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Optimal Pricing in Urban Expressway

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SYNOPSIS

A welfare economic approach is applied to investigate some implications of optimal pricing in urban expressway where two different groups of users are supposed to exist. On the assumption of a specified demand function, following implications are shown; (1) optimal prices must be such that the diversion ratios are the same and (2) the price rates must be set equal each other, where the price rate means the proportion of the price to the average user benefit. In connection with the results, the elasticity of the demand with respect to price is measured in Osaka area of Hanshin Expressway, where two different levels of price are flatly set for users according to the characteristics of their cars.

1. INTRODUCTION

The paper is concerned with a simultaneous decision of the price and stock level of urban expressway. Yamada[1] formulated the problem from view point of welfare economics. His formulation is "the price and stock level are optimal when consumers' (expressway users') surplus is maximized subject to a certain set or constraints."

In the problem, the price and stock level are interdependent because of a premiss set down for keeping the cost and toll revenues

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in a state of balance. By the way, the cost is assumed as an increasing function with respect to the level of stock of expressway and the revenues to be given by multiplication of the price and number of expressway users that is realized dependently upon the price and stock level.

On several assumptions for simplicity and in terms of a specified demand function, Yamada introduced an implicative finding as follows; set such a price and/or stock level as to make the marginal cost of stock per a unit of the marginal number of expressway users with respect to stock level and the user benefit gained (or the value of time saved) through using expressway by an average trip length meet.

The first author of the present paper showed a more general aspect of the optimal solution to the same problem[2],[3],[4]. He assumed such a general form of demand function that included the specified one used by Yamada as a special case. He, in terms of the general form, showed that the extremum condition to expressway users' surplus was equivalent to the one to the number of expressway users. He also showed that this finding covered the special case introduced by Yamada.

Two subjects are presented in this paper; a general aspect derived from relaxation of one of the assumptions made hitherto for simplicity and the measurement of the elasticity of the demand (the number of expressway users) with respect to price. The latter are taken in connection with the former.

2. MODEL

In stead of the assumption hitherto that the whole expressway users under study has a single demand curve, following assumption is made; the whole population of expressway users consists of two subgroups, each of which has its proper demand function. The reason of grouping users is that they are reasonably supposed to be different each other in the reaction to price. In the concrete, they have a variety of trip objectives, different trip length that will cause themselves to take different reactions to price. More accurately, each user has his proper demand curve. We, however, have only to investigate in the case of two subgroups in order to see the case of the many.

On the new assumption, the model is presented as follows.

Maximize the amount of expressway users' surplus (consumers' surplus)

$$S = \sum_{i=1,2} \int_0^{q_i} f_i(\xi,\ s)\,d\xi - C(s) \qquad (1)$$

subject to

$$C(s) - \sum_{i=1,2} p_i q_i = 0 \qquad (2)$$

where

$p_i = f_i(q_i, s)$, $i=1,2$; inverse demand functions for subgroups i, (3)

$C(s)$; cost function for supplying expressway stock,

q_i; the realized number of expressway users,

p_i; price set for subgroup i and

s ; level of stock of expressway.

Two premises are set down as hitherto; one is a request to keep the toll revenues and stock cost in a state of balance and the other is a pricing principle that price is set for each subgroup flatly in the region under study. By the way, the former is expressed by eq.(2). Both are working in the existing urban expressway public corporations in this country.

The followings are assumed as hitherto;

(1) static model is admitted of,

(2) congestion cost is disregarded and

(3) the cost of supplying expressway service is a function of the level s of stock of expressway alone, where s is measured by, for instance, a radius of the expressway network or the sum of length supplied.

3. SOME ASPECTS OF THE SOLUTION

The Lagrangian method of undetermined multiplier is applied to solve the problem. The Lagrangian is

$$L = \sum_i \int_0^{q_i} f_i(\xi,\ s)\,d\xi - C(s) + \mu\{\ C(s) - \sum_i p_i q_i\ \}, \qquad (4)$$

where μ is an undetermined multiplier. By partial differenciation of the Lagrangian with respect to q_i, s and μ, we have

$$\frac{\partial L}{\partial q_i} = f_i(q_i,\ s) - \mu(\ p_i + \frac{\partial p_i}{\partial q_i}\ q_i\ ),\ i=1,2 \tag{5}$$

$$\frac{\partial L}{\partial s} = \sum_i \frac{\partial}{\partial s}\int_0^{q_i} f_i(\xi,\ s)d\xi + \mu\{\ C'(s) - \sum \frac{\partial p_i}{\partial s}\ q_i\ \}, \tag{6}$$

$$\frac{\partial L}{\partial \mu} = C(s) - \sum_i p_i q_i, \tag{2}$$

where $C'(s) = dC(s)/ds$. The optimal solution is obtained by solving $\partial L/\partial q_i = 0$, $\partial L/\partial s = 0$ and $\partial L/\partial \mu = 0$. We, however, are concerned not with the optimal solution but with a certain aspect of the optimal solution.

By equating $\partial L/\partial q_i$ zero, we have

$$f_i(q_i,\ s) - \mu(\ p_i + \frac{\partial p_i}{\partial q_i}\ q_i\ ) = 0,\ i=1,2. \tag{7}$$

By putting $p_i = f_i(q_i,\ s)$ into and eliminating undetermined multiplier μ from the above equation, respectively, the following relationship is obtained.

$$\frac{p_1}{p_1 + (\partial p_1/\partial q_1)q_1} = \frac{p_2}{p_2 + (\partial p_2/\partial q_2)q_2} \tag{8}$$

from which we have

$$\frac{\partial p_1}{\partial q_1}\frac{q_1}{p_1} = \frac{\partial p_2}{\partial q_2}\frac{q_2}{p_2}\ , \tag{9}$$

or alternatively

$$\frac{\partial q_1}{\partial p_1}\frac{p_1}{q_1} = \frac{\partial q_2}{\partial p_2}\frac{p_2}{q_2}\ . \tag{10}$$

Eq.(9) or (10) implies that the elasticity of the number of expressway users belonging to each subgroup with respect to price must be equal each other in optimal point. The implication is easily generalized to the case of an arbitrary number of subgroups. Further inquiring, however, into general case of demand function is fruitless. In the following, investigation is tried in terms of a specified demand function.

Following functions are supposed.

$$p_i = \frac{1}{\alpha_i} \ln \frac{X_i(s)}{q_i} \ , \ i=1,2 \ , \tag{11}$$

where

α_i ; constant

$X_i(s)$; the population of subgroup i, which comes to use expressway only if it is of free use, a part of which is realized to be users otherwise.

This form of demand function was proposed by Yamada[1] and Sasaki[5], and the inverse of α_i is defined as the value of time saved when a user belonging to subgroup i travels along expressway just by the average trip length. In this sense the inverse of α_i is called the average user benefit (of subgroup i) hereafter in the present paper.

Partial differentiation of the functions specified above with respect to q_i gives, instead of eq.(9),

$$\alpha_1 p_1 = \alpha_2 p_2 \tag{12}$$

Eq.(12) is rewritten as

$$\frac{p_1}{1/\alpha_1} = \frac{p_2}{1/\alpha_2} \tag{13}$$

or

$$\frac{p_2}{p_1} = \frac{1/\alpha_2}{1/\alpha_1} \tag{14}$$

Following implications follow from eq's.(12), (13) and (14), respectively.

(1) The subgroups must be set such prices that the diversion ratios are the same. Here we define a diversion ratio of a subgroup by the proportion of the number of expressway users to the population of the subgroup.

(2) The price rates must be set equal each other. A price rate, here, is defined by the proportion of price to the average user benefit and

(3) The relative price must be set equal to the relative average user benefit. The relative price, here, is the proportion of price for a subgroup to the other and similar as for the relative average user benefit.

The first implication is easily obtained by putting $\alpha_1 p_1 = \alpha_2 p_2$ into demand function (11).

Another aspect is concerned with a condition on which the stock cost of expressway and the toll revenues are kept in a state of balance. The same demand function as specified previously is assumed. We have from the first implication above

$$r_1 = r_2 \tag{15}$$

where r_i is the diversion ratio of subgroup i defined by

$$r_i = \frac{q_i}{X_i(s)}, \quad i=1,2. \tag{16}$$

Substitution of eq's.(15) and (16) into eq.(2) yields

$$-r \ln r = \frac{C(s)}{X_1(s)/\alpha_1 + X_2(s)/\alpha_2}, \tag{17}$$

where

$$r \equiv r_1 = r_2 .$$

Since $0 \leqq r = q_i/X_i(s) \leqq 1$, we have inequality $0 \leqq -r \ln r \leqq e^{-1}$. Then the following condition must exist

$$\frac{C(s)}{X_1(s)/\alpha_1 + X_2(s)/\alpha_2} \leqq e^{-1}, \tag{18}$$

in order to keep the stock cost of expressway and toll revenues in a state of balance.

Each term of the denominator on the left-hand side of the above inequality is interpreted as the whole user benefit of each subgroup to be realized in case of free use of expressway. Accordingly, the balance will be lost unless an amount of the whole user benefit of subgroup is considerably large, as compared with the cost of stock of expressway. In the sense, the average user benefit is one of key factors for keeping the balance. Since the average user benefit is an increasing function with respect to both the average trip length and the time value, the longer the trip is in an average and/or the higher the user's time value is, the more likely the cost and revenues are to be kept in balance.

Fig.1 shows the states of and out of balance. In case of curve(1), there exists a certain region $s_1 \leqq s \leqq s_2$ where the cost and revenues are kept in a state of balance. In case of curve(2), on the other hand, no such a region is found.

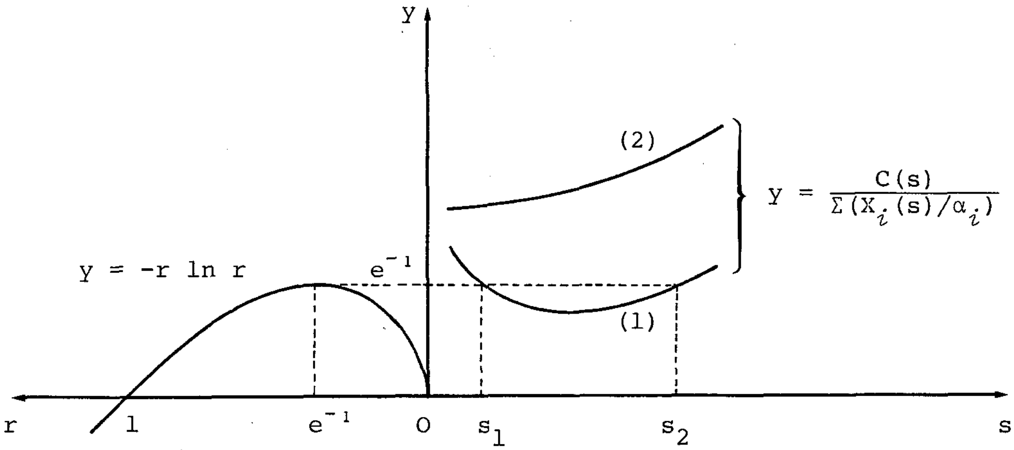

Fig.1 Balance of the Cost and Revenues

4. MEASUREMENT OF THE ELASTICITY

Eq.(9) or (10) interests us in studying the real aspect of the elasticity of the demand (the number of expressway users) with respect to price. The study was tried in Osaka area of Hanshin Expressway Network, where two different levels of price are flatly set for users according to the characteristics of their cars; one is for those driving lighter cars and the other for the heaviers. The former subgroup includes passenger cars and light trucks and the latter heavy trucks, buses and so on. Each is called the light and heavy group, respectively, hereafter in this paper.

Each group of users is further categorized by entrance ramp through which they enter expressway. The categorization of users is made on a likely presumption that the demand elasticity with respect to price is dependent largely upon the distance they travel along expressway[6]. The distance they travel along expressway is closely dependent upon which ramp they use to enter expressway. Fig.2 is the illustration of our categorization of users by entrance ramps.

Following formula[6] is applied to estimate the elasticity,

$$\eta_t = - \frac{Q_t^{P_2} - \hat{Q}_t^{P_1}}{\hat{Q}_t^{P_1}} \Big/ \frac{P_2 - P_1}{P_1} \qquad (19)$$

where

η_t; the demand elasticity with respect to price at period t,

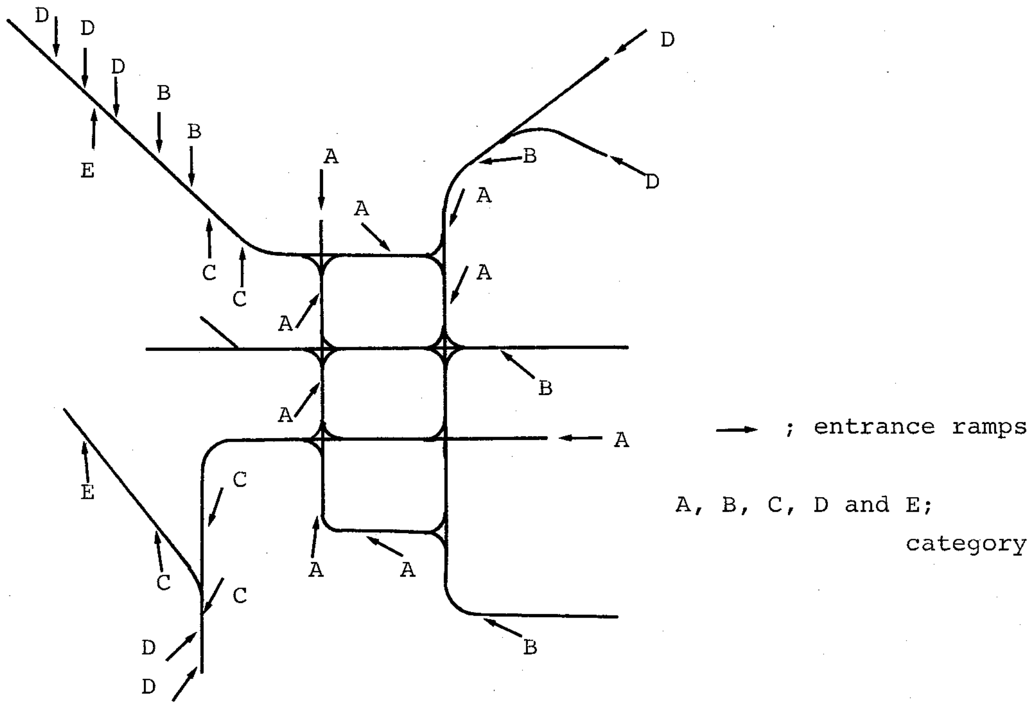

Fig.2 Hanshin Expressway Network (Osaka Area)

t ; a certain period after the price was changed,

P_1 and P_2; the price set before and after changing, respectively,

$\hat{Q}_t^{P_1}$; estimated demand that would have realized if the price P_1 was kept unchanged and

$Q_t^{P_2}$; demand observed after the price was changed to P_2.

The price was set changed as shown in Table 1 in April, 1980.

The elasticity was calculated every month between April, 1980 and March, 1981. By the way, $\hat{Q}_t^{P_1}$ was estimated by time series analysis using the demand observed monthly between April, 1978 and March, 1981.

The elasticity of the demand with respect to price is shown in Table 2 with the average travel

Table 1 Change in price

group / price	light	heavy
P_1	300	650
P_2	350	700

Table 2 Demand elasticity

group / category	light		heavy	
	elasticity	average travel distance	elasticity	average travel distance
		(km)		(km)
A	-0.372	14.2	0.778	14.8
B	-0.122	15.4	0.637	17.5
C	0.729	9.8	1.597	7.9
D	-0.036	17.6	0.766	19.9
E	0.322	2.7	1.542	3.0
Whole	-0.046	15.4	0.867	17.3

distance along expressway. The value in each cell on the columns to the elasticity is the average of those over twelve months between April, 1980 and March, 1981.

As a whole, the effect of the change in price is enough significant upon the demand of the heavy group, but little upon the light group. It, however, is not true to say that the light group is fully dull of reacting to pricing-up. The fact is that categories C and E of the light group have significant values of the demand elasticity with respect to price while the others have not. Clearly, the two categories travel shorter distances along expressway than the others. Accordingly, it is a matter of course that a significant reaction of pricing-up comes on the demand of the two categories but not on the other categories. Similar aspect is observed in the categories C and E of the heavy group, that is, the two categories have larger values of elasticity with shorter travel distance along expressway than the other categories of the same group.

There is another question as to the reason why the heavy group, as a whole, has a significant value of the elasticity while the light group has not, in spite of the fact that both travel nearly the same distances along expressway. Though a further research is necessary to answer the question, one of the factors influencing on the reaction of the users to pricing-up may consist in the ratio of the travel distance along expressway to the whole trip length between origin and destination. In the present study, the ratios, on an average, were

0.2 and 0.4 to the heavy and the light groups, respectively.

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REFERENCES

[1] H. Yamada, Expressways and Automobiles, XI(1968), 9, 17, (in Japanese).

[2] S. Myojin and K. Asai, Memoirs School Eng., Okayama Univ., 16(1981), 1, 105.

[3] S. Myojin and K. Asai, Expressways and Automobiles, XXV(1982), 4, 27, (in Japanese).

[4] S. Myojin and M. Ohnishi, Memoirs School Eng., Okayama Univ., 17(1983), 1, 73.

[5] T. Sasaki, Expressway and Automobiles, XI(1968), 2, 19, (in Japanese).

[6] H. Kohno, M. Yoshida and Y. Matsumura, Annual Report on Transprotation Economics, 1981, (in Japanese).