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Contraction Coefficients of Underflow Gates with a Vertical Lip

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Synopsis

In this paper the influence of a vertical lip upon the contraction coefficients of underflow gates is discussed. The coefficients of inclined gates with a vertical lip are investigated theoretically by using the method of conformal mapping. Solutions are obtained numerically for several values of the inclination of gate bottom. Theoretical solutions show that; as the length of a lip increases, the coefficient rapidly decreases from the value for the inclined gate, and when the length becomes of the order of a gate openning, it takes a value to be nearly equal to that for a vertical gate. These theoretical results are verified by experiments.

1. Introduction

Usually a vertical lip is attached to the lower end of shell type gate as in Fig.1.1. Such an attachment is convenient to works for sealing. On the other hand it has direct effects upon hydraulic characteristics of the gate such as stage-discharge relationships, pressure distributions and so on. Therefore, for the hydraulic design of this type of gate, it is necessary to know the effects of the lip on effluxes. Hydraulic characteristics of gates mentioned above can be nearly clarified by providing the contraction coefficient. Hence it is first important to reveal the influence of the lip on the contraction coefficient.

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Since the coefficient of shell type gate without a lip are considered to be nearly equal to those of an inclined gate with the same inclination δ, the contraction coefficient of inclined gate, instead of shell type gate, with a vertical lip as in Fig.1.2 are investigated in this study [1].

The coefficient of underflow gates placed in open channels is approximated by the theoretical solution based on the assumptions of two dimensional potential flow. This theoretical solution Cc is composed of two parts. The one, $(Cc)_0$, is the solution obtained by a free streamline theory neglecting the effects of gravity, and the other, $-\Delta Cc$, is the correction to the effects, that is;

$$Cc = (Cc)_0 - \Delta Cc \tag{1.1}$$

According to Marchi's investigation on the coefficient of inclined gate, it seems to be all right to consider that the effects of gate shapes on ΔCc is very small [2]. It shows that fundamental properties of the theoretical solution Cc to the change of lip length are determined by those of the solution $(Cc)_0$.

In the following investigation, $(Cc)_0$ will be obtained at first, and then the theoretical results will be verified by experiments.

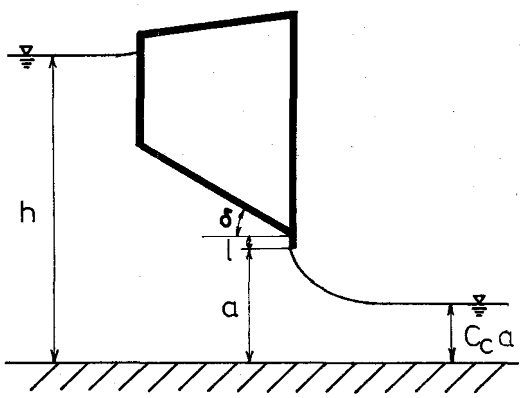

Fig.1.1 Shell type gate with a vertical lip

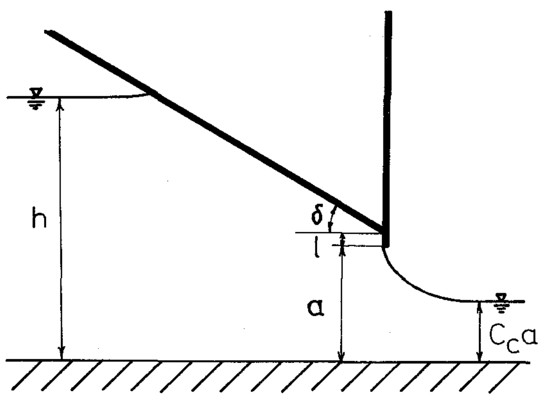

Fig. 1.2 Inclined gate with a vertical lip

2. Theoretical Investigation

2.1 Analysis by a Method of Conformal Mapping

To obtain $(Cc)_0$, the flow from the gate is analyzed by using the

Levi-Civita's method in conformal mapping techniques [3]. For convenience we adopt a two dimensional axial flow as the physical plane (Fig. 2.1). In this figure,notation are as follows;

- a : gate openning
- h : upstream water depth
- δ : inclination of gate bottom
- l : length of a lip
- $(Cc)_0$: contraction coefficient
- U : velocity at the section of contraction(=1 in the analysis)
- U_0 : velocity of the approaching stream
- x,y : coordinates
- ϕ : velocity potential
- ψ : stream function

Fig. 2.2 shows the complex potential plane(w-plane). The domain of an infinite strip in w-plane can be mapped onto the portion of the upper half of ζ-plane interior to the unit circle(Fig. 2.3). The relation between the w- and ζ-plane is given by

$$w = -\frac{2(Cc)_0 a}{\pi}\ln\left\{-\frac{1}{2}\left(\zeta+\frac{1}{\zeta}\right)\right\}+i(Cc)_0 a \,. \tag{2.1}$$

Now,we introduce the fuction ω as follows,

$$\frac{dw}{dz} = -e^{-i\omega} \quad , \quad \omega = \theta + i\tau \,. \tag{2.2}$$

That is,

$$q = e^{\tau} \quad , \quad \frac{u + iv}{q} = e^{i\theta} \quad , \tag{2.3}$$

where, q is an absolute value of velocity,and u and v are the components of velocity vector along the coordinates axes. If ω is defined as the function of ζ in the inner portion of the semi circle shown in Fig. 2.3, the complex variable of the physical plane z can be given by eq.(2.1) and eq.(2.2).

Using Villat's formula, ω is expressed as follows [4].

$$e^{i\omega(\zeta)}=\left\{\left(\frac{1-\zeta e^{i\sigma_1}}{e^{i\sigma_1}-\zeta}\right)\left(\frac{e^{i\sigma_1}+\zeta}{1+\zeta e^{i\sigma_1}}\right)\right\}^{\left(\frac{1}{2}-\frac{\delta}{\pi}\right)}\left\{\left(\frac{1-\zeta e^{i\sigma_2}}{e^{i\sigma_2}-\zeta}\right)\left(\frac{e^{i\sigma_2}+\zeta}{1+\zeta e^{i\sigma_2}}\right)\right\}^{\frac{\delta}{\pi}} \,, \tag{2.4}$$

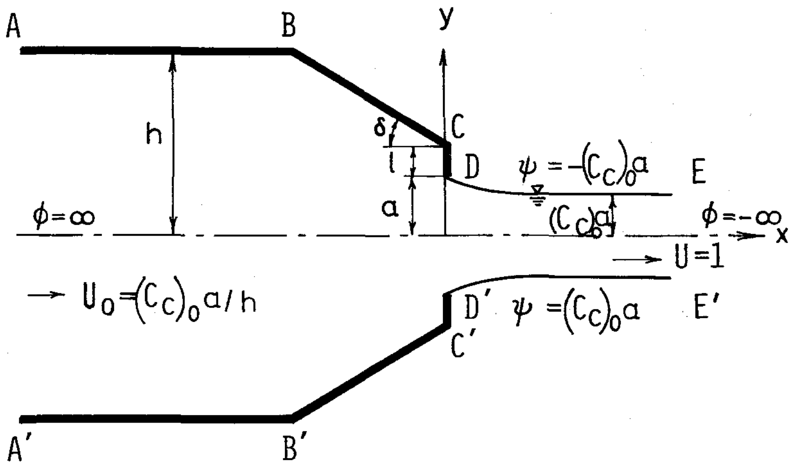

Fig.2.1 Physical plane (z-plane)

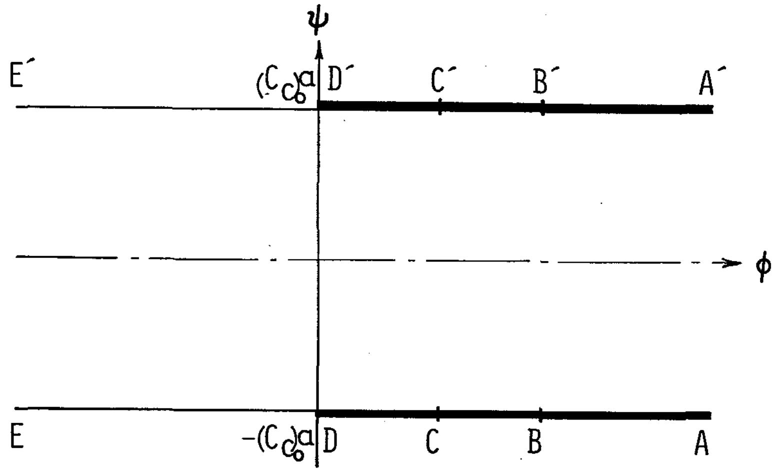

Fig.2.2 Complex potential plane (w-plane)

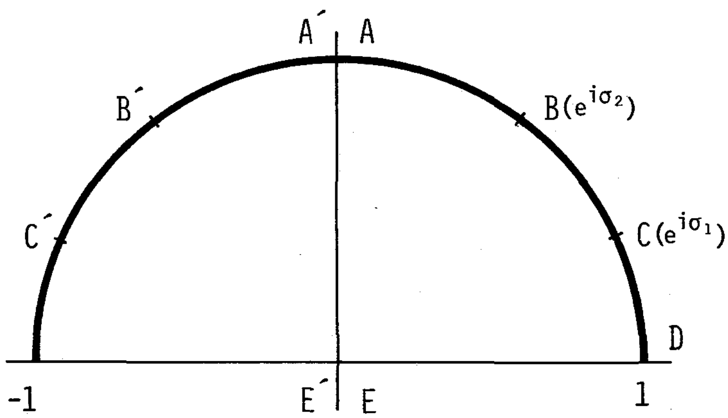

Fig.2.3 Auxiliary plane (ζ-plane)

where, σ_1 and σ_2 are arguments of ζ to the points C and B respectively. On the circumference ($\zeta=\exp(i\sigma)$), eq.(2.4) becomes,

$$e^{i\omega(\sigma)}=\left(\frac{\sin\sigma+\sin\sigma_2}{\sin\sigma-\sin\sigma_2}\right)^{\frac{\delta}{\pi}}\left(\frac{\sin\sigma+\sin\sigma_1}{\sin\sigma-\sin\sigma_1}\right)^{\left(\frac{1}{2}-\frac{\delta}{\pi}\right)}, \tag{2.5}$$

and eq.(2.1) is expressed as follows,

$$w=-\frac{2(Cc)_0 a}{\pi}\ln(-\cos\sigma)+i(Cc)_0 a\ . \tag{2.6}$$

Differentiating with respect to σ,

$$dw=\frac{2(Cc)_0 a}{\pi}\tan\sigma\, d\sigma\ . \tag{2.7}$$

From eq.(2.2),eq.(2.5) and eq.(2.7),the coordinate of the point on the boundary $\overline{ABCD}$ in Z-plane is obtained as follows,

$$z=-\frac{2(Cc)_0 a}{\pi}\int\left(\frac{\sin\sigma+\sin\sigma_2}{\sin\sigma-\sin\sigma_2}\right)^{\frac{\delta}{\pi}}\left(\frac{\sin\sigma-\sin\sigma_1}{\sin\sigma+\sin\sigma_1}\right)^{\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}\tan\sigma\, d\sigma+C_0\ , \tag{2.8}$$

where,C_0 is a constant of integration. Then,segments $\overline{CD}$ and $\overline{BC}$ are expressed as follows,

$$\overline{CD}=\frac{2(Cc)_0 a}{\pi}\int_0^{\sigma_1}\left(\frac{\sin\sigma_2+\sin\sigma}{\sin\sigma_2-\sin\sigma}\right)^{\frac{\delta}{\pi}}\left(\frac{\sin\sigma_1-\sin\sigma}{\sin\sigma_1+\sin\sigma}\right)^{\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}\tan\sigma\, d\sigma\ , \tag{2.9}$$

$$\overline{BC}=\frac{2(Cc)_0 a}{\pi}\int_{\sigma_1}^{\sigma_2}\left(\frac{\sin\sigma_2+\sin\sigma}{\sin\sigma_2-\sin\sigma}\right)^{\frac{\delta}{\pi}}\left(\frac{\sin\sigma-\sin\sigma_1}{\sin\sigma+\sin\sigma_1}\right)^{\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}\tan\sigma\, d\sigma\ . \tag{2.10}$$

Rewriting these equations by using the length of the lip,following two equations are obtained;

$$\frac{l}{a}=\frac{2(Cc)_0}{\pi}\int_0^{\sigma_1}\left(\frac{\sin\sigma_2+\sin\sigma}{\sin\sigma_2-\sin\sigma}\right)^{\frac{\delta}{\pi}}\left(\frac{\sin\sigma_1+\sin\sigma}{\sin\sigma_1-\sin\sigma}\right)^{\left(\frac{1}{2}-\frac{\delta}{\pi}\right)}\tan\sigma d\sigma\ , \tag{2.11}$$

$$\frac{l}{a}=\frac{1}{(a/h)}-1-\frac{2(Cc)_0}{\pi}\sin\delta\int_{\sigma_1}^{\sigma_2}\left(\frac{\sin\sigma_2+\sin\sigma}{\sin\sigma_2-\sin\sigma}\right)^{\frac{\delta}{\pi}}\left(\frac{\sin\sigma+\sin\sigma_1}{\sin\sigma-\sin\sigma_1}\right)^{\left(\frac{1}{2}-\frac{\delta}{\pi}\right)}\tan\sigma d\sigma. \tag{2.12}$$

Using the relation $\omega=i\tau$ at $\sigma=\pi/2$, following equation is obtained from eq.(2.3), eq.(2.5) and the equation of continuity,

$$(Cc)_0=\frac{1}{(a/h)}\left(\frac{1-\sin\sigma_2}{1+\sin\sigma_2}\right)^{\frac{\delta}{\pi}}\left(\frac{1+\sin\sigma_1}{1-\sin\sigma_1}\right)^{\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}. \tag{2.13}$$

Providing δ, l/a and a/h, $(Cc)_0$ are calculated by using eq.(2.11), eq.(2.12) and eq.(2.13).

In these equations, $\sigma_1=0$ gives the solution for an inclined gate with an inclination δ, and $\sigma_1=\sigma_2$ gives the solution for a vertical gate.

2.2 Results of Numerical Computations

$(Cc)_0$ has been obtained by numerical computations. The procedure of computations was reported in detail previously [5]. The results are shown in Figs. (2.4)~(2.8). In each figure an upper limit curve shows the value for an inclined gate and the lower one shows the value for a vertical gate. These figures show that;

(1) as the length of lip increases, $(Cc)_0$ decreases rapidly from the value for the inclined gate, and when the length becomes of the order of a gate openning, it takes a value to be nearly equal to that for a vertical gate,

(2) the amount of change in $(Cc)_0$ to the constant change in l/a increases as the inclination δ becomes small.

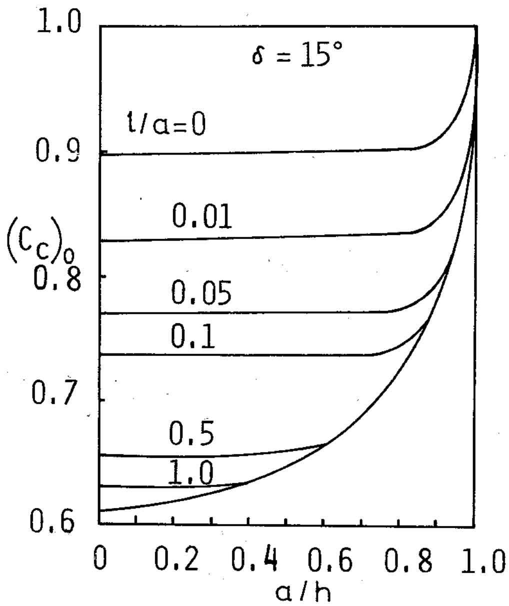

Fig.2.4 Contraction coefficient (δ= 15°)

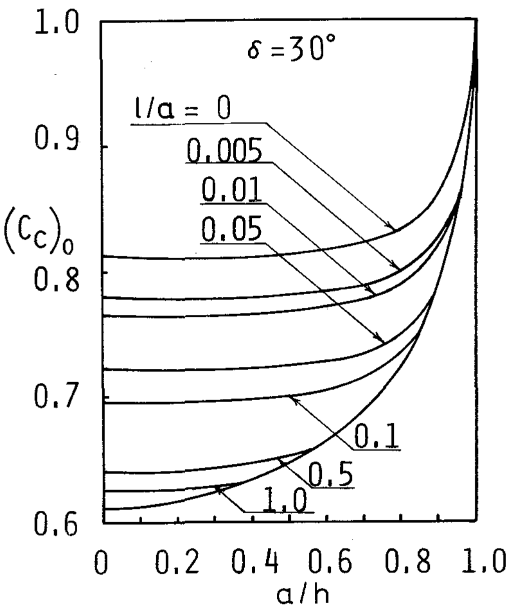

Fig.2.5 Contraction coefficient (δ = 30°)

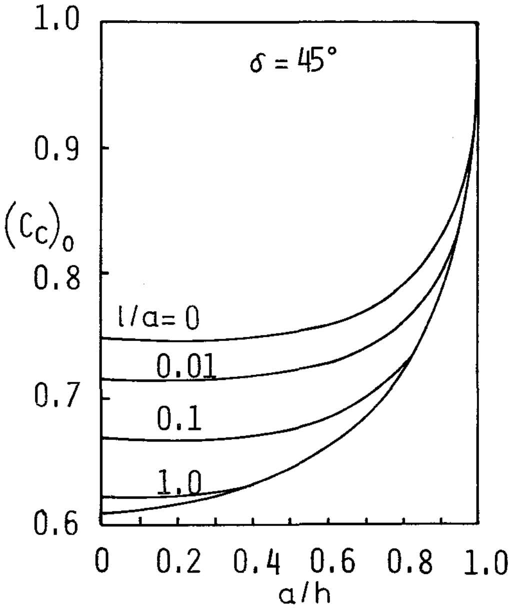

Fig.2.6 Contraction coefficient (δ = 45°)

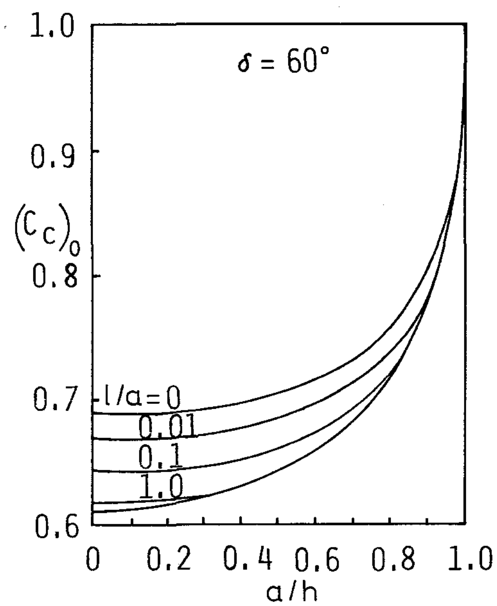

Fig.2.7 Contraction coefficient (δ = 60°)

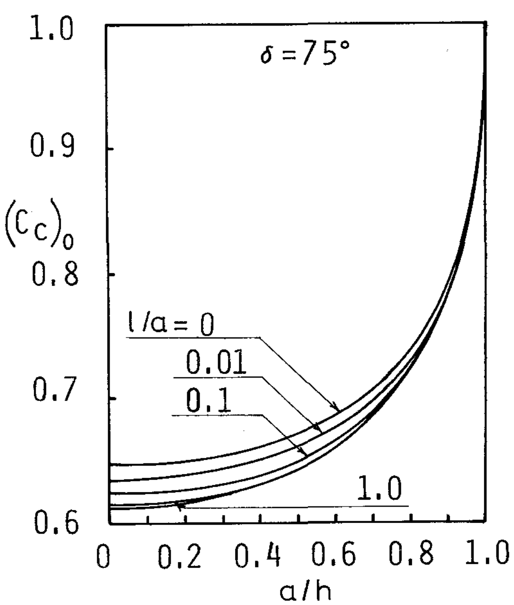

Fig.2.8 Contraction coefficient (δ = 75°)

3. Experimental Verifications

A horizontal channel with length 10 m,width 0.40 m and depth 0.60m was used for the experiment. The gate openning was fixed at 6.0 cm. Inclinations δ and lengths l used for the experiment are shown in Table 3.1. Water depth was measured by a point gauge with accuracy 0.1 mm.

Table 3.1 δ,l and l/a used for experiments

δ	l (cm)	l/a
30°	0.38	0.063
	0.88	0.146
	1.88	0.313
	2.88	0.479
	5.88	0.979
45°	0.30	0.050
	0.80	0.133
	1.80	0.300
	2.80	0.467
	5.80	0.967
60°	0.10	0.017
	0.60	0.100
	1.60	0.267
	2.60	0.433
	5.60	0.933

Experimental results are shown in Figs. (3.1) (3.3). In each figure the theoretical curves are shown for a comparison. These theoretical values are obtained by adding $-\Delta Cc$ to $(Cc)_0$,as shown in eq.(1.1). In

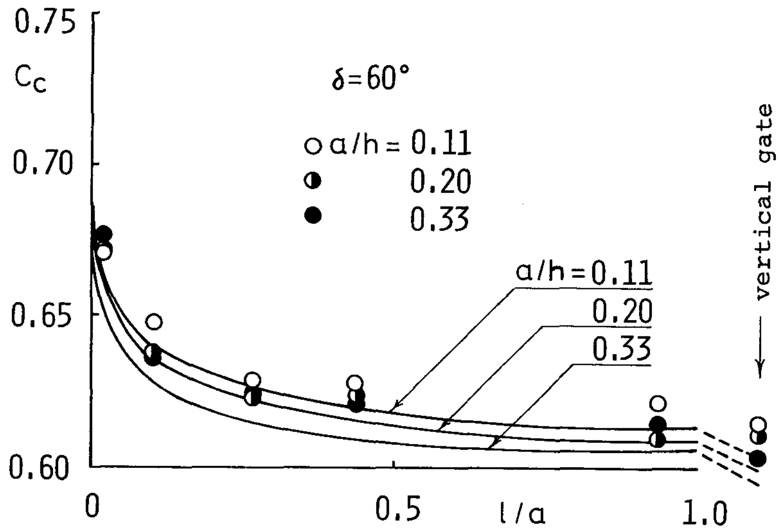

Fig.3.1 Contraction coefficient (δ=60°)

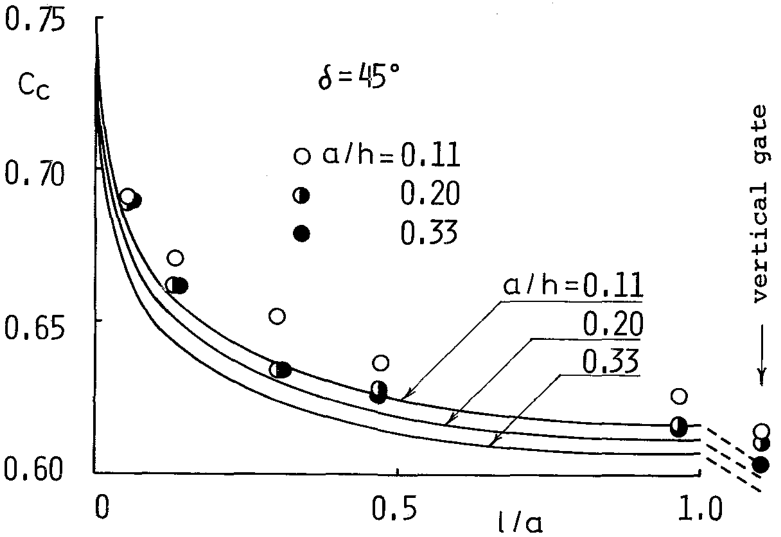

Fig.3.2 Contraction coefficient (δ=45°)

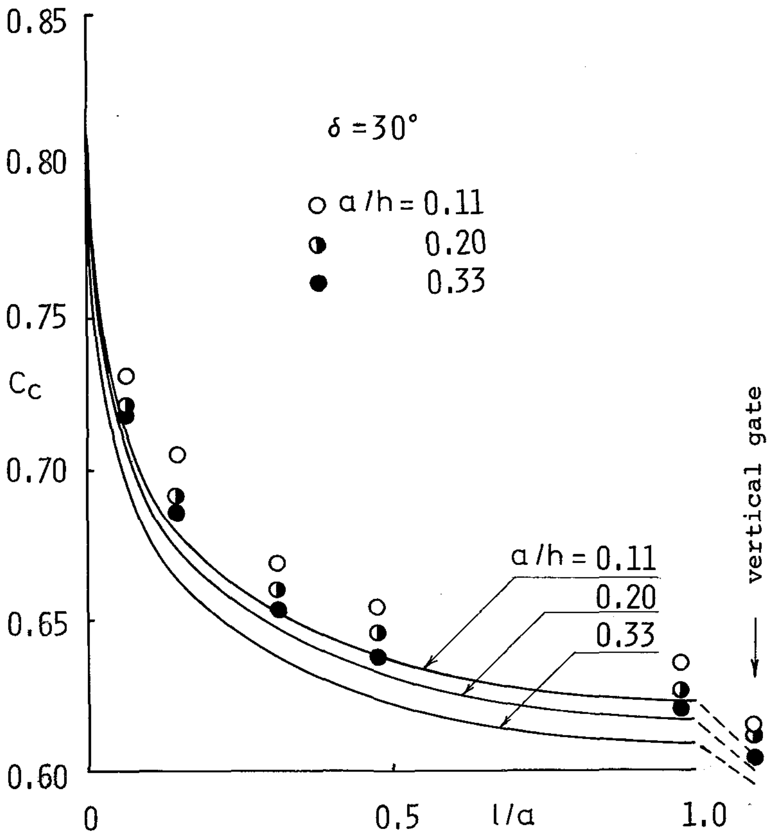

Fig.3.3 Contraction coefficient (δ=30°)

this occasion, as the correction ΔCc,the quantity calculated from the Fangmeier and Strelkoff's solution for a vertical gate, as in Fig.3.4, is adopted [6]. The tendency of experimental values to l/a is explained fairly well by the theoretical curves,though the former is slightly larger than the later. Quantitative differences must be considered to be a limitation of potential flow theory [7].

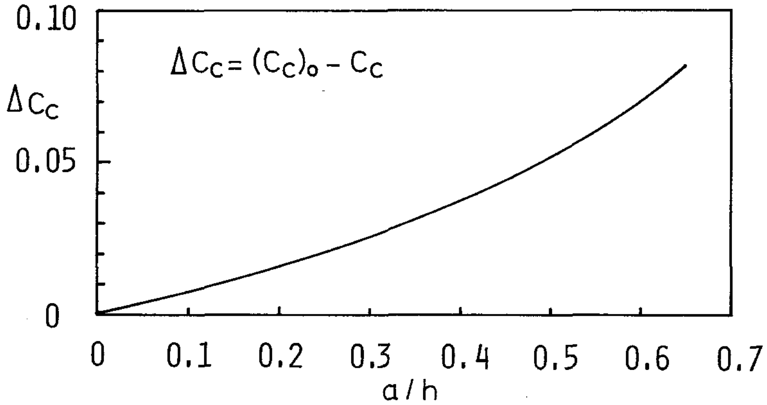

Fig.3.4 Correction for the effect of gravity, ΔCc

4. Conclusion

In this study the influence of a vertical lip on the contraction coefficient of underflow gates has been investigated. The theoretical solutions show that; as the length of a lip increases the coefficient decreases rapidly from the value for the inclined gate, and when the length becomes of the order of a gate openning, it takes a value to be nearly equal to that for a vertical gate. These theoretical results have been verified by experiments.

References

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