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Experimental and Theoretical Study on a Transient, Turbulent Free Hydrogen Gas Jet Issuing into Still Air

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SYNOPSIS

Distributions of hydrogen gas concentration in a suddenly started, single shot hydrogen gas jet issuing from a 1 mm diameter injector into still air were measured using laser interferometry method. This unsteady, turbulent free jet flow has also been calculated using the two-equation, high Reynolds number version of k-ε turbulence model and hybrid scheme for treating combined diffusion and convection in the SIMPLE algorithm. The injection pressure was 0.5 MPa for which predicted and measured temporal jet tip penetration distributions indicate that the jet discharged into still air at Mach 0.25. The level of agreement between present prediction and measurement is good in some regions and poor in others.

1. INTRODUCTION

Hydrogen gas is believed to be one of the important alternative fuels. Experimental and theoretical investigation of its perfomance characteristics during its injection and combustion in diffusion combustion systems like internal combustion engines, turbine combustion chambers, furnaces, domestic gas stoves, bunsen burners is therefore necessary for comprehensive understanding of these characteristics. Precise knowledge of the transient distributions of velocity, temperature, hydrogen concentration in the hydrogen-air

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mixture flow and the jet tip penetration is also needed for establishing sound design guide lines of such diffusion combustion systems which would use hydrogen gas as the fuel. In particular, the transient distributions of the fuel concentration in both transverse and axial directions, turbulence and jet penetration are of utmost importance in combustion analysis.

Fuel gases are generally injected into still air at high speed, usually at Mach number, Ma, greater than 0.3, in order to enhance turbulence and so mixing with the oxidant like oxygen in surrounding air. This implies that generally, the hydrogen jet binary mixture flow would also most probably be expected to be turbulent and compressible. This means that during jet propulsion, the flow gas mixture densities vary greatly with time and space as a result of high injection velocity, temperature changes, intense mixing between fuel gas and air. In the case of hydrogen gas diffusing in air these density variations near the jet boundary mixture are quite large because of the large mass diffusivity of the hydrogen gas-air medium. In addition, the large initial density difference between that of the injected hydrogen gas and ambient surrounding air which can be as high as 1400 % contributes to these large density variations.

Contemporary experimental work on measurement of transient concentration of hydrogen gas in transient, turbulent free round jets include that due to Tanabe et al.[1] and Hamamoto et al.[2]. Takayama et al.[3] have numerically predicted a sonic hydrogen jet spurted into still air.

2. MEASUREMENT

Measurement principles and procedures used in present experimental investigation of hydrogen gas concentration distributions in the transient jet by laser interferometry are outlined by Hamamoto et al.[2]. The injection pressure which was maintained very nearly constant during jet propulsion was 0.5 MPa while the injection duration was 11 ms after which the injection was stopped.

Schlieren photographs of the transient jet were also taken using stroboscope light for illumination and the knife edge optical arrangement. The jet tip penetration were determined from schlieren photographs and measured arrival time at pre-known axial distances of measurement locations.

3. NUMERICAL CALCULATION

The governing equations, boundary and initial conditions as well as numerical solution procedures employed in calculation of the present hydrogen gas jet issuing into still air are same or similar to those used in numerical calculation of the transient, free sonic methane gas jet discharging into quiescent atmosphere[4]. In the present case of hydrogen gas jet discharging at Mach 0.25 under atmospheric conditions, the initial temperature, T_o, and initial density, ρ_o, of injected hydrogen gas at the nozzle exit were estimated to be 298 K and 0.08267 kg/m³ respectively. The surrounding air temperature, T_s, was 301 K. The eddy Schmidt number, Sc_t, and Prandtl number, Pr_t, used were 0.7 and 1.0 respectively. The initial mass concentration, mf_o, at the nozzle exit was set equal to unity assuming pure hydrogen gas was injected. This means the initial mole fraction, mof_o, was also set to unity at the nozzle exit. The timestep for the prediction results presented herein was 0.2 ms. The low initial jet speed implies that the jet flow was incompressible.

Convergence was taken to be attained when the net efflux over all control volumes was practically small. However, use of practically small axial velocity difference of up to 0.8×10^{-3} m/s between adjacent iterations in the same timestep at all or several critical grid nodes as a convergence criterion also produced almost same numerical prediction results.

4. RESULTS

Typical measurement results of the injection control pulse and output transistor signal at the nozzle exit centre($x = 0$, $r_n = 0$) shown in Fig.1 were used for evaluating the precise initiation and end of

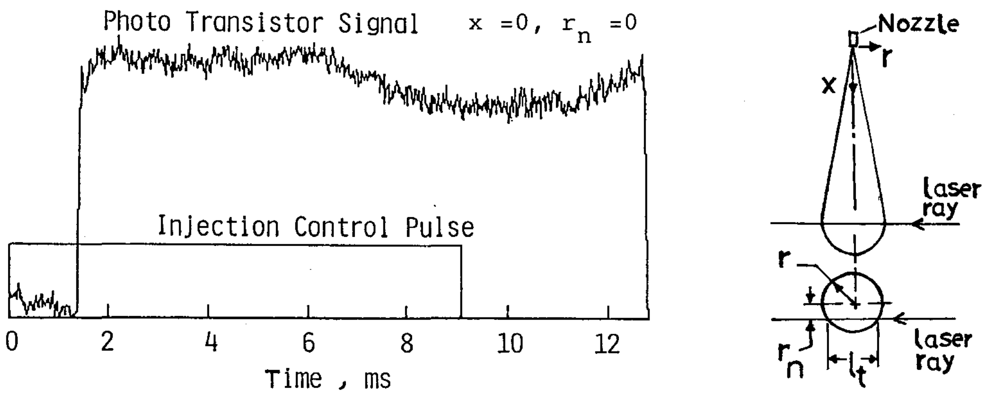
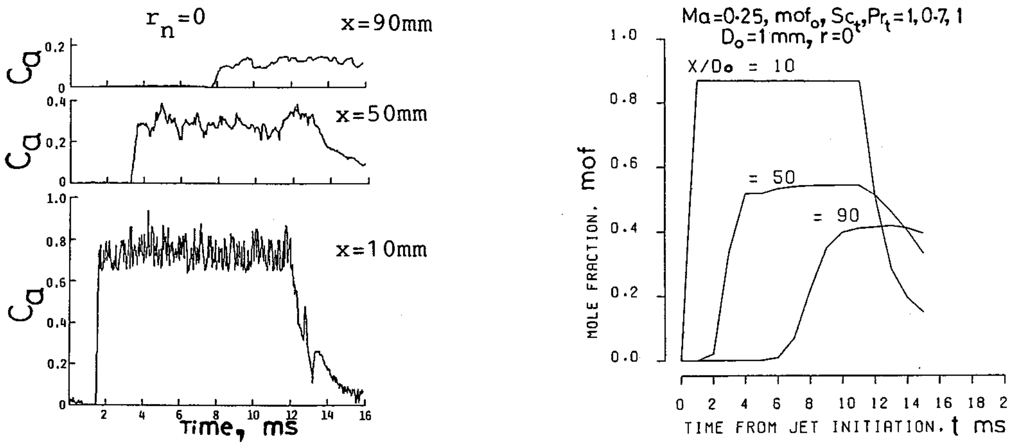


Fig.1 Signal at the nozzle exit

jet propulsion. It is seen that the starting and ending of the actual free jet propulsion were respectively delayed and prolonged by about 1.38 and 3.8 ms after the start and end of the injection control pulse. The actual jet propulsion period was about 11 ms. It is also seen from Fig.1 that throughout the jet propulsion period, the amplitude of the photo-transistor output signal was nearly constant. It was therefore inferred that jet propulsion was satisfactorily at uniform at the nozzle exit.

Figure 2(a) shows distributions of the measured instantaneous line averaged hydrogen gas concentration, C_a, on some jet axis locations(r_n = 0). It is noted that C_a is line-averaged over the total test distance traversed by the laser ray while crossing the jet flow. Here, r_n, is the perpendicular distance locating the laser ray from the jet axis. Figure 2(b) shows present prediction results of instantaneous point hydrogen gas concentration, mof, on some jet axis points at the same axial locations shown in Fig.2(a). Obviously C_a is



(a) Measurement-line mean (b) Prediction-point value

Fig.2 Transient hydrogen gas concentration on jet axis locations

expected to be lower than the predicted, mof, simply because C_a is averaged over the whole test distance, l_t, across the jet.

Upstream of the inner turbulent core of the jet represented by r_n = 0, x = 10 mm, the present measurement shows that the concentration rises suddenly and becomes nearly constant with respect to time due to strong fluctuation and mixing resulting from turbulent flow. In the mid-stream part(r_n = 0, x = 50 mm), measurement indicates that the concentration rises later and becomes nearly steady with weaker fluctuations than those present in the upstream part. The rise in concentration occurs much later in the downstream part (r_n = 0,x

=90 mm) where the trend and fluctuations are again remarkably uniform and steady with gentle and smaller fluctuations. Predicted transient concentration distributions show that the time rate of rise and fall of concentration to and from the quasi-steady level decrease with the axial distance from nozzle. General trends of the present prediction of hydrogen gas shown in Fig.2(b) compare reasonably well with measurement although there are some local regions of poor agreement.

Similarly, present measurement results of the instantaneous line-averaged hydrogen gas concentration distributions near the jet boundary ($r_n \simeq r_{max}$) at cross sections located at x = 10, 30, 50 mm are shown in Fig.3. Here also, present measurement shows that the intensity of concentration fluctuations decreases with the axial distance, x , from the nozzle. For instance at x = 10 mm, these fluctuations are stronger than those at x = 30 mm. These fluctuations exist partly because near the jet boundary-surrounding air interface, the jet flow occurs randomly and intermittently as well as alternately in space and time. This is mainly due to the shedding and propagation of vortices from the jet source in the nozzle. The difference in intensity of the fluctuations in the upstream and downstream regions

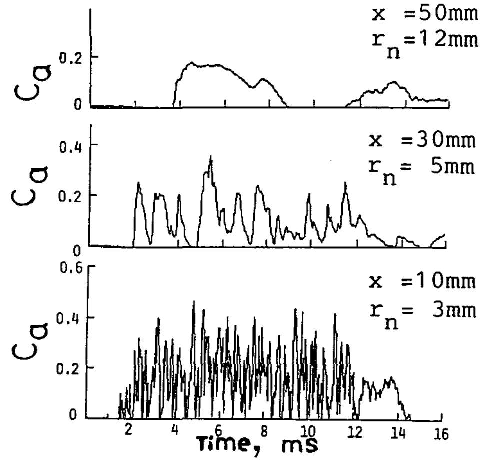


Fig.3 Transient hydrogen gas concentration near jet boundary

of the jet is probably due to the corresponding turbulence intensities which are stronger upstream. In addition, the different sizes of eddies so created and transmitted in these regions contribute to the discrepancy in concentration fluctuations.

A typical Schlieren photograph of the jet is shown in Fig.4. It is seen that upstream the jet flow, the flow is very smooth possibly due to the very small eddies produced near the jet flow in this region of the jet. However, further downstream the roughness of the jet boundary is clearly visible and the existence of much larger eddies which are also visible is verified by the photograph to a good extent. The photograph also shows that the jet flow is actually not symmetrical about the jet axis. It is important to note at this stage that in two-dimensional calculations of round or plane jets like in the numerical calculations and the onion peeling model used in the present study, symmetry of flow about the jet axis or symmetry line is assumed. However, this assumption is extremely difficult to realise in practice as is exhibited by the unsymmetrical actual hydrogen gas jet shown in the photograph of Fig.4.

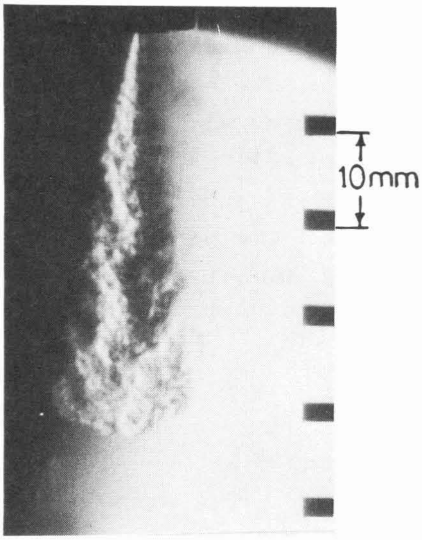


Fig.4 Schlieren photograph of hydrogen gas jet in still air

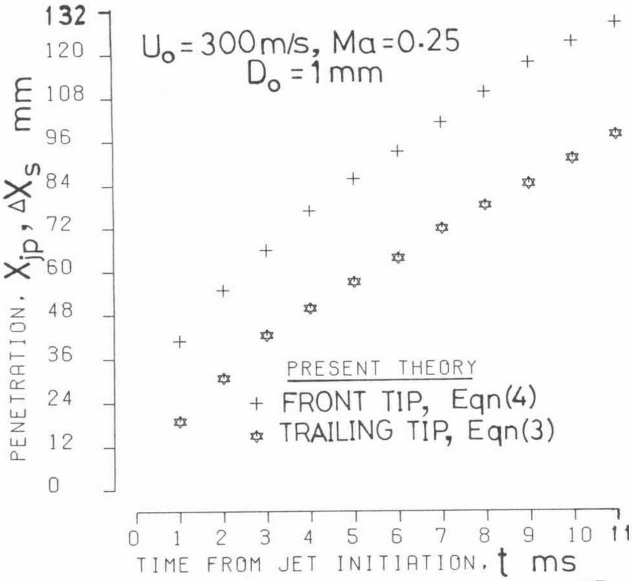
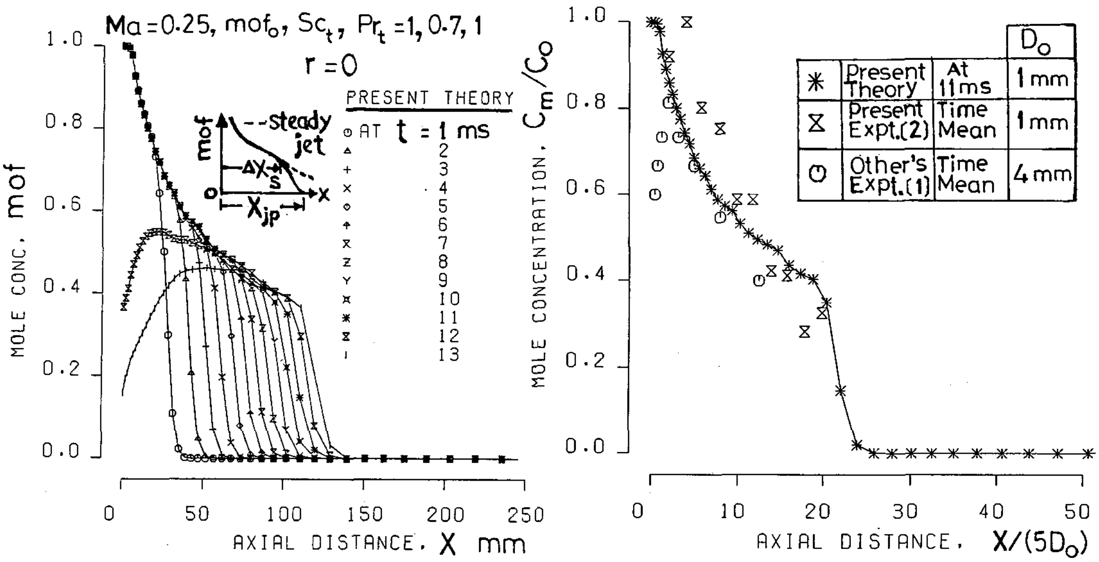


Fig.5 Trailing quasi-steady and total jet tip penetrations

Using the measured ensemble-averaged data of fringe shifts for the quasi-steady part of jet only, the onion peeling model and relevant equations, Hamamoto et al.[2] determined line-averaged hydrogen gas concentration in thin annular regions at different cross sections. Such predicted instantaneous and measured quasi-steady axial distributions of hydrogen gas concentration on the jet axis ($r = 0$) are shown in Fig.6(a) and (b) in dimensional and normalized forms respectively. In Fig.6(b), the concentration is normalized by the initial concentration, $C_o = 1.0$, at the nozzle exit while the axial distance is normalized by the potential core length, $X_{pc} = 5D_o$. Experimental data in the fully developed main region as obtained by Tanabe et al.[1] indicate a general trend similar to the present prediction and measurement. Present measurement results are rather high near the nozzle while those obtained by Tanabe et al.[1] in this same region seem relatively low. The trend of the axial decay of



(a) Dimensional (b) Normalised

Fig.6 Axial distribution of hydrogen gas concentration on jet axis

concentration on the jet axis is similar to that of the axial velocity, u, probably due to the fact that it is the same flow velocity, that is specific momentum, that transports matter represented by the chemical species concentration by way of convection and diffusion mechanisms.

Again present prediction results shown in Fig.6(a) indicate that the rear part(near nozzle) of the jet becomes quasi-steady almost soon after jet initiation while the front part of the jet continues to be transient right up to 11 ms after jet initiation. Beyond $x \simeq 50$ mm, there is instability in predicted concentration and density on the jet axis as the concentration rises and drops about a mean kind of profile. Both the present measurement and prediction show that in the rear quasi-steady part of the jet, concentration on the jet axis decays with the axial distance, x. After the end of hydrogen gas injection, the predicted concentration decreases suddenly near the nozzle, rises with the axial distance for a short distance and then follows the trend attained during injection as shown at time t =12, and 13 ms in Fig.6(a). It is also noted that the jet continues to penetrate the surrounding air even after the end of injection probably due to the remaining momentum still possessed by the collapsing jet.

In comparison to a heavier gas like methane used as the primary jet gas issuing into still air, the present measurement and prediction show that on the jet axis, hydrogen gas concentration in the binary gas mixture is strikingly and considerably higher particularly near the nozzle at corresponding axial locations. The present numerical calculation results show that in the rear quasi-steady region of the downstream main region the mole concentration, C_m, decays with the axial distance, x, according to the following relation;

$$\frac{C_m}{C_o} = 0.874e^{-0.009(x/D_o)} \quad \text{-------------(1)}$$

where C_o and D_o are the initial hydrogen gas concentration and initial jet diameter at the nozzle exit assumed to be equal to unity and the nozzle exit diameter respectively. This same predicted data of the decay of hydrogen gas concentration on the jet axis in the same down stream region may be represented by the following simpler and slightly less accurate expression;

$$\frac{C_m}{C_o} = 1.523\{\frac{D_o}{x}\}^{0.262} \quad \text{-------------(2)}$$

Although equations (1) and (2) are different in form from that given by Hinze[5] for some steady, turbulent gas jets discharging into dissimilar surrounding medium, they express concentration which is inversely proportional to the axial distance, x, from the nozzle exit in a similar way Hinze's equation does.

Present prediction results shown in Fig.6(a), indicate that the axial length of the quasi-steady rear part of the jet, ΔX_s, increases with time, t, after jet initiation for $1 \leqq t \leqq 11$ms according to the following relation;

$$\Delta X_s = 19.43\ t^{0.672} \quad \text{-------------(3)}$$

where ΔX_s is in mm and t in ms. This equation (3) represents the approximate instantaneous axial penetration of the rear quasi-steady part of the transient jet. Also from Fig.6(a), present prediction shows that at time $1 \leqq t \leqq 2$ ms the total jet tip penetration, X_{jp}, is nearly linear with time while at $t > 2$ ms penetration varies almost linearly with the square root of time as expressed by the following relation;

$$X_{jp} = 39.752\ t^{0.484} \quad \text{-------------(4)}$$

where X_{jp} is in mm and t in ms. These prediction results of both the trailing, rear quasi-steady and the total jet tip penetration distributions are shown in Fig.5. Many researchers like Komoda et al.[6], Witze[7], and Hamamoto et al.[8] found similar trends for the total jet tip penetration in their respective experimental work on transient, turbulent free sonic methane gas, air, and various gas jets issuing into still air or dissimilar gas. Others like Ha et al.[9] and Hiroyasu et al.[10] also found similar trends of in their measurement of diesel spray penetration in still air. Chiu et al.[11] have obtained similar distributions of diesel spray penetration in internal combustion engines using numerical calculation models.

The predicted and measured X_{jp} in the axial direction is shown in Fig.7. The experimental penetration was determined using two methods. In one method axial penetration distances were obtained from still Schlieren photographs taken at different times during jet flow development. The second method involved use of measured arrival times at accurately pre-known axial locations on the jet axis($r = 0$) and on some other radial locations offset from the jet axis by $r = r_f$ where r_f is the radial distance locating some measurement locations used in above interferometry tests. Assuming mass diffusion was negligibly small compared to mass convection in the axial direction during transient jet propulsion, the arrival time of the jet at these locations was taken to be the time at which the hydrogen gas concentration suddenly started rising. The theoretical distribution of jet tip penetration was determined from predicted instantaneous axial distributions of hydrogen gas concentration on the jet axis($r = 0$) shown in Fig.6(a). Thus in the present prediction, the theoretical jet penetration at a given time was taken to be axial distance at which the concentration practically dropped to zero. After using different values of initial axial velocity, U_o, in several trial and error numerical calculations, measured and predicted jet tip penetration distributions coincided when U_o was set at 300 m/s which corresponds to Mach number 0.25 approximately. From this low initial jet speed, it was therefore inferred that despite the seemingly high injection pressure in the gas reserviour, the experimental jet flow was most probably incompressible and subsonic. It seems the actual effective injection pressure inside the nozzle was considerably lower than the injection pressure.

For penetrations up to X_{jp} = 60 mm, the measurement obtained from Schlieren photographs and by arrival time methods are in satisfactory agreement although at X_{jp} greater than 60 mm penetration from photographs become gradually greater. Penetration measurements on the jet axis and offset locations are not identical probably due to the fact that the jet shape itself is not symmetrical about the jet axis as its photograph shown in Fig.4 indicates. In addition this unsymmetrical jet on the photograph indicates that the fastest part of actual jet is not necessarily on the jet axis all the time. The effect of deceleration of the mean jet flow down stream due to resistance of the frontal surrounding air also contributes to the discrepancy in jet

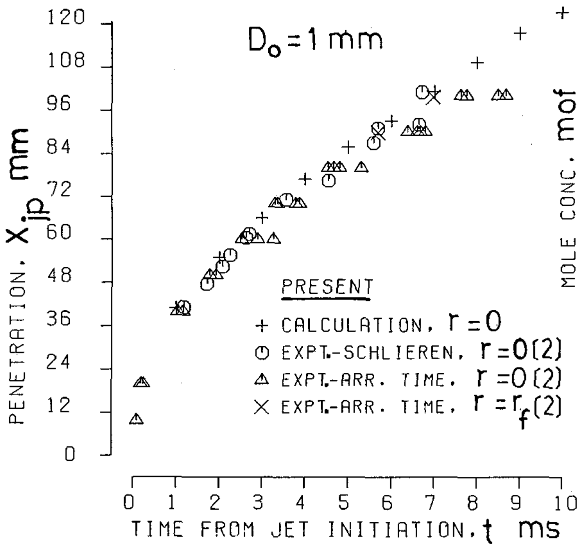


Fig.7 Jet tip penetration

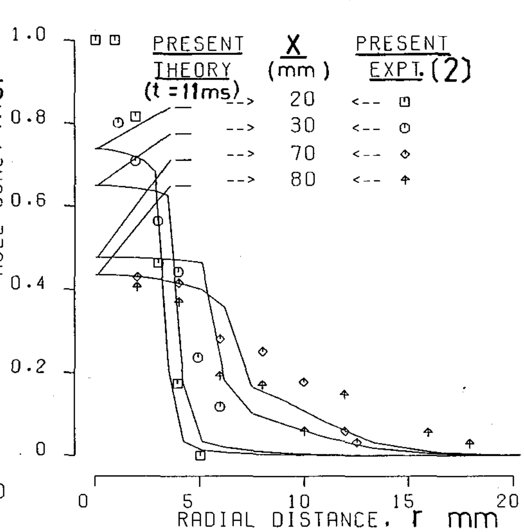


Fig.8 Radial profile of hydrogen gas concentration

penetration measured by the method of arrival time at the two locations, r = 0 and r = r_f, of a given cross section. At x = 90, 100 mm, penetration measurements by photography and arrival time with r =r_f techniques seem to be in good agreement. The level of agreement between predicted and measured general trends of the jet tip penetration is reasonably good.

Radial distributions of hydrogen gas mole concentration at some cross sections of the rear quasi-steady part of the jet as measured and predicted in the present study are shown in Fig.8. At upstream cross sections like x = 20 mm, both measured and predicted concentration gradients in the radial direction are much higher than those at cross sections located further downstream. This is possibly due to the much higher axial velocity near the nozzle. Further downstream, these concentration gradients gradually decrease with the axial location of the cross section measured from the nozzle. However, within the turbulent core of the jet, present prediction shows that the hydrogen gas concentration gradient is much lower than that for heavier primary jet gases like methane gas. This is probably due to the much higher mass diffusivity and so higher mass diffusion rate in the hydrogen gas-air medium than say in the methane gas-air medium. Outside the turbulent core, prediction shows that the concentration gradient in the radial direction increases almost suddenly. It seems general trends of these results are similar to those obtained by Tanabe et al.[1]. At cross sections located beyond x =50 mm, there seems to be instability in the predicted radial profiles. A similar kind of behavior is exhibited on the axial profile of concentration on the jet axis in this same region as shown in Fig.6(a). It is also noted that the shapes of the present prediction of radial distributions of hydrogen gas concentration

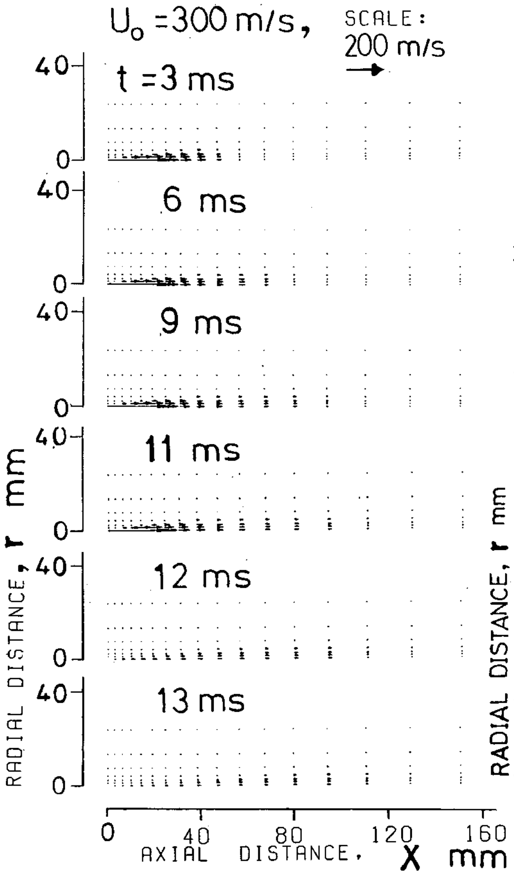


Fig.9 Transient velocity vector fields

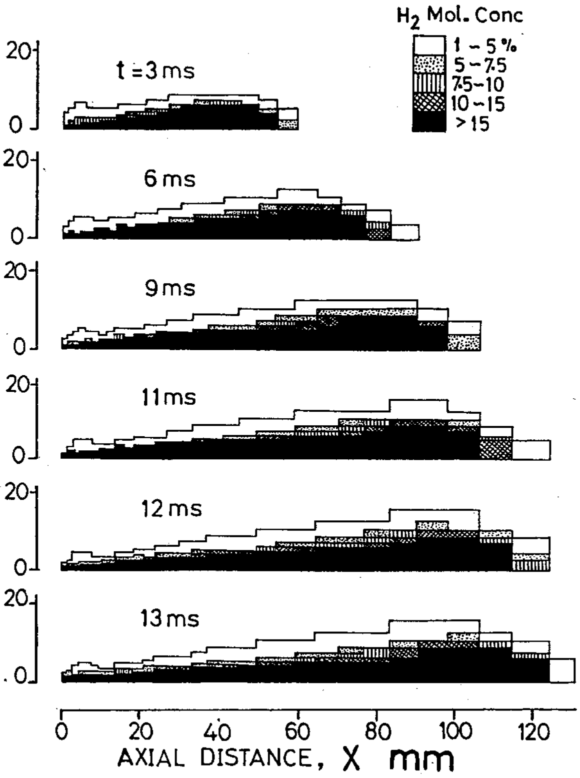


Fig.10 Transient stratified hydrogen gas-air mixture charge

deviate from that of the general universal curves represented by the 3/2 power law for concentration as given by Abramovich[12] for steady state jets issuing into dissimilar fluid or that obtained by Tanabe et al.[13] for helium gas. These predicted radial distributions shown in Fig.8 are somewhat different.

Transient velocity vector fields obtained by adding predicted velocity vectors $\bar{u}$ and $\bar{v}$ are shown in Fig.9. They indicate the approximate transient jet shapes formed during and after jet propulsion.

Present prediction results of the instantaneous stratified hydrogen gas-air mixture charge of the jet flow is shown in Fig.10. Such stratified charge distributions may be used with other relevant information for providing guidelines on design and performance of diffusion combustion systems particularly regarding ignition location and ignition timing aspects.

The predicted axial distributions of transient entrainment in the rear quasi-steady part of the main region of the jet(7 mm $\leqq$ x $\leqq$ ΔX_s) are shown in Fig.11. Here, m_{cs}, is the mass flow rate at a given cross section of the jet whose boundary was located at radii with 3 % of the the axial velocity on the jet axis while m_o is the initial mass flow rate at the nozzle exit. In the present calculations, m_o = 0.1948x10^{-4} kg/s, for U_o = 300 m/s while m_{cs} was evaluated from predicted axial velocity and jet flow mixture density fields by numerical integration.

The predicted data shown in Fig.11 fits the following relation which may be taken as an approximate entrainment law in the rear quasi-steady part of the transient, turbulent hydrogen gas jet issuing into still air;

$$\frac{m_{cs}}{m_o} = 0.261\{\frac{x}{D_o}\}^{1.087} \qquad \text{----------(5)}$$

In linear form, this same predicted entrainment data in the same region of the transient jet very nearly fits the following relation although equation (5) is more accurate;

$$\frac{m_{cs}}{m_o} = 0.42\frac{x}{D_o} \qquad \text{----------(6)}$$

It is noted that for hydrogen gas diffusing in air, the coefficient 0.42 in equation (6) is lower than 1.2 obtained by Ricou et al.[14] for steady, turbulent free hydrogen gas jet discharging into still air. This is partly due to the effects of the still developing transient jet which are difficult to comprehend with regard to entrainment. Although the rear part of the transient jet in the present prediction becomes quasi-steady soon after jet arrival at a cross section, entrainment of surrounding air and diffusion of hydrogen gas to the surrounding air continue to increase until the jet attains its final steady state and so lower entrainment may be expected in our still developing transient jet[14]. In addition to mass transfer by convection, it is noted that even in the almost stationary laminar hydrogen gas-air mixture near the jet boundary, molecular diffusion is significantly high as the heavier air tries to

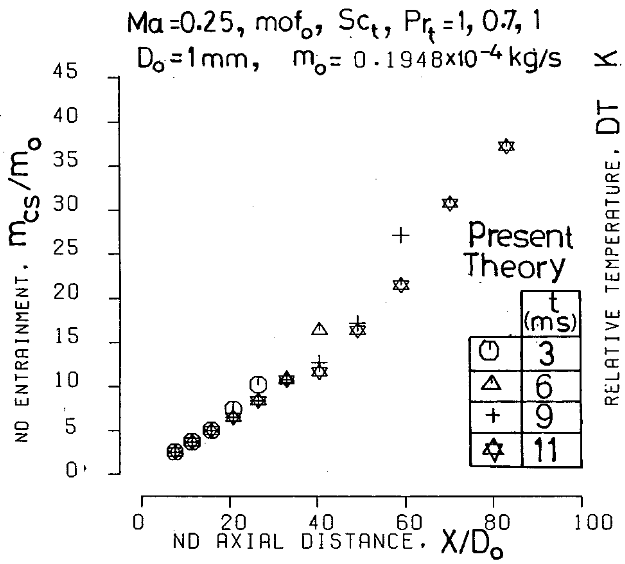


Fig.11 Axial distribution of entrainment

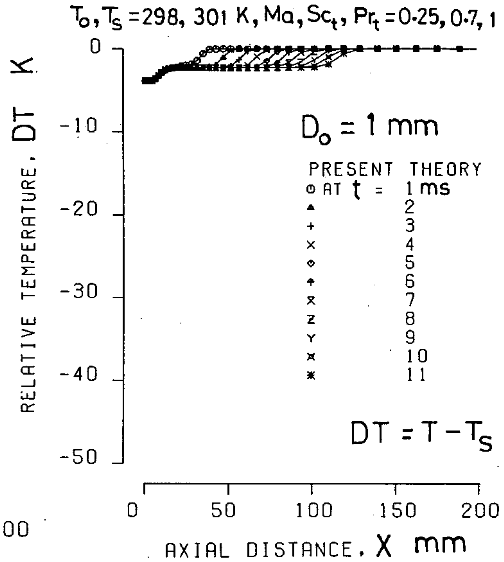


Fig.12 Axial profile of transient temperature on jet axis

displace the much lighter hydrogen gas. As Reid et al.[15] have pointed out, it is partly the large density difference between that of hydrogen gas and air that accounts for the high molecular mass diffusion coefficient of the hydrogen gas-air medium.

The predicted transient axial distribution of temperature on the jet axis is shown in Fig.12. Here, the excess temperature, $DT = T-T_s$, where T is the predicted temperature at the point and T_s is the ambient temperature of surrounding air. It seems that in the initial region, the temperature is slightly lower than the initial temperature perhaps due to the effect of the kinetic energy and turbulence energy included in the definition of the total enthalpy from which temperature was calculated. On the jet axis the temperature remains almost constant at 298.6 K approximately. This may be attributed to the high specific heat capacity of hydrogen gas whose concentration on the jet axis is considerably high. The predicted lower temperature showed little effect on density and other flow variables probably due to low initial excess temperature, ΔT_o, of only 3 degrees. This seems to be reasonable agreement with observations made by Pai[16], Abramovich[12] and Hinze[5] who say that experiments have indicated that low initial temperature differences of up to 50 degrees have insignificant effect on the main jet flow parameters like velocity and chemical species concentration even in high speed jets of up to Mach one. Moreover, density variations in the fully developed main region of jet are chiefly due to mixing between the surrounding air and hydrogen gas rather than compressible effects because of not only the rather low initial speed of the jet at the nozzle exit, but also the appreciable decay of the velocity which occurs in this region. According to Abramovich[12], this is also generally true even for high speed jets of up to sonic speed.

5. CONCLUSIONS

(1) Although it was very difficult to measure the hydrogen gas concentration in the front unsteady part of the jet and in the vicinity of the nozzle, the laser interferometry technique is a very useful experimental method of investigating transient density or chemical species concentration distributions in some turbulent flows like non-reacting, binary gas mixture flow.

(2) Accumulation of calculation errors associated with the onion peeling model particularly near the jet axis may have contributed to some errors in measured concentration distributions. In addition, despite ensemble averaging the experimental data, it is noted that photographs showed that real jet was actually unsymetrical unlike the axisymmetric jet assumed in the present numerical predictions and onion peeling mode. In the present numerical calculation, the k-ε turbulence model constants were not adjusted to suit the actual jet flow conditions. These three factors have to be taken into account when interpreting the present prediction and measurement results.

(3) Present prediction shows that the transient, turbulent hydrogen gas jet issuing into air is made up of a rear part which becomes quasi-steady almost soon after injection initiation and the front part which is remains unsteady. The present prediction also shows that axial and radial lengths of the rear quasi-steady part of the jet increase with time while that of the front part decrease with time. In fully developed main region, the radial spread is linearly proportional to quasi-steady axial lengths which compares reasonably well with data for steady jets.

(4) Despite seemingly high injection pressure in the hydrogen gas reserviour, the actual jet was most probably incompressible and subsonic. The level of agreement between measured and predicted jet tip penetration as well as global trends of the axial distribution of

hydrogen gas concentration on the jet axis is reasonably good particularly in the fully developed main region for the latter. These concentration levels are considerably higher than those of methane gas in a transient, turbulent methane gas jet issuing into still air under similar conditions. General trends of the predicted and measured instantaneous concentration distributions on some jet axis locations are also in reasonable agreement. There are some local regions of poor agreement.

(5) In the radial direction, the present prediction shows that the hydrogen gas concentration gradient is much lower within the turbulent core than it is outside this core where the gradient increases abruptly. The predicted shape of the radial distribution curve for hydrogen gas concentration deviates appreiciably from that of the universal curve represented by the 3/2 power law for concentration.

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