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Transmission Distance and Bit Rate in an Optical-Fiber Multi-Level PCM Transmission System

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Synopsis

Optical-fiber multi-level PCM transmission system is investigated on the transmission distance and bit rate. A communication channel is established as follows: a light signal is modulated in intensity with a completely-balanced M-level code at a transmitter, and is propagated over an optical glass fiber, and is demodulated at a receiver where the signal with an additive noise is processed through a PD, an equalizing amplifier, a matched filter, (M-1) comparators and a decision circuit. The relative power of the noise from the amplifier is increased in accordance with the reciprocal of fiber's transmittance, where shot noise and thermal noise are smoothed and decreased in power by the matched filter. The relation between the BER and the SNR leads the transmission distance. The product of the transmission distance and bit rate takes the large maximum at the large values of SNR and M.

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The large value of M is suitable for low-speed and high-rate system, and the small one for anti-interference system.

1. Introduction

Optical glass fiber is a high-quality communication medium, which guides the wide-band signals at a low attenuation rate without electromagnetic interference (EMI). The slender, light and limp structure of the fiber is suitable to forming the multi-wire cable of very large channel capacity.

On the other hand, digital technology is penetrating into all the electronic and communication systems. Optical-fiber digital communication can play an important role in the information network system (INS) (1),(2), where the tremendous information terminals of multiple functions are to communicate with each other in a domestic or world-wide area. A home television (TV) set at present, is seemed to be replaced by the optical-fiber two-way video terminal, to multiplex and demultiplex video, stereophonic, teletext, facsimile and other signals with pulse code modulation (PCM) in future(3).

The equipment for transceiving these signals with ordinary 2-valued PCM must be constructed from high-speed or ultra-high-speed digital devices, but such device nowadays corresponds to emitter-coupled logic (ECL), whose power consumption is by far the highest in every other logic. For customer's use, a high-speed and low-power logic device or a low-speed system needs to be developed. The light signals on the low-speed system may be propagated over the considerably longer fiber without intermediate repeater.

The low-speed digital optical-fiber transmission system can be made by means of wavelength multiplexing(4), spatial (multi-fiber) multiplexing, multi-level PCM(5), and/or data compression(6). This paper deals with multi-level PCM transmission over an optical fiber, where a transmission rate and a non-repeating transmission distance are related to the transmittance over the fiber, the manners of modulation and demodulation, the signal-to-noise ratio (SNR) at a receiver, and the desired bit-error rate (BER)(7). A communication channel is established as follows: the light signal modulated in intensity with a completely-balanced multi-level code at a transmitter, passes through a graded-index multi-mode fiber, and at a receiver is demodulated to the electrical signal of an object waveform for decision.

2. Transmittance of light wave over optical glass fiber

Optical fiber is classified into the following three types according to the propagation mode of light wave: step index multi-mode fiber, step-index single-mode fiber, and graded-index multi-mode fiber. These fibers are illustrated in Fig. 1. The ratio of core's

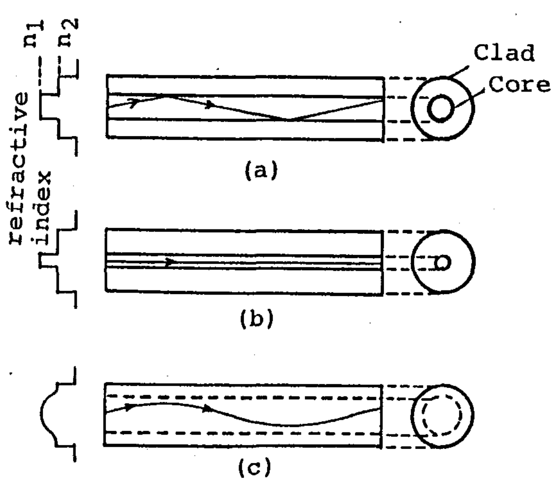


Fig.1. (a) Step-index multi-mode fiber. (b) Step-index single-mode fiber. (c) Graded-index multi-mode fiber.

refractive index n_1, to clad's n_2 is about 1.02 to 1.04. The standard cross-sectional size of step- or graded-index multi-mode fiber is 50 μm in core diameter/125 μm in clad diameter, and that of step-index single-mode fiber is 10 μm/125 μm.

Light power transmittance over a fiber depends on wavelength, fiber length and signal frequency. The transmittance of a sharp-spectral light from a laser diode (LD) is approximately represented by

$$H(d,f)=\exp[-\{\alpha(\frac{d}{d_0})+\beta(\frac{f}{f_0})^2(\frac{d}{d_0})^{2\gamma}\}] \qquad (1)$$

,

where d is the fiber length and f is the signal frequency and d_0, f_0, α, β, γ are the constants. The value γ takes about 0.5 in the step-index fiber and 0.75 in the graded-index fiber(8). The constant f_0 denotes the fiber band or the cut-off frequency at $d=d_0=1$ km when the light power becomes 3 dB-down at $\beta \fallingdotseq 0.7$. The cut-off frequency at d is given by

$$f_c=f_0(\frac{d_0}{d})^{\gamma} \qquad (2)$$

where $d_0=1$ km. The value α, for example, is 0.23 at the attenuation rate 1 dB/km. The values of the attenuation rate and the fiber band, range over respectively 3 to 5 dB/km and 20 to 40 MHz in the step-index multi-mode fiber, 0.7 to 3.5 dB/km and 200 to 1000 MHz in the graded-index multi-mode fiber, 0.5 to 3.5 dB/km and 3 to 10 GHz in the step-index single-mode fiber.

Electrical power transmittance over a transmission line is given by $H^2(d,f)$, so that the transmittance -3dB of light signal corresponds to the transmittance -6dB of electrical signal. A received signal waveform is distorted by the transmittance at high frequencies. A receiving amplifier equalizes the waveform to an object one, but this process intensifies the noise power at these frequencies.

3. Transmission of completely-balanced multi-level signal

We consider a completely-balanced M-level code illustrated in Fig. 2, where the weight a takes one of the M values $\{-1, -(1-\frac{2}{M-1}), \ldots, 1-\frac{2}{M-1}, 1\}$ (M=2, 3,...) at the time interval $2T_c$. This code is easy to be amplified because of having no DC component, and can transmit information at the following transmission rate or bit rate

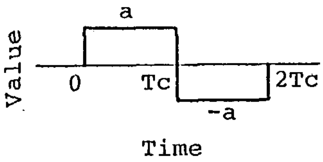


Fig.2. Completely-balanced code.

$$v = \frac{1}{2T_c}\log_2 M \quad . \tag{3}$$

At the transmitter, the electrical signals composed of the code sequence, modulate the continuous-wave (CW) light of the power P_0 at the intensity-modulation factor m (≤ 1). Hence the light signals with the power $p_0(1+am)$ are transmitted, and the signals weakened through an optical transmission line reach the receiver. At the receiver, shot noise is mainly added from a photo-diode (PD) or avalanche photo-diode (APD) and thermal noise from resistor and amplifier. The power of shot noise is proportional to the received light power and becomes dominant as the light power decreases.

Let the shot-noise power be σ_s^2 at m=0 and the thermal-noise power be σ_t^2 , when they are normalized by the maximum squared amplitude of the demodulated signal. A total noise power on demodulating is given by

$$\sigma'^2 = \sigma_s^2(1+a\cdot m)+\sigma_t^2 \quad , \tag{4}$$

which varies at every time slot. This variation, undesirable for signal detection, can be improved by the matched filter shown in Fig. 3. Let the input signal of time t be expressed as

$$x(t) = a\{u(t)-2u(t-T_c)+u(t-2T_c)\} \tag{5}$$

where u(t) is the unit step function, and let the input noise have the power σ'^2. The output signal is given by

$$y(t) = \frac{x(t-T_c)-x(t)}{2} \quad , \tag{6}$$

which takes the value a at the sampling time $t=\frac{3}{2}T_c$. The output noise power at this time becomes

$$\sigma^2 = \frac{\sigma_s^2(1+am)+\sigma_s^2(1-am)+2\sigma_t^2}{4} = \frac{\sigma_s^2+\sigma_t^2}{2} \quad . \tag{7}$$

Thus, the noise power is smoothed and decreased.

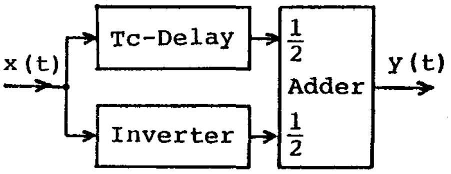


Fig.3. Matched filter.

The filtered M-level signal is compared with the M-1 threshold levels made from a CW light component. The minimum distance among the possible signal levels and the threshold levels is 1/(M-1). The SNR in the M-level signal detection is defined as

$$\mathrm{SNR}_M = \frac{\mathrm{SNR}_0}{(M-1)^2} \;;\; \mathrm{SNR}_0 = \frac{1}{\sigma^2} \;, \tag{8}$$

where increasing M makes the signal detection badly interfered by noise. The noise amplitude η has a Gaussian distribution given by the following density

$$p(\eta) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\eta^2}{2\sigma^2}} \;. \tag{9}$$

The transmitter sends a signal representing one of M digits, at the probability 1/M. The digit error rate DER is calculated as

$$\begin{aligned}
\mathrm{DER} = & \frac{1}{M}\,\mathrm{Prob}\left(-1+\frac{1}{M-1} < a+\eta \;\middle|\; a=-1\right) \\
& + \frac{1}{M}\sum_{k=1}^{M-2} \mathrm{Prob}\left(a+\eta < -1+\frac{2k-1}{M-1},\; -1+\frac{2k+1}{M-1} < a+\eta \;\middle|\; a=-1+\frac{2k}{M-1}\right) \\
& + \frac{1}{M}\,\mathrm{Prob}\left(a+\eta < 1-\frac{1}{M-1} \;\middle|\; a=1\right) \\
= & \frac{M-1}{M}\,\mathrm{erfc}\left[\frac{1}{\sqrt{2}(M-1)\sigma}\right] \;,
\end{aligned} \tag{10}$$

where erfc(•) is the co-error-function:

$$\begin{aligned}
\mathrm{erfc}(z) &= \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-z'^2}\, dz' \\
&\simeq \frac{e^{-z^2}}{\sqrt{\pi}\, z} \;;\; z > 2 \;.
\end{aligned} \tag{11}$$

Therefore, the BER in the M-level signal detection is connected with the SNR as follows:

$$BER_M = \frac{M-1}{M\log_2 M} \operatorname{erfc}\left[\frac{1}{(M-1)} \sqrt{\frac{SNR_0}{2}} \right] \tag{12}$$

Fig. 4 shows a few curves of the BER versus the SNR on M=2,3,4. The 10^{-9} BER usually required for data transmission is obtained at SNR_0=15.6, 21.5, 25.0 in dB or at SNR_2=15.6, SNR_3=15.5, SNR_4=15.5 in dB on M=2,3,4, respectively.

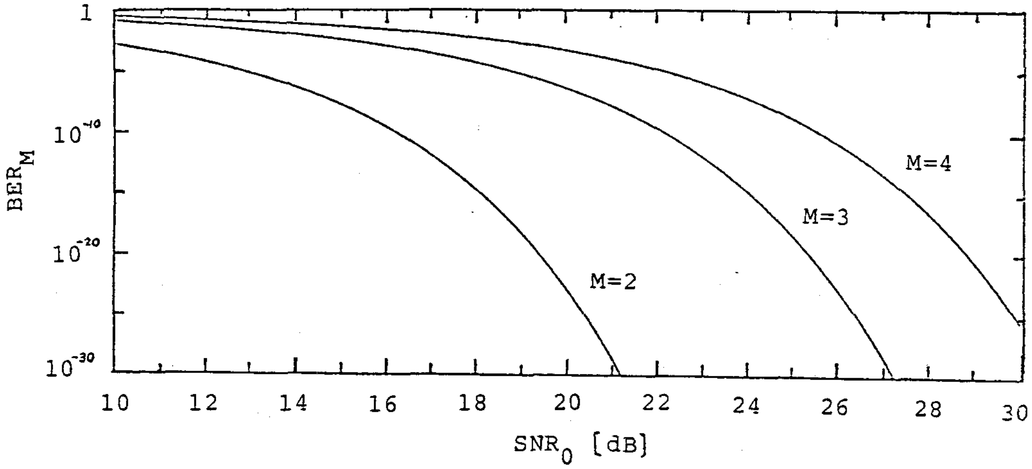


Fig.4 BER vs. SNR.

4. Relation between waveform equalization and SNR

The practical receiving amplifier also acts as a waveform equalizer, which compensates the transmission characteristic on the line for fine signal detection. Therefore, the SNR is related to the transmittance mainly on the fiber. Fig. 5 shows a model of the transmission line. The waveform generator contains a pulse generator, and an electrical-to-optical (E/O) converter with a light-emitting diode (LED) or an LD. The equalizing amplifier represents the receiving amplifier containing a PD for O/E conversion. These blocks can be considered as linear electrical systems.

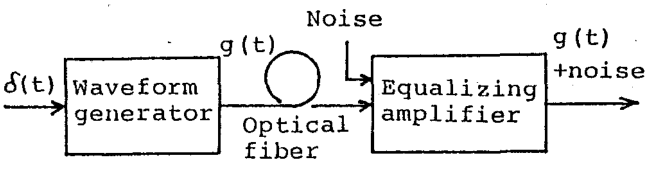


Fig.5. Model of the transmission line.

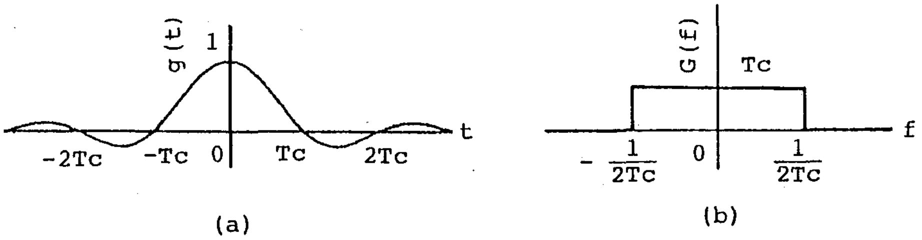
For a simple calculation, let both the waveform generator and the equalizing amplifier produce the identical waveform:

$$g(t)=\frac{\sin \pi t/T_c}{\pi t/T_c} \tag{13}$$

when an impulse $\delta(t)$ is applied to the generator input. The waveform is also the object waveform satisfying Nyquist's first condition that no intercode interference occurs at the sampling time of every T_c. Its amplitude spectrum on the frequency f is expressed as

$$\begin{aligned} G(f) &= T_c \;;\quad |f| \le \frac{1}{2T_c} \\ &= 0 \;;\quad |f| > \frac{1}{2T_c} \end{aligned} \tag{14}$$

where $f_\nu=1/(2T_c)$ corresponds to the Nyquist frequency. Fig.6 shows the waveform and the spectrum. Signal transmission is made by



(a) (b)

Fig.6. (a) Object waveform. (b) Its Fourier spectrum.

applying $a\{\delta(t)-\delta(t-T_c)\}$ instead of $\delta(t)$. Thereby, the received $x(t)$ and the filtered signal $y(t)$ as before, are respectively replaced by

$$x'(t)= a\{g(t)-g(t-T_c)\} \tag{15}$$

and

$$y'(t)= \frac{x'(t)-x'(t-T_c)}{2} \quad . \tag{16}$$

The equalizing amplifier can produce the desired waveform when it has a transfer function as

$$\begin{aligned} E(f) &= \frac{1}{H(d,f)} \quad ; \quad |f|\le f_\nu \\ &= 0 \qquad\qquad ; \quad |f|>f_\nu \end{aligned} \quad . \tag{17}$$

At this time, the variance σ^2 of the noise sampled at the matched-filter output can be rewritten by an integral form as follows:

$$\sigma^2= \frac{\sigma_\nu^2}{f_\nu} \int_0^\infty |E(f)|^2 df \quad , \tag{18}$$

where σ_ν^2 is its variance at d=0 in the Nyquist band f_ν. Equations (8) and (18) mean that the SNR is decreased with increasing the fiber length and/or the transmission rate. The SNR is rewritten by

$$SNR_M= \frac{1}{(M-1)^2\sigma_\nu^2 e^{2\alpha D} \quad \Phi(2\beta F_\nu^2 D^{2\gamma})} \quad , \tag{19}$$

where $F_\nu=f_\nu/f_0$, $\alpha D=d/d_0$ and

$$\Phi(z) = \int_0^1 e^{z \cdot z'^2} dz'$$

$$= \sum_{k=0}^{\infty} \frac{z^k}{(2k+1)k!} \quad . \qquad (20)$$

The index $\Phi(2\beta F_\nu^2 D^{2\gamma})$ denotes the degree of increase in noise power, dependent on the ratio of the Nyquist frequency f_ν to fiber's cut-off frequency f_c. Fig. 7 shows the curve of $\Phi(z)$. The index takes the value 1.75 at $f_\nu=f_c$, and 1.3 at $f_\nu=f_c/\sqrt{2}$, and 1.1 at $f_\nu=f_c/2$. Similar results are obtained even by using the other object waveform with spectrum at non-zero roll-off rate.

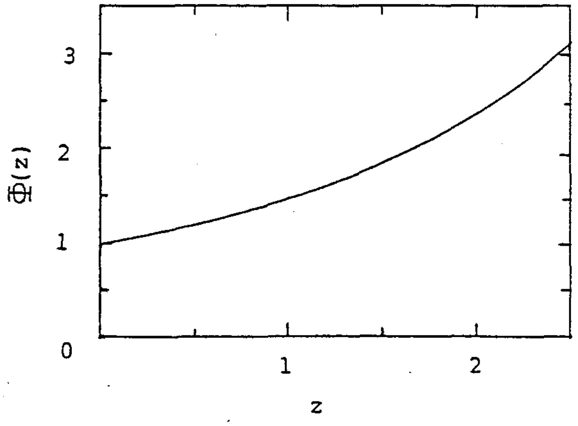


Fig.7. The index of increase in noise power.

5. Transmission rate and transmission distance

Transmission distance without repeating is obtained by solving Equation (19). Here, the value of the distance is approximated on M=2 through 10 at the low Nyquist frequency as satisfying $\Phi(z) \simeq 1$. The normalized distance available for every fiber is

$$\alpha D \simeq \ln\frac{1}{(M-1)K\sigma_\nu}\quad, \tag{21}$$

where $K=\sqrt{SNR_M}=5.96$: every SNR_M (M=2 through 10) takes the value about 35.5 (15.5 dB) in order to keep the 10^{-9} BER. Taking M from Equation (21) makes

$$M= 1+\frac{e^{-\alpha D}}{K\sigma_\nu}\quad . \tag{22}$$

Substituting Equation (22) to Equation (3) produces the normalized transmission rate available for every f_ν as

$$\frac{v}{f_\nu}= \log_2 M= \log_2 (1+\frac{e^{-\alpha D}}{K\sigma_\nu}) \tag{23}$$

which is also the normalized channel capacity in optical-fiber transmission. However, it is considered that such capacity should be replaced by the product of the normalized transmission distance and transmission rate as follows:

$$\alpha D\cdot\frac{v}{f_\nu}= \{\ln\frac{1}{(M-1)K\sigma_\nu}\}\log_2 M\quad . \tag{24}$$

Figs. 8 and 9 respectively show the normalized distance and the product of the normalized transmission distance and transmission rate, in the curves at several fixed SNR's where M is treated as a continuous variable and $1/\sigma_\nu^2$ is identical to the signal-to-noise ratio $SNR_{0\nu}$ at d=0 at f_ν:

$$SNR_{0\nu}= \frac{1}{\sigma_\nu^2}\quad . \tag{25}$$

The normalized channel capacity in Fig. 8 takes 3.4 at $SNR_{0\nu}$=70 dB,

$\alpha D=4$, where the absolute value of the transmission distance and the channel capacity are respectively $d=17.2$ km and $v=34$ Mb/s at $\alpha=0.23$(1dB/km), $f_\nu=10$ MHz. The product of the normalized transmission distance and transmission rate in Fig. 9 takes the maximum 14.4 at $M=21.9$ at $SNR_{0\nu}=70$ dB($\alpha D=3.2, v/f_\nu=4.5$). The maximum point moves into $M=2.9$ at $SNR_{0\nu}=35$dB ($\alpha D=1.6, v/f_\nu=1.5$). Under such a bad noise-environment, M=3 is recommended. Equation (21) also shows little difference between the distance at M=2 and that at M=3, under a high-SNR condition.

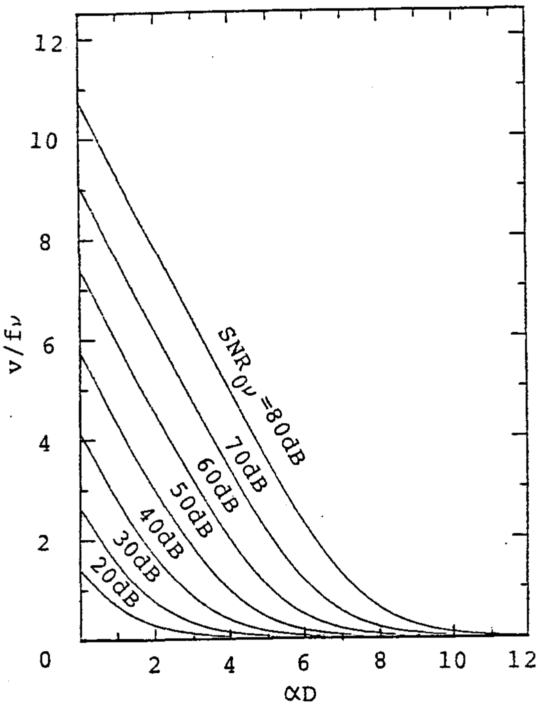


Fig.8. Normalized transmission rate.

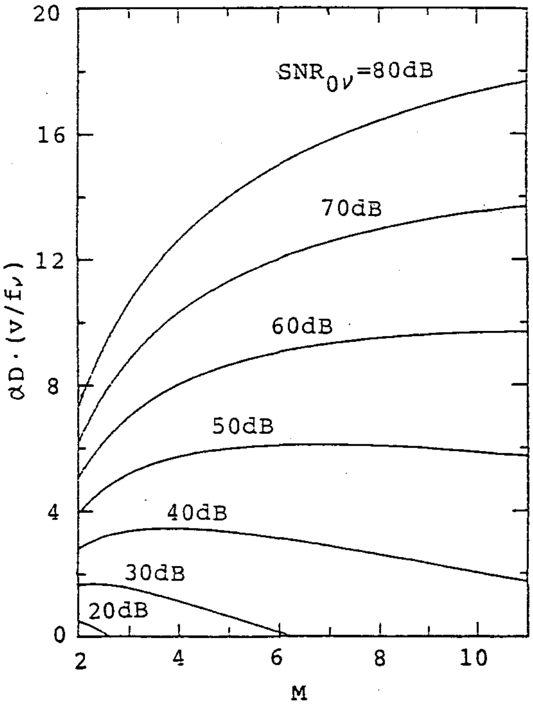


Fig.9. Product of the normalized transmission distance and transmission rate.

Fig. 10 shows the photographs of the reformed pictures in an optical-fiber two-way PCM-TV transmission system built as a trial(6). The picture in Fig. 10(a) is transmitted over a 5-dB/km 50-m multi-mode fiber by the completely-balanced three-level code at the 19-Mb/s transmission rate. The picture in Fig. 10(b) is transmitted over the other 50-m fiber by the same code at the rate equivalent to 23Mb/s, on the basis of block quantization.

(a) (b)

Fig. 10. (a) A picture by 19-Mb/s optical-fiber transmission.
(b) A picture by 23-Mb/s optical-fiber transmission.

6. Conclusion

An optical-fiber multi-level PCM transmission is proposed where a completely-balanced multi-level code is used for low-speed operation and noise smoothing. The low-speed operation makes the fiber be effectively utilized: an index is given which represents the ability of system as the product of the transmission distance and the transmission rate under a desired bit-error rate.

Such system is effective for constructing the consumer's terminals in the information network system.

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