is the same of the Theis' solution. Therefore, Eqs.(2.7),(2.13) are defined as general solutions for pumping test in a confined aquifer.

For relatively large values of time, the function $f(u_r,r/b,\ell/b,z/b)$ can for all partical purposes, be replaced by $2K_0(n\pi r/b)$ in which case Eq.(2.13) becomes independent of time as follows:

$$\zeta=\frac{Q}{4\pi kb}[\ln(t/r^2)-\ln(S_s/2.25k)+f_s(r/b,\ell/b,z/b)] \tag{2.16}$$

$$\zeta=\frac{2.3Q}{4\pi kb}[\log(t/r^2)-\log(S_s/\exp(f_s)2.25k)] \tag{2.17}$$

$$f_s=\frac{4b}{\pi\ell}\sum_{n=1}^{\infty}\frac{1}{n}K_0\left(\frac{n\pi r}{b}\right)\sin\left(\frac{n\pi\ell}{b}\right)\cos\left(\frac{n\pi z}{b}\right) \tag{2.18}$$

in which K_0 is the zero-order modified Bessel function of the second kind.

2.2 The Effects of Partial Penetration

The effecs of partial penetration on the drawdown around a pumping well is shown in Figs.2.2a,2.2b and2.2c. The variation is for a vicinity of well in an isotropic aquifer(k_r=k_z).Regardless of the location of the wells and the space position of their screens, the time-drawdown curves, at relatively large values of time($t>S_s/k\ r^2$), will have approximately the same slope. This slope is the same as would obtained if the pumping well completely penetrates the aquifer.In other words, the effect of partial penetration has attained its maximum value.

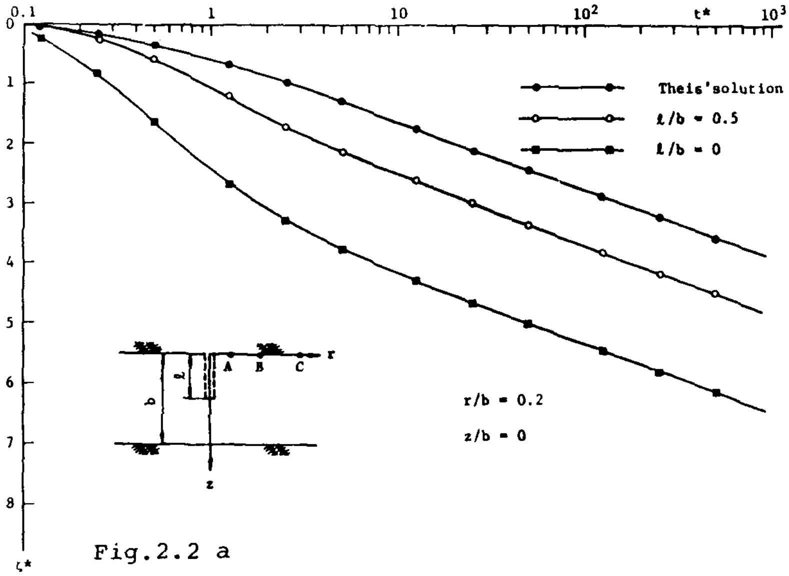

Fig.2.2 a

Drawdown characteristics for partially penetrating well in a confined aquifer

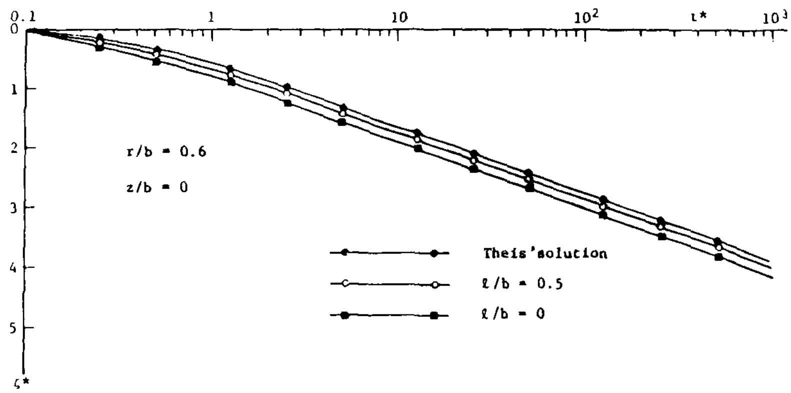

Fig.2.2 b

Drawdown characteristics for partially penetrating well in a confined aquifer.

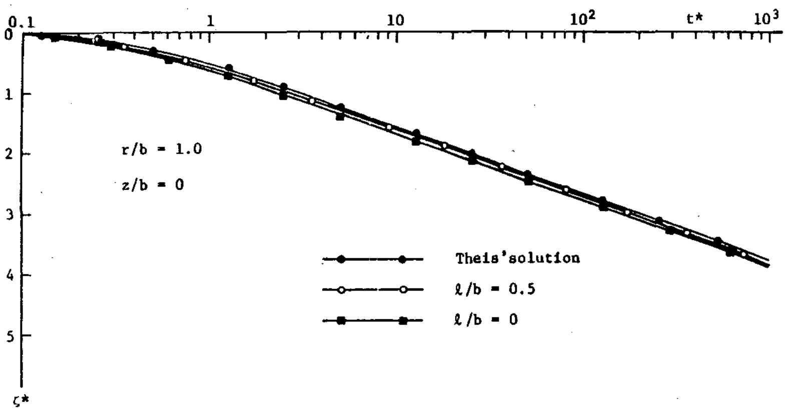

Fig.2.2 c

Drawdown characteristics for partially penetrating well in a confined aquifer.

If the observation well is relatively apart($r/b>0.5$), the drawdown is given by the Theis' formula. In other words, the drawdown in such a well is not affected by partial penetration; it is the same as though the pumping well completely penetrated the aquifer. Hantush has also discussed the wells located at $r/b>1.5$, regardless of the space position of its screen.[1]

Fig.2.3 compares the drawdowns observed in two equally distance wells, one of zero penetration and the other screened throughout its depth of penetration. Noticing it shows that two wells equally distant from a partially penetrating pumping well may register two different drawdowns. In fact, depending on the length and the relative positions of the screens, it is possible for a more distant well to reflect a greater drawdown.

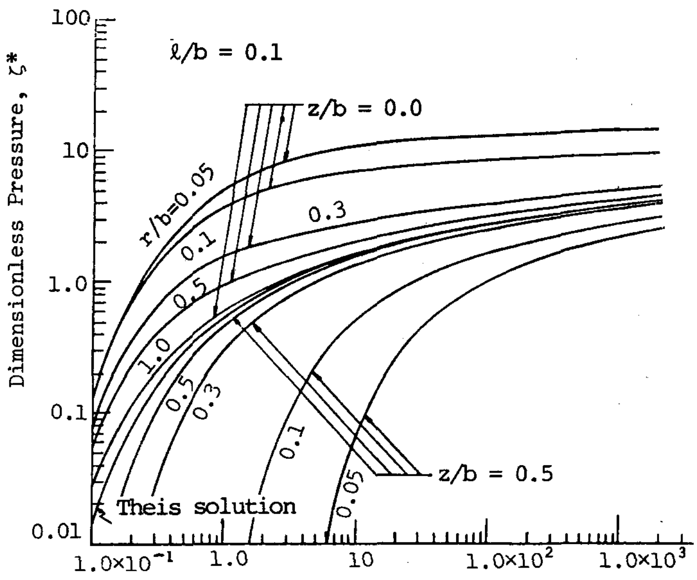

Fig.2.3

ζ* versus t* from Eq.(2.13) for infinite radial case with tenth penetration in pumping well.

A partially penetrating well will discharge less than a completely penetrating well if the two are operated at the same pumping level, other conditions controlling the flow remaining constant. If they are pumped at the same rate, however, the pumping level of the former will be lower than that of the latter.

Pumping at the same level, the yield of partially penetrating well in an anisotropic aquifer($k_r \neq k_z$) will decrease with decreasing k_z/k_r, other conditions being the same. The effect of the anisotropy decreases as the well penetration incrrease,as shown in **Fig.2.3**.Note in **Fig.2.3** that the following dimensionless time is used for the abscissa.

$$t^* = \frac{1}{4u} = (k/S_s)(t/r^2) \qquad (2.19)$$

If k_r/k_z does not differ greatly from unity, the anisotropy will not be of particular consequence except for very small penetration. On the other hand, should k_z/k_r be very small, the anisotropy of the aquifer may cause an appreciable decrease in the yield of the partially penetrating well If k_z should actually vanish, the flow toward the well will become purely radial, confined to the part of the aquifer in which the well is screened.

2.3. Methods of Analyzing Field Data

In evaluating the resuls of a pumping test where partial penetration must be considered i.e. where **r/b<0.5**, one needs to know the geological conditions of the aquifer under investigation. In drilling the exploration wells, a considerable number of cores will often be taken, and an analysis of the porosity of these samples provides valuable data on the nature of the aquifer. Such cores, of course, provide only a very small sample of the total aquifer system. Thus, the results of a pumping test can be very helpful in providing an additional source of reliable data.

In the field of hydrology, basic methods of analyzing field data have been developed; Log-Log Method,Log-Log Distance Drawdown Method, which will be discussed below. In addition, Jacob's Method Ajusted for Partial Penetration, which is a variation of Jacob's Method, will also be presented.

All of the above methods require data that are measured in observation wells at some distance from the pumping well. If one could measure fluid levels in the pumping well itself, similar analysis could be made but it is rarely possible to keep the pumping rate exactly constant. Thus, the fluid levels may fluctuate rapidly,making it difficalt to get reliable data. The depth to the pumping fluid level will, of course, be much greater than in the observation wells, and this may make it difficult to obitain accurate measurements.For these reasons, an analysis of the drawdown data in the pumping well is not often made.

2.3.1 Log-Log Method

In the Log-Log Method, one can use graphical methods similar to Theis' Method. Knowing the values of ℓ/b and z/b one can prepeare a graph of log ζ^* versus log $t^*(t^*=1/4u)$ for the appropriate r/b between pumping and observation wells. As is evident from Fig.2.3, separate curves will have to be prepared for each observation well, unless the values of the three ratios($\ell/b, z/b, r/b$) are identical.

When the drawdown data from each observation well have been plotted on log-log paper with the same dimensions per cycle as used above, one can match the field results to the theoretical curve in the same manner as is done when using the Theis' curve. When the curves are matched, one can read the dimensionless parameters that correspond to each point of field data. It will be found that one can also choose any point of the curve of field data and still obtain the same result by using the appropriate values of ζ^* and t^* for that particular point.

An equivalent value ζ^* can be determined for any ζ measured in the observation well and an equivalent value of t^*, for the corresponding value of real time, t. The permeability can be calculated from Eq.(2.14)

$$k=\frac{Q\zeta^*}{4\pi b\zeta} \qquad (2.20)$$

and the compressibility factor from Eq.(2.19)

$$S_s=kt^*\left(\frac{t}{r^2}\right) \qquad (2.21)$$

The compressibility result obtained in this manner should give a value that is of the same order as the compressibility of water. At the reservoir conditions that will generally prevail in water storage operations, the compressibility of water is about $4.6\times10^{-11}cm^{-1}$.

2.3.2. Log-Log Distance Drawdown Method

When the aquifer is cosidered anisotrpic, the permeability must be evaluated k_r and k_z respectively. In this case, it is necessary to develop a new method and the " Log-Log Distance Drawdown Method " is a variation of the log-log method. Knowing the values of ℓ/b and z/b, one can prepare a graph of log ζ_r^* versus log t_r^* similar to that shown in Fig.2.4, and

$$t_r^*=\frac{1}{4u_r}=(S_s/k_r)(t/r^*)^2 \qquad (2.22)$$

As is evident from Fig.2.4, separate curves will have to be prepared for each r*/b. Here, as r* is dependent on a horizontal conductivity k_r and a vertical conductivity k_z, estimating the values of $\sqrt{k_z/k_r}$, separate curves of r*/b will have to be prepared for each observation well.

When the drawdown data from each observation well have been plotted on log-log paper with the same dimensions per cycle as used above, one can match the field results to the theoritical curve in the same manner as is done when using the Theis' curve. One obtains a graphical solution by placing the field results on top of the theoretical solution and shifing the plots, keeping the axes parallel, until the field data fall on the theoretical curves, when the curves are matched, one can read the dimensionless parameters that correspond to each point of field data and still obtain the same result by using the appropriate of ζ_r^* and t_r^* for that particular point.

At any datum point, one therefore reads the drawdown ζ, and its corresponding value of ζ_r^*. The horizontal conductivity k_r, of the aquifer being pumping may be calculated from

$$k_r = \frac{Q\zeta_r^*}{4\pi b\zeta} \qquad (2.23)$$

Having obtained the horizontal conductivity, one reads from the same data point used above, the radial distance r,of a observation well and the dimensionless parameter r*. The vertical conductivity k_z, may be calculated from

$$k_z = k_r\left(\frac{r^*}{r}\right)^2 \qquad (2.24)$$

The compressibility factor may be calculated using the elapsde pumping time t and its corresponding value of t_r^*,and the above determined vertical permeability from:

$$S_s = k_z \frac{t}{t_r^*} \qquad (2.25)$$

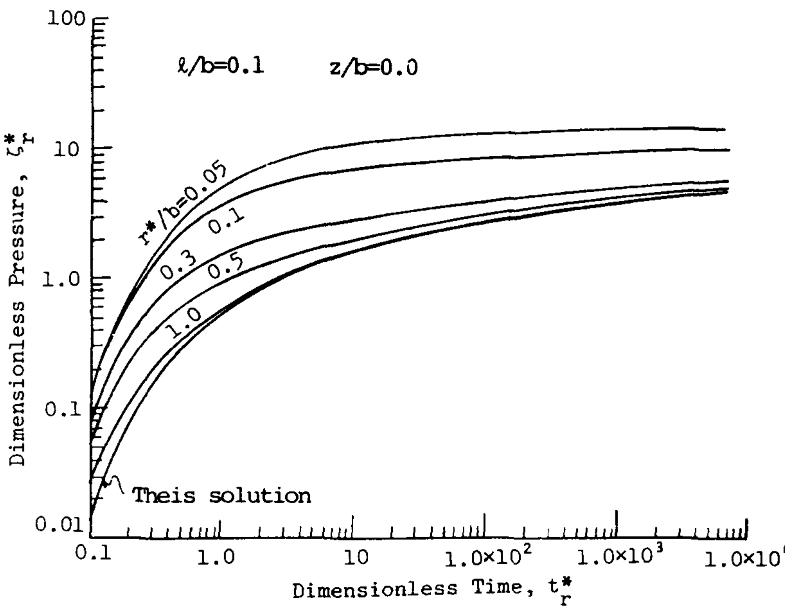

Fig.2.4 ζ_r^{**} versus t** from Eq(2.7) for each r*/b.

2.3.3 Jacob's Method Ajusted for Partial Penetration

During the time period in which the ultimate semilogarthmic straight line forms, the drawdown is given, depending on the well observed, by Eq.(2.16) or Eq.(2.17). Because the second term of these equations are constant with time, it is clear that Jacob's method can be applied if the numerical value of this constant can be obtained. The procedure is as follows:

(1) On the observed semilogarithmic plot, construct the ultimate straight line and extend it to the zero-drawdown axis.

(2) Obtain the slope, ($m=\Delta\zeta$/cycle) of this line and its time intercept, (t/r^2) on the zero-drawdown axis.

(3) the permeability can be calculated from Eq.(2.17)

$$k=\frac{2.3Q}{4\pi bm} \qquad (2.26)$$

(4) Compute $\exp(f_s)$ from Eq.(2.18) interpolating r/b, ℓ/b, z/b as shown in Fig.2.5.

(5) Then calculate the compressibility factor from

$$S_s=[\frac{2.25kt}{r^2}]\cdot\exp(f_s) \qquad (2.27)$$

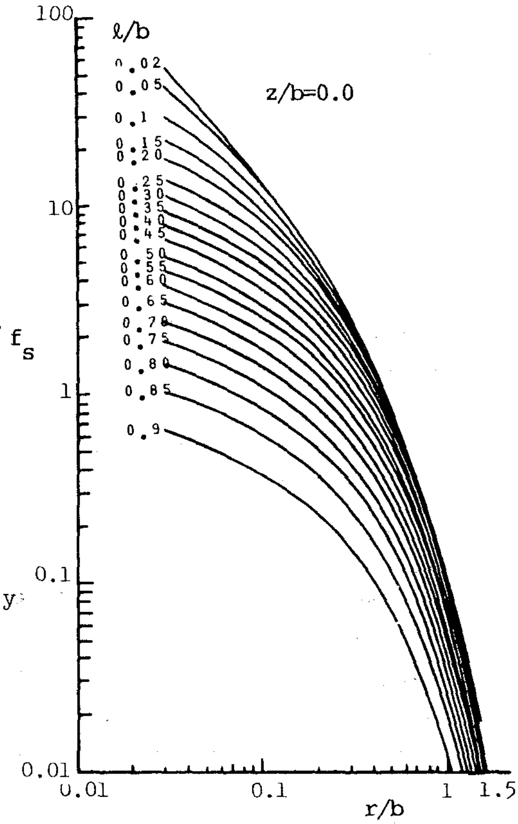

Fig.2.5

The relation of f_s versus r/b .

2.3.4 Trial and Error Method for Unknown Aquifer Thickness

In general, the thickness of aquifer is assumed from the boring logs that have been obtained from boring a pumping well and observation wells. But sometimes one problem that has arisen in connection with the aquifer being pumped is that the total thickness may not be known precisely from the boring logs. The question has therefore been raised how pump test results can be used to determine the total thickness of the aquifer that is responding to the pressure disturbances caused by the

the water removal.

Since there are normally several wells available for observation purposes at the time of the pump test, it is quite likely that one or more of these wells will be located for enough from the pumping well that the distance r will exceed 0.5 times the aquifer thickness **b**. In this event, one should first analyze the drawdown behavior of the distant observation wells where it is reasonably certain that **r>0.5b**. In this case, the Theis' solution can be employed directly because the effects of partial penetration should be nil. Once one has obtained a match between field data and the Theis' curve, the tatal effective permeability- thickness can be calculated from

$$T=kb=\frac{Q}{4\pi}\frac{\zeta^*}{\zeta} \qquad (2.28)$$

On the basis of the core analysis results from wells that have been completed in the aquifer, one should have an approximate idea of the average permeability, and thus the first estimate of b can be determined from the value of **kb** obtained in Eq.(2.28). Appropriate curves of ζ^* versus **t*** can then be prepared for each well where **r<0.5b** since the necessary ratios (ℓ/b,z/b,r/b) can all be calculated. If the observed field data make a satisfactory fit to these curves of dimensionless values, one can again calculate **kb** and compare with the results previously obtained for wells with **r>0.5b**. However,if the field data do not give a good match because they lie above (or below) the theoretical curve, the assumed value of **b** must be reduced (or increased). This process can be repreated on a trial and error basis until a satisfactory match is obtained. In this manner, both **b** and **k** can be betermined.

2.4 Analysis of Pump Test Data

The following discussion gives example calculations for the methods given above,except " Log-Log Distance Drawdown Method". The pump test data are taken from a real aquifer project that is located in Okayama City. The geologic conditions obtained from wells logs is shown in Fig.2.6. The water level of the sand-gravel layer under G.L.-13m is differnt from that of the upper gravel layer; therefore,the sand-gravel layer is revealed a confined aquifer. A test well to check the thickness of the sand-gravel layer was penetrated into the depth of G.L.-30m, thickness could not be ascertained though. The pump test data are taken from a hypothetical case where both pumping and observation wells partially penetrate the aquifer of unknown thickness. The depth of penetration in each observation well is 20m and that of penetration in the pumping well is 25m, as shown in **Fig.2.6**.

The pump test was performed on this project using an average rate of $7\times10^3 cm^3/sec$ for a period of 30 hours. Fluid levels were measured in observation wells.

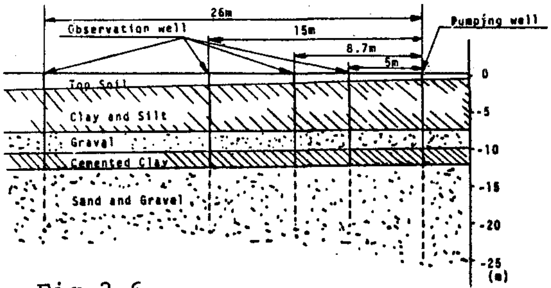

Fig.2.6
The geologic conditions of the field

2.4.1 Log-Log Method and Trial and Error Method

As a first trial in analyzing the drawdown data of this punping test, **b=100m** was assumed, trial curves were compared with the drawdown data. Trial curves did not give a satisfactory fit to the field data, indicating the assumed aquifer thickness of 100m is too high.

A second trial curve of ζ^* versus **t*** was constructed on the assumption that **b=30m**. Trial curves either did not fit the field data.

A third trial of **b=50m** was assumed.The parameter **ℓ/b** is obtained for the pumping well

$$\ell/b=25/50=0.5$$

wells also have penetrations of 20m, then **z/b** is obtained

$$z/b=20/50=0.4$$

and radial distances of each well to obtain

No.1	$r_1/b=5.0/50=0.1$
No.2	$r_2/b=8.7/50=0.17$
No.3	$r_3/b=15/50=0.3$
No.4	$r_4/b=26/50=0.52$

One can interpolate the results to construct the curve of ζ^* versus **t*** for **ℓ/b=0.5,z/b=0.4** and each **r/b** curves are compared to the drawdown data as shown in **Fig.2.7**, they can be matched satisfactorily to the field data.

At the match point where $\zeta^*=3.1, t^*=1.3\times10^3$, one reads $\zeta=1\times10^2 cm$ and $t/r^2=1\times10^{-3} sec/cm^2$. From Eq.(2.20), the permeability can be calculated

$$k=\frac{Q\zeta^*}{2\pi b\zeta}=\frac{7.0\times10^3\times3.1}{(2\times3.14)\cdot(5.0\times10^3)\cdot(1.0\times10^2)}=6.91\times10^{-3} cm/sec$$

From Eq.(2.21), the compressibility factor can be calculated

$$S_s = kt^*\left(\frac{t}{r^2}\right) = (6.91\times10^{-3})\times(1\times10^{-3})/(1.3\times10^{3}) = 5.32\times10^{-9}\text{cm}^{-1}$$

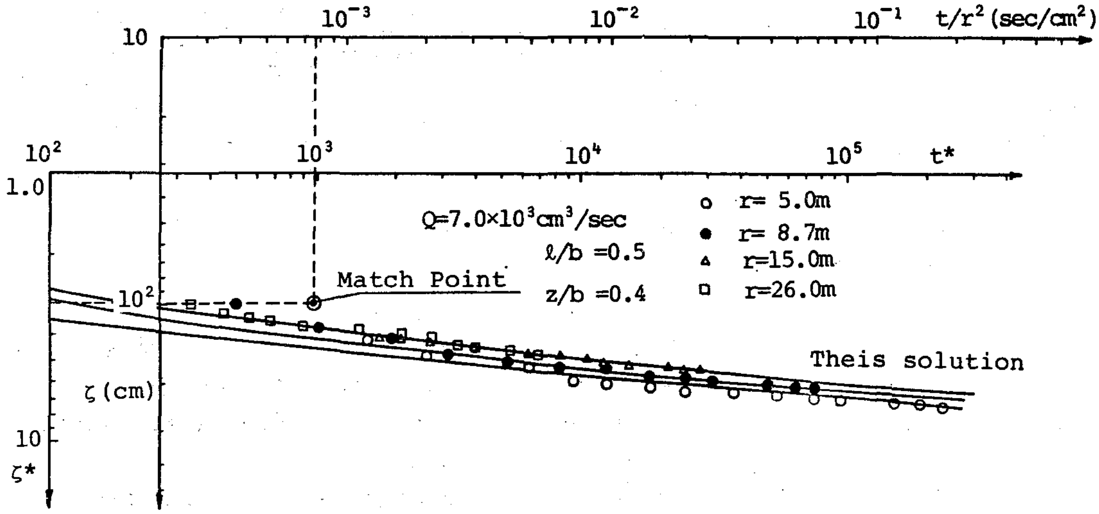

Fig.2.7 Analysis of drawdown test data for partially penetrating wells in the confined aquifer.

2.4.2 Jacob's Method Ajusted for Partial Penetration

The same data are analysised by the Jacob's Method Ajusted for Partial Penetration, the values of the slope for each observation well are nearly same as shown in Fig.2.8, therefore the permeability can be obtained from the slope of straight lines and Eq.(2.26)

$$k=\frac{2.30Q}{4\pi bm}=\frac{2.30\times7.0\times10^3}{4\times3.14\times5.0\times10^3\times40}=6.41\times10^{-3}\text{cm/sec}$$

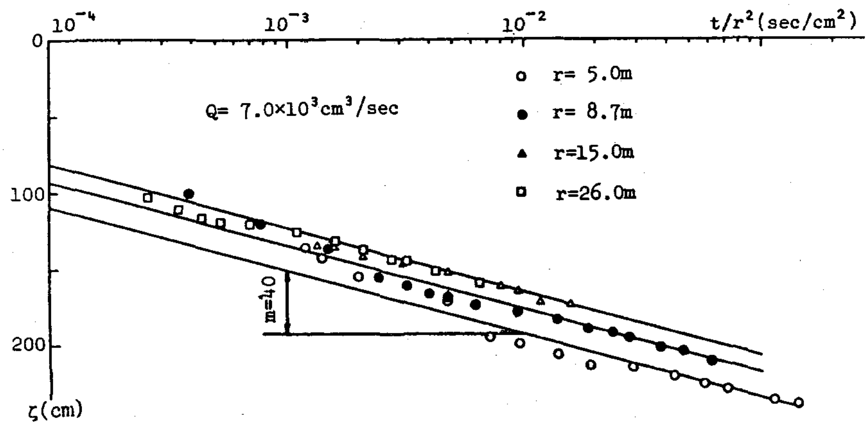

Fig. 2.8 Semi-log plot of drawdown data for partially penetrating wells in the confined aquifer (for Q=7.0×10³cm³/sec).

The compressibility factor can be calculated from the time(t/r^2) intercept on the zero-drawdown axis, but for the effect of partial penetration, different values of t/r^2 are obtained from respective data of the observation well, as indicated in **Table 2.1**.

Table 2.1 (for $Q=7.0\times10^3 cm^3/sec$)

r (m)	5.0	8.7	15.0	26.0
r/b	0.10	0.17	0.30	0.52
ℓ/b	0.5	0.5	0.5	0.5
z/b	0.4	0.4	0.4	0.4
f_s	1.486	0.833	0.392	0.147
t/r^2	2.3×10^{-7}	6.3×10^{-7}	1.5×10^{-6}	1.5×10^{-6}
S_s (cm^{-1})	1.47×10^{-8}	2.09×10^{-8}	3.20×10^{-8}	2.51×10^{-8}

Interpolating the parameter ℓ/b,z/b,r/b in Eq.(2.18), the values f_s(r/b,ℓ/b,z/b) are obtained for each observation well by numerical calculation. Using Eq.(2.27), the compressibility factor can be calculated as indicated in **Table 2.1** and its average value is $2.32\times10^{-8}cm^{-1}$.

In the same field the pump test was performed using an average rate of $Q=4.33\times10^3 cm^3/sec$ for a period of 30 hours.In this case, the permeability can be obtained from the slope of straight lines in **Fig.2.9**.

$$k=\frac{2.30Q}{4\pi bm}=\frac{2.30\times 4.33\times10^3}{4\times3.14\times5.0\times10^3\times33}=4.81\times10^{-3}cm/sec$$

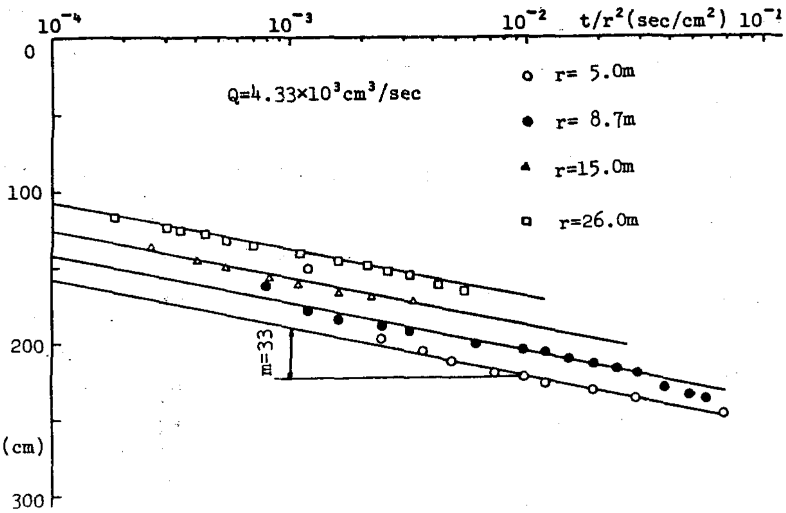

Fig.2.9 Semi-log plot of drawdown data for partially penetrating wells in the confined aquifer (for $Q=4.3\times10^3cm^3/sec$).

The compressibility factor can be calculated as indicated in Table 2.2, its average value is $1.97\times10^{-10}cm^{-1}$. This value is vary small compared to the value of the pump test using an average rate of 7.0×10^{3} cm^{3}/sec. This discrepancy is discussed in paragraph 2.5.

Table 2.2(for $Q=4.333\times10^{3}cm^{3}$ /sec)

r(m)	5.0	8.7	15.0	26.0
r/b	0.10	0.17	0.30	0.52
ℓ/b	0.5	0.5	0.5	0.5
z/b	0.4	0.4	0.4	0.4
f_s	1.486	0.833	0.392	0.147
t/r^2	2.0×10^{-9}	5.0×10^{-9}	1.2×10^{-8}	3.0×10^{-8}
S_s (cm^{-1})	9.56×10^{-11}	$1.24\ 10^{-10}$	$1.92\ 10^{-10}$	$3.76\ 10^{-10}$

2.5 Discussion of Analysis of Pump Test Data

In section 2.3 some methods of Analyzing Field Data are given and in section 2.4 the example calculations for the methods are shown. Comparing these methods with Theis' and Jacob's methods, which can evluate an anisotropy of permeability and the aquifer thickness,proved that they would be more effective than Theis' and Jacob's methods.

Using Jacob's method, the various values of compressibility factor for each observation well are obitained. Therefore the constant compressibility factor can not be calculated.Sometimes, Jacob's method results a big variation for a compressibility factor of second to third power in difference.

In section 2.4, different compressbilityfactors are obtained for each average pumping rate,that is, $S_s=2.32\times10^{-8}cm^{-1}$, $S_s=1.97\times10^{-10}cm^{-1}$ are are obtained from $Q=7.0\times10^{3}cm^{3}/sec$, $Q=4.33\times10^{3}cm^{3}/sec$ respectively. This discrepancy is considered as follow.

General behavior between of the effective stress(σ')-volumetric strain(ε_v) is shown in Fig.2.10. From Fig.2.10 it is evident that the comprssion factor is small as the average pumping rate is small.

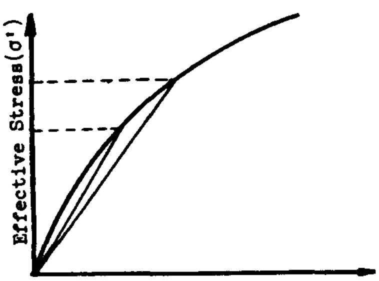

Fig.2.10 The behavior of $\sigma'-\varepsilon_v$

A pumping test aims at finding coefficients of an aquifer befor a ground excavation. In the actual excavation the drainage rate of water is larger than the pumping test. Thus, the compression factor must be assumed a larger value than that obitained from pumping test.

For example consider the confined aquifer as shown in **Fig. 2.11.** In this case the drawdown for unsteady state is obtained as follows;[4]

$$\frac{\zeta}{\zeta_o}=\mathrm{erfc}(\xi) \qquad (2.29)$$

where

$$\xi=x/2\sqrt{k/S_s t} \qquad (2.30)$$

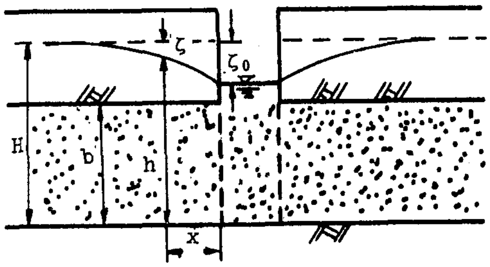

Fig.2.11

Excavation trench in a confined aquifer.

and ζ is drawdown, ζ_o is drawdown at the face of excavation, and **erfc(ξ)** is the complementary function of the error function i.e

$$\mathrm{erfc}(\xi)=1-\mathrm{erf}(\xi) \quad (2.31)$$

The numerical result of Eq. (2.29) is shown in **Fig.2.12.** If a large S_s is given in Eq.(2.30) the value of ξ becomes a small value, then from **Fig.2.12** the drawdown ζ at the distance **x** from the excavation face becomes small.

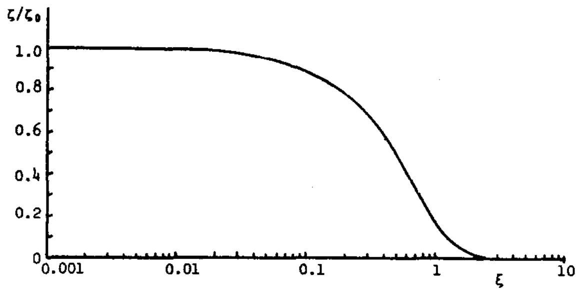

Fig.2.12

The numerical solution of erfc(ξ)

For this reason if the value S_s obtained from small pumping rate is used in the analysis of the ground excavation, the larger result of the analysis must be obtained comparing with the actual drawdown.

In the actual excavation analysis, sometimes this discrepancy has happened. For this cause in traditional notion,it has been cosidered that the permeability coefficient obtained from pumping test is larger than that of the aquifer. Therefore the compressibility factor has not been watched. However, from the above evaluation, it becomes clear that the drawdown in the actual excavation is smaller than that of the analysis.

On the other hand, it can be considered that the permeability coefficient is independent of the pumping rate from the analysis in section 2.4.

3. Analytic Solution for Partially Penetrating Well in an Unconfined Aquifer

The subject of this investigation is the flow toward a single well, partially penetrating an unconfined aquifer that in infinite in lateral extent. The saturated region is unconfined, possesing a free surface that is initially horizontal. The velocity distribution in time and space within the porous aquifer is obtained for various pizometric head functions in the well. Then it is possible to relate quantities, such as free surface drawdown, well discharge flow rate,and piezometric head with aquifer parameters and answer some pertinent questions regarding the behavior and characteristics of this physical system.

First, certain physical assumptions about the problem need to be made. Each one restricts in some way the applicability of the final solution to the real physical problem and determines the nature of the mathematical model of the situation.

(1) Flow within the porous medium obeys Darcy's Law.

(2) The water is assumed to be incompressible and the porous matrix rigid. This assumption is suitable for unconfined aquifers where storage yield corresponds to a lowering of the free surface with essentially no compression of the porous matrix or volumetric expansion of water.

(3) Only single phase (or saturated) flow occurs in the aquifer.

(4) Capillarity is neglected at the free surface.

(5) The porous medium has homogeneous, constant, anisotropic permeability.

(6) The effective porosity or specific yield of the aquifer is assumed to be uniform and constant.

(7) The well is assumed to have no surface of seepage.

(8) Head losses through the well screen are neglected.

3.1 Basic Equation and Solution

The physical situation is one of three-dimensional flow with axial symmety as shown in Fig.3.1. Thus cylindrical coordinates are the natural selection. The origin is taken to be on the well axis at the level of the horizontal free surface at time zero.

・Darcy's Law gives

$$V_r=-k_r\frac{\partial h}{\partial r}\ ,\quad V_z=-k_z\frac{\partial h}{\partial z}$$

Applying the equation of cotinuity for incompressible flow yields Laplace's equation for the potential,

$$k_r\frac{\partial^2 h}{\partial r^2} + k_r\frac{1}{r}\frac{\partial h}{\partial r} + k_z\frac{\partial^2 h}{\partial z^2} = 0 \qquad (3.1)$$

at all points of the saturated aquifer.

The boundary conditions for this problem as follows:
(1) The free surface is a boundary whose location in space and time is unknown before the problem is solved. Therefor, let $z=\zeta(r,t)$ designate the free surface. Since atomospheric pressure over the free surface is taken to be zero, the defining equation for h becomes

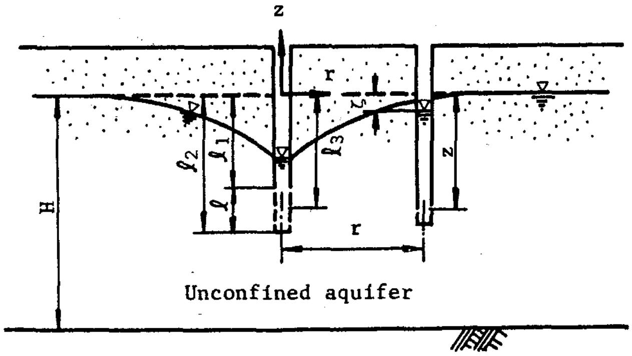

Fig.3.1
Partially penetrating wells in an confined aquifer.

$$h(r,z,t)_{z=\zeta}=\zeta \qquad (\text{at } z=\zeta) \qquad (3.2)$$

The kinematic boundary condition comes from the fact that particle initially on the free surface remains on the free surface as the surface moves. Mathematically, this means that the derivative following the motion of the equation defining the free surfsce must equal zero. Thus,the non-linear, kinematic, free-surface boundary condition,

$$S_y\frac{\partial h}{\partial t} + k_z\frac{\partial h}{\partial z} + k_r\left(\frac{\partial h}{\partial r}\right)^2 + k_z\left(\frac{\partial h}{\partial z}\right)^2 = 0 \qquad (\text{at } z=\zeta) \qquad (3.3)$$

where S_y is the effective porosity or specific yield.
(2) On the no flow across lower boundary,

$$\frac{\partial h}{\partial z} = 0 \qquad (3.4)$$

(3) A log the well $(-\ell_2<z<-\ell_1)$,

$$\lim_{r\to 0} 2\pi k_r r \int_{-\ell_2}^{-\ell_1} \frac{\partial h}{\partial r}dz = -Q \qquad (3.5)$$

Initially the free surface is horizontal. Thus,

$$h(r,t)=H \qquad (\text{at } t=0) \qquad (3.6)$$

$$\zeta^{**} = \frac{1}{\pi \ell^*}\Big[\frac{1}{4}\log\frac{\ell_\xi^* + \ell^*/2 + z^* + \{(\ell_\xi^* + \ell^*/2 + z^*)^2 + r^{*2}\}^{1/2}}{\ell_\xi^* - \ell^*/2 + z^* + \{(\ell_\xi^* - \ell^*/2 + z^*)^2 + r^{*2}\}^{1/2}} \cdot \frac{\ell_\xi^* + \ell^*/2 - z^* + \{(\ell_\xi^* + \ell^*/2 - z^*)^2 + r^{*2}\}^{1/2}}{\ell_\xi^* - \ell^*/2 - z^* + \{(\ell_\xi^* - \ell^*/2 - z^*)^2 + r^{*2}\}^{1/2}}$$
$$-\int_0^\infty \frac{\cosh\lambda(1+z^*)\cdot\sinh\lambda(\ell^*/2)\cdot\cosh\lambda(1-\ell_\xi^*)}{\lambda\sinh\lambda\cdot\cosh\lambda}\exp(-\lambda t^{**}\tanh\lambda)\cdot J_0(\lambda r^*)d\lambda$$
$$+\int_0^\infty \frac{\sinh\lambda(\ell^*/2)\cdot\cosh\lambda\ell_\xi^*\cdot\cosh\lambda z^*}{\lambda\sinh\lambda}\cdot\exp(-\lambda)\cdot J_0(\lambda r^*)d\lambda\Big] \tag{3.17}$$

where

$$\zeta^{**} = \zeta^*/Q^* = \zeta k_r H/Q \tag{3.18}$$

$$t^{**} = \varepsilon t^* = k_z t/S_y H \tag{3.19}$$

Substituting $k_r = k_z = k$ in Eq.(3.17), the solution for an isotropic aquifer is obtained as the same form of Eq.(3.17).

3.2 The Effects of Partial Penetration

Considering the shallow the penetration of a pumping is the more superior the effect of partial penetraton becomes, the effect of partial penetration on the drawdown around a pumping well for $\ell/H=0.2$ is shown in **Fig.3.2.** The variations are around a well in an isotropic aquifer ($k=k_r=k_z$). If the observation well is relatively large distances($r/b>1.2$) the time-drawdown is give by the Theis'formula. In other words, the drawdown in such well is not affected by partial penetration; it is the same as though the pumped well completly penetrated the aquifer. The same result is obtained from **Fig.3.3** in the case of $\ell/H=0.4$.

In **Figs.3.4a,3.4b**, the effect of pumping well penetration is shown. It is appear that for the observation well setted in relatively small distances($r/h=0.3$), the effects of partial penetration is striking. On the other hand,for the observation well setted at $r/b=0.6$ its effect is not so striking.

This is mixed boundary-value problem with a non-linear boundary condition at the free surface. Note that time appears only in the boundary conditions and not in the partial differential equation.

It is now appropriate to introduce dimensionless variables

$$h^*=h/H,\quad z^*=z/H,\quad \zeta^*=\zeta/H,\quad t^*=t/t_0,\quad Q^*=Q/(k_r H^2) \tag{3.7}$$

$$\ell^*=\ell/H$$

and

$$r^*=\frac{r}{H}\sqrt{k_z/k_r} \tag{3.8}$$

t_0 is the time scale factor.

Writing the system equations in dimensionless from gives the following set:

Differential Equation

$$\frac{\partial^2 h^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial h^*}{\partial r^*} + \frac{\partial^2 h^*}{\partial z^{*2}} = 0 \tag{3.9}$$

Boundary Conditions

$$\frac{\partial h^*}{\partial t^*} + \varepsilon\frac{\partial h^*}{\partial z^*} + \varepsilon\left(\frac{\partial h^*}{\partial r^*}\right)^2 + \varepsilon\left(\frac{\partial h^*}{\partial z^*}\right)^2 = 0 \qquad (\text{at } z^*=\zeta^*) \tag{3.10}$$

$$\zeta^*=h^*(r^*,z^*,t^*)\quad z^*=\zeta^* \tag{3.11}$$

$$\frac{\partial h^*}{\partial z^*}=0 \tag{3.12}$$

$$\lim_{r^*\to 0} r^*\frac{\partial h^*}{\partial r^*}=\frac{Q^*}{2\pi\ell^*} \tag{3.13}$$

$$h^*=1 \qquad r^*\to\infty \tag{3.14}$$

Initial Condition

$$h^*=1 \quad (\text{at } t^*=0) \tag{3.15}$$

The parameter ε is defined as

$$\varepsilon=\frac{t_0 k_z}{S_y H} \tag{3.16}$$

When ε is small, perturbation expansion techniques may be used to linearize the problem. The dimensionless drawdown ζ^{**} is solved[5,6]

3.3 Methods of Analyzing Field Data

In evaluating the results of a pumping test where partial penetration must be considered,i.e. where $r/b<1.2$, one needs to know the geological conditions of the aquifer under investigation.

In the field of hydrology, an basic methods of analyzing field data have been developed; Log-Log Method for an isotropic aquifer and Log-Log Distance Drawdown Method for an anisotropic aquifer.

Both of the above methods require data that are measured in observation wells at some distances from the pumping well.

3.3.1 Log-Log Method

In the log-log method, one can use graphical method similar to Theis' Method. Knowing the values of ℓ/H, ℓ_3/H and z/H, one can prepeare a graph of $\log\zeta^{**}$ versus $\log t^{**}$ for the appropriate r/H between pumping and observatiom wells from Eq.(3.17). As is evident from **Fig.3.5** separate curves will have to be prepared for each observation well, unless the values of the three ratios (ℓ/H, z/H, r/H) are identical.

When the drawdown data from each observation well have been plotted on log-log paper with the same dimensions per cycle as used above,

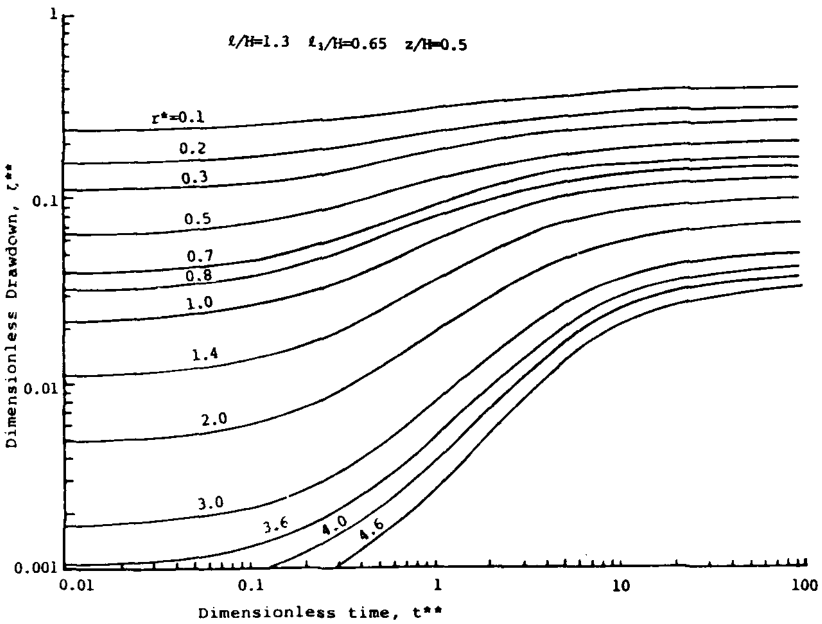

Fig.3.5 Relation of log ζ^{**} versus log t^{**} from Eq.(3.17)

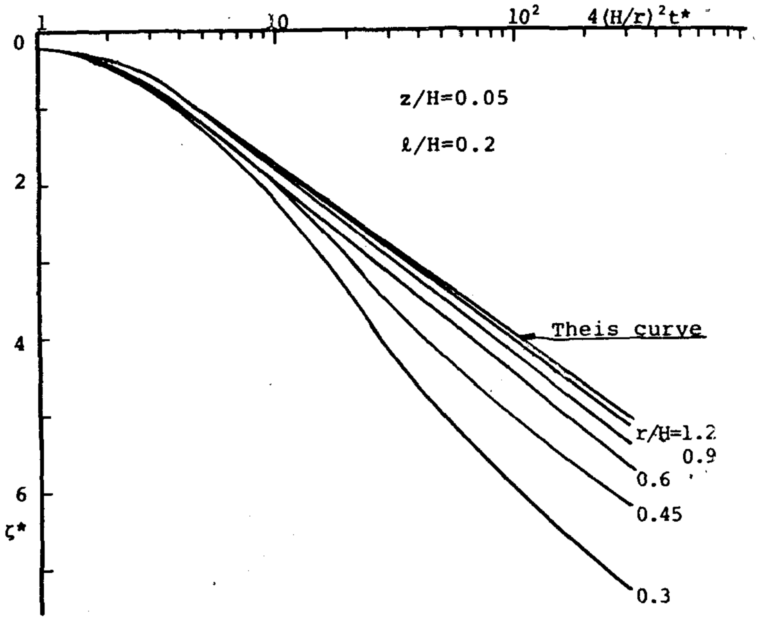

Fig. 3.2
Drawdown characteristics for partially penetrating well in an confined aquifer

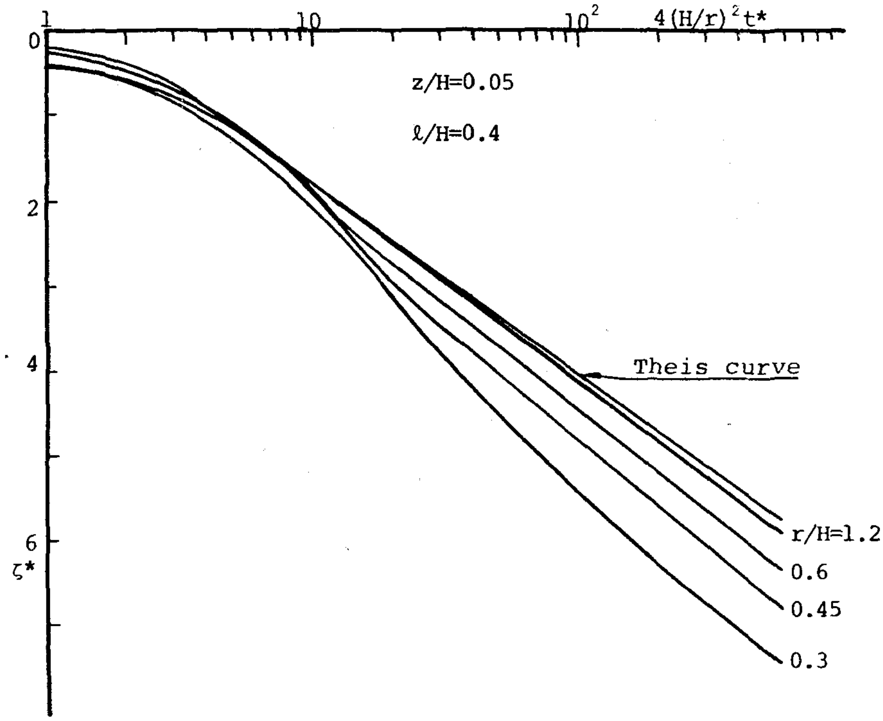

Fig. 3.3

Drawdown characteristics for partially penetrating well in an confined aquifer

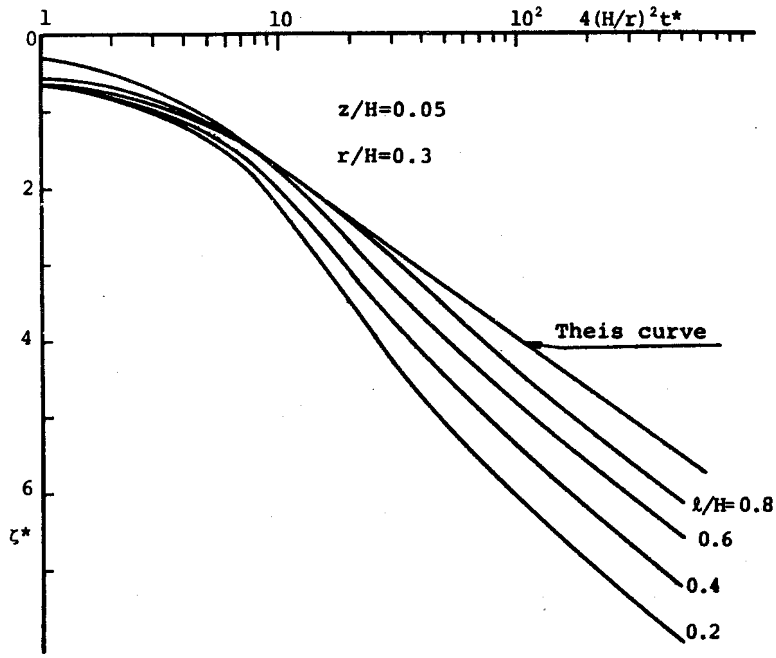

Fig. 3.4a

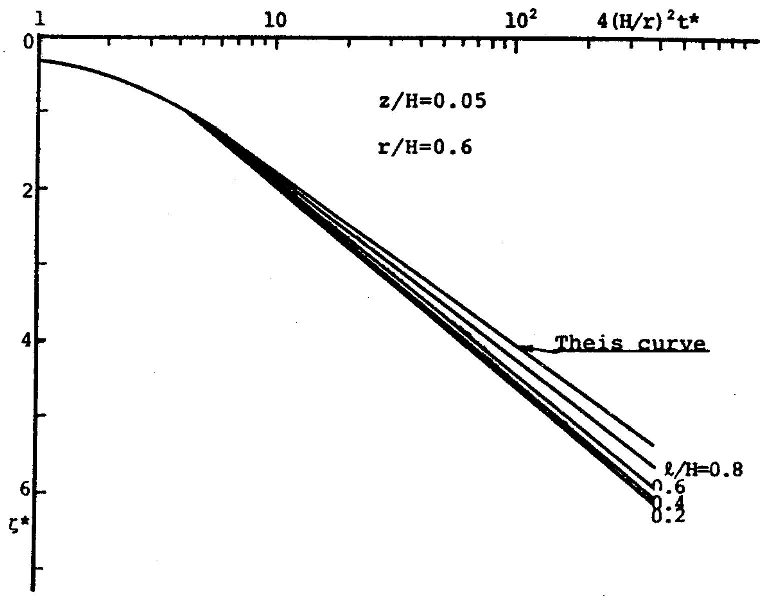

Fig. 3.4b

Drawdown characteristics for partially penetrating well in an confined aquifer one can match the field results to the theoretical curve in the same manner as is done when using the Theis' curve and chapter 2.

When the curves are matched, one can read the dimensionless parameters that correspond to each point of field data. It will be found that one can also choose any point of the curve of field data and still obtain the same result by using the appropriate values of ζ^{**} and t^{**} for that particular point.

An equivalent value of ζ^{**} can be determined for any ζ, measured in the observation well and an equivalent value of t^{**}, for the corresponding value of real time, t. The permeability can be calculated from Eq.(3.18)

$$k=\zeta^{**}Q/\zeta H \tag{3.20}$$

and the effective porosity from Eq.(3.19)

$$S_y=kt/t^{**}H \tag{3.21}$$

3.3.2 Log-Log Distance Drawdown Method

" Log-Log Distance Drawdown Method " is a variation of " Log-Log Method" .

The charcteristics of the aqufer k_r, k_z and S_y may be obtained by matching the measured drawdowns and the theoretical curve from Eq.(3.17).

For this purpose the values of ℓ^*, ℓ_ξ^* (pumpimg well) and z^* (observation well) have to be inserted in Eq.(3.17); ζ^{**} becomes a function of t^{**} and r^*. A set of curves $\zeta^{**}-t^{**}$ for different constant r^* are to be drawn on a log-log paper. Since most anisotropic aquifers have $k_z/k_r<1$, r^* has to be smaller than r/H.

The measured drawdowns have to be repreasented on a similar logarithmic paper. By matching them with one of the curves of the set, five values are obtained. r^* is obtained from the best fitting curve, an equivalent value of ζ^{**} can be ditermined for any ζ, measured in the observation well and an equivalent value of t^{**}, for the corresponding value of real time t. Since r, H and Q are known, the values of k_r, k_z and S_y may be easily found from next equations.

$$k_r=\zeta^{**}Q/\zeta H \tag{3.22}$$

$$k_z=(r^*H/r)^2 k_r \tag{3.23}$$

$$S_y=tk_z/t^{**}H \tag{3.24}$$

3.4 Analysis of Pump Test Data

The following discussing gives example calculations for the methods discussed above. The pump test data are taken from real aquifers project that is located in Kyoto City.

3.4.1 Log-Log Method

The geologic conditions obtained from wells logs is shown in **Fig.3.6.** The water level in the sandy layer is G.L.-11m and the aquifer of sandy layer is revealed an unconfined aqufer. A test well to check the thickness of the sandy layer was penetrated into the depth of G.L. -30m, however, the thickness could not be ascertained. The pump test data are taken from a hypothetical case where both pumping and observation wells partially penetrate the aquifer of unknown thickness. The depth of penetration in each observstion well is 5m and that of penetration in the pumping well is **13m**, as shown in **Fig.3.6.**

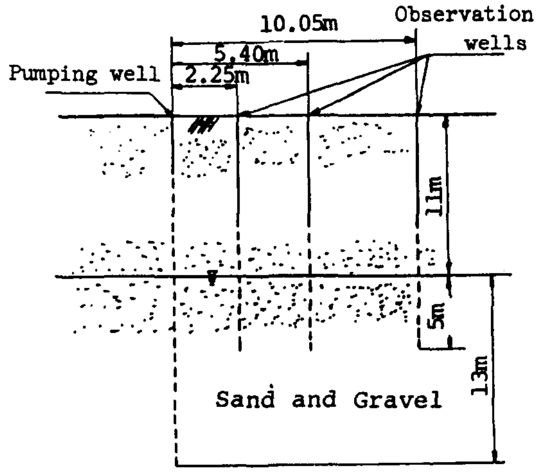

Fig.3.6
The geologic condition of the field.

The pump test was performed on this project using an average rate of $7.0\times10^3 cm^3/sec$ for a period of 5 hours.

As a first trial in analyzing the drawdown data of this pumping test, **H=50m** was assumed, trial curves were compared with the drawdown data. Trial curves did not give a good match.

A second trial curves of ς^{**} versus t^{**} was constructed on the assumption that H=10m. Trial curves either did not fit the field data.

A third trial of H=20m was assumed, the parameter ℓ^*, ℓ_3^* are obtained for the pumping well

$$\ell^* = \ell/H = 13/20 = 0.65$$
$$\ell_3^* = \ell_3/H = 6.5/20 = 0.325$$

Observation wells also have penetration of 5m,

$$z^* = z/H = 5/20 = 0.25$$

and radial distances of each well to obtain

No.1 $r_1^*= 2.25/20 =0.113$

No.2 $r_2^*= 5.40/20 =0.270$

No.3 $r_3^*=10.05/20 =0.50$

One can interpolate the results to obtain the curve of ζ^{**} versus t^{**} for ℓ^*, ℓ_3^*, z^* and each r_i^*. Curves are compared to the drawdown data as shown in Fig.3.7, they can be mached satisfactorily to the field data.

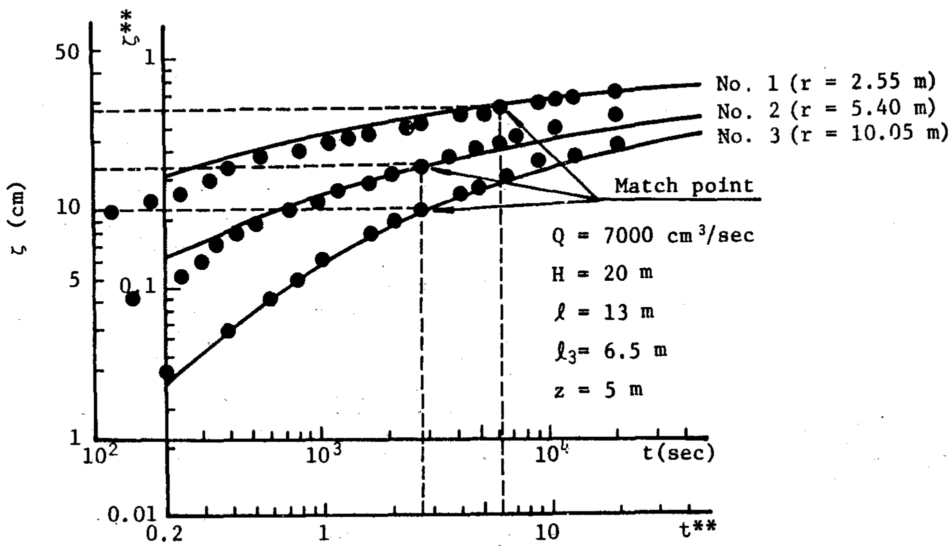

Fig.3.7
Analysis of drawdown test data for partially penetrating wells in the unconfined aquifer(isotropic)

Using the match point and Eqs.(3.20),(3.21), the permeability and the effective porosity can be calculated as indicated in Table 3.1.

Table 3.1
Analysis of drawdown test data for partially penetrating wells in the unconfined aquifer(isotropic)

r (m)	2.55	5.40	10.05
ζ (cm)	27.5	15.5	10.0
t (sec)	6.2×10^3	2.75×10^3	2.75×10^3
ζ^{**}	0.60	0.34	0.22
t^{**}	6.00	2.65	2.65
k (cm/sec)	7.64×10^{-2}	7.68×10^{-2}	7.63×10^{-2}
S_y	3.95×10^{-2}	3.99×10^{-2}	3.96×10^{-2}

By the way, the same pump test data was analyzed by Jaccb's method as shown in **Fig.3.8**. The permeability obtained from the slop of straight lines is as follows:

$$k=\frac{2.30\times7.0\times10^3}{4\times3.14\times2.0\times10^3\times13.5}=4.75\times10^{-2}\ \text{cm/sec}$$

But the various effective porosity for each observation well are obtained as indicatec in **Table 3.2**. The major cause for this discrepancy must be the effect of partial penetration.

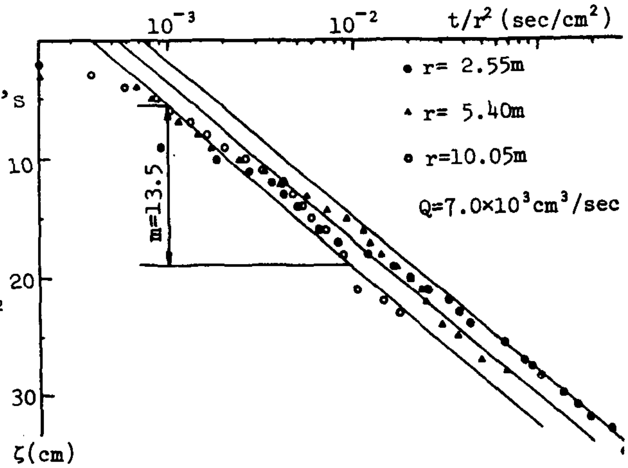

Fig.3.8 Semi-log plot of drawdown data for partially penetrating wells in the unconfined aquifer.

Table 3.2
Effective porosity from Jacob's Method

r(m)	2.55	5.40	10.05
t/r^2	3.9×10^{-4}	5.4×10^{-4}	7.2×10^{-4}
S_y	8.3×10^{-2}	1.2×10^{-1}	1.5×10^{-1}

3.4.2 Log-Log Distamce Drawdown Method

The geological conditions obtained from wells logs is shown in **Fig.3.9**. The water level in the sandy layer is G.L.-11.47m and the aquifer of sandy layer is revealed an unconfined aquifer. A test well to check the thickness of the sandy was penepreted into the depth of G.L.-40.4m. From the result of well logs, the sand layer thickness is revealed H=23.2m. This assumption is based on that the clay layer that is regarded as an impermiable layer lies at G.L.-34.67m.

The depth of penetration in each observation well is 4.23m and that of penetration in the pumping well is 6.93m, as shown in **Fig.3.9**.

The pumping test was performed on this project using an average rate

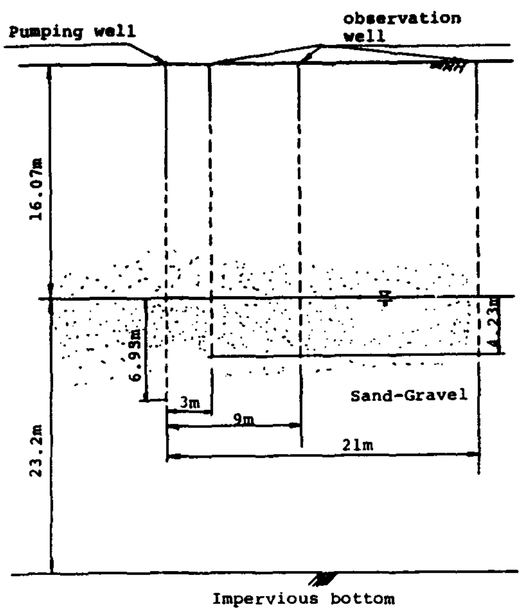

Fig.3.9
The geological conditions of the field.

of $Q=3.23\times10^3 cm^3/sec$ for a period of 24 hours.

The parameters ℓ^*, ℓ_3^* are obtained

$$\ell^* = \ell/H = 6.93/23.2 = 0.299$$

$$\ell_3^* = \ell_3/H = 3.46/23.2 = 0.149$$

Observation wells also have penetration of 4.23m

$$z^* = z/H = 4.23/23.2 = 0.182$$

One can interpolate these parameters in Eq.(3.17), curves relating ζ^{**} $(=\zeta k_r H/Q)$ versus t^{**} $(=tk_z/S_y H)$ at different value of r^* $(=(r/H)\sqrt{k_z/k_r})$ have been computed numerically on a logarithmic paper as shown in **Fig. 3.10**. The drawdown ζ as function of **t** have drawn on similar logarithmic paper and the measured points and theoretical curves have been match in **Fig.3.10**. The values of $r^*, \zeta^{**}, t^{**}, r, \zeta$ and **t** at the matching points are presented in **Table 3.3**. The results show an average anisotropy of $k_z/k_r = 0.32$ and $k_r = 1.92\times10^{-1}$cm/sec, $k_z = 6.04\times10^{-2}$cm/sec.

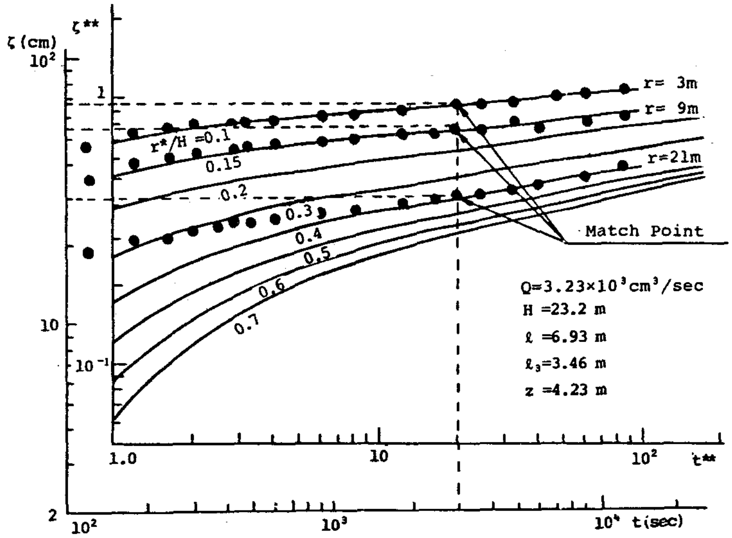

Fig.3.10
Analysis of drawdown test data for partially penetrating wells in the unconfined aquifer(anisotropic)

Table 3.3
Analysis of drawdown test data for partially penetrating wells in the unconfined aquifer(anisotropic)

r (cm)	300	900	2100
r*	0.10	0.15	0.40
ζ**	0.91	0.75	0.42
t**	20	20	20
ζ (cm)	66	54	30.5
t (sec)	2.90×10^{3}	2.90×10^{3}	2.90×10^{3}
k_r (cm/sec)	1.92×10^{-1}	1.93×10^{-1}	1.92×10^{-1}
k_z (cm/sec)	1.15×10^{-1}	2.89×10^{-2}	3.74×10^{-2}
S_y	7.18×10^{-3}	1.81×10^{-3}	2.34×10^{-3}

Same data are analysised by the Jacob's method, the values of the slops for the each observstion well are nearly same as shown in **Fig.3.11**, therefore the permeability can be obtained from the slope of the straight lines;

$$k=\frac{2.30Q}{4\pi H\ m'}=\frac{2.30\times3.23\times10^{3}}{4\times3.14\times2.32\times10^{3}\times\ 12}=2.12\times10^{-1}\text{cm/sec}$$

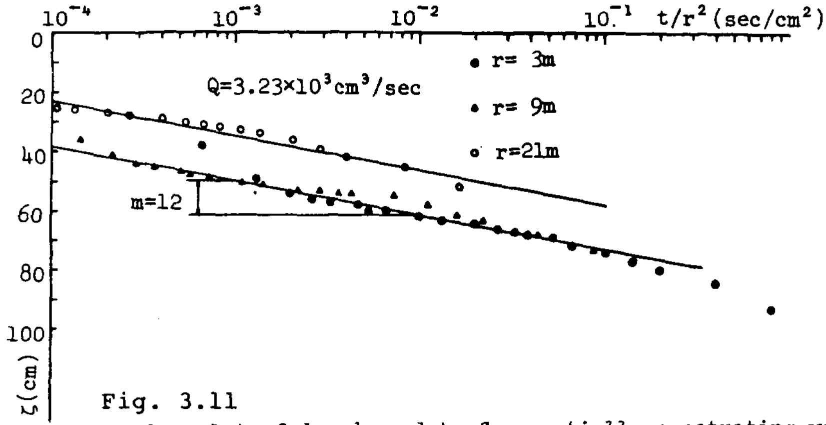

Fig. 3.11
Semi-log plot of drawdown data for partially penetrating wells in the unconfined aquifer.

The effective porosity can be calculated from the time(t/r^2) intercept on the zero-drawdown axis, but for the effect of partial penetration, different values of t/r^2 are obtained from respective data of the observation well, as indicated in **Table3.4**. This discrepancy is very large.

Table 3.4
Effective porosity from Jacob's Method

r (m)	3	9	21
t/r^2	4.30×10^{-8}	4.30×10^{-8}	1.00×10^{-6}
S_y	4.76×10^{-5}	4.76×10^{-5}	1.11×10^{-3}

3.5 Discussion of Analysis of Pump Test Data

In section 3.3 teo methods of Analyzing Field Data are given and in section 3.4 the example calculations for the methods are shown.

Comparing these methods with Theis' and Jacob's methods, as these methods can also evaluate an anisotropy of the permeability and the aquifer thickness as same as the methos for a confined aquifer shown in section 2.3,2.4, therefore, they would be more effective than Theis' and Jacob's method.

4. Conclusions

Analysises of pump test data for partial penetrating well in confined or unconfined aquifer have been shown to determine the anisotropy of permeability and to evaluate the storage coefficient and the thickness of aquifer.

The conclusions obtained in this paper are as follows;

1) The unsteady analytical solution of phreatic flow to partially penetrating wells in confined aquifer is shown.
2) Using this solution, three methods are provided to evaluated the anisotropy of permeability and the compressibility factor (or specific storage) of confined aquifer.
3) The unsteady analytical solution of phreatic flow to partially penetrating wells in unconfined aquifer is drived.
4) Using this solution, two methods are provided to evaluate the anisotropy of permeability and effective porosity (or storage yield) of unconfined aquifer.
5) The limitations of adapting Theis' and Jacob's methods for partially penetrating well test in confined and uncofined aquifer are given.

Acknowledgement

The autors would like to acknowledge the helpfull advices and suggestions of Professor Koichi Akai of Kyoto University.

References

[1] M.S.Hantush: Drawdown around a Partially Penetrating Wells,Trans.ASCE, 127,1962,pp.268-283.

[2] M.S.Hantush: Aquifer tests on Partially Penetrating Wells,Trans.ASCE, 127,1962,pp.284-308.

[3] I.Javandel: Verification of Analytic Solution for Partial Penetration Wells by Mathematical and Heat Transfer Models,M.S.Thesis,U.C. Berkeley,1965.
[4] H.S.Carslaw and J.C.Jaeger: Condaction of heat in solids,Oxford Univ. Press,1959,p75.
[5] G.Dagan: A Method of Determining Permeability and Effective Porosity of Unconfined, Anisotropic Aquifers, Water Resources Res.,3,4,1967, pp.1059-1071.
[6] K.Akai,et al.: The Unsteady Analysis of drawdown due to pumping in vartical and horizontal well,10th Conf.JSSMFE,1975,pp.619-622. (in Japanese)