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The Dependency of Stress on the Diffraction Plane in the Polycrystalline Metals

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The loading and residual stresses measured by using X-ray stress measurment depend on diffraction plane. In order to make clear its cause, the several models on elastic and plastic deformations are developed and the theoretical values are compared with measured ones. It was found that the dependencies of measured stress on the diffraction plane can be explaned by accepting Reuss's model for elastic deformation and Taylor's model for plastic deformation.

§ 1. Introduction

It is well known that the measurment of stress in polycrystalline metals by using X-ray is a unique and effective method of nondestructive stress measurment in both micro and macro region. So it is applied in wide fields of material engineering studies.

The stress measured by its method, however, depends on the kind of diffraction plane[1]. In this connection, there are some problems regarding the character of X-ray stress measurment, for example, the generation of the lattice strain obtained from the peak shift of certain diffraction line is closely related to the crystal grain size, the deformation mechanism, the crystal anisotropy and other complicated facter[2].

In generally, the stresses are calculated from the change of lattice spacing by Hook's law as follow ;

$$\varepsilon_\varphi = \frac{1+\nu}{E}\sigma_x \sin^2\varphi - \frac{\nu}{E}(\sigma_x+\sigma_y)$$

This equation can be developed with following two assumptions, that is, each crystal in polycrystalline metal is isotropy, and polycrystalline metal is homogeneous. However, it is well known that the all kinds of crystals are elastic and plastic unisotropic bodies[3], It seems that the strain measured on certain plane of certain oriented grain is affected by anisotropy.

In this paper, it is the purpose to discuss about the causes of the dependency of stress on the diffraction plane, and the mechanisms of elastic and plastic deformations.

§ 2. Experimentals

The chemical compositions and the mechanical properties in annealed state of the metals used as specimens are tabled in table 1.

Table 1 Chemical compositions and mechanical properties of Carbon steel, Aluminum and Copper.

	Chemical compositions				
	C	Si	Mn	P	S
Carbon steel	0.12	0.30	0.76	0.02	0.01
	Cu	Si	Fe	Mg	Zn
Aluminum	0.11	0.12	0.55	Tr	Tr
	Fe	Sb	As	Bi	S
Copper	0.03	0.03	0.02	0.02	0.03

	Mechanical properties			
	$\sigma_{(0.2)}$ (kg/mm²)	σ_C (kg/mm²)	Elongation (%)	H_{RB}
Carbon steel	27.0	41.5		32
Aluminum	5.6	10.3	35.0	28
Copper	6.0	31.5	42.0	32

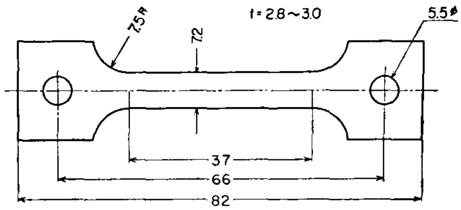


Fig. 1 The shape of specimen

After being machined to the shape as shown in Fig. 1, the specimens were polished by emery paper, and fully annealed in a vacuum at 620 °C for 2hr. (carbon steel), 300°C for 2hr. (aluminum) and 400°C for 2hr. (copper).

The specimens were loaded by using a tensile testing machine which was furnished with X-ray stress measurement and the lattice strains were measued directly. The details of equipment are same as those previously repoted.[4] The conditions of X-ray diffraction are shown in table 2.

Table 2 Conditions of X-ray diffraction technique

Target	Cu, Co, Cr and Fe
Divergence angle	0.25 degree
Radiation area	2×5 mm^2
Tube voltage	30 KV
Tube current	10 mA
Full scale	200 cps
Time constant	10 sec
Goniometer speed	1/4 deg/min
Chart speep	20 mm/min

§ 3. The Lattice Strain and Stress on X-ray Stress Measurment

In a grain in the specimen, let the co-ordinate O-x, y, z be in crystal axis and let Z direction (l_3 m_3 n_3), D direction (a b c), Y direction (l_2 m_2 n_2) and X direction (l_1 m_1 n_1) be respectively in normal of specimen surface, in normal of diffraction plane, intersection of the specimen surface and the diffraction plane and in direction perpendicular Y and Z (when D and Z are parallel direction, Y is any direction on the surface).

In the cubic lattice, Hook's low is expressed in completely terms as a linear relationship between the six strain components and six stress components, as[5];

$$\left.\begin{aligned} \varepsilon_x &= S_{11}\sigma_x + S_{12}(\sigma_y+\sigma_z) \\ \varepsilon_y &= S_{11}\sigma_y + S_{12}(\sigma_z+\sigma_x) \\ \varepsilon_z &= S_{11}\sigma_z + S_{12}(\sigma_x+\sigma_y) \\ \gamma_{yz} &= S_{44}\,\tau_{yz} \\ \gamma_{zx} &= S_{44}\,\tau_{zx} \\ \gamma_{xy} &= S_{44}\,\tau_{xy} \end{aligned}\right\} \quad (1)$$

where S_{11}, S_{12} and S_{44} are the elastic compliances. Describing the stress components σ_X, σ_Y, σ_Z, τ_{YZ}, τ_{ZX} and τ_{XY} in the co-ordinate O-X, Y, Z as σ_{11}, σ_{22}, σ_{33}, τ_{23}, τ_{31} and τ_{12} respectively, the stress components in equations (1) are expressed as ;

$$\left.\begin{aligned} \sigma_x &= \sum\sum l_i l_j \sigma_{ij} \\ \sigma_y &= \sum\sum m_i m_j \sigma_{ij} \\ \sigma_z &= \sum\sum n_i n_j \sigma_{ij} \\ \tau_{yz} &= \sum\sum m_i n_j \sigma_{ij} \\ \tau_{zx} &= \sum\sum n_i l_j \sigma_{ij} \\ \tau_{xy} &= \sum\sum l_i m_j \sigma_{ij} \\ & i, j = 1, 2, 3 \end{aligned}\right\} \quad (2)$$

The normal stress in the diffraction plane are :

$$\varepsilon_D = a^2\varepsilon_x + b^2\varepsilon_y + c^2\varepsilon_z + bc\gamma_{yz} + ca\gamma_{zx} + ab\gamma_{xy} \quad (3)$$

Substituting equations (1) and (2) into equation (3), ε_D are rearranged as ;

$$\begin{aligned} \varepsilon_D = &\sum_{i=1}^{3}\sum_{j=1}^{3}(S_{11}-S_{12}-\frac{1}{2}S_{44})\,M_{ij}\sigma_{ij} + S_{12}(\sigma_{11} \\ &+\sigma_{22}+\sigma_{33}) + \frac{1}{2}S_{44}(\sigma_{11}\sin^2\varphi + \sigma_{33}\cos^2\varphi \\ &+\tau_{13}\cos\varphi\sin\varphi) \end{aligned}$$

$$M_{ij} = a^2 l_i l_j + b_2\, m_i m_j + c^2\, n_i n_j \quad (4)$$

where φ is an angle between D and Z direction. When the specimen is rolling sheat, the Z direction is parallel to normal of rolling plane, and let R direction (l_0 m_0 n_0) be in rolling direction. The l_0, m_0 and n_0 are expressed as ;

$$\left.\begin{aligned} l_0 &= l_1\cos\beta + l_2\sin\beta \\ m_0 &= m_1\cos\beta + m_2\sin\beta \\ n_0 &= n_1\cos\beta + n_2\sin\beta \end{aligned}\right\} \quad (5)$$

where β is an angle between T and X direction.

Let p be the volume fraction of crystals whose orientation indicated by (l_0 m_0 n_0) and (l_3 m_3 n_3) are parallel to rolling direction and nolmal direction of the rolling plane respectively. In the metal deformed by the another working, p can be determined by same way.

The lattice strain measured by using X-ray stress measurment is ;

$$\bar{\varepsilon}_D = \int_0^{2\pi}\left\{\sum_{i=1}^{3}\sum_{j=1}^{3}(S_{11}-S_{12}-\frac{1}{2}S_{44})M_{ij}\sigma_{ij}\right.$$

$$+S_{12}(\sigma_{11}+\sigma_{22}+\sigma_{33})+\frac{1}{2}S_{44}(\sigma_{11}\cos^2\varphi$$
$$+\sigma_{33}\sin^2\varphi+\sigma_{13}\cos\varphi\sin\varphi)\Big\}$$
$$\times pd\alpha\Big/\int_0^{2\pi}pd\alpha \qquad (6)$$

where α is an angle between Y direction and certen fixed direction on diffraction plane. And the dispersion of strain measured from broadening on the X-ray diffraction line is as follow ;

$$\delta^2(\varepsilon_D)=\int_0^{2\pi}\Big\{\sum_{i=1}^{3}\sum_{j=1}^{3}(S_{11}-S_{12}-\frac{1}{2}S_{44})M_{ij}\sigma_{ij}$$
$$+S_{12}(\sigma_{11}+\sigma_{22}+\sigma_{33})+\frac{1}{2}S_{44}(\sigma_{11}\cos^2\varphi$$
$$+\sigma_{33}\sin^2\varphi+\sigma_{13}\cos\varphi\sin\varphi)$$
$$-\overline{\varepsilon_D}\Big\}^2Pd\alpha\Big/\int_0^{2\pi}Pd\alpha \qquad (7)$$

It appears in equations (6) and (7) that the relationship between σ_{ij} and orientation and the value of p must be know to obtaine the average stresses or loading stresses by X-ray stress measurment. And p can be determined by the measuring texture, but the relationship between σ_{ij} and orientation is still not clear in spite of many works. Therefore following models are discussed.

§ 3.1. The Models of Elastic Deformation

(1) The Uniform Local Stress (Rruss's Model)[6]

The model of uniform local stress is an assumption which means that the stresses in each grain equal to loading stresses and the strains in each grain differ each other but its average values are equal to strains in bulk. They can be described as follow ;

$$\sigma_{ij}=\overline{\sigma_{ij}}=\sigma_{ij}^{b} \qquad (8)$$

$$\overline{\varepsilon_{ij}}=\varepsilon_{ij}^{b} \qquad (9)$$

where σ_{ij} are stresses in a grain, $\overline{\sigma_{ij}}$ and $\overline{\varepsilon_{ij}}$ are average stresses and strains and σ_{ij}^b and ε_{ij}^b are loading stresses and strains in bulk. Substituting σ_{ij} in to equations (6) and (7), $\overline{\varepsilon_D}$ and $\delta^2(\varepsilon_D)$ are as ;

$$\overline{\varepsilon_D}=\sum_{i=1}^{3}\sum_{j=1}^{3}(S_{11}-S_{12}-\frac{1}{2}S_{44})\overline{M_{ij}}\,\overline{\sigma_{ij}}$$
$$+S_{12}(\overline{\sigma_{11}}+\overline{\sigma_{22}}+\overline{\sigma_{33}})+\frac{1}{2}S_{44}(\overline{\sigma_{11}}\sin^2\varphi$$
$$+\overline{\sigma_{33}}\cos^2\varphi+\overline{\sigma_{13}}\cos\varphi\sin\varphi) \qquad (10)$$

$$\delta^2(\varepsilon_D)=\sum_{i=1}^{3}\sum_{j=1}^{3}(S_{11}-S_{12}-\frac{1}{2}S_{44})(\overline{M_{ij}^2}$$
$$-\overline{M}_{ij}^2)\overline{\sigma_{ij}} \qquad (11)$$

where

$$\overline{M_{ij}}=\frac{\int_0^{2\pi}M_{ij}Pd\alpha}{\int_0^{2\pi}Pd\alpha}$$

$$\overline{M_{ij}^2}=\frac{\int_0^{2\pi}M_{ij}^2Pd\alpha}{\int_0^{2\pi}Pd\alpha}$$

(2) The Uniform Local Strains (Voigt's and Nagashima's Model)[7,8]

Voigt's model can be described corresponded with Reuss's model as follow ;

$$\left.\begin{aligned}\varepsilon_{ij}&=\overline{\varepsilon_{ij}}=\varepsilon_{ij}^{b}\\ \overline{\sigma_{ij}}&=\sigma_{ij}^{b}\end{aligned}\right\} \qquad (12)$$

where ε_{ij} are strain components in each grain. Using the coordinate in equations (1), the stress components are expressed as ;

$$\sigma_{ij}=\sum_{k=1}^{3}\sum_{l=1}^{3}\Big\{(C_{11}-C_{12}-2C_{44})M_{ijkl}+C_{12}B_{ij}B_{kl}$$
$$+C_{44}(B_{ik}B_{jl}+B_{il}B_{jk})\Big\}\varepsilon_{kl} \qquad (13)$$

where

$$M_{ijkl}=l_il_jl_kl_l+m_im_jm_km_l+n_in_jn_kn_l$$
$$B_{ij}=l_il_j+m_im_j+n_in_j$$
$$\varepsilon_{ij}=\varepsilon_i\,(i=j)$$
$$=\frac{1}{2}\gamma_{ij}\,(i\neq j)$$

From equation (13), the average stresses are calculated as follow ;

$$\overline{\sigma_{ij}}=\sum_{k=1}^{3}\sum_{l=1}^{3}\Big\{(C_{11}-G_{12}-2C_{44})\,\overline{M_{ijkl}}+C_{12}B_{ij}B_{kl}$$
$$+C_{44}(B_{ik}B_{jl}+B_{il}B_{jk})\Big\}\,\overline{\varepsilon_{kl}} \qquad (14)$$

where

$$\overline{M_{ijkl}}=\frac{\int_S\int_0^{2\pi} M_{ijkl}\, P d\alpha ds}{\int_S\int_0^{2\pi} P d\alpha ds}$$

($\int_S(\ )ds$ is represented integral in all combination of l_3, m_3 and n_3 in the stereo triangle.)

Six similar equations about stress components $\overline{\sigma_{ij}}$ are contained in equations (14), and the strain components $\overline{\varepsilon_{ij}}$ can be obtained by solving equation (14). Therefore the stress components σ_{ij} in each grain can be calculated by substituting the solutions of equations (14) into equations (13), also $\overline{\varepsilon_D}$ and $\delta^2(\varepsilon_D)$ can be obtained by substituting σ_{ij} into equations (6) and (7).

However, in the surface layer, Voigt's model must be modified by concidering the balance of stresses as suggested by Nagashima et al..

The equations (11) and (12) are rewritten as;

$$\left.\begin{array}{l}\sigma_{33}=\sigma_{13}=\sigma_{23}=0\\ \overline{\varepsilon_{33}}=\varepsilon^b_{33},\ \ \overline{\varepsilon_{13}}=\varepsilon^b_{13},\ \ \overline{\varepsilon_{23}}=\varepsilon^b_{23}\end{array}\right\} \qquad (15)$$

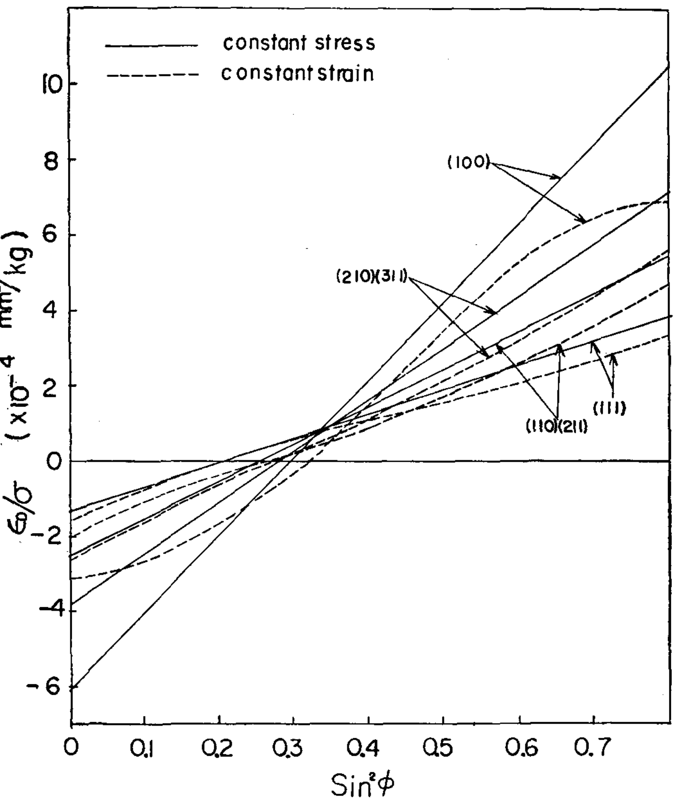


Fig. 2 Theoretical results of ε_D/σ versus $\sin^2\varphi$ in copper.

and other components are same with equations (11) and (12). In this case, the stress and strain components are also calculated by substituting the solutions of equations (14) and (15) into equation (13).

Fig. 2 and table 3 show some examples of $\overline{\varepsilon_D}$ and $\delta^2(\varepsilon_D)$ in surface layers.

§ 3.2. The experimental results of elastic deformation

Fig. 2 shows the lattice strain in a unit of stress (ε_D/σ) which calculated by Reuss's and Voigt's models. For Reuss's model, $\overline{\varepsilon_D}/\sigma$ and $\sin^2\varphi$ are linear relationship in full range, but for uniform local strain model, the deviation liner relationship is considerable at small range of $\sin^2\varphi$. Fig. 3 shows also $\overline{\varepsilon_D}/\sigma$ calculated by Reuss's model in the specimen being no texture and very strong (100)-[100] texture. It is clear in fig. 3 that ε_D/σ and $\sin^2\varphi$ are linear relationship for no texture but not for texture without (100) and (111) diffraction plane. The (100) and (111) diffraction planes are not generally affected by textures. In order to consider experimentaly the models of elastic deformation, therefore, the using (100) and (111) diffraction planes is a favorable method. Fig. 4 shows measuring value in annealed copper, and these results mean that the mechanism of elastic deformation is very close on Reuss's model.

The brodening of X-ray diffraction lines is expressed by Hall[11];

$$\frac{\beta\cos\theta}{\lambda}=\frac{1}{\eta}+\frac{2\varepsilon\sin\theta}{\lambda}+\frac{\beta_1\cos\theta}{\lambda} \qquad (16)$$

where β is the observed integral line breadth, β_1 is the instrumental integral line breadth, λ is X-ray wavelength, θ is Bragg angle, η is the size of particle and ε is the local strain. In the elastic deformation, since η does not change, eq. (16)

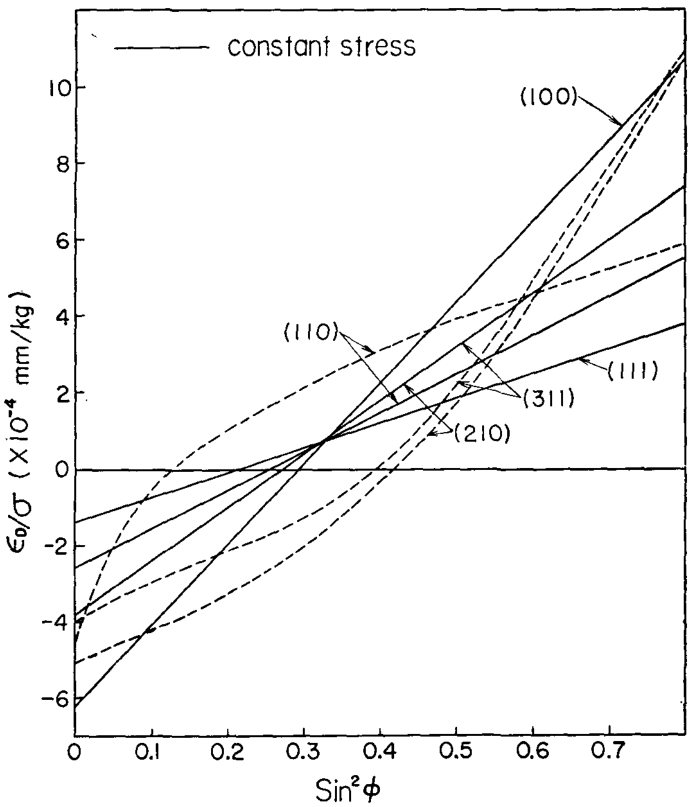


Fig. 3 Theoretical results of ε_D/σ versus $\sin^2\varphi$ in copper, solid lines; no texture, dotted line; (100)-[100] texture. The solid and dotted lines of (100) and (111) overlap each other.

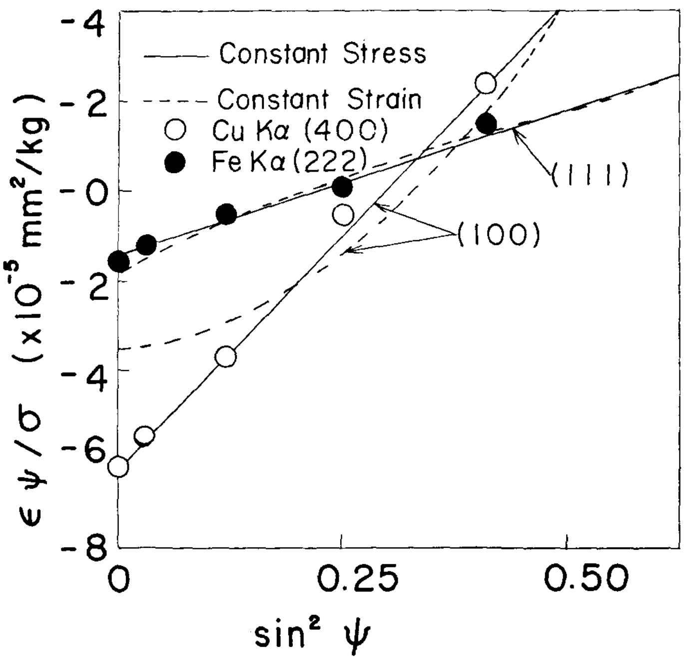


Fig. 4 Correlation of theoretical and experimental result in copper.

can be rewritten as

$$\varepsilon = \frac{(\beta-\beta_0)\cot\theta}{2} \qquad (17)$$

where $\beta_0 = \frac{\lambda}{\eta\cos\theta} + \beta_1$, β_0 is inlegral line breadth of annealed specimen. Therefore the dispersion of strain can be get by measuring β and β_0. Table 3 shows dispersions of local strain which are calculated by Reuss's model and uniform local strain model and measured. This result also points out that the mechanism of elastic deformation is close on Reuss's model.

Table 4 shows the elastic constant for X-ray stress measurment. The second column shows the measured equivalents of $(1+\nu)/E$, the third shows the values calculated by Reuss's model in no texture and the fouth shows mechanical values of $(1+\nu)/E$ which use usualy. It is appear that the main facter of dependency of stress on diffraction plane is the term of the elastic unisotropy.

§ 4. The Residual Stress in the Plastic Deformation

The residual stresses are induced by following two causes as fig. 5.

1) The stress in each grain is different in loading state.

2) The change of stress in each grain is different during unloading process.

The latter cause induces a little difference of stress, because this process are elastic deformation. Therefor, the residual stress are equal to deviation from average stress in loading state.

Table 3 Calculated values and measured values of the dispersion of lattice strain for Aluminum and steel.

hkl	Aluminum (×10⁻(mm²/kg)			Steel (×10⁻⁵mm²/kg)		
	Reuss's model	Voigt's model	measured value	Reuss's model	Voigt's model	measured value
100	0.00	0.00	0.03	0.00	0.00	0.03
110	0.67	0.27	0.64	1.06	0.52	1.12
111	0.00	0.00	—	—	—	—
210	0.43	0.14	0.41	—	—	—
211	0.22	0.12	0.25	0.35	0.23	0.28
221	0.41	0.17	—	—	—	—
310	—	—	—	0.38	0.14	0.36
311	0.18	0.08	0.14	—	—	—
331	0.54	0.22	—	—	—	—

Table 4 Experimental results in carbon steel, aluminium and copper

hkl	$(1+\nu)/E$ measured (×10⁻⁵mm²/kg)	$(1+\nu)/E$ calculated (×10⁻⁵mm²/kg)	$(1+\nu)/E$ mechanical (×10⁻⁵mm²/kg)	$(1+\nu)/E$ measured / $(1+\nu)/E$ calculated	$(1+\nu)/E$ measured / $(1+\nu)/E$ mechanical
Steel					
(310)	7.23	8.58		0.84	1.19
(211)	5.21	5.71	6.09	0.90	0.86
(220)	4.97	5.71		0.86	0.81
Al					
(422)	18.90	18.45		1.02	1.01
(420)	19.35	19.50		0.99	1.02
(400)	19.45	21.30	19.85	0.92	1.03
(222)	17.89	17.50		1.02	0.95
(311)	19.00	19.50		0.97	1.01
(220)	18.28	18.45		0.99	0.97
Cu					
(420)	13.80	13.95		0.98	1.28
(400)	20.35	20.88		0.97	1.93
(222)	6.40	6.50	9.30	0.98	0.59
(311)	14.30	13.95		1.02	1.33
(220)	8.05	8.50		0.95	0.79

§ 4.1. The Models of Plastic Deformation

(1) The Model That Each Grain Deforms Independently of Its Neighbors

The shear stress acting as a given slip system is related to the applied stress as ;

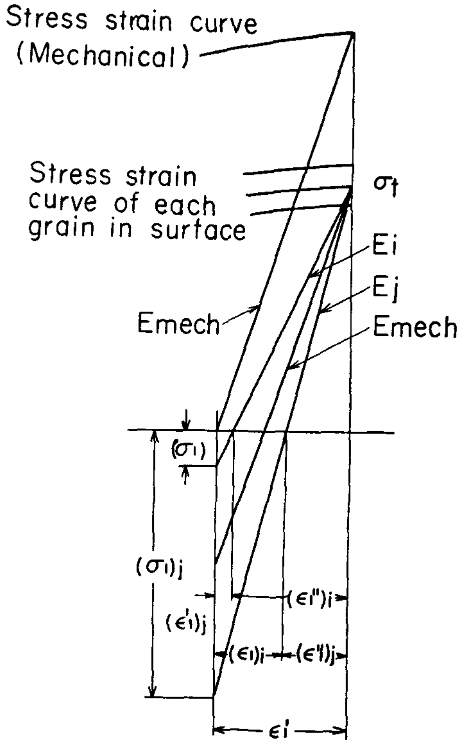


Fig. 5 Illustrating the change of stress in each grain during the reloading.

$$\tau^k = \sum_{i=1}^{3}\sum_{j=1}^{3}(l_i p_1^k + m_i q_1^k + n_i r_1^k)(l_j p_2^k + m_j q_2^k + n_j r_2^k)\,\sigma_{ij} \tag{18}$$

where $(p_1^k\ q_1^k\ r_1^k)$ and $(p_2^k\ q_2^k\ r_2^k)$ are the normal direction of slip plane and slip direction of kth slip system. Sachs[10] proposed that in tensile deformation, there is only tensile stress, other stress components are zero, and the maximum τ^k equal critical resolved shear stress. In order to be realized for this assumption in multiaxial stress, it must be meaned that the stress components in each grain are proportional to applied stress components. Therefore ;

$$\sigma_{ij} = \sigma_{ij}^b \cdot \sigma_{11}/\sigma_{11}^b \tag{19}$$

eq. (18) is rearranged as ;

$$\sigma_{ij} = \frac{\tau_c}{\left\{\sum_{i=1}^{3}\sum_{j=1}^{3}(l_i p_1^k + m_i q_1^k + n_i r_1^k)(l_j p_2^k + m_j q_2^k + n_j r_2^k)\dfrac{\sigma_{ij}^b}{\sigma_{11}^b}\right\}_{max}} \cdot \frac{\sigma_{ij}^b}{\sigma_{11}^b} \tag{20}$$

Therefore $\overline{\varepsilon_D}$ and $\delta^2(\varepsilon_D)$ are calculated by substituting eq. (20) into eqs. (6) and (7).

(2) Uniform Local Plastic Strain[9]

The strain components in plastic deformation are expressed as follow ;

$$\varepsilon_{ij}=\frac{1}{2}\sum_{k=1}^{m}\Big\{(l_i p_1^k+m_i q_1^k+n_i r_1^k)(l_j p_2^k+m_j q_2^k+n_j r_2^k)+(l_i p_2^k+m_i q_2^k+n_i r_2^k)(l_j p_1^k+m_j q_1^k+n_j r_1^k)\,\gamma^k \quad (21)$$

where m is number of slip system, $\gamma^k>0$ when $\tau^k=\tau_c$ or $\gamma^k=0$ when $\tau_c>\tau^k$, γ^k is shear strain in kth slip system. Taylor[9] proposed that the polycrystal is deformed uniformly plastic strain in each grain. In this assumption in order to satisfy the eq. (18) and (21), some slip system must become active. The energy expended during a small strain of unit volume in a grain is given by

$$E=\sum_{k=1}^{m}\tau_c^k\,d\gamma^k \quad (22)$$

where τ_c^k and $d\gamma^k$ refer to resolved shear stress and increment of shear strain on the kth slip system, respectively. This energy must equal the work done by the external stresses in producing strains ;

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\sigma_{ij}\cdot\varepsilon_{ij}=\sum_{k=1}^{m}\tau_c^k\,d\gamma^k \quad (23)$$

The stress components are computed by finding the combination of slip system that gives the minimum value of $\sum_{k=1}^{m}\tau_c^k\,d\gamma_i^k$ and satisfied eqs. (18) and (21) under following condition.

1) in the inside of specimen

$$\varepsilon_{ij}=\varepsilon_{ij}^b$$

2) on the surface

$$\varepsilon_{11}=\varepsilon_{11}^b \quad \varepsilon_{22}=\varepsilon_{22}^b \quad \varepsilon_{12}=\varepsilon_{12}^b$$
$$\sigma_{33}=\sigma_{13}=\sigma_{33}=0$$

This problem is same type of dual problem of linear programing, therefor it can be resolved by using simplex method.

§ 4.2. The Experimental Results of Plastic Deformation

Fig. 6 and 7 show the values of σ_1/τ_c and σ_2/τ_c calculated by Sacks's model and Taylor's model in the grains which have the typical orientation, when the specimen is applied tensile stress in X direction. The residual stress in each grain can be calculated by substrating the average of stress in all grains from the values shown fig. 6 and 7. The average values of residual stress calculated and measured by using X-ray on the typical planes as (100), (110) and (111) plane are shown in table 3. In Taylor's model, since the residual stresses in the surface layer and in the inside

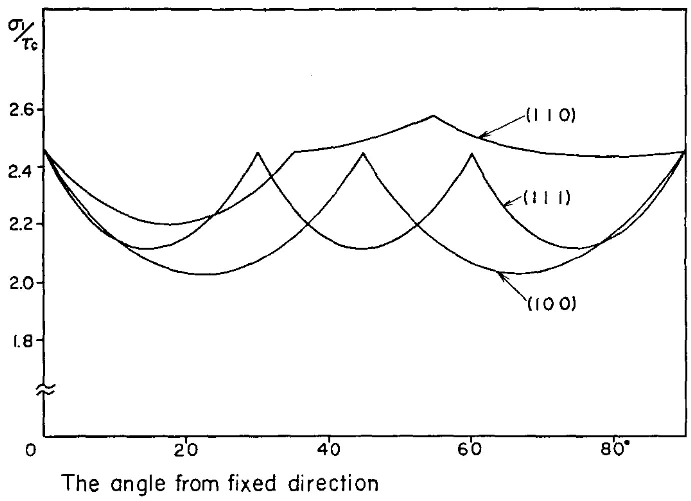
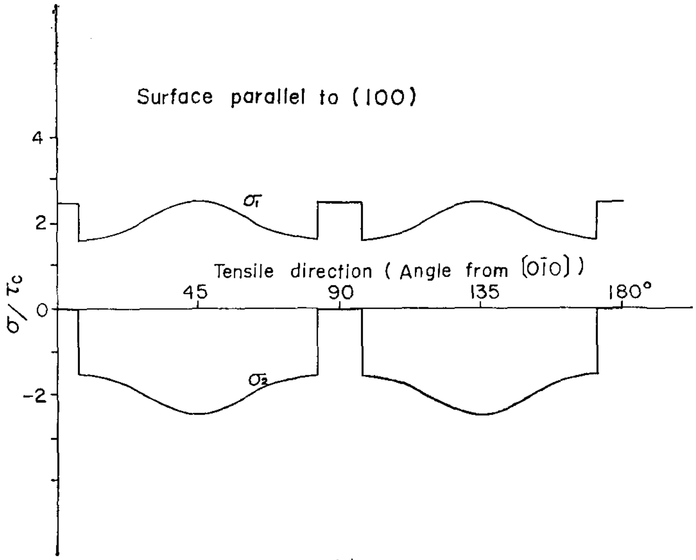
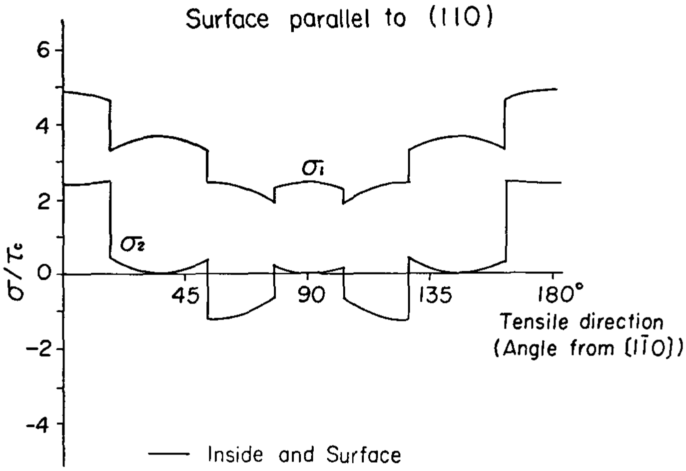


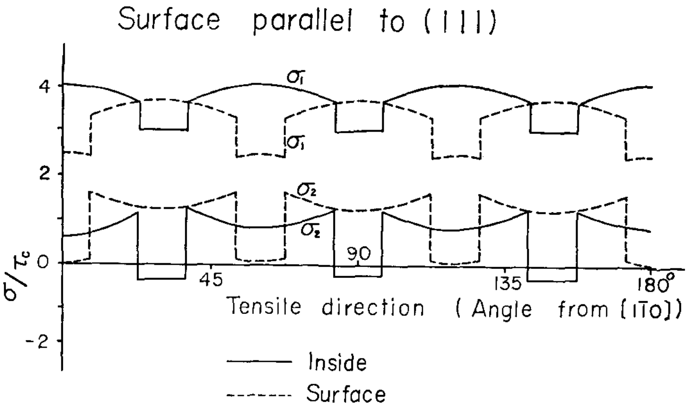
Fig. 6 Theoretical flow stress in Sacks's model. Fixed directions are [001] for (100) and (110) and [011] for 111).



(a)



(b)



(c)

Fig. 7 Theoretical flox stress in Taylor's model.
(a) ; on (100) plane.
(b) ; on (110) plane.
(c) ; on (111) plane.

are different, those in the surface layer are shown in table 5 for easy coparison with measured stress. It appear in table 5 that the

Table 5 The residual stress calculated and measured

	hkl	100	110	111
Sabks's model	σ_1/τ_c	−0.08	0.22	−0.01
	σ_2/τ_c	0.00	0.00	0.00
tayler's model	σ_1/τ_c	−1.18	0.02	0.24
	σ_1/τ_c	−1.77	0.15	0.12
measured residual stress in copper	σ_1/τ_c	−1.49	0.00	−0.39
	σ_2/τ_c	−1.40	−0.33	0.79

measured residual stresses on each diffraction plane are close on calculated values by Tayler's model. And in this model, the avelage of stress in all grains in the surface layer is usually less than that in the inside, this result agrde qualitatively with the fact that compressive stress is generated in plastic elongated specimen.

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