Liquefaction of Highly Saturated Sand Layer under Oscillating Water Pressure

Hiroshi NAGŌ*

Synopsis

In this paper the vertical distribution of pore water pressure in the highly saturated sand layer under the oscillating water pressure are treated theoretically and experimentally.

By the experiments it is shown that the water pressure acting on the sand surface propagates into the sand layer with the damping in amplitude and the lag in phase, and that the liquefaction, the state that the effective stress becomes zero, occurs under certain conditions. These experimental results are explained fairly well by the same theoretical treatment as for the ground water problems in the elastic aquifer.

The main characteristics of liquefaction clarified by the analysis are as follows.
1) The depth of the liquified layer increases with the increase of the amplitude and the frequency of the oscillating water pressure.
2) The increase of the volume of the water and the air in the layer increases the liquified depth. Especially the very small amount of the air affects the lique-faction significantly.
3) The liquified depth decrease rapidly with the increase of the compressibility coefficient of the sand.
4) In the range beyond a certain value of the perme-ability coefficient the liquified depth decrease with the increase of the coefficient.

* Department of Civil Engineering

1. Introduction

    Under the oscillating water pressure normal to the surface
of the highly saturated sand layer the pore water pressure in
the layer changes with time, and according to circumstances
the excess pore pressure occurs. The increase of the excess.
pore pressure brings to the decrease of the effective stress.
Then, under certain conditions it is expected that the
effective stress becomes to be equal to zero, that is, the
state of liquefaction occurs. The liquified sand is removed
easily by the flow tangential to the surface, and the sand
layer will be scoured successively. Therefore, the clarifi-
cation of the dynamical mechanism of such liquefaction is
very important to design of hydraulic structures arounded by
the sand layer.

    In this report as an basic study for the explanation of
the local scouring mechanism around the hydraulic structures
and for the establishment of the prevention works against
the scouring the influence of the properties of the oscillating
water pressure and the sand layer on the characteristics of
the liquefaction is investigated theoretically and experi-
mentally.

2. Derivation of fundamental equation for pore water pressure

    In the theoretical treatment we consider the vertically
one-dimensional sand layer as shown in Fig.1. The layer is
highly saturated with water and is placed on the impermeable
bed. The depth of the layer is D. The oscillating water
pressure $\rho g h_S$ acts on the surface of the layer.

    The motion of the water and the sand in the layer are
analized by the same method as for the ground water problems
in the elastic aquifer[1], [2]. That is, it is assumed that
the sand and the water are compressible, and then that the
density of water, the porosity of the sand and the thickness
of the layer are variable. But lateral strain of the sand is
assumed to be negligible. The void in the layer is occupied
by the water and the air. Then, it is assumed that the
porosity λ is composed of the part for the water λ_w and the

part for the air λ_a.

We consider the flow of
water through the faces of
an elemental volume as shown
in Fig.1. By the law of
conservation of matter the
net inward flux must equal
the rate at which water is
accumulating within that
volume. That is,

$$\frac{\partial}{\partial t} \, (\rho\lambda_w\delta y\delta A)$$

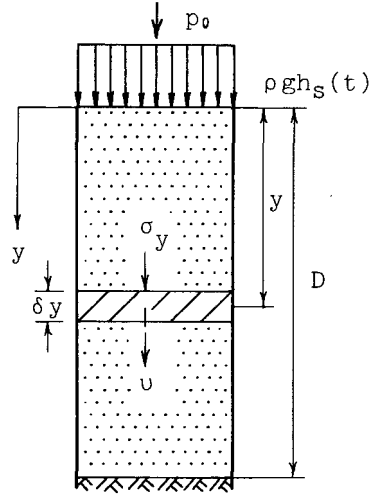$$= -\frac{\partial}{\partial y} \, (\rho\upsilon\delta A)\delta y \quad (1)$$

Fig.1 Sand layer under
Oscillating water
pressure

where, ρ: density of water,

υ: velocity of flow,

δA: horizontal cross sectional area.

The left hand side of the above equation is transformed
as follows,

$$\frac{\partial}{\partial t} \, (\rho\lambda_w\delta y\delta A) = (\lambda_w\frac{\partial\rho}{\partial t}\delta y + \rho\frac{\partial\lambda_w}{\partial t}\delta y + \rho\lambda_w\frac{\partial\delta y}{\partial t})\delta A \quad (2)$$

The vertical dimension of the elemental volume δy
varies with the vertical component of compressive stress σ_y
in accordance with,

$$d(\delta y) = -\alpha\delta yd\sigma_y \qquad (3)$$

That is,

$$\frac{\partial(\delta y)}{\partial t} = -\alpha\delta y\frac{\partial\sigma y}{\partial t} \qquad (4)$$

where, α: vertical compressibility of the sand.

Since the compressibility of the individual sand grains
is small compared to the compressibility of the water and
the change in the porocity, it will be permissible to
consider that the volume of solid material in the volume

remains constant. That is,

$$\{1 - (\lambda_w + \lambda_a)\} \, \delta y \delta A = \text{constant}.$$

Differentiating the above equation by time variable t, we obtain the following equation.

$$\frac{\partial(\lambda_w + \lambda_a)}{\partial t} = -\alpha\{1 - (\lambda_w + \lambda_a)\} \frac{\partial \sigma_y}{\partial t} \qquad (5)$$

If the inertia force of the sand is negligible small, it is permissible to consider that the vertical component of compressive stress and the pore water pressure can be equated to the downward acting force on the plane of contact of the sand. That is,

$$\sigma_y + \rho g h = \gamma_s y + \rho g h_s \qquad (6)$$

The pore water pressure $\rho g h$ and the weight of the sand column above the plane of contact $\gamma_s y$ are represented as follows.

$$\rho g h = \rho g(h_s + y + h') \quad, \qquad (7)$$

$$\gamma_s y = \rho_s g y(1 - \lambda) + \rho g y \lambda_w \qquad (8)$$

where, h': excess pore water pressure,
$\rho_s g$: weight of unit volume of the individual sand
grain,
g: accelaration due to qravity.
Substituting Eqs.(7), (8) into Eq.(6), and considering $\lambda_w \fallingdotseq \lambda$,

$$\sigma_y + \rho g h' = (\rho_s - \rho) \, g y \, (1 - \lambda) = \text{constant.} \quad (9)$$

Accordingly,

$$\rho g \, \frac{\partial h'}{\partial t} = - \frac{\partial \sigma_y}{\partial t} \qquad (10)$$

Then, the following equation is obtained from Eq.(4) and Eq.(10).

$$\frac{\partial \delta y}{\partial t} = \rho g \alpha \delta y \frac{\partial h'}{\partial t} \tag{11}$$

From Eq.(5) and Eq.(10),

$$\frac{\partial (\lambda_w + \lambda_a)}{\partial t} = \rho g \alpha \{1 - (\lambda_w + \lambda_a)\} \frac{\partial h'}{\partial t} \tag{12}$$

It is assumed that the volume of the air in the layer changes in accordance with Boyle's law. That is,

$$\lambda_a \delta y \delta A (\rho g h + p_0) = \overline{\lambda}_a \delta y \delta A (\rho g \overline{h} + p_0) = \text{constant}, \tag{13}$$

where, p_0: atmospheric pressure,

$\overline{\lambda}_a$: porosity occupied by the air under the standard pressure $(\rho g \overline{h} + p_0)$.

Differentiating Eq.(13) by t, and using Eq.(10), the following equation is obtained.

$$\frac{\partial \lambda_a}{\partial t} = -\frac{\lambda_a}{(h + p_0/\rho g)} \frac{\partial h}{\partial t} - \rho g \alpha \lambda_a \frac{\partial h'}{\partial t} \tag{14}$$

From Eqs.(12), (14),

$$\frac{\partial \lambda_w}{\partial t} = \rho g \alpha (1 - \lambda_w) \frac{\partial h'}{\partial t} + \frac{\lambda_a}{(h + p_0/\rho g)} \frac{\partial h}{\partial t} \tag{15}$$

The density of water changes in accordance with the following equation,

$$\frac{d\rho}{\rho} = \beta dp, \quad p = \rho g h + p_0 \tag{16}$$

where, β: compressibility of water,

p: absolute pressure.

Accordingly,

$$\frac{\partial \rho}{\partial t} = \beta \rho^2 g \frac{\partial h}{\partial t} \tag{17}$$

Substituting Eqs.(11), (15), (17) into Eq.(2),

$$\frac{\partial}{\partial t} (\rho\lambda_W \delta y \delta A) = \rho^2 g\{(\beta\lambda_W + \frac{\lambda a}{\rho g h + p_0}) \frac{\partial h'}{\partial t} + \alpha\frac{\partial h'}{\partial t}\}\delta y \delta A$$

(18)

It is assumed that the velocity of water in the layer is represented by Darcy's law,

$$\upsilon = -k \frac{\partial h'}{\partial y}$$

(19)

where, k: permeability coefficient.

Substituting Eqs.(18), (19) into eq.(1), finally we obtain the fundamental equation for the water pressure in the layer as follows.

$$(\beta\lambda_W + \frac{\lambda a}{\rho g h + p_0} + \alpha) \frac{\partial h'}{\partial t} + (\beta\lambda_W + \frac{\lambda a}{\rho g h + p_0})\frac{\partial h_S}{\partial t}$$

$$= \frac{k}{\rho g} \frac{\partial^2 h'}{\partial y^2}$$

(20)

By solving the above equation under the following boundary conditions, the pore water pressure and the effective stress in the sand layer are obtained.

$$\left.\begin{array}{llll} h' = 0 & at & y = 0 \\[2mm] \frac{\partial h'}{\partial y} = 0 & at & y = D \end{array}\right\}$$

(21)

The liquefaction occurs under the following condition,

$$\frac{\sigma_y}{(\rho_S - \rho)gy(1 - \lambda)} = 0$$

(22)

For the case that the water pressure acting on the surface changes sinusoidally, that is,

$$h_S = h_0 + a_0 \sin 2\pi f t,$$

(23)

Eq.(20) becomes,

$$(\beta\lambda_W + \frac{\lambda a}{\rho g h + p_0} + \alpha)\frac{\partial h'}{\partial t} + 2\pi f a_0 (\beta\lambda_W + \frac{\lambda a}{\rho g h + p_0})\cos 2\pi f t$$

$$= \frac{k}{\rho g} \frac{\partial^2 h'}{\partial y^2}$$

(24)

where, h_0:   head of mean water pressure on the sand surface,

   a_0:   amplitude of oscillating water pressure,

   f :   frequency of oscillating water pressure,

   Eq.(20) and Eq.(24) are nonlinear differential equations. Since it is difficult to solve these equations analytically, generally they are solved numerically.

3.   Experiments on liquefaction

3.1   Experimental procedure

   For the experiment the vertical cylinder as shown in Fig.2 was used. The inside diameter of the cylinder is 8.9cm. Its height is 1.5m. It is filled with the highly saturated sand to about 43 % of its height. The water depth from the sand surface is about 80cm. The periodically oscillating air pressure acts on the water surface. Its amplitude is 95cm in water head. The frequency changes from 1.37 Hz to 5.66 Hz. The nearly uniform sand which 50% grain size is 0.25mm was used. Two kinds of the sand sample concerning to the volume of the air in the layer was used. That is, the one sample which $\overline{\lambda}_a$ is about 0.5% at $\overline{h}$ = 0.3m and the another which $\overline{\lambda}_a$ is very small comparing with the former sample. In the following description each sample is called the aerated and the deaerated sample. The value of λ_w is about 40%. The permeability coefficient is about 2×10^{-2}cm/s. The water pressure in the water and sand layer are picked up by the semi-conductor pressure transducer attached to the side of the cylinder.
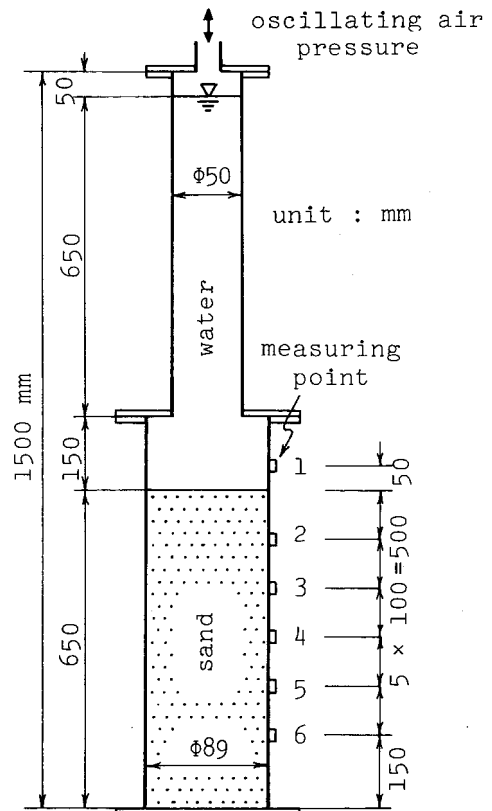

Fig.2 Model in experiments

3.2 Experimental results

In Fig.3 (a), (b) the changes of the pore water pressure
for the deaerated and the aerated sands are shown. These
figures show that the water pressure on the sand surface
propagates into the sand layer accompanied with the damping
in amplitude and the lag in phase. The rate of the damping
and the lag in the aerated sand is larger than that in the
deaerated one.

Fig.4 (a), (b) show the damping ratio of the amplitude
for both samples. The symbol a in the figures represents
the amplitude in the sand layer. These figures show that
the amplitude decreases with increasing frequency.

Fig.5 (a), (b) show the change of the effective stress.
It is shown in these figures that the state of the lique-
faction ($\sigma_y = 0$) appears at the point 1 for the deaerated
sample and at all measuring points for the aerated one.

Figs.6, 7 and 8 show the calculated curves of the pore
water pressure, the damping ratio and the effective stress
corresponding to Figs. 3, 4 and 5 respectively. In the
calculation it is assumed that the oscillating water pressure
on the sand surface changes sinusoidally, and that the value
of the compressibility of the sand α is $1.8 \times 10^{-6} \, cm^2/g$.
Comparing with experimental results, the calculated curves
explain the experimental results fairly well. The differ-
ences between the experimental values and the calculated
curves are considered to be mainly due to the approximation
of the oscillating water pressure and the compressibility in
the calculation.

4. Characteristics of liquefaction

4.1 Factors affecting liquefaction

The characteristics of the liquefaction depend on the
properties of water, sand layer, oscillating water pressure
and the field. For convenience adopting the sinusoidal
excitation like Eq.(23) as the oscillating water pressure,the
characteristic parameters influencing on the phenomenon are as
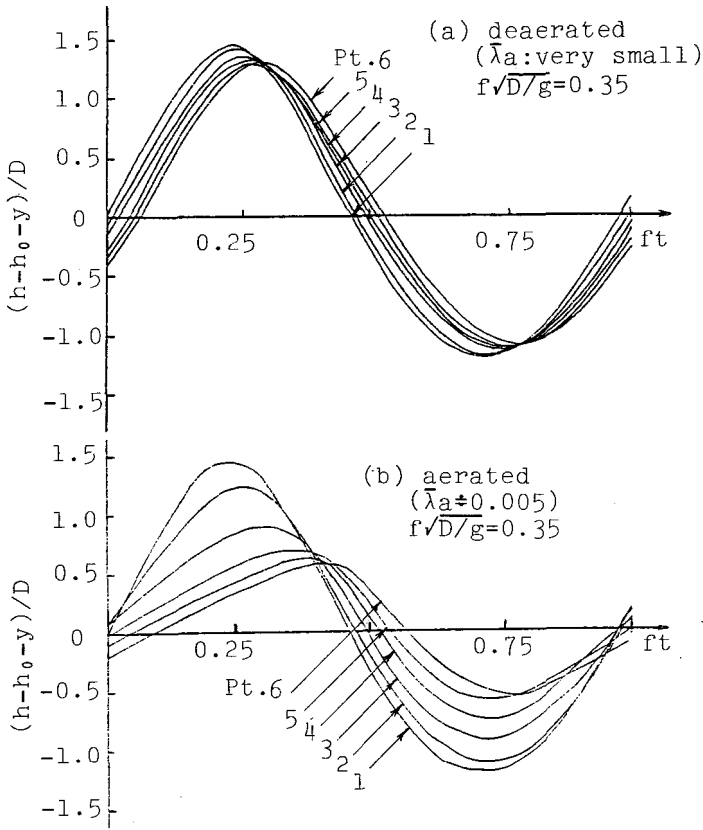
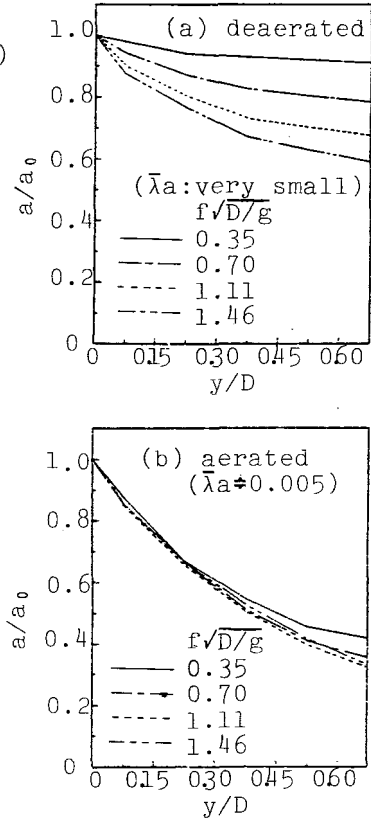Fig.3 Pore water pressure (observed)
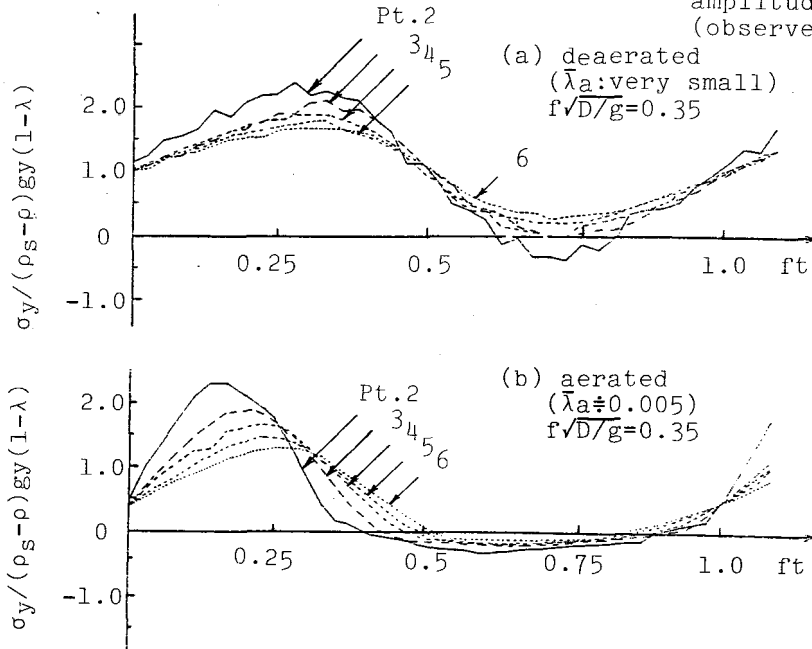
Fig.4 Damping ratio of amplitude (observed)

Fig.5 Effective stress (observed)
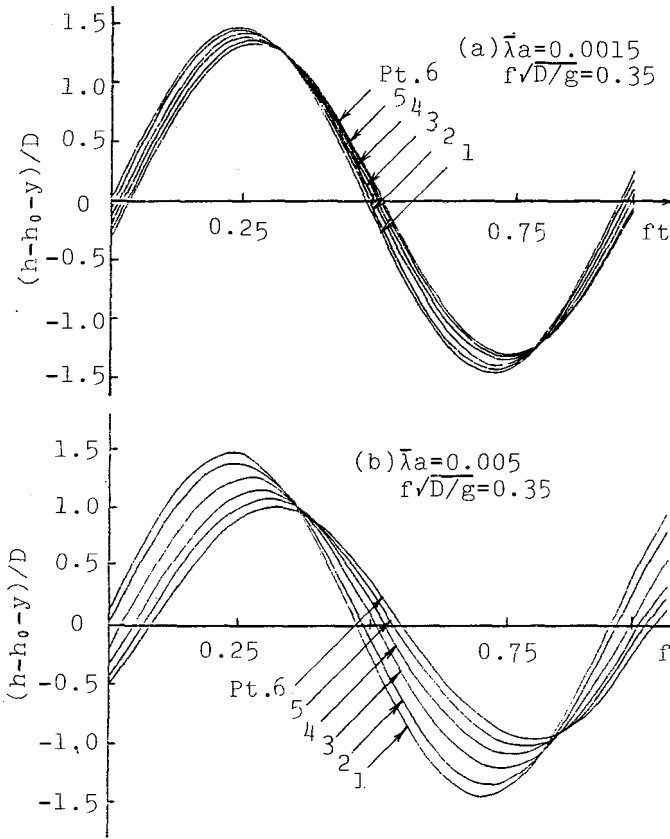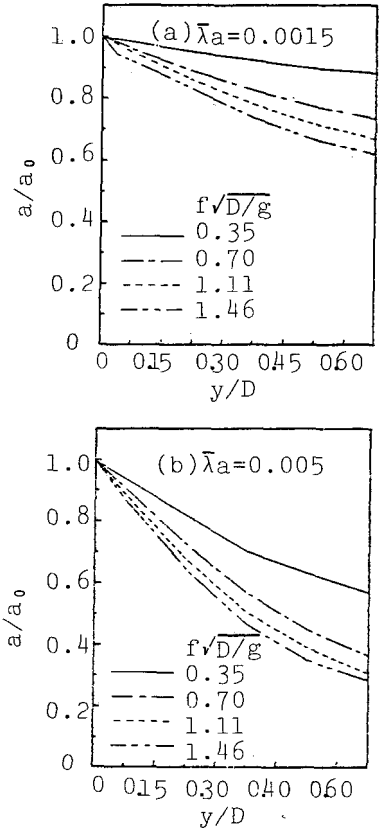
Fig.6 Pore water pressure (calculated)

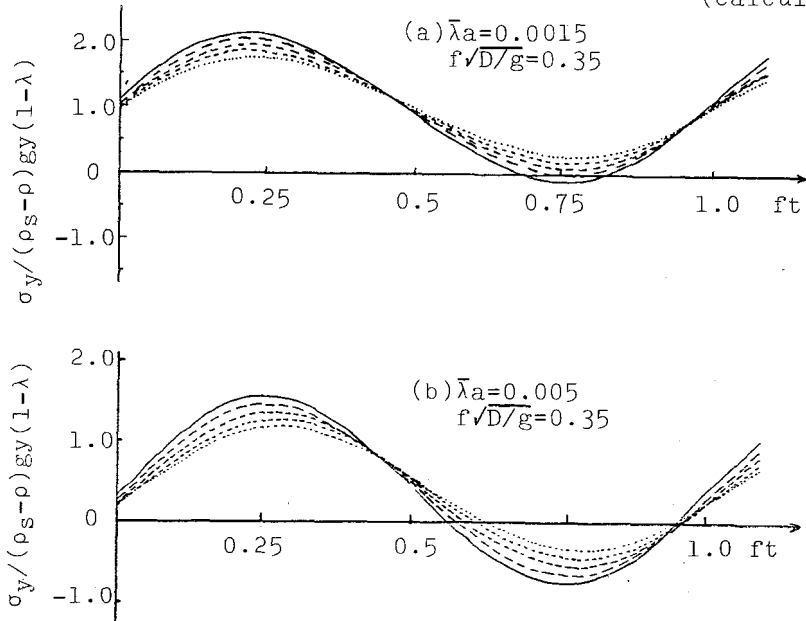Fig.7 Damping ratio of amplitude (calculated)

Fig.8 Effective stress (calculated)

follows.

1) on water   : ρ, β
2) on sand layer   : ρ_s, α, λ_w, $\overline{\lambda}_a$, k, D, $\overline{h}$
3) on water pressure : a_0, f, h_0
4) on the field   : g, p_0

Accordingly, the quantitative property A of the liquefaction is given by the following functional relation.

$$A = F(a_0, \ f, \ h_0, \ \rho_s, \ \alpha, \ \lambda_w, \ \overline{\lambda}_a, \ k, \ D,$$
$$\rho, \ \beta, \ g, \ p_0, \ \overline{h} \ ; \ y, \ t) \qquad (25)$$

By the dimensional analysis using ρ,g and D as the fundamental quantities of dimensions the above equation is transformed to,

$$\pi_A = \Phi(\frac{a_0}{D}, \ f\sqrt{\frac{D}{g}}, \ \frac{h_0}{D}, \ \frac{\rho_s}{\rho}, \ \alpha(\rho g D), \ \lambda_w, \ \overline{\lambda}_a,$$
$$\frac{k}{\sqrt{gD}}, \ \beta(\rho g D), \ \frac{p_0}{\rho g D}, \ \frac{\overline{h}}{D}; \ \frac{y}{D}, \ t\sqrt{\frac{g}{D}}) \qquad (26)$$

4.2 Maximum depth of liquified layer

Here we deal with the maximum depth of the liquified sand layer y_* as one of the properties of liquefaction. It is analized theoretically under the following conditions,

$$\rho_s/\rho = 2.65, \ \overline{h}/D = 0.3, \ p_0/\rho g D = 10.3, \ \beta(\rho g D) = 0.41 \times 10^{-5}.$$

In Figs.(9) to (14) the changes of the maximum depth y_* with a_0/D, $\overline{\lambda}_a$, $\overline{\lambda}_w$, $k/\sqrt{gD}$, $\alpha(\rho g D)$ and h_0/D are shown with the parameter $f\sqrt{D/g}$.

From Figs.(9), (10) and (11) it is shown that the depth y_* increases with the increase of a_0/D, $\overline{\lambda}_a$ and λ_w, and that the high frequency stimulates the liquefaction. Especially a very small amount of air in the sand affects the liquefaction considerably.

Fig.12 shows that y_* decreases with the increase of the permeability coefficient in the range beyond certain value of the coefficient.

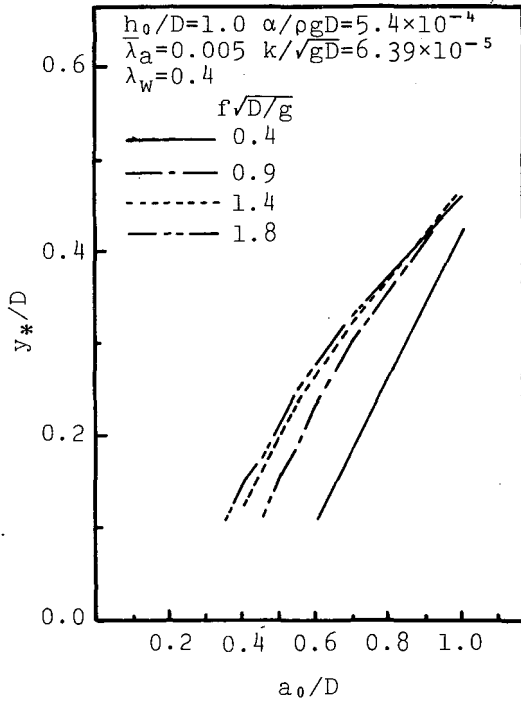Fig.13 shows that y_* decreases rapidly with the increase
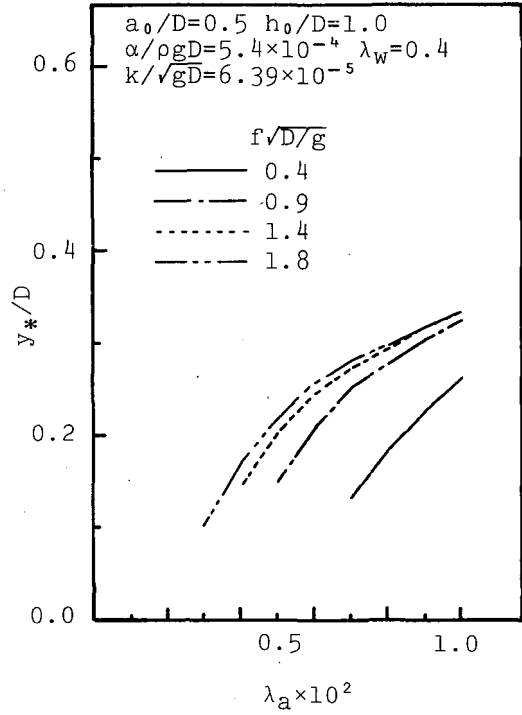
Fig.9 $a_0/D \sim y_*/D$
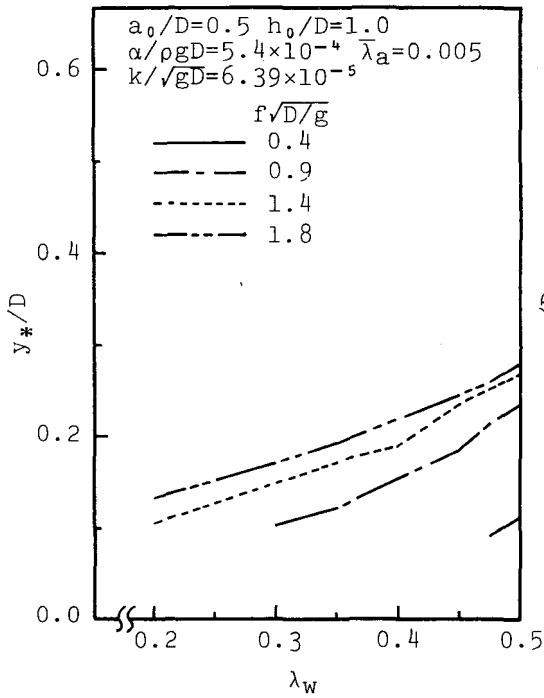

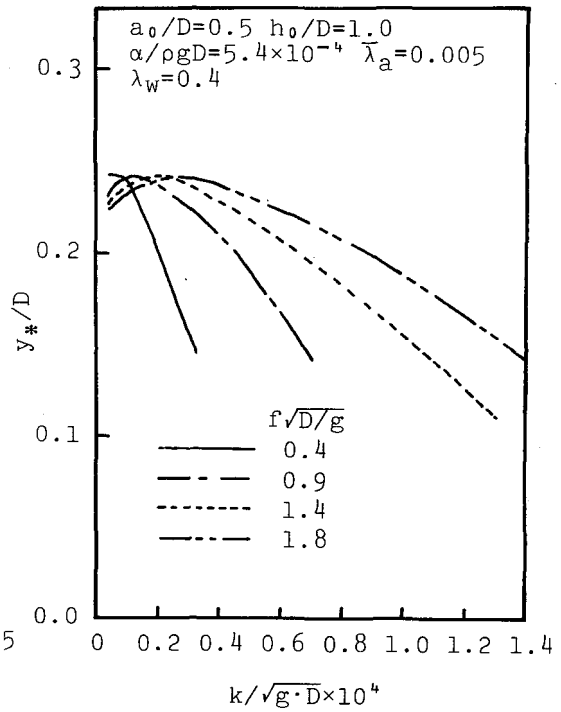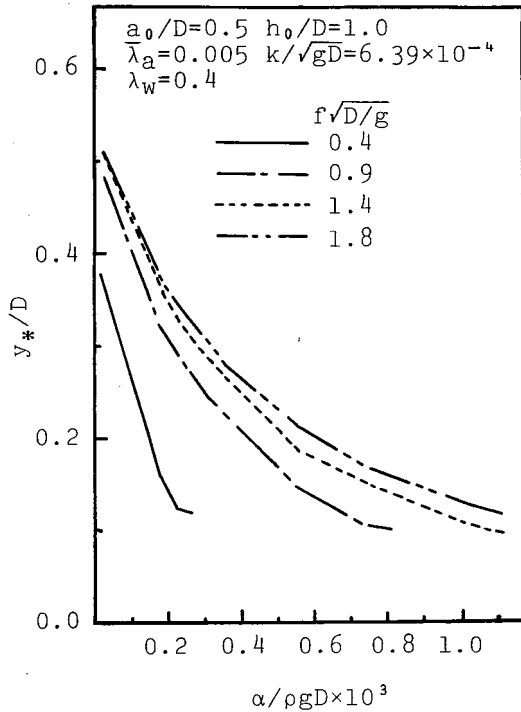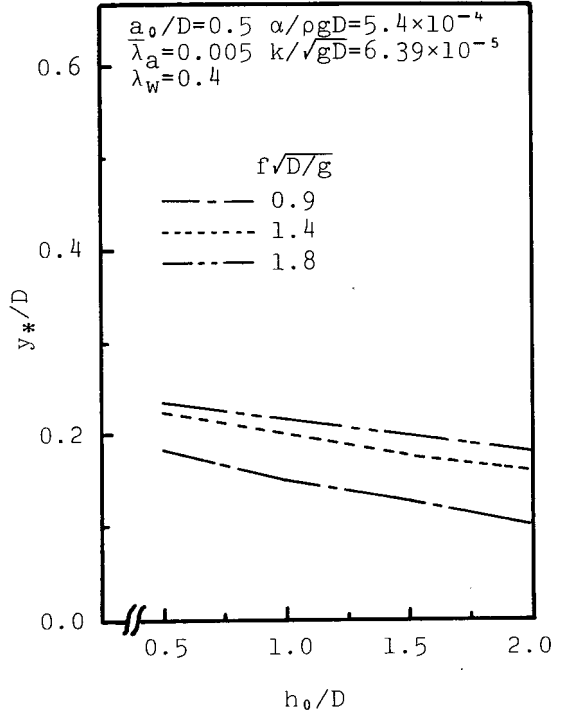Fig.10 $\lambda_a \sim y_*/D$

Fig.11 $\lambda_W \sim y_*/D$

Fig.12 $k/\sqrt{g \cdot D} \sim y_*/D$

Fig.13 $\alpha/\rho gD \sim y_*/D$

Fig.14 $h_0/D \sim y_*/D$

of the compressibility of the sand.

Fig.14 shows that the increase of the mean water pressure decreases the liquified depth. But its degree is very small.

5. Concluding remarks

From the experiments and the theoretical analysis described above, we obtain following conclusions.
1) By the experiments it is clarified that the oscillating water pressure acting on the surface of the highly saturated sand layer propagates into the layer with the damping in amplitude and the lag in phase, and that under certain codi-tions the liquefaction appears. Further it becomes obvious that the damping ratio and the phase lag depend largely on the volume of the air involved in the layer.

These experimental results are explained fairly well by the theoretical analysis.

2) By the theoretical analysis several properties on the
liquefaction are obtained as follows.

The maximum depth of liquified layer increases with the
increase of the amplitude and the frequency of the oscillating
water pressure and the volume occupied by the air and the
water in the sand layer. Especially a very small amount of
the air affects the liquefaction properties significantly.
The increase of the permeability coefficient in the range
beyond certain value decreases the liquified depth. With
the increase of the mean water pressure and the compressibility
of the sand the liquified depth decreases. Especially in
the small range of the compressibility the slope of the
decreasing curve is very steep.

Acknowledgement

References

1) Hunter Rouse: Engineering Hydraulics, John Willey &
    sons, New York, 1950, p.p. 326-331.
2) Reisaku Inoue: Propagation of pore water pressure in
    sand layer of high degree of saturation, Proc. JSCE,
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