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On a Study of the Empirical Formula to Explain the Work Amount ★

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Synopsis

This paper deals with the empirical formula to explain the work amount curve of a worker during a work. The empirical formula $y_t = at^b + c$ was used to explain this phenomenon until now. This formula has been used mainly to approximate to the monotonous trend of the work amount curve. But it was made clear that if the work amount curve showed the polynomial trend, it could not be done so.

Then the authors attempt to establish the empirical formula $y_t = a/\{\exp(\sum b_i t^i) - 1\} + c$, which was the general form of the logistic curve in order to explain not only the monotonous trend but also the polynomial trend of the work amount curve. And it was made clear from the results of the approximation that this formula was the one of the most usuful formula in order to explain the work amount curve.

1. Introduction

The relations between the working load and the working ability of a worker have been examined mainly until now.[1)2)] It was made clear from these analyses that the work amount which was conducted by the worker within given time interval showed the some trend with the lapse of time. It was pointed out by many authors[3)4)5)] that the logarithm of the work amount was directly proportioned to the logarithm of the time if the work was the repitition of the simple monotonous task. But this relation did not always come into existence if the work was the combination of the complex task.

Then the authors attempt to establish the more usuful empirical formulas in order to explain the various trend of the work amount curve.

2. Analytical Method

The work amount curve was shown usually by the following empirical formula.

$$y_t = at^b + c \quad \text{.........} \quad (1)$$

y_t : work amount at the time t

t : time

If the work amount increased or decreased monotonously with the lapse of time, formula (1) could be used to explain the work amount curve exactly. But if it did not increase or decrease monotonously, formula (1) can not be done so.

Therefore the following empirical formula is established to explain the various trend of the work amount curve.

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$$y_t = a / \exp(\sum_{i=1}^{k} b_i t^i) + c \quad \ldots\ldots\ldots (2)$$

If the degree k of the polynomial in this formula is odd, y_t amounts to the extreme value at t_0 which is the solution of $d(\sum b_i t^i)/dt = 0$. And if t tends to positive infinite, y_t converges to c. Then this formula can be used to approximate to the various trend of the work amount curve. But formula (2) has the following dimerit that y_t exsists even if t becomes negative. That is, if the time becomes negative, the work amount exsists.

Therefore the following empirical formula which is the improved model of formula (2) is established to approximate to the various trend of the work amount curve.

$$y_t = a / \{\exp(\sum_{i=1}^{k} b_i t^i) - 1\} + c \quad \ldots\ldots (3)$$
$$k = 2m + 1 \;, \; m=0,1,2,\ldots$$

Formula (3) has the merits of formula (1) and (2). Then formula (3) can be used to approximate to not only the monotonous trend of the work amount but also the polynomial trend. This formula is the general form of the logistic curve.

If c = 0 in formula (1) and (2), the estimation of the parameters a and b or b_j (j = 1,2,...,k) can be obtained from the usual least sqare method.

The data of the time and the work amount is shown (t_i , y_i) i=1,2,...,n .

The estimated value $\hat{a}$ and $\hat{b}$ of the parameters a and b in formula (1) are obtained from the following equation.

$$(\log_{10}\hat{a} \;,\; \hat{b}) = \begin{pmatrix} n & T_i \\ T_i & T_i^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^{n} Y_i \\ \sum_{i=1}^{n} Y_i T_i \end{pmatrix} = A^{-1}\vec{e}$$

where $T_i = \log_{10} t_i$ and $Y_i = \log_{10} y_i$.

Further the estimated values $\hat{a}$ and $\hat{b}_j$ of the parameters in formula (2) are obtained by the following way .

$$(\log \hat{a}, \hat{b}_1, \hat{b}_2, \ldots, \hat{b}_k) = \begin{pmatrix} s_{11} & \cdots\cdots & s_{1\;k+1} \\ s_{21} & \cdots\cdots & s_{2\;k+1} \\ \vdots & & \vdots \\ s_{k+1\,1} & \cdots\cdots & s_{k+1\,k+1} \end{pmatrix}^{-1} \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_{k+1} \end{pmatrix} = S^{-1}.\vec{d}$$

where $s_{j1} = -\sum_{i=1}^{n} t_i^{j-1}$, $s_{jp} = \sum_{i=1}^{n} t_i^{j+p-2}$, j=1,2,...,k+1 p=2,3,...,k+1

$d_j = \sum_{i=1}^{n} y_i t^{j-1}$ j=1,2,...,k+1

The relation between t and y_t in formula (3) is non-linear even if c = 0. Then the usual least square method can not be used to estimate the parameters. Therefore the least square linear Taylor differential correction technique[6] is used to estimate the parameters a and b_j (j=1,2,..k).

The function relating variables t and y_t is put as follows.

$$y_t = f(t, c, a, b_1, b_2, \ldots, b_k) = a / \{\exp(\sum_{i=1}^{k} b_i t^i) - 1\} + c$$

where c, a, b_j are unknown parameters.

Further this function is shown by the following short form.

$$f_i = f(t_i, c, a, b_1, b_2, \ldots, b_k)$$

${}^{o}c$, ${}^{o}a$, ${}^{o}b_j$ are the initial estimated values of the parameters c , a , b_j . Then the residuals in the case of $c = {}^{o}c$, $a = {}^{o}a$, $b_j = {}^{o}b_j$ in formula (3),

$$Q_i = f(t_i, {}^{o}c, {}^{o}a, {}^{o}b_1, {}^{o}b_2, \ldots, {}^{o}b_k) - y_i \;,\; i=1,2,\ldots,n$$

can be calculated.

The improved values $\delta^{o}c$, $\delta^{o}a$, $\delta^{o}b_j$ of the initial estimated values in order to minimize $\sum_{i=1} Q_i^2$ are obtained from the following euation.

$$(\delta^0 c, \delta^0 a, \delta^0 b_1, \ldots, \delta^0 b_k) = \begin{pmatrix} a_{11} \cdots\cdots & a_{1\,k+2} \\ a_{21} \cdots\cdots & a_{2\,k+2} \\ \vdots & \vdots \\ a_{k+2\,1} \cdots\cdots & a_{k+2\,k+2} \end{pmatrix}^{-1} \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_{k+2} \end{pmatrix}$$

where

$$a_{11} = \sum_{i=1}^{n} (\partial f_i/\partial c)_0 \ , \qquad a_{21} = a_{12} = \sum_{i=1}^{n} (\partial f_i/\partial c)_0 (\partial f_i/\partial a)_0$$

$$a_{22} = \sum_{i=1}^{n} (\partial f_i/\partial a)_0^2$$

$$a_{I+2\,1} = a_{1\,I+2} = \sum_{i=1}^{n} (\partial f_i/\partial c)_0 (\partial f_i/\partial b_I)_0$$

$$a_{I+2\,2} = a_{2\,I+2} = \sum_{i=1}^{n} (\partial f_i/\partial a)_0 (\partial f_i/\partial b_I)_0 \qquad l=I,2,3,\ldots,k$$

$$a_{I+2\,m+2} = a_{m+2\,I+2} = \sum_{i=1}^{n} (\partial f_i/\partial b_I)_0 (\partial f_i/\partial b_m)_0$$

$$I, m = 1,2,3,\ldots,k$$

$$p_1 = -\sum_{i=1}^{n} (\partial f_i/\partial c)_0 Q_i \ , \quad p_2 = -\sum_{i=1}^{n} (\partial f_i/\partial a)_0 Q_i$$

$$p_{I+2} = -\sum_{i=1}^{n} (\partial f_i/\partial b_I)_0 Q_i \qquad I = 1,2,3,\ldots,k \ .$$

Further $\sum = \sum_{i=1}^{n}$. And $(\ )_0$ is the value of the partial derivative in the parenthesis in the case of $c = {}^0c$, $a = {}^0a$, $b_l = {}^0b_l$ $l=1,2,\ldots,k$.

From this solution, the improved values of the initial values are obtained from the following equation. The first estimated values

$${}^1c = {}^0c + \delta^0 c \quad , \quad {}^1a = {}^0a + \delta^0 a \ ,$$

$${}^1b_j = {}^0b_j + \delta^0 b_j \quad j = 1,2,3,\ldots,k$$

can be calculated.

The same process of the calculation is repeated regarded the first estimates as the initial estimates. Then the second estimates ${}^2c$, ${}^2a$, ${}^2b_j$ are obtained . The repetition is stopped by the following criterions.

1) Criterion 1

m+1 th improved values are put $\delta^{m+1}c$, $\delta^{m+1}a$, $\delta^{m+1}b_j$.

If for any $\varepsilon \geqq 0$, $|\delta^{m+1}c| < \varepsilon$, $|\delta^{m+1}a| < \varepsilon$, $|\delta^{m+1}b_j| < \varepsilon$, then the repetition is stopped. The final estimates of the parameters are ${}^mc$, ${}^ma$, ${}^mb_j$.

2) Criterion 2

In each repetition, the residuals

$${}^mQ_i = f(t_i, {}^mc, {}^ma, {}^mb_1, {}^mb_2, \ldots, {}^mb_k) - y_i$$

$$i = 1,2,3,\ldots,n \ , \ m = 0,1,2,3,\ldots\ldots$$

can be calculated.

The mean square residual

$$\sigma_m^2 = \sum_{i=1}^{n} {}^mQ_i^2 / \{n - (\text{number of parameters})\}$$

can be calculated in each repetition. The estimates ${}^mc$, ${}^ma$, ${}^mb_j$ converges to some values if the same process is repeated many times. Then σ_m^2 converges to the some value. Therefore

if for any $\varepsilon \geqq 0$, $|\sigma_m^2 - \sigma_{m+1}^2| < \varepsilon$, then the repetition is stopped.

$\hat{y}_t$ is put as the estimated value calculated by formula (1) or (2) in which the parameters a and b or b_j are replaced by the estimates $\hat{a}$ and $\hat{b}$ or $\hat{b}_j$.

Then the mean square residual

$$\sigma^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 / \{n - (\text{number of parameters})\}$$

can be calculated.

3. Results

The data used here is the frequency of miss to the color signal of experiment using the driving simulator. The experimental conditions are shown by C1, C2 and C3. The subjects are classified into three or four classes(P1,P2,P3 or R1,R2,R3,R4) by the degree of the working load in each experimental condition. The frequency of miss in each class is regarded as the work amount. Formula (1), (2) and (3) are used to approximate to the trend of the work amount curve in each class. The frequency of miss converges logically to zero. Then the parameter c in formula (1),(2) and (3) is put as zero.

3.1 Approximation of $y_t = at^b + c$

As n=10 , $t_1=1$,$t_2=2$,.....,$t_{10}=10$, then

$$A = \begin{bmatrix} 10 & 6.55976 \\ 6.55976 & 5.21516 \end{bmatrix}$$

is the same in all estimation . Therefore $\hat{a}$ only is calculated in each estimation. For example, on P1 of C1, $\sum_{i=1}^{10} Y_i = 14.6243$, $\sum_{i=1}^{10} T_i Y_i = 9.3676$. Then $\hat{a} = 42.138$, $\hat{b} = -0.247$. Table 1 shows $\hat{b}$ and $\hat{\sigma}^2$ in each class.

experimental condition	class	approximation of formula (1)			approximation of formula (2) of k=1		
		$\hat{b}$	σ^2	F_0	$\hat{b}_1$	σ^2	F_0
C1	P1	-0.247	6.3826		0.051	18.9927	2.976*
	P2	-0.040	2.1894		0.007	2.9680	1.356
	P3	-0.157	9.4614		0.031	18.3947	1.944*
C2	P1	-0.109	4.0799	1.205	0.030	3.3856	
	P2	-0.072	4.8881		0.006	5.9436	1.011
	P3	-0.035	1.9650		0.008	1.9766	1.001
C1	R1	-0.185	4.3724	1.257	0.046	3.4791	
	R2	-0.164	5.4658		0.032	12.8528	2.344*
	R3	-0.093	5.4843		0.016	10.1614	1.853*
	R4	-0.080	32.7200		0.008	37.5098	1.146
C2	R1	-0.261	3.9840		0.061	5.5418	1.391
	R2	0.032	4.5657		-0.012	4.3083	
	R3	-0.155	1.3960		0.029	2.4596	1.762*
	R4	-0.016	3.8703		-0.0004	3.9262	
C3	R1	-0.515	1.4245		0.107	2.6898	1.888*
	R2	-0.058	4.2986		0.014	4.3047	1.001
	R3	-0.017	1.9452	1.005	0.005	1.9349	
	R4	-0.156	2.6284		0.030	3.9571	1.506

* : rejects at level 25%

Table 1 , $\hat{b}$ or $\hat{b}_1$, and F_0 of formula (1) and formula (2) of k=1.

The frequency usually decreases with the lapse of time. Then the estimated value $\hat{b}$ must be negative. As $\hat{b}$ except R2 of C2 becomes negative, it is made clear that this formula can be used to explain the work amount curve. But as the mean square residual on R4 of C1 is the largest in these results, it is made clear that this formula can not explain the work amount curve of this class exactly. Further as $\hat{b}$ on R2 of C2 becomes positive, then $\hat{y}_t$ becomes positive infinite even if the work amount converges some value with lapse of time. Then it is made clear that this formula can not be used to explain the work amount of R4 of C1 and R2 of C2.

3.2 Approximation of $y_t = a/\exp(\sum_{i=1}^{k} b_i t^i) + c$

First it is examined whether there is significant difference between the degree of the approximation of formula (1) and that of formula (2) of k=1. The following statistical F-test is used to examine the difference among the degrees of the approximations of various formulas.

σ_a^2 and σ_b^2 are put as the mean square residuals of formula A and B.

If $\sigma_a^2 \geqq \sigma_b^2$ ($\sigma_a^2 \leqq \sigma_b^2$),

then $F_o = \sigma_a^2 / \sigma_b^2$ ($F_o = \sigma_b^2 / \sigma_a^2$)

can be calculated.

And if $F_o \geqq F_\alpha(m_1, m_2)$ where m_1 is the degree of freedom of the numerator of F_o . m_2 is that of the denominater of F_o , then there is significant difference between the degree of the approximation of formula A and that of formula B.

Further if $F_o \leqq F_\alpha(m_1, m_2)$, then there is not difference between them.

As c = 0 in formula (1) and (2), then number of the parameter of formula (1) and formula (2) of k=1 is two. As n = 10 , then $m_1 = m_2 = 8$. Table 1 shows $\hat{b}_1$ and σ^2 of formula (2) of k=1 in each class. Further the result of F-test between σ^2 of formula (1) and that of formula (2) of k=1 is shown in Table 1.

As $\hat{b}_1$ except R2 and R4 of C2 is positive, the formula (2) of k=1 can approximate to the work amount. But the degree of the approximation of formula (1) is better than that of formula (2) on P1,P3,R2,R3 of C1, R3 of C2 and R1 of C3. In the other cases the degree of the approximation is statistically equal in each other. Then it is made clear that the degree of the approximation of formula (1) is better than that of formula (2) of k=1. Therefore it is examined whether there is significant difference between the degree of approximation of formula (1) and that of formula (2) of k = 3 or 5.

Table 2 shows the mean square residuals of these formulas in each class and F_o among them. The sign " — " in this table denotes the case that the estimated value of the parameter does not satisfy the condition.

experimental condition	class	approximation of formula (1)		approximation of formula (2)			
				k = 3		k = 5	
		σ^2	Fo	σ^2	Fo	σ^2	Fo
C1	P1	6.3826	1.938*	3.2929		—	
	P2	2.1894	2.611*	0.8385		—	
	P3	9.4614	2.863*	3.3051		—	
C2	P1	4.0799	2.661*	1.5330		—	
	P2	4.8881	2.353*	2.0775		2.9062	1.399
	P3	1.9650		2.5393	1.292	—	
C1	R1	4.3724	1.825*	2.3959		—	
	R2	5.4658	3.535*	1.5463		—	
	R3	5.4843	4.593*	1.4062	1.178	1.1940	
	R4	32.7200	2.470*	13.2470		—	
C2	R1	3.9840	1.517	2.6254		—	
	R2	—		4.4364		—	
	R3	1.3960	3.217*	0.9245	2.130*	0.4340	
	R4	3.8703		—		5.6139	1.451
C3	R1	1.4245	2.394*	—		0.5951	
	R2	4.2986		5.1078	1.188	—	
	R3	1.9452	1.125	1.7285		2.4357	1.409
	R4	2.6284	2.336*	2.7478	2.442*	1.1253	

* : rejects at level 25%

Table 2 , Comparision of the degree of approximation between formula (1) and formula (2) of k=3 or k=5 .

It is made clear that the degree of the approximation of formula (2) of k=3 or 5 is better than that of formula (1) in 12 classes out of 18 classes. On R2 of C2 only formula (2) of k=3 can be used to approximate to the work amount curve. And on the rest five classes the degree of the approximation is statistically equal in each other. In the latter cases the formula which contains the lowest degree of the polynomial can be used to explain the work amount. Therefore as the degree of the approximation is equal among formulas, the formula containing the lowest degree of the polynomial can be used to approximate to the work amount curve.

Then it is made clear that the equation (2) of k=3 or 5 can be approximated exactly to the various trend of the work amount curve. Table 3 shows the estimated value of the parameter of formula which has the minimum mean square residual in each class.

exp. cond.	class	formula		estimated values of parameters							σ^2
		num	k	a	$b \times 10^{-1}$	$\hat{b}_1 \times 10^{-1}$	$\hat{b}_2 \times 10^{-2}$	$\hat{b}_3 \times 10^{-3}$	$\hat{b}_4 \times 10^{-3}$	$\hat{b}_5 \times 10^{-3}$	
C1	P1	2	3	63.31		3.81	-5.31	2.4			3.293
	P2	2	3	47.75		1.56	-2.75	1.5			0.839
	P3	2	3	68.65		3.95	-6.89	3.7			3.305
C2	P1	2	3	25.87		2.10	-4.80	3.3			1.533
	P2	2	3	28.22		3.40	-5.37	2.4			2.078
	P3	1		15.09	-0.36						1.965
C1	R1	2	3	49.10		1.70	-2.80	1.8			2.396
	R2	2	3	55.92		3.10	-4.45	2.0			1.546
	R3	2	3	59.06		2.28	-3.52	1.7			1.406
	R4	2	3	78.72		6.84	-13.4	7.6			13.247
C2	R1	1		20.83	-2.61						3.984
	R2	2	3	25.59		2.12	-4.58	2.7			4.436
	R3	2	5	40.52		13.46	-49.8	85.1	- 6.8	0.201	0.434
	R4	1		16.77	-0.16						3.870
C3	R1	2	5	29.25		17.28	-87.1	212.8	-22.5	0.840	0.595
	R2	1		14.22	-0.58						4.299
	R3	1		13.17	-0.17						1.945
	R4	2	5	44.87		12.34	-40.0	53.7	- 2.7	0.026	1.125

exp. cond. = experimental condition , num = formula number

Table 3 , The best formula in each class in order to explain the work amount curve

3.3 Approximation of $y_t = a/\{\exp(\sum_{i=1}^{k} b_i t^i) - 1\} + c$

Table 4 shows results of the repeated calculation of the least square linear Taylor differetial correction technique on P1 of C1.

m	$\delta^m a$	$\delta^m b$	$^m a$	$^m b$	σ^2_m
0			- 2.0381	-0.0511	380.2773
1	-38.0503	-0.9708	-40.0883	-1.0219	240.0213
2	13.6328	0.1664	-26.4556	-0.8555	4.8554
3	0.0892	0.0411	-26.3663	-0.8143	4.6201
4	0.0118	-0.0007	-26.3546	-0.8150	4.6195
5	0.0001	-0.0000	-26.3547	-0.8150	4.6195
6	0.0000	-0.0000	-26.3547	-0.8150	4.6195

Table 4 , Approximation of formula (3) of k = 1 to the work amount of P1 of C1 .

The estimated parameter values in formula (1) and (2) are used in the initial values of the parameters in formula (3) in each class. Then it is made clear from the results of approximation that only formula (3) of k = 1 can approximate to the work amount on R3 of C2 and R4 of C3, and formula (3) of k = 3 can do to that of P1,R1,R2 of C1 and P1,R1 of C2. Further formula (3) of k =5 can do to that of R1 of C3. On the rest ten classes formula (3) of k = 1 can do to the work amount sufficiently.

Table 5 shows results of F-test between the minimum residuals of formula (3) and those of formula (1) and (2) in each class.

Then it is made clear that the degree of the approximation of formula (3) is equal to that of formula (1) and (2). Then Formula (3) can approximate to the various trend of the work amount curve by only the degree of the polynomial being exchanged. The other word, if formula (1) and (2) are used to explain the work amount ,it is judged from the trend of the work amount whether formula (1) or (2) is used.

Then it is made clear that formula (3) is the one of the most useful formula in order to explain the various trend of the work amount curve.

Table 6 shows the estimated value of parameter of formula (3) in each class. Further Figure 1 ,2 ,3 show the approximation of formula (1) or (2) or (3) to the data in some class.

experimental condition	class	formula (1),(2)		formula (3)	
		σ^2	Fo	σ_m^2	Fo
C1	P1	3.2929	1.616	2.0380	
	P2	0.8385		1.4945	1.782*
	P3	3.3051	1.482	2.2296	
C2	P1	1.5330		2.4774	1.616
	P2	2.0775		2.7429	1.320
	P3	1.9650		2.0087	1.022
C1	R1	2.3959		3.5067	1.464
	R2	1.5463	1.710	0.9045	
	R3	1.4062	1.297	1.0839	
	R4	13.2470		21.1636	1.598
C2	R1	3.9840	2.703*	1.4739	
	R2	4.4364		4.7808	1.078
	R3	0.4340	1.291	0.3362	
	R4	3.8703		3.8819	1.003
C3	R1	0.5951	1.278	0.4658	
	R2	4.2986		4.3708	1.017
	R3	1.9452	1.040	1.8696	
	R4	1.1253		1.2226	1.086

* : rejects at level 25%

Table 5 , Comparition of the degree of approximation between formula (1),(2) and formula (3).

exp. cond.	class	m	k	$\hat{a}$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3 \times 10^{-1}$	$\hat{b}_4 \times 10^{-3}$	$\hat{b}_5 \times 10^{-4}$	σ_m^2
C1	P1	16	3	-26.2	-1.47	0.79	-1.81			2.0380
	P2	9	1	-37.2	-2.21					1.4945
	P3	8	1	-33.7	-1.09					2.2296
C2	P1	6	3	-15.5	-1.49	0.38	-0.28			2.4774
	P2	8	1	-15.9	-1.36					2.7429
	P3	6	1	-14.2	-2.59					2.0087
C1	R1	7	3	-27.2	-1.21	0.30	-0.23			3.5067
	R2	5	3	-28.7	-1.98	1.10	-2.50			0.9045
	R3	9	1	-38.2	-1.46					1.0839
	R4	5	3	-16.1	-0.55	0.11	-0.06			15.2509
C2	R1	4	3	- 7.4	-0.52	0.11	-0.08			1.4739
	R2	11	1	-20.9	-4.11					4.7808
	R3	8	1	-10.3	-1.05					0.3362
	R4	15	1	-16.4	-3.29					3.8819
C3	R1	8	5	2.7	0.43	-0.25	0.71	-8.2	3.2	0.4658
	R2	10	1	-13.0	-2.39					4.3708
	R3	9	1	-12.8	-2.72					1.8696
	R4	7	1	-11.9	-1.05					1.2226

exp. cond. = experimental condition

Table 6 , The best formula in each class in order to explain the work amount curve using formula (3) .

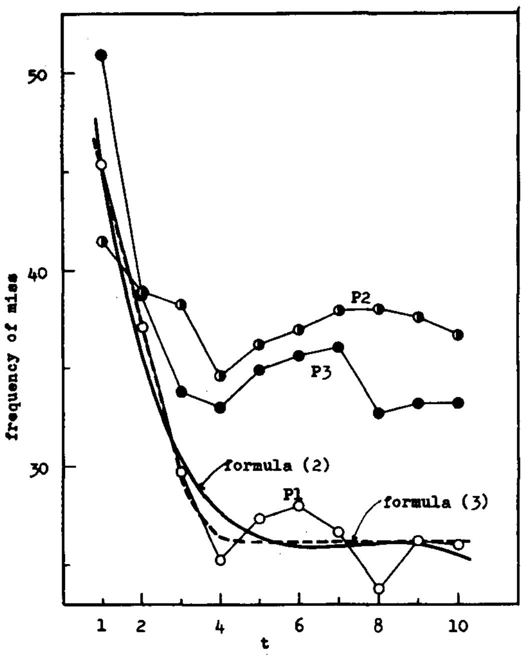

Figure 1 , Approximation of formula (2) in Table 3 and formula (3) in Table 6 on P1 of C1

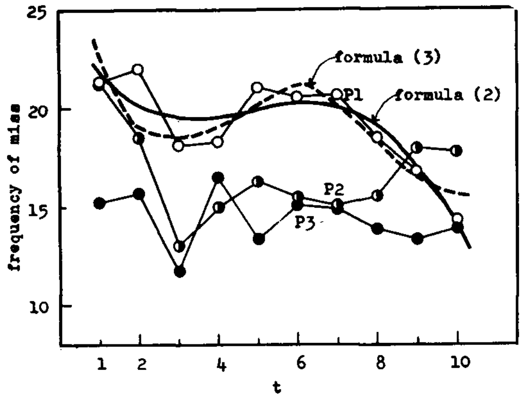

Figure 2 , Approximation of formula (2) in Table 3 and formula (3) in table 6 on P1 of C2

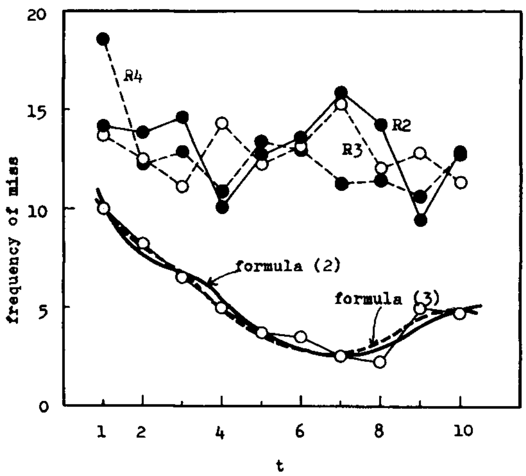

Figure 3 , Approximation of formula (2) in Table 3 and formula (3) in Table 6 on R1 of C3

4. Conclusion

The work amount has been explained by formula $y_t = at^b + c$ until now. This formula can explain exactly the monotonous trend of the work amount. But if the trend of the work amount is not monotonous, this formula can not be used to explain the trend.

Then the various formulas are established in order to explain the various trend of the work amount curve. And it has been examined whether these formulas can approximate to the data of the frequency of miss to the color signal of the experiment using the driving simulator. Then the following results are obtained.

1) $y_t = at^b + c$ which has been used until now is more accurate than $y_t = a/\exp(b_1 t) + c$ in order to explain the work amount curve. But if k = 3 or 5 in $y_t = a/\exp(\sum_{i=1}^{k} b_i t^i) + c$, the degree of the approximation of these formulas to the work amount is more accurate than that of $y_t = at^b + c$. Then it is made clear that these two formulas can approximate to the various trend of the work amount curve. But the formula should be interchanged from one to the other according to the trend of the work amount.

2) $y_t = a/\{\exp(\sum_{i=1}^{k} b_i t^i) - 1\} + c$ can be used to approximate to the work amount curve on the same level of the approximation with formula $y_t = at^b + c$ and $y_t = a/\exp(\sum_{i=1}^{k} b_i t^i) + c$. Then it is made clear that this formula can approximate to the various trend of the work amount by only the degree of the polynomial being exchanged.

5. References

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