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A Logical Problem Decomposing Method for Decision Makers

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Decision Support Systems (DSS) have been taken as hopeful support tools for decision making for more than 20 years. There are a lot of literatures on DSS, but most of them are not so practical as the designers expected. This paper points out the crux of this situation and argues that the research on DSS should pay some more attention to the decision making activities before the model using stage. A method named "Problem Situation Decomposing Graph (PSDG)" is presented in this paper for helping the decision maker(DM) elicit the decision making problems. A PSDG is an acycle AND/OR logical directed graph, and which includes all the factors affecting the problem situation based on the DM's knowledge. The logical nodes and parameter determining methods in PSDG can reflect the DM's decision making style. This paper introduces some basic concepts of PSDG, discusses some of its characteristics, and proposes a logical adjacency matrix for PSDG representation and analysis.

1. INTRODUCTION

Automated support for managerial decision making has been a focus of research efforts since the early 1970s. DSS, originally termed management decision systems by Scott Morton, were defined as "interactive computer-based systems that help decision makers (DMs) use data and models to solve unstructured problems"[1]. While most of the literatures on DSS have been considering a model management system (MMS) that supports the development of decision models and their subsequent use as a curial to the success of DSS [2][3][4][5][6], DSS represent a point of view on the role of the computer in the management decision making process. Decision support implies the use of computer to

a. assist managers in their decision processes in semistructured tasks;

b. support the DM rather than replace him to make managerial judgement;

c. improve the effectiveness of decision making rather than its efficiency.

Thus DSS only play a role of supplementary tools for DMs. In order to improve the effectiveness of DSS, they should not be cut from the real decision problems.

MacCrimmon and Taylor [7] related the problem ill-structured to the solver's unfamiliarity with the relevant states and transformations. Mason and Mitroff [8] defined an unstructured problem as one for which one or more of the sets of actions, values, outcomes or states of nature are unknown. Eilon [9] differentiated structured from unstructured decisions in terms of "the degree of clarity with which the DM conceives his task". And Schwenk [10] reduced ill structuredness to the solver's uncertainty concerning problem states and transformation. So decision support also implies that the first stage in DSS' development be the support for decision analysis and for the DM to define the key decision problems. The DM can best recognize the particular

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aspects that can be improved and the components that have the most overall impact on the effectiveness of decision making with the system analyzer's aid, or with the help of DSS to stimulate his own potentiality.

The Problem Situation Decomposing Graph (PSDG in short) proposed in this paper is in general a computationally explosive process, and as we give the great number of constraints and facts, it can present most of illstructured decision problems. The PSDG method is decompositional, can reflect the fact that real world problems are internally complex. Every problem may involve a set of tasks or demands, and each of these tasks or demands may be structured, structurable or unstructured. After points of ill structuredness have been identified, the methodology matches structurable tasks to relevant techniques, and unstructured problem elements are analyzed to identify promising heuristic strategies, which are provided by MMS in the DSS. The PSDG method is based on Simon's [11] analysis of architectural design, which attributes the lack of a solution strategy to problem complexity, and attempts to reduce intractably large problems into manageable small ones. Most managerial problems are consist of subproblems that can be further decomposed. Subproblems can involve diverse content domains and require various cognitive activities for their solution. Available solution techniques tend to be focused, responsive to a relatively narrow set of demands." A structuring methodology must provide a means of decomposing complex real world problems into subproblems that match up more directly to solution techniques" [12], and the PSDG method proposed in this paper is such a methodology, and it is a highly creative method that depends on the DM's experience and insights. It is also dynamic since it must taken into consideration the constantly changing decision environment. Automating the structuring process of a PSDG is a challenging task that explores the limits of current AI technology.

The next section illustrates some basic concepts of PSDG; the third section discusses some characteristics of PSDG; the fourth section presents a so called "Logical Adjacency Matrix" for PSDG representation and analysis; and the last section gives a brief conclusion.

2. SOME BASIC CONCEPTS OF PSDG

DEFINITION 1 *A acyclic directed graph P: $A \times N \times L$, in which A means the set of arcs, N means the set of nodes and $L=\{AND, OR\}$, is called a PSDG of a problem P, if this graph satisfies such conditions as:* ① *$n_i \in N$ is a node related to P, and arc $a_{jk} \in A$ from node n_j to n_k shows the causaulity between nodes;* ② *for $\forall\, n_i$, $\exists$ subset $N_i \subset N$, and $\exists$ arc a_{iq_l}, for all $n_{q_l} \in N_i$, $(l=1, 2, \cdots, m)$ then N_i is the subproblem of n_i in P;* ③ *for $\forall\, n_i$, existing subset N_i, the relationship among arcs a_{iq_l}, for $n_{q_l} \in N_i$ is shown by AND ($\wedge$) or OR ($\vee$) $\in L$. The relationship shows as follows using the nodes: $n_i \models f_L(n_{q_1}, n_{q_2}, \cdots, n_{q_m})$, and there are only three kinds of operation: "( )", " $\wedge$ " and " $\vee$ " in the function $f_L(n_{q_1}, n_{q_2}, \cdots, n_{q_m})$* (see Figure 1).

DEFINITION 2 *A node n_i in PSDG P is called* ① *an OR−node, if there is only one arc directing to it or all of the arcs directing to it have an OR relationship;* ② *an AND−node, if all of the arcs directing to it have an AND relationship;* ③ *an AND/OR−node, if the relationship of the arcs directing to it do not satisfied* ① *and* ② .

For an example, node n_i has a subproblem, that is, $N_i=\{n_{q_1}, n_{q_2}, n_{q_3}\}$. The relationship can be shown as one in Figure 1.

An example of PSDG is shown in Figure 2, in which the node PROFITS means the problem of profit shortfall in a company, the node PRODUCTION−COST means the problem of high production cost, etc.. We will use n_1 to stand for the node PROFITS, and n_2 to stand for the node PRODUCTION−COST, etc. in this paper (as shown in Figure 4).

There are some other basic concepts about a PSDG:

According to the constitution of PSDG, we are assumed that PSDG has only one sink node n_1, which is the main problem situation itself.

DEFINITION 3 A node in a PSDG is called a sink node if there is no arc leaving from it.

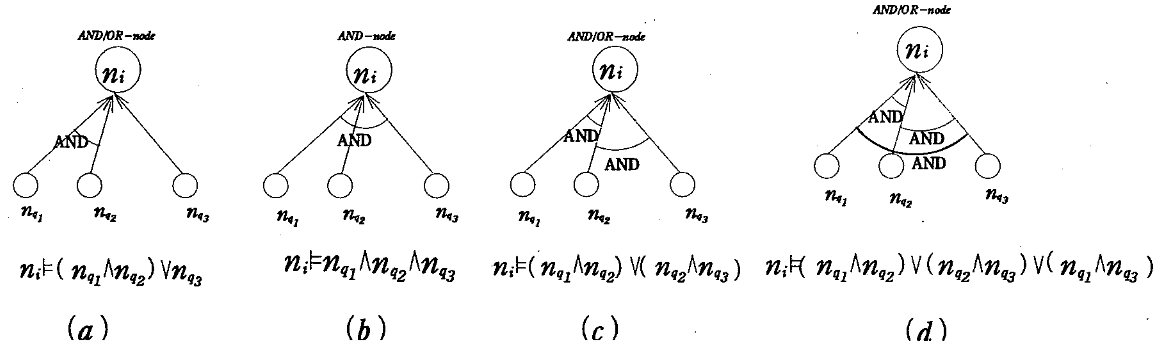


Figure 1 The relationship among arcs of node n_i

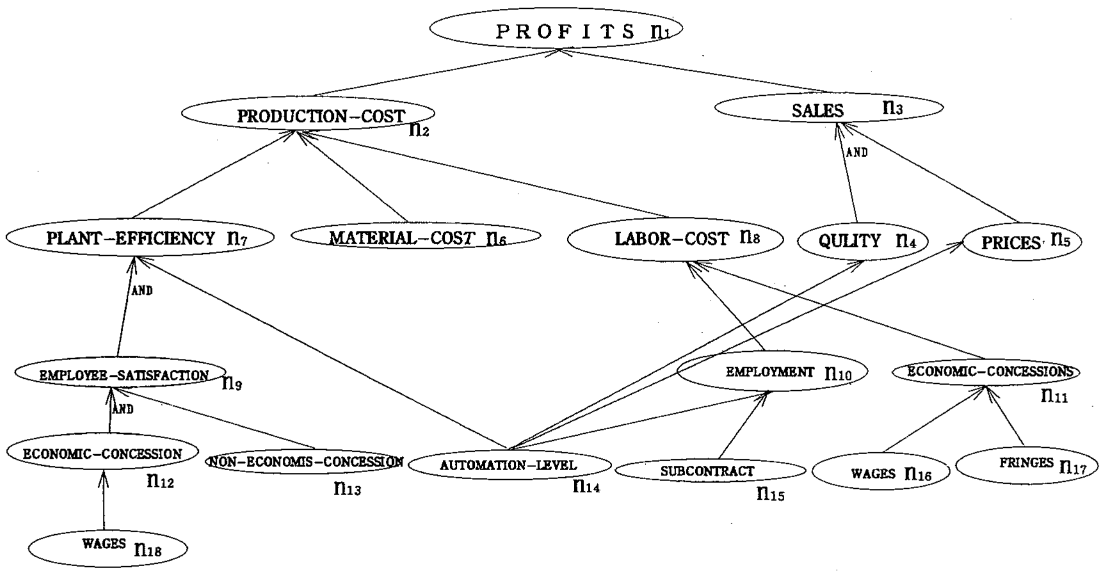


Figure 2 A DM's PSDG

DEFINITION 4 A node in a PSDG is called a source node if there is no arc directing to it.

DEFINITION 5 In PSDG, the distance from node n_i to the sink node n_1, i.e. the length of the path from n_i to n_1, is called the level of n_i.

Since DSS are designed for the DMs with high level in an organization and the PSDG is for their decision makings on how to solve their problems, the concept "level" could be thought as coinciding with the level of the organization, which is responsible for the solution of the problem/subproblems in the PSDG.

In most theoretical discussions, to solve a problem is to find a sequences of operations that transform some source nodes to the sink node.

DEFINITION 6 Let P: $A \times N \times L$ is a PSDG. Let n_m is a node of P. P': $A' \times N' \times L$ is called n_m's decomposed subgraph when (a) $n_m \in N'$ and the source nodes for n_m are contained in N' and also the source nodes in N, and (b) for $\forall$ arc $a_{ij} \in A-A'$ and arc $a_{ij}' \in A'$, the relationship between a_{ij} and a_{ij}' is not AND.

For example, a decomposed subgraph of PROFITS in Figure 2 can be something like Figure 3, which means: "Other things being equal, increasing AUTOMATION-LEVEL and WAGES and NON-ECONOMIC-

CONCESSION result in increasing PLANT−EFFICIENCY which causes a decrease in PRODUCTION−COST and an increase in PROFITS."

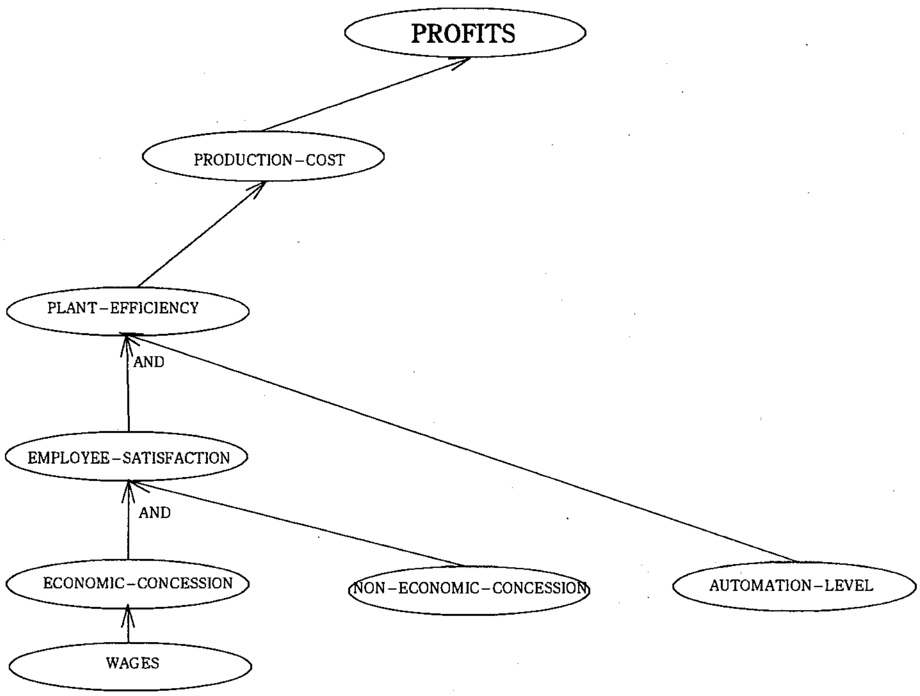


Figure 3 A decomposed subgraph of Figure 2

3. SOME CHARACTERISTICS OF PSDG

(a) Nilson [13] regards the distinction between searches in an ordinary simple graph for problem representation and searches in AND/OR graphs as fundamental. He called simple graphs "state−space representations" and AND/OR graphs "problem−reduction representations". Consider Figure 2, the problem is proposed by unsatisfactory profits of a company. Solution activity or decision making has to consider the appropriateness of relevant profit objectives, diagnose the cause of the sales shortfall, research the production cost, design potential remedial actions, predict the outcomes of such actions, and select the most promising option, and so on. Most managerial problems are similarity complex, and consist of subproblems that can be further decomposed. Subproblems can involve diverse content domains and require various cognitive activities for their solution. Available solution techniques tend to be focused, responsive to a relatively narrow set of demands. No existing procedure can solve the profit shortfall problem, but there are forecasting techniques and market research methods for determining the material cost etc.. A PSDG provides a means of decomposing complex real world problems into subproblems that match up more directly to solution techniques (or models in MMS) and identifying the "deep structure" of the problem [11].

(b) A PSDG can reflect the DM's decision making style. The basic objective of the PSDG is to decompose the ill−structured problem into comparatively structured subproblems. It is based on the DM's knowledge/experience on the domain. For example, the node and arc selection can reflect the DM's viewpoint and knowledge, e.g. the DM would select an affair as a subproblem based on his understanding according to his distinctive decision making training of thought. On the other hand, the AND/OR logical relationship can reflect the DM's emotion and his decision making style. Suppose the DM is a risk averting manager, he might take more nodes in a PSDG as AND−nodes according to the risk decentralizing principal, while the risk neutral DMs might take them as OR−nodes. On the contrary, if the DM is a risk taking manager, he might take some nodes as OR−nodes which are usually taken as AND−nodes. Because he could not understand the node's situation fully, he would rather to think that some of the subproblems of this node would result in a better solution based on his experience or his intuition. Sometimes, the constraints of problem solving resources, such as time, fund etc., would also force the DM to take some nodes as OR−nodes, and the emotion, e.g. the DM's partiality for the solutions of some nodes might result in his taking some nodes as AND−nodes.

(c) A PSDG can be easily changed into a cognitive map. The nodes in a PSDG regard as the variable, and the arcs can obviously be taken as causal connections. We take an arc from node n_i to node n_j as positive when n_i casually increases n_j (for example, the arc from SALES to PROFITS in Figure 2), and an arc from n_i to n_j as negative when n_i casually decreases n_j (for example, the arc from PRICE to SALES in Figure 2) and take all the nodes as OR−nodes, the PSDG is changed into an ordinary cognitive map, which can be called as the ***partner cognitive map*** of the PSDG. Thus, some causal reasoning methods used in cognitive maps can also be used in PSDG.

For example: During real problem analyzing process, as the causality is fuzzy, causality admits of degrees, and vague degrees at that, e.g. in Figure 2, the causal relationship between QUALITY and SALES is usually said as *"very much"*, *" to some extent"*, *"a lot"* etc.. On the other hand, the arcs stand for the causalities. Causality is more complicated than logical implication. Consider causal increase, or positive causality, if "n_i causes n_j" is represented as "n_i implies n_j" in logical implication, then by contraposition, "n_i implies n_j" is everywhere replaceable with "not $-n_j$ implies not $-n_i$". But in causality, though smoking causes lung cancer, not having lung cancer does not cause non−smoking. Rather what can be inferred is that not smoking tend to cause lung non−cancer (we call lung non−cancer is the dis−concept of lung cancer). And this is a quite general relationship among causally increasing quantities (positive correlates). If n_i causes n_j, the increasing n_i increases n_j, and decreasing n_i decreases n_j, we use arc $n_i \overset{\rightarrow}{+} n_j$ to represent this relationship. If n_i causally decreases n_j, the increasing n_i decreases n_j and decreasing n_i increases n_j, we use arc $n_i \overset{\rightarrow}{-} n_j$ to represent this relationship. Kosko [14] defines negative causality with the same fuzzy quantities and relationship as positive causality, i.e. negative causality is eliminable. He gives a general rule of replacement in a fuzzy cognitive map:

Rule *Replace every* $n_i \overset{\rightarrow}{-} n_j$ *with* $n_i \overset{\rightarrow}{+} -n_j$, *where* $-n_j$ *means the dis−concept of* n_j

Henceforth the negative causal arrow $\overset{\rightarrow}{-}$ will not occur. The unsigned arrow $\rightarrow$ will mean positive causality. Suppose the fuzzy cognitive map represented by Figure 2 is transformed into Figure 4 and Figure 5, and the causal values are given by a partial order *"none"* $\lesssim$ *"some"* $\lesssim$ *"much"* $\lesssim$ *"a lot"* $\lesssim$ *"very much"*, then the indirect effects of n_i on n_j can be derived by searching a path from n_i to n_j with the smallest causal value based on fuzzy set conjunct calculation. The result is shown in Table 1, in which (+/−) means the indirect effect of node n_i on the sink node is positive/negative which is determined with the rule: if all the paths from node n_i can finish at the dis−concept node of the sink node, n_i's indirect effect is negative; else all the paths from node n_i can finish at the sink node, n_i's indirect effect is positive; if there are paths from n_i finish both at the sink node and at its dis−concept node, the causal value of n_i 's indirect effect is the difference between the minimum values of both the paths finishing at the sink node and the paths finishing at its dis−concept node, of course it is also a fuzzy value.

(d) The DM can be helped to find his decision goals clearly with the aid of setting some suitable parameters on the nodes and/or the arcs of the PSDG. In a real complex decision making environment, as source nodes of the PSDG might be too many for the DM to deal with simultaneously, he might have to choose the most economical or the simplest solvable source nodes etc. to set about. For example, in the situation represented by

Table 1 The indirect effect of each node in Figure 4 on node n_1

node	n2	n3	n4	n5	n6	n7	n8	n9	n10
effect	much(-)	a lot(+)	much(+)	a lot(-)	much(-)	much(+)	some(-)	much(+)	some(-)

node	n11	n12	n13	n14	n15	n16	n17	n18
effect	some(-)	much(+)	some(+)	some(+)	some(+)	some(-)	some(-)	much(+)

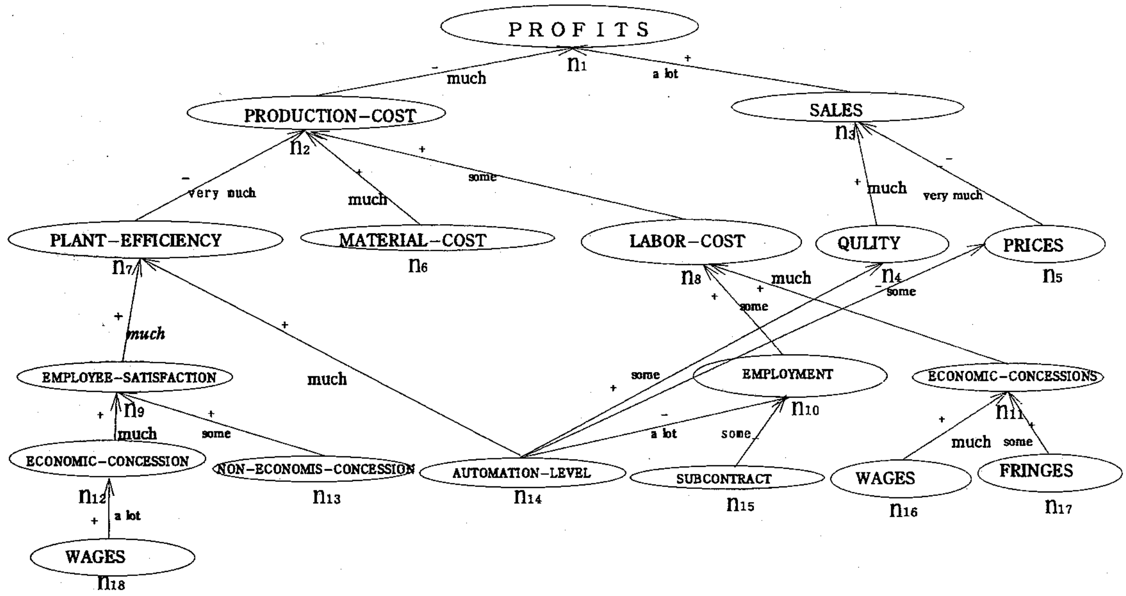


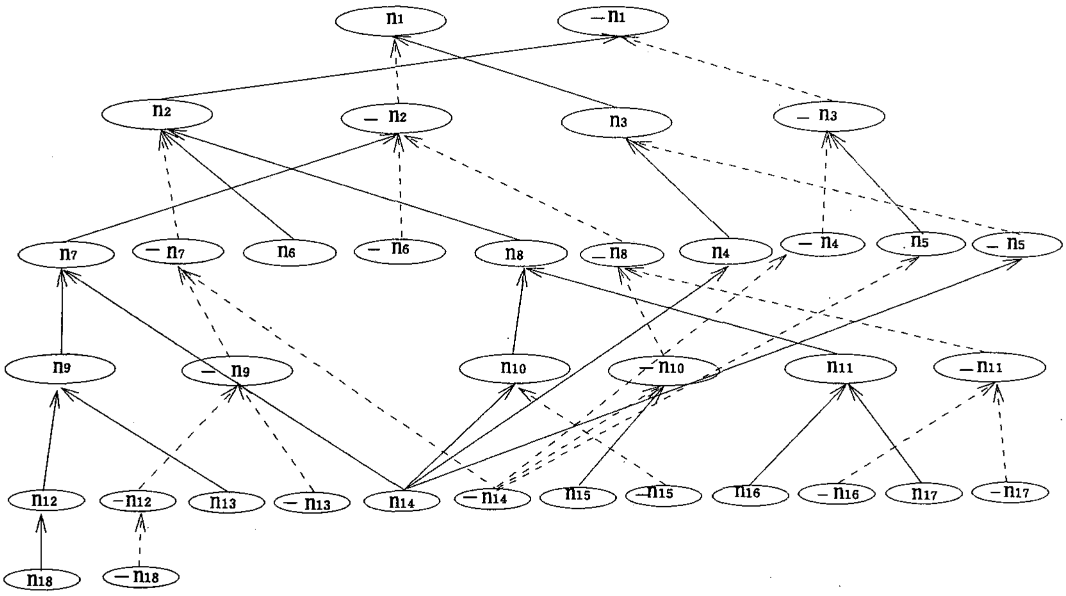
Figure 4 The partner cognitive map of Figure 2

Figure 2, the DM is required to increase the profits within a rational time period. Based on his knowledge, or sometimes on his intuition, the DM might gives some estimation in fuzzy words such as "*soon*", "*some time*", "*a period*", "*long*" or "*very long*" etc. to each node except the sink node in Figure 2. Obviously, it is also a fuzzy decision making problem. We can use a fuzzy function $\tilde{f}(n_i)$ to represent to how extent the solution of node n_i can make contribution to the problem solving within a "rational time period". In this situation, we can take $\tilde{f}(n_i)=\tilde{f}(\tilde{c}_i, \tilde{t}_i)$, where $\tilde{c}_i$ is the indirect effects of n_i on the node PROFITS (node n_1), $\tilde{t}_i$ is the time the node n_i would take, $\tilde{c}_i$ and $\tilde{t}_i$ are both fuzzy variables. We can use some kinds of methods to define $\tilde{f}(n_i)$, e.g. we can take $\tilde{f}(n_i)$ as the utility function of n_i. Therefore, the DM could give the nodes in a decomposed subgraph with the largest utility value priority for solving [15].

4. LOGICAL ADJACENCY MATRIX: A TOOL FOR PSDG REPRESENTATION AND ANALYSIS

PSDG can be easily constructed with the aid of AI approaches such as semantic network methods or for more reasonable, situation semantic method [16] for that PSDG is to represent the problem/subproblem situations and the parameters/variables can be mostly represented by nouns, adjectives and adverbs [17]. Furthermore, the nonmonotonous reasoning and parallel processing methods [18] can also be transplanted into PSDG for its knowledge reasoning and validating etc.

However, on the other hand, a PSDG is a kind of special system structure diagram. The nodes in it can be taken as the elements interacted with some relations (the arcs) toward a goal (i.e. the main problem solving) in



Where all arcs indicate positive causality (causal increase). Arcs from new dis−concepts (node $-n_i$, $i=1,2, \cdots ,18$) are dashed. The fuzzy weight of each arc(the weights of the arcs form node $\pm n_i$ to $\pm n_j$ is thought as the same as the weight of the arc from n_i to n_j in Figure 4), is omitted to simply.

Figure 5 Positive−causality representation of Figure 4.

the system. Therefore, it is reasonable and necessary for us to find another tool for analyzing the system represented by the PSDG. It is well known that a most promising analysis tool for dealing with graph/digraph problems is matrix, such as adjacency matrix, connection matrix etc.. In the PSDG situation we can also try a matrix as a powerful analysis tool.

Lemma *If G: A × N is an acyclic digraph, in which A is the set of G's arcs, N is the set of G's nodes, , it is possible to order the nodes of G so that its adjacency matrix is lower triangular.*

Proof If *G* has no cycles, then it has at least one sink node. We label one such node n_1. Now since *G* has no cycles, $N-\{n_1\}$ has a node n_2 which is a source node. By continuing the process we find in $N-\{n_1, n_2\}$ a node n_3 and in general in $N-\{n_1, n_2, \cdots, n_k\}$ a node n_{k+1} is a source node. With the nodes of *G* ordered in this way, let $\overline{G}=[g_{ij}]$ be its adjacency matrix, there is no arc a_{ij} in *G* if $i \leqq j$ the corresponding entry g_{ij} is 0. [Q. E. D]

We can realize this representation by setting the number of a node according to its level simply, e.g. the number from the smallest to the largest while the level form the smallest to the largest.

Based on *Lemma*, we can easily draw a conclusion as follow:

Any PSDG $P{:}A \times N \times L$, *can be represented by a matrix* $£_{M\times M}$, *which is called a logical adjacency matrix, where M is the number of the nodes in the PSDG,* $£_{M\times M}$ *has such a form as shown in Figure 6, in which* $\tilde{f}(n_i)$ *is used to show the state of node* n_i*, such as to how extent the decision maker is safisfied with the situation of* n_i*, etc.,* b_{ij} $(i > j)$ *is shown the state of an arc* a_{ij}*, such as the influence degree of node* n_i *on* n_j *etc., and* R_{ij} $(i < j)$ *is a set used to stand for the relationship among the arcs, and* R_{ij} *has such characteristics:* ① *if there is no arc* a_{ji} *in P,* $R_{ij} = \Phi$ ② *if* R_{ij} *and* R_{ik} $(j, k > i)$ *are not* Φ *, and they have the same element, arc* a_{ji} *and* a_{ki} *have an AND relationship, else*

a_{ji} and a_{ki} have an OR relationship.

$$
\pounds_{M\times M}=
\begin{array}{c}
 \\ n_1 \\ n_2 \\ \vdots \\ \vdots \\ n_M
\end{array}
\begin{array}{c}
\begin{array}{cccc} n_1 & n_2 & \cdots\cdots & n_M \end{array} \\
\left[\begin{array}{cccc}
\tilde{f}(n_1) & & & \\
 & \tilde{f}(n_2) & R_{ij} & \\
 & b_{ij} & \ddots & \\
 & & & \tilde{f}(n_M)
\end{array}\right]
\end{array}
$$

Figure 6 A logical adjacency matrix of a PSDG

For example, the logical adjacency matrix of Figure 1 (c) might be something as

$$
\pounds\,'=
\begin{array}{c}
 \\ n_i \\ n_{q_1} \\ n_{q_2} \\ n_{q_3}
\end{array}
\begin{array}{c}
\begin{array}{cccc} n_i & n_{q_1} & n_{q_2} & n_{q_3} \end{array} \\
\left[\begin{array}{cccc}
\tilde{f}(n_i) & \{1\} & \{1,2\} & \{2\} \\
b_{q_1 i} & \tilde{f}(n_{q_1}) & & \\
b_{q_2 i} & & \tilde{f}(n_{q_2}) & \\
b_{q_3 i} & & & \tilde{f}(n_{q_3})
\end{array}\right]
\end{array}
$$

The logical adjacency matrix has many useful characteristics, such as its upper triangular matrix can represent the partner cognitive map corresponding with the PSDG and we can use revised Floyd Shortest Path Algorithm etc. [19] to find the indirect effect of every node on the sink node directly from the matrix, and any its left−corner submatrix is a logical adjacency matrix of a subsystem i.e. a decomposed subgraph of the PSDG , which can be used for system structure analysis etc. [20], and in a complex decision making environment, the logical adjacency matrix is usually a scatter matrix. We can use the methods such as strong/weak component finding algorithm etc. to decompose the problem system into some subsystems easily [21] and so on.

5. CONCLUSION

The PSDG method is a highly creative process that depends on the DM's experience and insights. It is also non−deterministic since it must take into consideration the constantly evolving decision making situation. The logical relationship and the parameter representing ways are changeable according to the DM's decision making style.

Research on DSS has traditionally been concerned with decision making, but from an explicitly normative actually behave. The PSDG method proposed in this paper tries to remedy this defect. The characteristics and the logical adjacency matrix of PSDG put forward a wide prospect for further research on this area.

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